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The Mystery of Electroweak Symmetry Breaking

1. Electroweak Symmetry Breaking in the Standard Model

It is often said that the discovery of the Higgs boson in July 2012 "completed" the Standard Model of particle physics. It is correct that all of the particles predicted by the Standard Model (SM) have now been discovered. The classic 1976 paper by Ellis, Gaillard, and Nanopoulos that worked out the SM predictions for the decay rates of the Higgs boson ended with the statement that the Higgs boson "decays predominantly to as yet conjectural massive new particles." Today, those particles—W, Z, b, t, and g—are all known and experimentally well characterized. The precision electroweak program at LEP and SLC in the 1990's measured the weak interaction couplings of the quarks and leptons to part-per-mil accuracy. So, the particles and forces of the SM have not only been discovered by also tested in careful ways.

However, I feel, the model is still incomplete. The SM is expected to stand on three legs. The first of these is the particle content, identified in terms of specific quantum numbers. The second is the set of gauge couplings required by the gauge group $SU(3) \times SU(2) \times U(1)$. The third is the principle that leads to the generation of masses for the quarks, leptons, and gauge bosons. As I will review in this lecture, none of these particles are permitted to obtain mass unless some symmetries of the model are spontaneously broken.

I find it troubling that the SM seems to be incapable of giving a real understanding of this symmetry breaking. I will argue, in this lecture and the next, that the explanation of symmetry breaking in the SM is *ad hoc* and, also, a dead end in the search for answers to other important questions in particle physics.

Correcting this deficit requires major additions to the SM. It requires that there must be new particles and forces in nature that we will encounter at higher energies. In the course of these lectures, I will give examples of new phenomena that can explain the spontaneous symmetry breaking of the SM and, plausibly, are within our reach experimentally. The search for new particles and forces that give rise to these phenomena has become, in my opinion, the most important problem in elementary particle physics.

A brief overview of this subject, with references to the literature on many of the points I will discuss, can be found in my recent paper arXiv:1506.08185.

To begin, I will discuss the problem of generating masses for the particles of the SM. In non-relativistic physics, there is no problem in giving masses to particles. Stuff has mass. For a scalar field in relativistic quantum theory, there is similarly no difficulty. The Lagrangian for a massless scalar field is

$$\mathcal{L} = \frac{1}{2}(\partial_m \phi)^2$$

To give this field a mass, we need only add an extra term

$$\mathcal{L} = \frac{1}{2}(\partial_m \phi)^2 - \frac{1}{2}m^2\phi^2$$

The new term does not violate any symmetry of the original model, so there is no principle that prohibits its appearance.

For fields of spin greater than 0, the situation is completely different. For spin $\frac{1}{2}$, a massive field is described by the Dirac equation

$$(i\gamma \cdot \partial - m)\Psi = 0$$

To analyze this, choose the representation of the γ matrices

$$\gamma^m = \begin{pmatrix} 0 & \sigma^m \\ \bar{\sigma}^m & 0 \end{pmatrix} \qquad \begin{array}{l} \sigma^m = (1, \vec{\sigma})^m \\ \bar{\sigma}^m = (1, -\vec{\sigma})^m \end{array}$$

and decompose

$$\Psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$$

Then

$$i\bar{\sigma}\cdot\partial\,\psi_L - m\,\psi_R = 0$$

$$i\sigma\cdot\partial\,\psi_R - m\,\psi_L = 0$$

When $m = 0$, this system has a higher symmetry. The fermion numbers of ψ_L and ψ_R are separately conserved. This separate fermion number conservation, and its associated $U(1)$ symmetry, are broken only by the mass term. When the mass is zero, the equations for ψ_L and ψ_R become decoupled,

$$i\bar{\sigma}\cdot\partial\,\psi_L = 0 \qquad\qquad i\sigma\cdot\partial\,\psi_R = 0$$

and we can even consistently keep one of these fermion fields without the other.

The physical content of ψ_L is a left-handed (helicity $= -\frac{1}{2}$) massless fermion and its antiparticle, a right-handed (helicity $= +\frac{1}{2}$) antifermion. The physical content of ψ_R is a right-handed massless fermion and a left-handed antifermion. These fermions transform under the Lorentz group according to the elementary representations

$$\left(\tfrac{1}{2}, 0\right) \qquad \text{and} \qquad \left(0, \tfrac{1}{2}\right)$$

Neither representation, separately, can be the representation of a *massive* particle. If a particle is massive, we can go to its rest frame, then boost or rotate it arbitrarily, turning a left-handed state into a right-handed state or vice versa

This implies that generation of the mass in the Dirac equation requires mixing of two fields ψ_L and ψ_R. In order for this to be possible, the two fields should have the same quantum numbers under all conserved symmetries.

