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The Mystery of Electroweak Symmetry Breaking

2. Technicolor

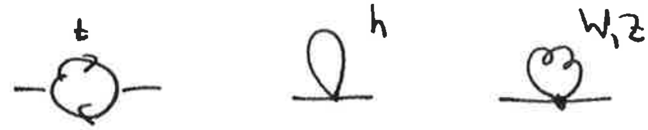
In the previous lecture, I explained that spontaneous breaking if the $SU(2) \times U(1)$ symmetry is an essential part of the SM. I reviewed the model in which the symmetry is broken by a single Higgs scalar field φ and pointed out some of its good properties. You have also heard at this school that the properties of the Higgs boson predicted in this model are in good accord with the first experimental results from the LHC. So it is reasonable to ask whether there is anything wrong with this model that would keep us from saying that we understand the underlying mechanism of EWSB.

Unfortunately, the model with a single Higgs field gives us *no* understanding of the origin of EWSB. What is the *reason* that electroweak symmetry is broken in this model? The most general renormalizable potential for the field φ is

$$V(\varphi) = \mu^2 |\varphi|^2 + \lambda |\varphi|^4$$

The point $\varphi = 0$ is unstable if $\mu^2 < 0$. This is the complete extent of the explanation for EWSB in the SM. The parameters μ^2 and λ are renormalizable coupling that must be specified in advance to define the theory. There is no asking why these parameters have the values that the are measured to have.

I have discussed the renomalization of λ in the previous lecture. If λ is set at at some high mass scale such as the Planck scale, it changes slowly with $\log Q$ to its value at the TeV scale (though, if the SM is taken literally, the value at the Planck scale would be negative). The renormalization of μ^2 has a different structure. This parameter receives *additive* divergent corrections. At one loop order, the formula for μ^2 is



$$\mu^2 = \mu^2_{\rm bare} - \frac{3y_t^2}{8\pi^2}\Lambda^2 + \frac{3\lambda}{8\pi^2}\Lambda^2 + \frac{9\alpha_w + 3\alpha_w'}{16\pi}\Lambda^2$$

The radiative corrections to μ^2 are of both signs and are extremely sensitive to the ultraviolet cutoff. It is very difficult to understand why μ^2 has its observed order of magnitude of about $(100\ \text{GeV})^2$. If the cutoff is much larger than 1 TeV, this can only come about through a cancellation of large corrections. Also, the sign of μ^2 cannot be predicted. Which set of diagrams dominate depends on exactly how the various contributions are regularized. If we naively put $\Lambda = m_{\rm Pl}$ into all three terms, the observed value of μ^2 requires a cancellation of these terms against the bare value of μ^2 in the first 30 decimal places.

A way to describe the results of the previous paragraph is that the SM has no prediction for the presence of EWSB or for the magnitude of the symmetry-breaking vacuum expectation value. EWSB is put in by hand. This is not the sort of explanation usually deemed acceptable in physics.

How can we do better? The concept of spontaneous symmetry breaking came to particle physics from condensed matter physics, where there are concrete examples of this phenomenon. In many condensed matter systems, there is a symmetry of the Hamiltonian that is not a symmetry of the ground state. These include magnets, superconductors and superfluids, binary alloys, and liquid crystals. Each of these systems has fascinating properties resulting from the symmetry breaking. The laws of physics that apply in each case are just the simple laws of the non-relativistic quantum mechanics of atoms. But, in each, case, there is a particular, quite nontrivial, mechanism by which the minimization of the ground state energy leads to symmetry breaking.

Particle physicists should know these systems better. I would like to discuss, in particular, the origin of superconductivity.

A superconductor is a metal that conducts electricity with zero resistance. Most metals are superconducting at sufficiently low temperature. This was a major puzzle in solid state physics through the first half of the twentieth centure. Very low temperatures are required, of order 1° K while the Fermi energy of electrons in a metal is of order $10^4\ ^\circ$ K. Both parts of the phenomenon needed explanation.