For clarity as proceed, I will refer to ψ_L and ψ_R as having left- and right-handed *chirality*. A state of a massive particle with left-handed helicity (spin opposite to the direction of motion) is a mixture of states created by ψ_L and ψ_R. This mixture becomes a pure ψ_L state only in the limit $v \to c$.

For an electron, it seems almost obvious that e^-_L and e^-_R have the same quantum numbers. However, if we regard the gauge symmetries of the SM as fundamental symmetries, this statement is not correct. In particular, the elementary weak interactions couple only to the lepton and quark fields of left-handed chirality. This is seen clearly in the measured polarizations of electrons emitted in β decay (Figure 1). Different β emitters produce electrons at different energies and velocities. Always, the polarization is preferentially left-handed. To be more quantitative, note that a massive electron moving in the $\hat{3}$ direction has the wavefunction

$$u(p) = \begin{pmatrix} \sqrt{E-p} \\ 0 \\ \hline \sqrt{E+p} \\ 0 \end{pmatrix} (\text{spin} \uparrow) \qquad \begin{pmatrix} 0 \\ \sqrt{E+p} \\ \hline 0 \\ \sqrt{E-p} \end{pmatrix} (\text{spin} \downarrow)$$

An interaction coupling to ψ_L only gives the polarization

$$\mathcal{P} = \frac{(E-p) - (E+p)}{E-p + E+p} = -\frac{p}{E} = \frac{v}{c}$$

exactly as observed.

More generally, the structure of the SM is such that the weak interaction $SU(2)$ symmetry acts only on the ψ_L component of quarks and leptons. The coupling of the Z boson to fermion chirality states is