We can make a model of zero resistance flow of current by imagining that the metal contains a scalar field with nonzero electric charge. The $U(1)$ gauge symmetry of electromagnetism transforms the electron field as

$$\psi(x) \rightarrow e^{-i\alpha}\psi(x)$$

If the scalar field has charge Q, it transforms as

$$\Phi(x) \rightarrow e^{iQ\alpha}\Phi(x)$$

If $\Phi(x)$ obtains a vacuum expectation value, the $U(1)$ symmetry is broken. The $U(1)$ current is

$$\vec{J} = \frac{+ie}{2m}\left(\psi^{\dagger}\vec{\nabla}\psi - \vec{\nabla}\psi^{\dagger}\psi\right) - i\frac{Qe}{2M}\left(\Phi^{\dagger}\vec{\nabla}\Phi - \vec{\nabla}\Phi^{\dagger}\Phi\right)$$

where m is the mass of the electron and M is a phenomenologically determined mass for the Φ field. Assume that the ground state of the Hamiltonian has

$$\langle\Phi\rangle = \Phi_0 \neq 0$$

Equally well, by symmetry, we could have any of the ground states

$$\langle\Phi\rangle = e^{i\beta}\Phi_0$$

Now consider the state with

$$\langle\Phi(x)\rangle = e^{ikz}\Phi_0$$

This state has a very low energy, proportional to k^2, and it has nonzero current

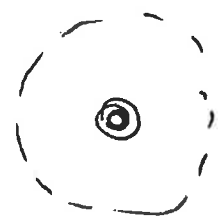
$$\vec{J} = eQ \, \frac{k}{M} \, \hat{z}$$

It can be shown thata, for sufficiently small k, this is the state of lowest energy for momentum k. Then it cannot decay, and the current flows frictionlessly.

There is another interesting phenomenon associated with the field Φ. In a 2-dimensional slab of superconductor, there are field configurations of finite energy with

$$\Phi(x) \to \Phi_0 \, e^{i\phi}$$

$$\text{as} \quad |x| \to \infty$$



The energy of Φ contains the term

$$\int d^3x \, |\vec{D}\Phi|^2 \qquad \vec{D}\Phi = (\vec{\nabla} - ieQ\vec{A})\Phi$$

The radial component is

$$(D\Phi)_\phi = \left(\frac{1}{r}\frac{\partial}{\partial\phi} - ieQ A_\phi\right)\Phi$$

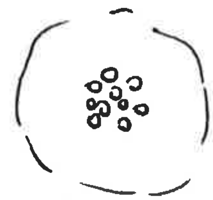
$$\to \left(\frac{i}{r} - ieQ A_\phi\right)\Phi_0 \, e^{i\phi}$$

so if $|D\Phi|^2$ is to go to zero at large r,

$$A_\phi \to \frac{1}{eQ\, r}$$

Then, around a large circle at infinity,

$$\oint d\vec{x}\cdot\vec{A} = \frac{2\pi}{Qe} = \int d^2s\, \hat{n}\cdot\vec{B}$$



More explicitly, restoring factors of $\hbar$ and c, the magnetic flux piercing the superconductor is quantized, in units of

$$\frac{2\pi\hbar c}{Qe}$$

The quantization of magnetic flux in a superconductor was observed in 1961 by Deaver and Fairbank; they found $|Q| = 2$, that is, the condensate field Φ has the charge of 2 electrons.

In 1950, Ginzburg and Landau wrote a phenomenological theory of superconductivity based on the field $\Phi(x)$. Essentially, they described Φ by an effective free energy of the form

$$\int d^3x \left\{ |\vec{D}\Phi|^2 + V(\Phi) \right\}$$

where

$$V(\Phi) = \mu^2 |\Phi|^2 + \lambda |\Phi|^4$$

with

$$\mu^2 = A\,(T - T_c)$$

in order to make μ^2 negative at $T = 0$. This leads to a very useful theory that allows one to calculate many properties of a superconductor, including the structure of the magnetic flux tubes described above, the critical current, the thermodynamics of the phase transition, and many other properties. However, it does not explain why metals are superconducting at low temperatures.

Fortunately, there was more physics to be learned. An important realization was that there is a small attractive interaction between electrons in a metal. The repulsive Coulomb interaction between conduction electrons is mainly screened by inner atomic electrons. However, electrons can interact by scattering from nuclei and creating lattice vibrations (phonons) that then affect other electrons. The interaction is of the form

$$\sim (ig)^2 (\psi^\dagger\psi) \frac{i}{(q^0)^2 - c^2|\vec{q}|^2} \psi^\dagger\psi$$

where c is the speed of sound. For small q^0 and finite $\vec{q}$, there is an attractive potential, as usual for scalar boson exchange. A typical momentum transfer will be of the order of the Fermi momentum in the metal

$$k_F \sim \mathring{A}$$

It is a good approximation to treat this as a small attractive local interaction

$$\Delta\mathcal{L} = +i\lambda (\psi_a^\dagger\psi_a)(\psi_b^\dagger\psi_b)$$

where $a, b = \uparrow, \downarrow$ are spin indices.

In the presence of this interaction, the Schrödinger equation for electrons in the metal takes the form

$$\left(i\frac{\partial}{\partial t} + \frac{1}{2m}\nabla^2\right)\psi_\uparrow - \lambda(\psi_\downarrow^\dagger\psi_\downarrow)\psi_\uparrow = 0$$

$$\left(i\frac{\partial}{\partial t} + \frac{1}{2m}\nabla^2\right)\psi_\downarrow - \lambda(\psi_\uparrow^\dagger\psi_\uparrow)\psi_\downarrow = 0$$

noting that $(\psi_\uparrow)^2 = (\psi_\downarrow)^2 = 0$. We can take advantage of the nonlinear term in the following way, imagined by Bardeen, Cooper, and Schrieffer (BCS): If there were a reservoir of electrons, allowing electron number to be indefinite, we could have a state with a vacuum expectation value for an electron-number-changing operator

$$\langle \psi_\uparrow \psi_\downarrow \rangle \neq 0$$

Specifically, write

$$\lambda \langle \psi_\uparrow(k)\, \psi_\downarrow(-k) \rangle = \Delta = \lambda \langle \psi_\downarrow^\dagger(k)\, \psi_\uparrow^\dagger(-k) \rangle$$

The quantity Δ would be nonzero if electrons for zero-momentum pairs (*Cooper pairs*) and these pairs form a Bose condensate in which the total number of pairs was indefinite. This condensate would allow us to freely add a pair of electrons just above the Fermi surface or remove a pair of electrons just below the Fermi surface. Since the Fermi surface is sharp at low temperatures, this costs very little free energy

The equation for ψ is now

$$\left(E - \frac{p^2}{2m}\right) \psi_\uparrow(E,p) + \Delta\, \psi_\downarrow^\dagger(E,-p) = 0$$

$$\left(E - \frac{(-p)^2}{2m}\right) \psi_\downarrow^\dagger(E,-p) - \Delta\, \psi_\uparrow(E,p) = 0$$

Linearize about the Fermi energy,

$$\varepsilon = E - E_F \qquad \frac{p^2}{2m} = \frac{k_F^2}{2m} + \frac{k_F}{m} \cdot p + \cdots$$

Then

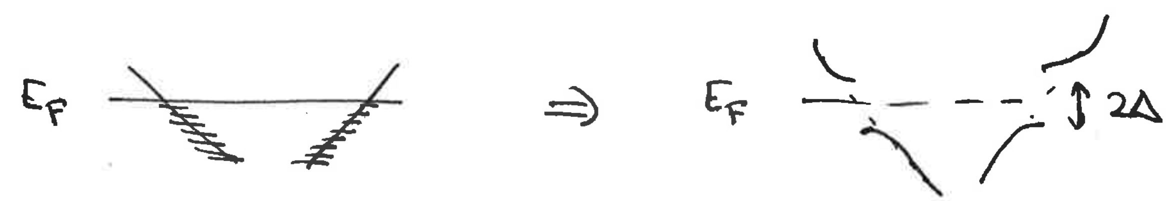
$$(\varepsilon - v_F p)\,\psi_\uparrow + \Delta \psi_\downarrow^\dagger = 0$$

$$(\varepsilon + v_F p)\,\psi_\downarrow^\dagger - \Delta \psi_\uparrow = 0$$

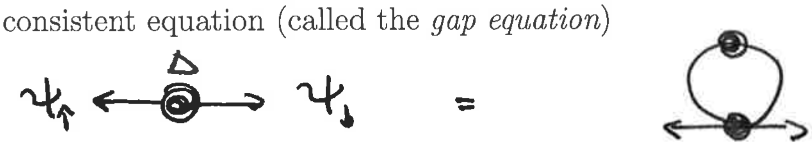
so that

$$\varepsilon^2 - (v_F p)^2 - \Delta^2 = 0$$

The system opens up an energy gap at the Fermi surface



decreasing the energy of the system. The gap parameter Δ is found by the self-consistent equation (called the *gap equation*)



$$\Delta = i\lambda \int \frac{d\varepsilon\, d^3p}{(2\pi)^4} \frac{i\,\Delta}{\left[\varepsilon^2 - (v_F p)^2 - \Delta^2 + i\epsilon\right]}$$