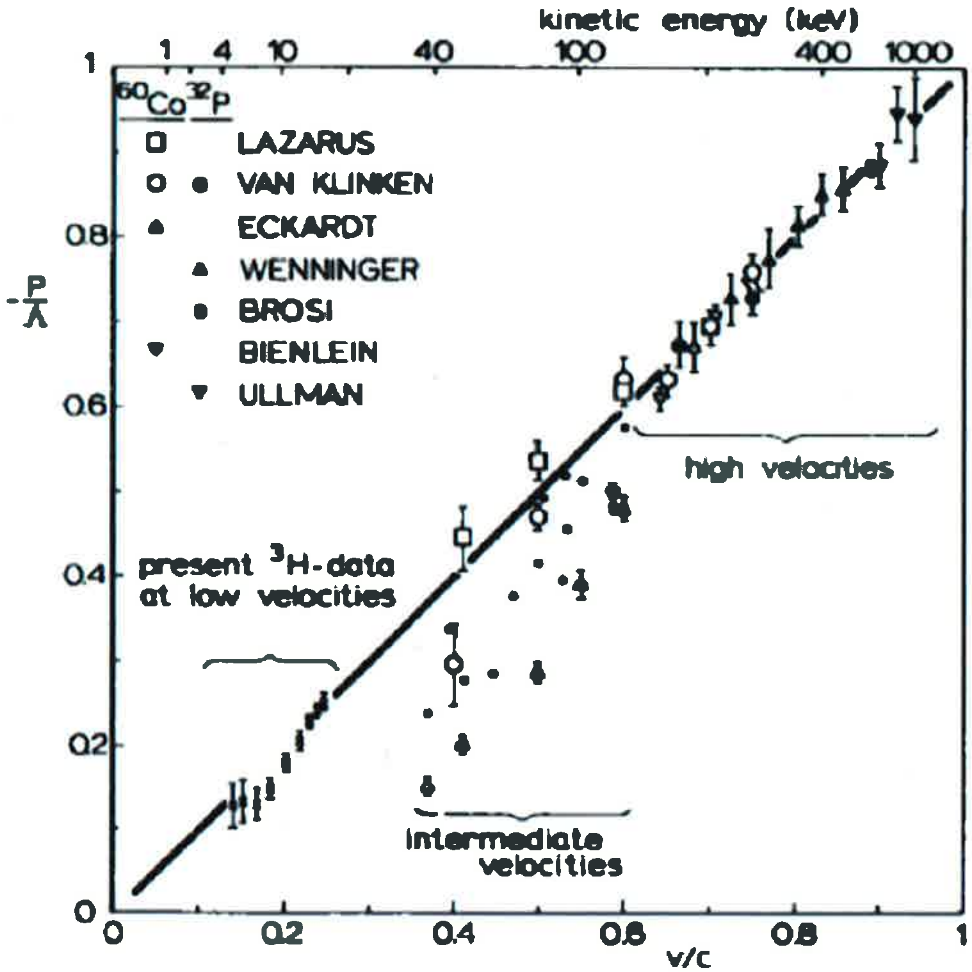


Figure 1

compilation by Koks and van Klinken

$$Q_Z = I^3 - \sin^2\theta_w Q$$

where Q is the electric charge and

$$I^3 = \begin{cases} \pm\frac{1}{2} & \text{for } q_L\ \ell_L \\ 0 & \text{for } q_R\ \ell_R \end{cases}$$

This gives rise to parity asymmetries in the Z couplings

$$A_e, A_\tau = 15\% \qquad\qquad A_b = 94\%$$

which are in good agreement with observations (Figs. 2,3). If ψ_L and ψ_R had the same couplings to the Z, the asymmetries that are obvious in these figures would be zero.

There is a similar issue with the generation of mass for vector bosons. You learned in undergraduate electrodynamics that Maxwell's equations predict precisely two physical modes of polarization for light—or, in the quantum theory, for photons. These have the field description

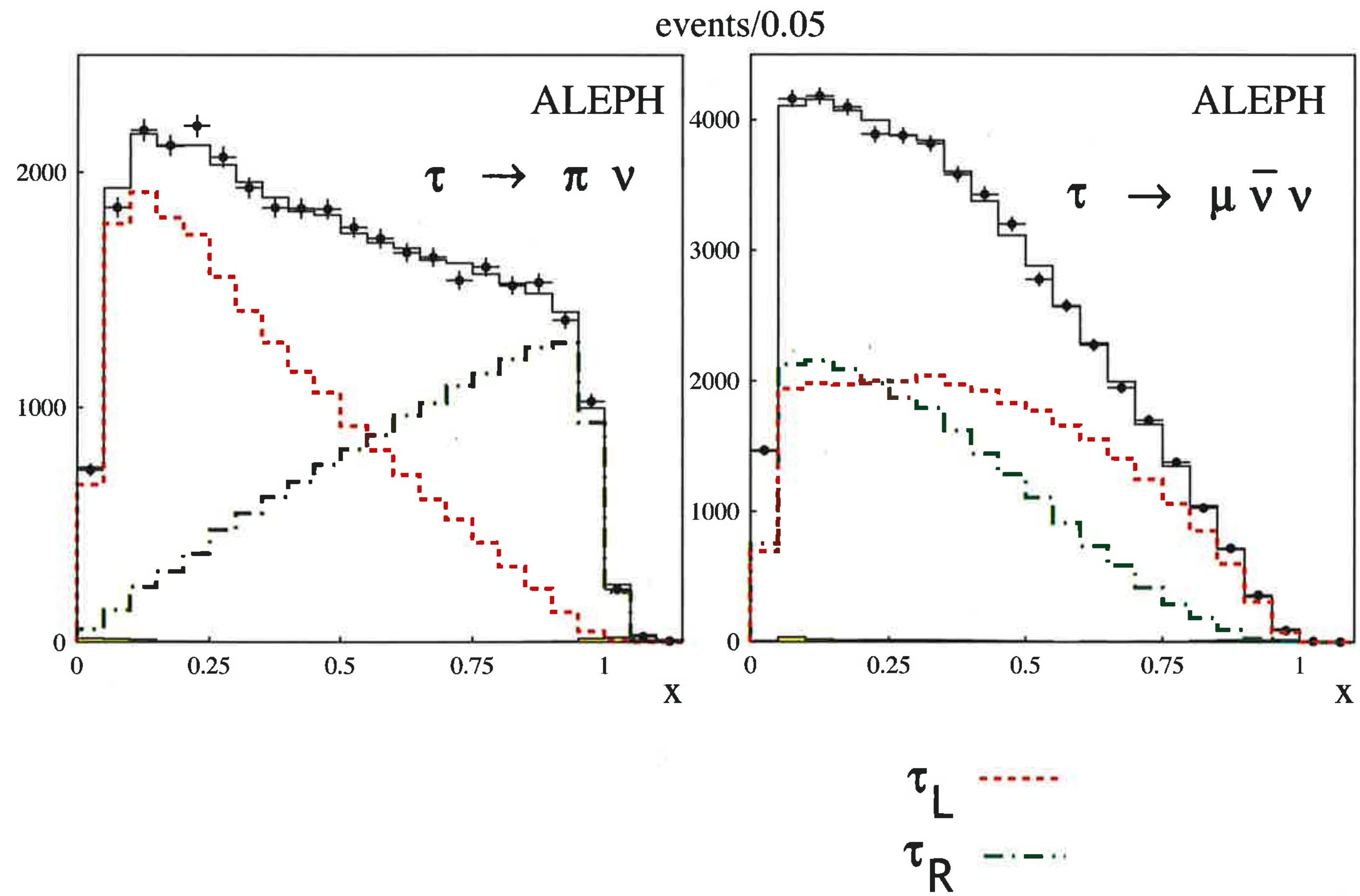
$$A_m(x) = \epsilon_m(p)\, e^{-ip\cdot x}$$