Evaluating the $d\epsilon$ integral by contours,

$$\Delta = \lambda \Delta \int \frac{d^3p}{(2\pi)^3} \frac{1}{\left[(v_F p)^2 + \Delta^2\right]^{1/2}}$$

$$\Delta = \Delta \cdot \lambda \frac{k_F^2}{2\pi^2} \int_{-\infty} dP \, \frac{1}{\left[v_F^2 P^2 + \Delta^2\right]^{\frac{1}{2}}}$$

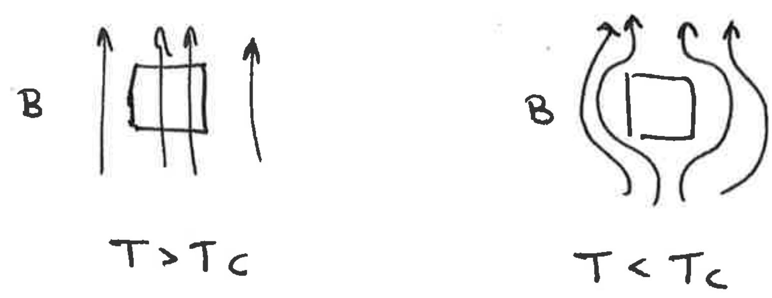
$$= \Delta \cdot \lambda \frac{k_F^2}{2\pi^2 v_F} \log \frac{\Lambda}{\Delta}$$

The integral seems divergent, but, in a metal, the integral is cut off by the Debye energy ω_D, the largest possible energy of a phonon. We find

$$\Delta = \omega_D \exp\left[- \frac{2\pi^2}{m_e k_F} / \lambda \right]$$

Remarkably, the gap equation can be satisfied with nonzero Δ for any arbitrarily weak attractive interaction among electrons. The form of this equation explains why Δ can be nonzero but extremely small compared to the typical energies in a metal.

An expectation value $\Delta \neq 0$ is incompatible with the original $U(1)$ symmetry of electron number. This symmetry is spontaneously broken. This is also the gauge symmetry of electromagnetism. So, the photon must obtain a mass in a superconductor through the Higgs mechanism. This gives rise to the exclusion of magnetic flux by superconductors, the *Meissner effect.*



There is much more to say about superconductivity. A good starting point is Tinkham's book *Introduction to Superconductivity.* Now I would like to apply these ideas to make a theory of EWSB.

The discussion I have just given of symmetry breaking in superconductivity has a direct analog for massless fermions in 4-dimensional quantum field theory. This was first recognized by Nambu and Jona-Lasinio in a classic paper, Phys. Rev. 122, 345 (1961). I recommend this paper highly.

The analogy is as follows: If we have a theory with massless fermions and attractive interactions, the fermions might pair up, form a condensate in the vacuum, and open a gap in their energy spectrum. A gap at zero energy is exactly the generation of a fermion mass. The pairs that condense must have zero total momentum and angular momentum. This is satisfied for pairs such as

that is, pairs of $f_L \overline{f}_L$ and $f_R \overline{f}_R$. Remember that the antiparticle of f_L is $\overline{f}_R$ and vice versa. So these pairs have nonzero charges under the separate fermion number symmetries of ψ_L and ψ_R. The condensation of pairs breaks these symmetries and allows the generation of mass for the fermions.

I will discuss this pair condensation first in the more familiar setting of 2-flavor QCD. The masses of the u and d quarks are small, and it is a good first approximation to neglect them. In this limit, the QCD Lagrangian is

$$\mathcal{L} = -\frac{1}{4}(F_{mn})^2 + Q_L^\dagger i\overline{\sigma}\cdot D Q_L + Q_R^\dagger i\sigma\cdot D Q_R$$

where

$$Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \qquad Q_R = \begin{pmatrix} u \\ d \end{pmatrix}_R$$

This has $U(2) \times U(2)$ symmetry

$$Q_L \to U_L Q_L \qquad Q_R \to U_R Q_R$$

The overall $U(1)$ symmetry is associated with baryon number. It can be shown that the axial $U(1)$ symmetry

$$Q_L \to e^{i\alpha} Q_L \qquad Q_R \to e^{-i\alpha} Q_R$$

is broken explicitly by non-perturbative effects of the QCD gluon field. Then we have the symmetry group

$$U(1) \times SU_L(2) \times SU_R(2)$$

called *chiral symmetry.*

Condensation of quark-antiquarks as discussed above give nonzero vacuum expectation values

$$\langle Q_{La} Q^{\dagger}_{Rb} \rangle = -\Delta \delta_{ab} = \langle Q_{Rb} Q^{\dagger}_{La} \rangle$$

and breaks the chiral symmetry to

$$U(1) \times SU(2)$$

which is known to be a good approximate symmetry of QCD. The 3 broken generators of the original symmetry group give rise to 3 Goldstone bosons π^a, created from the vacuum by the broken symmetry currents

$$j^{\mu a5} = Q^{\dagger}_R \sigma^{\mu} \frac{\sigma^a}{2} Q_R - Q^{\dagger}_L \bar{\sigma}^{\mu} \frac{\sigma^a}{2} Q_L = \bar{Q} \gamma^{\mu} \gamma^5 \frac{\sigma^a}{2} Q$$

The matrix elements for creation or annihilation of Goldstone bosons can be written

$$\langle 0 | j^{\mu 5 a}(0) | \pi^b(p)\rangle = i f_\pi p^\mu \delta^{ab}$$

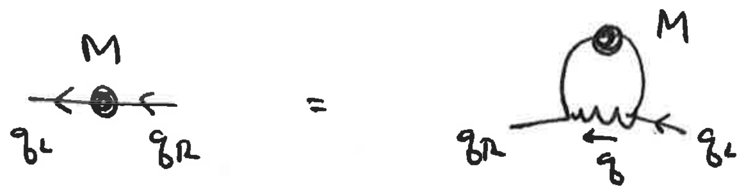
where f_π is a nonzero constant called the *pion decay constant.* In QCD, $f_\pi = 93$ MeV. The π^a form a parity -1, $I = 1$ multiplet of states that is naturally identified with the three *pi* mesons.

If we turn to u and d quark masses back on, it can be shown that we find the relation

$$m_\pi^2 = \frac{(m_u + m_d)\,\Delta}{f_\pi^2}$$

which explains why $m_\pi^2/m_p^2 \sim 1\%$. The complete phenomenological picture of π meson interactions can be explained by their identification as Goldstone bosons. You can find a detailed discussion of this in the book of Donoghue, Golowich, and Holstein, *Dynamics of the Standard Model*, Chapter VI.

The major difficulty with this understanding of QCD is that it s not straightforward to compute Δ from first principles. The leading order form of the gap equation is



where M is the generated mass for the u and d quarks. Roughly,

$$M = g^2 C_2(r) \int \frac{d^4k}{(2\pi)^4} \frac{M}{q^2\left[(k-q)^2 - M^2\right]}$$

Unfortunately, this equation does not have the same excellent infrared singularity that we saw in BCS theory. For $q^2 \to 0$, the integral over k behaves as

$$\int d^3k \frac{1}{[k^2+M^2]^4} \sim \int \frac{dk \cdot k^2}{[k^2+M^2]^4}$$

which is nonsingular as $M \to 0$. So we do not have symmetry breaking at arbitrarily weak coupling but only for

$$g^2 C_2(r) = \frac{4}{3} g^2 > \mathcal{O}(1)$$

Presumably, due to the opposite of asymptotic freedom, this criterion is satisfied in QCD for sufficiently small moment transfer. Lattice QCD simulations confirm that $\left\langle Q_L^\dagger Q_R \right\rangle$ is indeed nonzero in the numerically generated QCD vacuum state.

The picture of spontaneous breaking of chiral symmetry is thus very successful in the application to QCD. In 1978, Weinberg and Susskind suggested that this idea could also be used as a mechanism for EWSB. Imagine that, in addition to QCD, there is another strongly interacting gauge theory, just like 2-flavor QCD but at a higher mass scale. We can call this new gauge theory *technicolor.* Technicolor will have 2 flavors of massless techniquarks (U, D) and a global symmetry

$$U(1) \times SU_L(2) \times SU_R(2)$$

To build a theory of EWSB, couple (U, D) to the SM exactly as we couple (u, d)

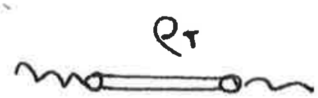
$$Q = \begin{pmatrix} U \\ D \end{pmatrix}_L \qquad U_R \qquad D_R$$

$$I = \tfrac{1}{2} \quad Y = y_T \qquad I = 0 \quad Y = y_T + \tfrac{1}{2} \qquad Y = y_T - \tfrac{1}{2}$$