with, for $\hat{p} \parallel \hat{3}$,

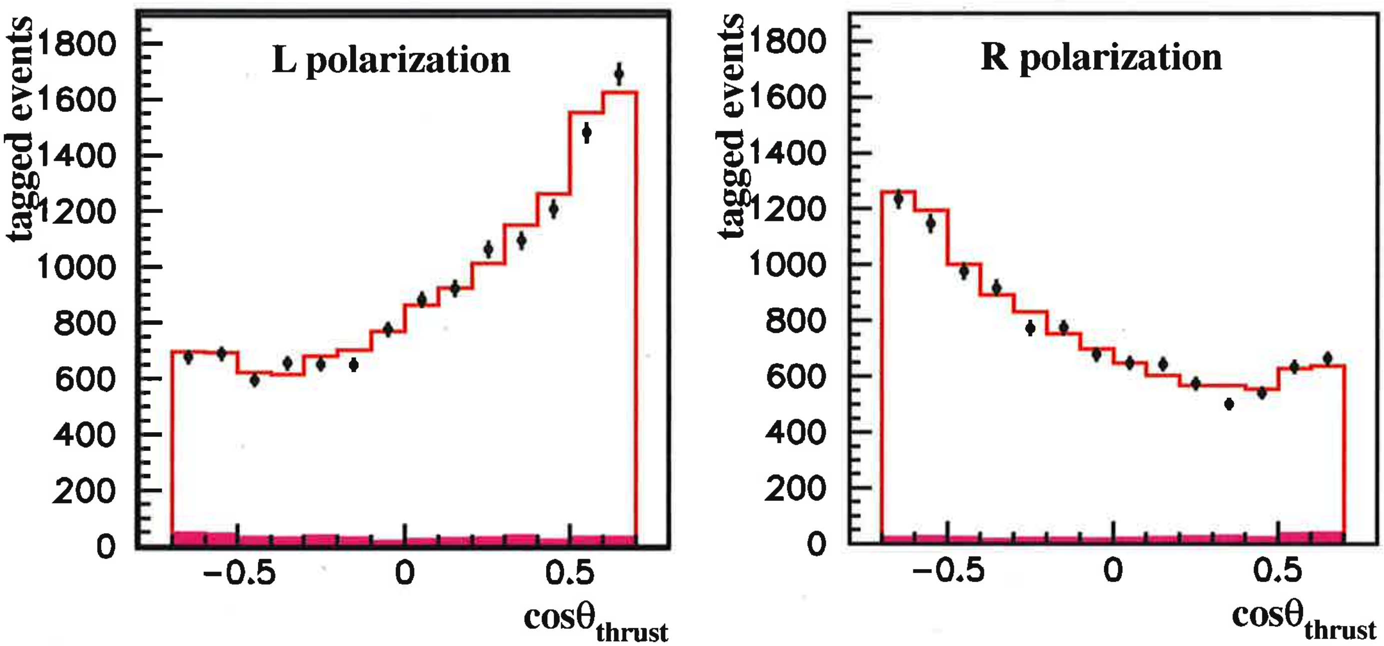
$$\epsilon^m(p) = \begin{cases} \frac{1}{\sqrt{2}}(0, 1, -i, 0) & L \\ \frac{1}{\sqrt{2}}(0\ 1, i, 0) & R \end{cases}$$

These correspond to the (1,0) and (0,1) elementary representations of the Lorentz group. A massive vector boson can be boost to rest and then rotated arbitrarily.