Before we turn on the SM gauge couplings, the technicolor sector generates 3 Goldstone bosons Π^a. These have a pion decay constant given by

$$\langle 0 | J^{m5a} | \pi^b(p)\rangle = i F p^m \delta^{ab}$$

The technicolor theory also has a large number of $Q\overline{Q}$ bound states whose characteristic mass is $4\pi F$. (In QCD, $4\pi f_\pi \sim 1$ GeV.) In particular, there is a techni-ρ meson that can be created by a virtual photon



The breaking of the technicolor chiral symmetry implies mass generation for the SM gauge bosons. Under the identified gauge symmetries of $SU(2) \times U(1)$, the expectation value $\left\langle Q_L^\dagger Q_R \right\rangle$ transforms as

$$\langle Q_L^\dagger Q_R\rangle \to e^{i\vec{\alpha}\cdot\vec{\sigma}/2} e^{i\beta y_T} \langle Q_L Q_R^\dagger\rangle \begin{pmatrix} e^{-i\beta(y_T+1/2)} & \\ & e^{-i\beta(y_T-1/2)} \end{pmatrix}$$

If we put

$$\langle Q^\dagger_{La} Q_{Rb}\rangle = -\Delta \delta_{ab}$$

the symmetries are not respected. However the specific transformation with $\alpha^3 = -\beta$ leaves the expectation value invariant, so one linear combination of A^3 and B remains massless to give a massless photon.

There is a way to compute the mass matrix of the weak bosons more explicitly. Consider the gauge boson self-energy

$$+i\Pi^{ab}_{mn}(q) =$$

The Ward identity implies

$$q_m \Pi^{ab}_{mn} = 0$$

so Π^{ab} must have the form

$$\Pi^{ab}_{mn} = \left(g_{mn} - \frac{q_m q_n}{q^2} \right) \Pi^{ab}(q^2)$$

Any singularity of Π^{ab}_{mn} signals a massless boson that can be created by the gauge currents. If there are no such massless particles, then

$$\Pi^{ab}(q^2) \sim q^2 \quad \text{as} \quad q^2 \to 0$$

and no mass is generated for the gauge bosons. On the other hand, if

$$\Pi^{ab}(q^2) \to m^2 \neq 0 \quad \text{as} \quad q^2 \to 0$$

we find

$$= \frac{-i}{q^2 - m^2} \left(g_{mn} - \frac{q_m q_n}{q^2} \right)$$

and the gauge bosons acquire a mass.

In this case, there are massless particles created by the gauge currents, namely, the Goldstone bosons. For the $SU(2)$ currents

$$J_A^{ma} = Q_L^\dagger \bar{\sigma}^m \frac{\sigma^a}{2} Q_L = \frac{1}{2}\left(J^{ma} - J^{m5a}\right)$$

so

$$\langle 0 | \, i g J_A^{ma} \, | \pi^b(q) \rangle = (+ig)(iFq^m)\left(-\tfrac{1}{2}\delta^{ab}\right)$$

and

$$= \left(\frac{gFq_m}{2}\right) \frac{i}{q^2} \left(-\frac{gFq_n}{2}\right) \delta^{ab}$$

$$= -i \left(\frac{gF}{2}\right)^2 \frac{q_m q_n}{q^2} \delta^{ab}$$

The contact piece of the self-energy is harder to evaluate directly, but it is constrained by gauge invariance. Then

$$= i\left(g_{mn} - \frac{q_m q_n}{q^2}\right) \left(\frac{gF}{2}\right)^2 \delta^{ab}$$

and we recognize that this produces a mass for the $SU(2)$ gauge bosons. Similarly, the hypercharge current is

$$J_Y^m = y_T \, Q_L^\dagger \bar{\sigma}^m Q_L + Q_R^\dagger \sigma^m \begin{pmatrix} y_T + \frac{1}{2} & \\ & y_T - \frac{1}{2} \end{pmatrix} Q_R$$

$$= y_T \, \bar{Q}\gamma^m Q + \frac{1}{2} \bar{Q}\gamma^m \gamma^5 \sigma^3 Q$$

so that

$$\langle 0|\, ig' J_Y^m \,|\pi^b(q)\rangle = (ig')\,(iFq^m)\cdot\frac{1}{2}$$

We find the gauge boson mass matrix

$$m^2 \begin{pmatrix} A^1 \\ A^2 \\ A^3 \\ B \end{pmatrix} = \left(\frac{F}{2}\right)^2 \left(\begin{array}{ccc|c} g^2 & & & \\ & g^2 & & \\ & & g^2 & -gg' \\ \hline & & -gg' & (g')^2 \end{array}\right) \begin{pmatrix} A^1 \\ A^2 \\ A^3 \\ \hline B \end{pmatrix}$$

This is the same formula that we cound for the mass matrix of vector bosons in the simple one-Higgs-scalar model. Actually, it is required that we obtain a mass matrix of the same form, because we have satisfied the basic axioms leading to this result. The theory has an unbroken $U(1)$ gauge symmetry, and it has an unbroken custodial $SU(2)$ symmetry which, in this case, is the technicolor isospin symmetry. We can now determine F by identifying it with the Higgs field vacuum value needed to get the correct W and Z masses

$$F = 246 \text{ GeV}$$

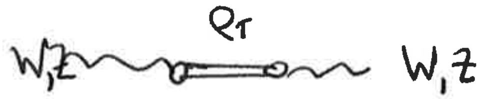
Then the mass scale of technicolor bound states should be

$$M \sim 4\pi F \sim 2 \text{ TeV}$$

When we get access to such a high mass scale—and we are just now accessing this energy at the LHC—we should see the techni-ρ as a resonance in WW and WZ production.

This is a beautiful dynamical explanation of EWSB. It gives a physical explanation for why electroweak symmetry is broken in the context of a testable model involving new particles. Unfortunately, the model is not correct. There are three compelling arguments against it.

First, the model predicts signficant corrections to precisely measured weak-interaction couplings and masses that are not observed in the LEP and SLC experiments. In particular, the techni-ρ constributions



generate a pattern of shifts of observables from their SM values. For example,

$$\Delta\left(\frac{m_W}{m_Z}\right) \sim -0.3\%$$

$$\Delta\,(A_e) \sim +3\%$$

This is quantified by the S and T parameters of electroweak physics (which unfortunately I do not have time to discuss in detail). The technicolor model I have described predicts

$$S \sim 0.8$$

while the precision electroweak experiments give

$$S < 0.14 \qquad 95\%\ \text{conf.}$$

as shown in Figure 1. The techni-ρ decouples as $1/m_{T\rho}^2$, so to avoid this problem we need to move the techni-ρ to above about 5 TeV. Unfortunately, we cannot do this without also raising F and thus raising the W mass.

Second, it is very difficult to give mass to quarks and leptons in this model. In the one-Higgs-scalar model, we generated masses through couplings such as