$e^+e^- \to Z^0 \to \tau^+\tau^-$



$e^+e^- \to Z^0 \to b\bar{b}$



$A_b = 0.94$ at the Z^0 SLD

This implies that a massive spin 1 boson must have $3 = (2J + 1)$ physical states. To turn the massless bosons described by Maxwell's equations into massive vector bosons, we must add a third state to the two already present. This state must have chirality 0 before mixing with the photon states of chirality ± 1.

In electrodynamics, the photon is massless and this problem does not arise. But in the SM, there are also massive vector bosons W^+, W^-, and Z. These obey the field equations of Yang-Mills theory, which are just Maxwell's equations plus some nonlinear interactions. So the problem discussed in the previous paragraph does arise here. We need to find chirality 0 fields to mix with the elementary Yang-Mills fields in order to generate masses. The obvious choice of using the longitudinal polarization state of the vector potential $A_m(x)$ does not work. If this state is physical, the timelike polarization state of $A_m(x)$ must also be physical, and emission of this state generates negative probabilities.

So we have the two related problems that the symmetries of the SM prohibit the generation of mass for SM fermions and gauge bosons. One possible solution is that the gauge symmetries of the SM are not fundamental. This could have been an acceptable explanation in the 1970's, but it is not tenable today. The gauge symmetries of the SM lead to Ward identities that imply that the coupling constants of the weak interactions are universal. If these constraints were not in place, the weak interaction couplings would be sensitive to large, species-dependent, radiative corrections. The precision electroweak experiments have verified the formula

$$Q_Z = I^3 - \sin^2\theta_W Q$$

to high accuracy, separately for light quarks, c and b quarks, and the leptons e, μ, τ. This checks the universality of the weak interaction couplings g and g' across species. At the end of this lecture, I will discuss tests of the $WW\gamma$ and WWZ couplings. These agree at the few-percent level with the predictions from the nonlinear terms of the Yang-Mills Lagrangian. So the gauge theory origin of W and Z is well established experimentally.

The other possible solution is that the $SU(2) \times U(1)$ symmetry of the SM is a symmetry of the Lagrangian but not a symmetry of the vacuum state. That is, the symmetry is spontaneously broken. From here on, I will refer to the spontaneous symmetry breaking of $SU(2) \times U(1)$ as Electroweak Symmetry Breaking or EWSB.

The simplest model of EWSB is to introduce a scalar field $\varphi(x)$ that transforms under $SU(2) \times U(1)$ and can acquire a constant value throughout space. In the SM, we introduce the Higgs field

$$\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} \qquad \text{with } I = \tfrac{1}{2},\ Y = -\tfrac{1}{2}$$

Using the $SU(2)$ gauge freedom, we can rotate any constant expectation value for φ to

$$\langle \varphi \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

The full content of $\varphi(x)$ is

$$\varphi(x) = \begin{pmatrix} \pi^+(x) \\ \frac{1}{\sqrt{2}}\left(v + h(x) + i\pi^0(x)\right) \end{pmatrix}$$

The fields π^+, π^-, π^0 are Goldstone bosons, since they are generated by acting

$$j^a(x) \cdot \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

where $j^a(x)$, $a = 1, 2, 3$ are the $SU(2)$ symmetry currents.

Local gauge invariance allows us to rotate these fields away. In this gauge, called the *unitarity gauge*, the kinetic term of the Higgs field takes the form

$$|D_m \varphi|^2 = \left|\left(\partial_m - ig A_m^a \frac{\sigma^a}{2} - ig' B_m \cdot \tfrac{1}{2}\right)\varphi\right|^2$$

where A_m^a, B_m are the $SU(2) \times U(1)$ gauge fields. Putting in $\langle \varphi \rangle$, we find

$$|D_m \varphi|^2 = \Big[g \frac{A^1_m - i A^2_m}{2} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} + g \frac{A^1_m + i A^2_m}{2} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$+ g A^3_m \begin{pmatrix} \frac{1}{2} & \\ & -\frac{1}{2} \end{pmatrix} + g' B \begin{pmatrix} \frac{1}{2} & \\ & \frac{1}{2} \end{pmatrix} \Big] \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

$$= \left| \frac{g}{\sqrt{2}} \left(\frac{A^1_m - i A^2_m}{\sqrt{2}} \right) \frac{v}{\sqrt{2}} \right|^2 + \left| \left(-\frac{1}{2} g A^3_m + \frac{g'}{2} B_m \right) \frac{v}{\sqrt{2}} \right|^2$$

This is the mass term for the gauge fields

$$= m_W^2 W^+_m W^{-m} + \frac{1}{2} m_Z^2 Z^2$$

with the identifications

$$W^\pm_m = \frac{A^1_m \mp i A^2_m}{\sqrt{2}} \qquad Z_m = \frac{g A^3_m - g' B_m}{\sqrt{g^2 + g'^2}}$$

and

$$m_W^2 = \frac{g^2 v^2}{4} \qquad m_Z^2 = \frac{(g^2 + g'^2) v^2}{4}$$

To obtain the observed values of the W and Z masses

$$v = 246 \text{ GeV}$$

So the three Goldstone bosons disappear, but they are transmuted into the chirality 0 components need to make the three bosons W^+, W^-, and Z massive.

The last linear combination of fields is massless

$$A_m = \frac{g' A^3_m + g B_m}{\sqrt{g^2 + g'^2}}$$

This is the photon field of the SM.

It is a standard exercise to plug the mass eigenstate fields into the covariant derivative with which the SM acts on a chiral fermion

$$D_m \psi = \left(\partial_m - i g A^a_m \frac{\sigma^a}{2} - i g' B Y\right)\psi$$

and work out the fermion couplings. With the identifications

$$\cos\theta_w = c_w = \frac{g}{\sqrt{g^2+g'^2}} \qquad \sin\theta_w = s_w = \frac{g'}{\sqrt{g^2+g'^2}}$$

we find

$$D_m \psi = \Big[\partial_m - i \frac{g}{\sqrt{2}} (W^+_m \sigma^+ + W^-_m \sigma^-) - i e A_m Q - i \frac{e}{s_w c_w} Z_m Q_Z \Big]\psi$$

with

$$e = \frac{g g'}{\sqrt{g^2+g'^2}}$$

and the Z charge

$$Q_Z = I^3 - s_w^2 Q$$

that I have already noted above.

The gauge boson mass matrix has two important properties. First, there is a massless boson. This reflects the fact that there is one gauge symmetry that is not spontaneously broken by $\langle\phi\rangle$. The $SU(2) \times U(1)$ transformation is

$$\varphi \to e^{i\vec{\alpha}\cdot\vec{\sigma}/2} e^{i\beta/2} \varphi$$

For $\vec{\alpha} = (0, 0, \beta)$, so that

$$\vec{\alpha}\cdot\frac{\vec{\sigma}}{2} + \beta/2 = \begin{pmatrix} \beta & 0 \\ 0 & 0 \end{pmatrix}$$

we find

$$\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \to \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

The model has to be built in this way so that it can contain electromagnetism.

Second, the W and Z masses are related by

$$m_W / m_Z = c_w$$

a relation satisfied experimentally to better than 1% accuracy after radiative corrections due to the top quark are taken into account. This is actually a symmetry relation: For $g' = 0$,

$$m_W = m_Z$$

More generally, the gauge boson mass matrix for the bosons $(A^1, A^2 A^3, B)$ has the form

$$m^2 \begin{pmatrix} A^1 \\ A^2 \\ A^3 \\ \hline B \end{pmatrix} = \frac{v^2}{4} \left(\begin{array}{ccc|c} g^2 & & & \\ & g^2 & & \\ & & g^2 & -gg' \\ \hline & & -gg' & (g')^2 \end{array}\right)$$

The off-diagonal entry is such that there is a zero eigenvalue. Then the relation $m_W = m_Z c_w$ follows from the fact that the three elements on the diagonal are equal. This corresponds to an unbroken $SO(3)$ or $SU(2)$ symmetry among (A^1, A^2, A^3). This is called *custodial symmetry.* For this to be present, the original theory should have an $SO(4) = SU(2) \times SU(2)$ global symmetry, for which, first, one $SU(2)$ is the weak interaction gauge group, and, second, spontaneous symmetry breaking breaks $SO(4)$ to a global $SO(3)$ or $SU(2)$.

What is the origin of the custodial symmetry? In the theory of EWSB based on a single Higgs field, the custodial symmetry is a special property of this field. The Higgs field has 4 real components

$$\varphi = \begin{pmatrix} \frac{1}{\sqrt{2}}(\phi^1 + i\phi^2) \\ \frac{1}{\sqrt{2}}(h + i\phi^0) \end{pmatrix}$$

and

$$|\varphi|^2 = \frac{1}{2}\left[h^2 + (\phi^0)^2 + (\phi^1)^2 + (\phi^2)^2\right]$$

is $SO(4)$ invariant. A gauge invariant Higgs potential then has the form

$$V(\phi) = V(|\varphi|)$$

and so is $SO(4)$ invariant. The Higgs field vacuum expectation value break the $SO(4)$ to an $SO(3)$ that rotates (ϕ^1, ϕ^2, ϕ^3) and (A^1, A^2, A^3).