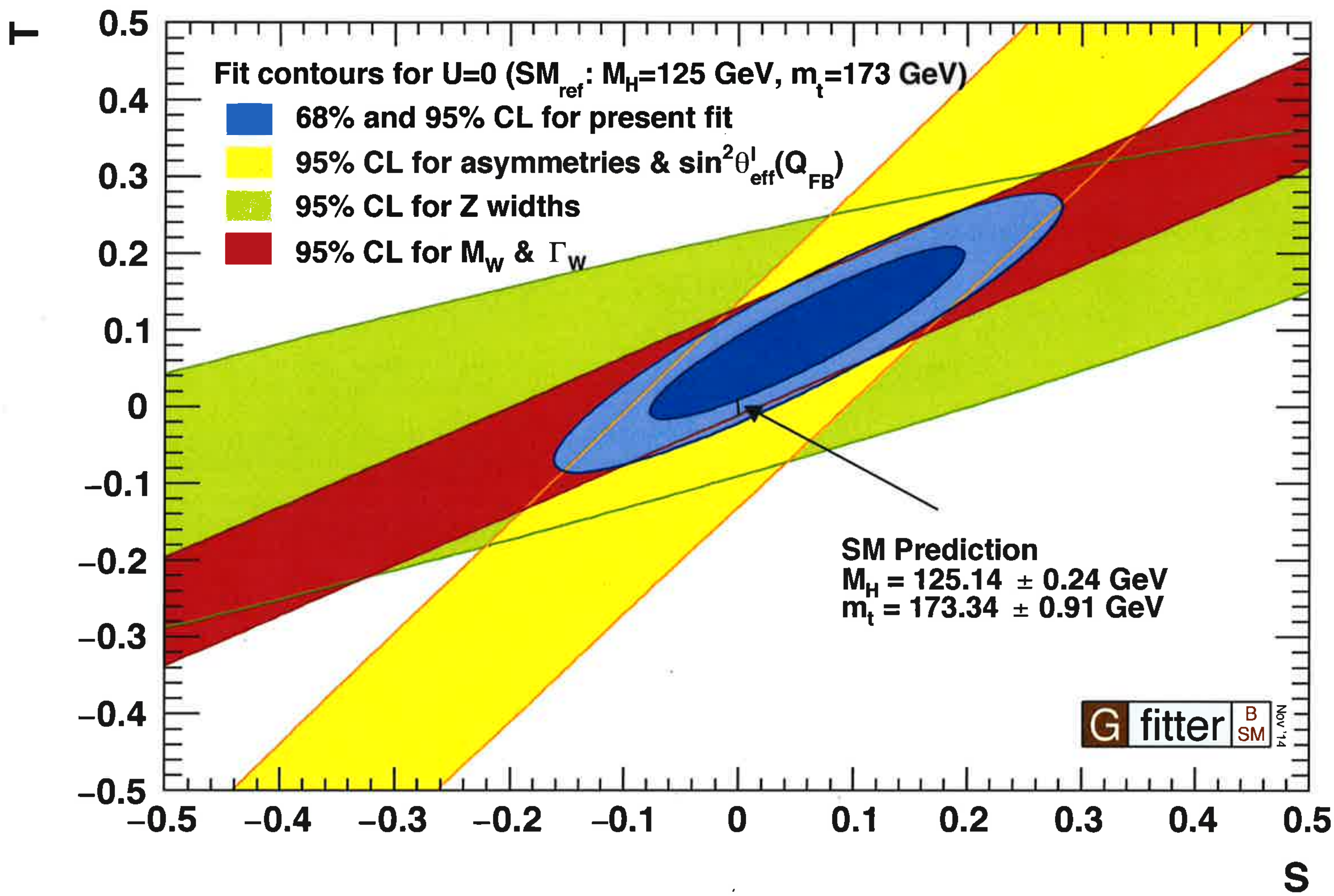


Figure 1

Gfitter limits on S, T from precision electroweak measurements.

$$y_e \, \phi \cdot L^{\dagger} e_R$$

This operator has dimension 4 and so the coupling constant y_e is dimensionless. The term is a renormalizable interaction that could be generated at an arbitrarily high mass scale.

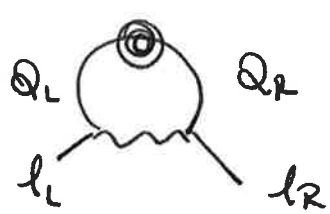
In technicolor, there is no fundamental field φ. The symmetry is broken by the expectation value of the operator $Q_R^{\dagger} Q_L$. in principle, we could generate masses by adding the Lagrangian a term

$$G \cdot \; L^{\dagger} \cdot Q_R \, Q_L^{\dagger} e_R$$

But this is a dimension 6 operator, and so the coefficient $G \sim (\mathrm{mass})^{-2}$. A quark or lepton mass generated by this term would be of the order of

$$m_\ell \sim G \Delta \sim 1\,\mathrm{GeV} \cdot \frac{G}{(30\,\mathrm{TeV})^{-2}}$$

Eichten and Lane and Dimopoulos and Susskind suggested that we could generate this term by postulating additional gauge interactions (ETC) linking the SM fermions and technifermions, so that the mass for quarks and leptons is generated by a diagram



However, this structure also requires ETC interactions among 4 SM fermions. These generate dangerous flavor-changing neutral currents that contribute to processes such as $K \to \mu^+ \mu^-$ and the K^0, D^0, and B^0 mass differences.

It is possible that these problems could be solved if the dynamics of the technicolor model differed in some way from that of QCD. In planetary physics, when we had one example of a solar system, we assumed that all other solar systems in the universe were similar. This turned out not to be correct. In the same way, strong interactions can behave differently for different gauge groups or representations, or even for different values of n_f/N_c within QCD. Maybe there is a way out.

However, there is now a third argument that is an absolute killer. In QCD, the pseudoscalar mesons are the only bound states with have masses well below $4\pi f_\pi$. These are the states that become Goldstone bosons when the masses of u, d, and s are taken to zero. The lightest scalar bound state of $q\overline{q}$ must be P-wave and has no reason to be light. In QCD phenomenology, the lightest 0^+ state is the f_0 or σ, an extremely broad resonance at about 500 GeV barely visible as a feature in the S-wave $\pi\pi$ phase shift. Probably, this is a multiquark state, with the lightest $q\overline{q}$ state being the $f_0(1500)$. There is no candidate in strongly coupled fermion theories for a light and extremely narrow 0^+ state similar to the observed boson at 125 GeV.

So, we need to give up on this highly motivated dynamical theory of EWSB. We have to find some alternative in which EWSB comes from the vacuum value of a scalar field. In the next lecture, I will discuss a way to do that.