As these lectures proceed, I will discuss models of EWSB that are more complex than the one based on a single scalar field. To prepare for this, we should ask what properties of the SM are unique to the assumption of one scalar field, or to EWSB due to fundamental scalar doublets. We will see in the next lecture that custodial symmetry is not a unique property of this system. In the next lecture, we will meet other theories of the Higgs potential that also give rise to custodial symmetry in a natural way.

The most important property of the scalar field model of EWSB is that it is straightforward to give mass to the quarks and leptons. Notate the SM fields of one generation as

$$
\begin{array}{cccccc}
L = \begin{pmatrix} \nu \\ e^- \end{pmatrix}_L & e^-_R & q = \begin{pmatrix} u \\ d \end{pmatrix}_L & u_R & d_R \\
Y = \quad -\frac{1}{2} & -1 & +\frac{1}{6} & +\frac{2}{3} & -\frac{1}{3}
\end{array}
$$

Then, using the Higgs field φ, we can simply write the renormalizable Lagrangian term

$$
\delta\mathcal{L} = -y_e \, L^\dagger \cdot \varphi \, e_R + h.c.
$$

where I introduce

$$
L^\dagger \cdot \varphi = L^\dagger_a \varphi_a \qquad L^\dagger * \varphi^* = L^\dagger_a \epsilon_{ab} \varphi^*_b
$$

$$
a = 1, 2
$$

The dimensionless coupling y_e is called a *Yukawa coupling.* This interaction is invariant under $SU(2) \times U(1)$. In particular, the total hypercharge is $Y = +\frac{1}{2} + \frac{1}{2} - 1$. Inserting

$$
\varphi \to \langle \varphi \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}
$$

we find

$$\delta\mathcal{L} = -\frac{y_e v}{\sqrt{2}}\left(e_L^\dagger e_R + e_R^\dagger e_L\right)$$

which is just the electron mass term required in the Dirac Lagrangian. The field φ precisely bridges the gap between the different quantum numbers of e_L^- and e_R^-. In a similar way, we can generate masses for u and d by writing

$$\delta\mathcal{L} = -y_d\, q^\dagger \cdot \varphi\, d_R - y_u\, q^\dagger * \varphi^* u_R$$

The various fermion masses are

$$m_e = \frac{y_e v}{\sqrt{2}} \qquad m_d = \frac{y_d v}{\sqrt{2}} \qquad m_u = \frac{y_u v}{\sqrt{2}}$$

The model in which all fermion masses come from a single Higgs field has a quite nontrivial property called *natural flavor and CP conservation.* Consider the generalization of the model just described to the case of 3 generations. This requires, at least, a Yukawa coupling for each species. But, instead, we might write the most general renormalizable Yukawa interaction consistent with $SU(2) \times U(1)$,

$$\delta\mathcal{L} = -Y_e^{ij}\, L^{\dagger i} \cdot \varphi\, e_R^j - Y_d^{ij}\, q^{\dagger i} \cdot \varphi\, d_R^j - Y_u^{ij}\, q^{\dagger i} * \varphi^* u_R^j$$

where Y_e, Y_d, Y_u are general complex 3×3 matrices. This looks like a disaster, since these matrices seem to induce large flavor and CP violating terms.

However, it is possible to make changes of variables that remove these dangerous terms. The key to this is that any 3×3 matrix can be written in the form

$$Y = U_L y U_R^\dagger$$

where U_L and U_R are unitary and y is real, diagonal, non-negative. Then, in the first term, for example, we can define

$$e_R = U_{Re} e_R' \qquad L = U_{Le} L'$$

and find that this term simplifies to

$$- y_{ei} \, L^i \cdot \varphi \, e_R^i$$

The real, positive lepton masses correspond to the real diagonal elements of y_e. The change of variables must be made in the entire SM Lagrangian. But, since the L and e_R kinetic terms each include only one chirality,

$$e_R^\dagger i\sigma \cdot D e_R + L^\dagger i\bar{\sigma} \cdot D L = e_R'^\dagger i\sigma \cdot D e_R' + L^{\dagger\prime} i\bar{\sigma} \cdot D L'$$

The matrices U_{Le} and U_{Re} disappear from the Lagrangian. Similarly for quarks, we set

$$d_R = U_{Rd} d_R' \qquad u_R = U_{Ru} u_R'$$
$$d_L = U_{Ld} d_L' \qquad u_L = U_{Lu} u_L'$$

The matrices U_L, U_R cancel out in the same way, except in two places. First, the W interaction transforms

$$\frac{g}{\sqrt{2}} u_L^{+} \sigma W^{+} d_L \rightarrow \frac{g}{\sqrt{2}} u_L^{+} \sigma W^{+} (U_{Lu}^{+} U_{Ld}) d_L$$

giving a nontrivial unitary matrix that allows flavor transformation and CP violation in charge-changing weak interactions. This is the Cabibbo-Kobayashi-Maskawa matrix

$$V_{CKM} = U_{Lu}^{+} U_{Ld}$$

needed for the phenomenology of s, c, and b decays. It is now known that this matrix is sufficient to explain CP violation in the weak decays of these quarks. In a similar way, if we generalize the SM to produce neutrino masses, the neutrino mass terms will have nontrivial generation mixing and, possibly, CP violation.

The second residue of the U_L and U_R arises from axial vector anomaly in the strong interactions. The phase

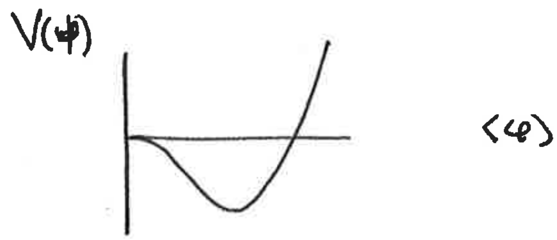
$$\phi = \arg \det \left[U_{Ld}^{+} U_{Rd} U_{Lu}^{+} U_{Ru} \right]$$

contributes to the θ-term of the QCD Lagrangian. This potentially introduces P and T violation in the strong interactions, for example, inducing a neutron electric dipole moment. This is a problem for the SM that requires some extension. There are possible solutions. In particular, it is possible to remove the effects of θ by introducing a very weakly coupled light boson called the *axion*.

There is one more interesting prediction that is unique to the EWSB model with one scalar field. In this model, the most general renormalizable Higgs potential is

$$V(\varphi) = +\mu^2 |\varphi|^2 + \lambda |\varphi|^4$$

We take $\mu^2 < 0$ to give the shape of the Higgs potential



in which the point $\varphi = 0$ is unstable and the potential is minimized at a nonzero value. I will have more to say about this in the next lecture, but, so far, so good. Now, assume that the SM with one Higgs field is unchanged up to very high energies. The potential is modified at high energies or large field values by radiative corrections. To leading order, this effect is described by the renormalization group running of the coupling constant λ

$$V(\varphi) = -\mu^2 |\varphi|^2 + \lambda(\varphi) |\varphi|^4$$

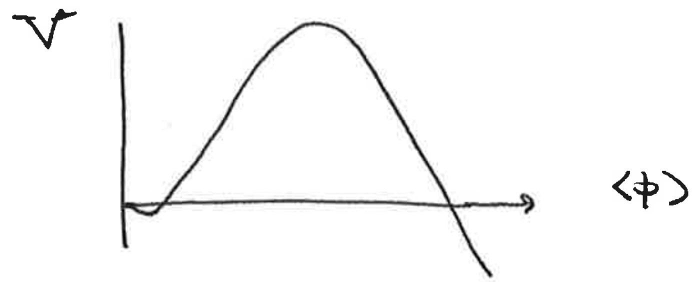
The renormalization group equation for λ is

$$\frac{d\lambda}{d\log Q} = \frac{3}{2\pi^2}\left[\lambda^2 + \frac{1}{2}\lambda y_t^2 - \frac{y_t^4}{4}\right]$$

Putting in

$$y_t^2 = \frac{2m_t^2}{v^2} \qquad \lambda = \frac{1}{2}\frac{m_h^2}{v^2} \qquad \begin{array}{l} m_t = 173 \text{ GeV} \\ m_h = 125 \text{ GeV} \end{array}$$

it turns out that the term involving the top quark Yukawa coupling y_t dominates, and so λ receives negative corrections as Q becomes large. The equation predicts that $\lambda(Q)$ goes through zero and becomes negative near $Q \sim 10^{11}$ GeV, so that the Higgs potential actually has the form



The unvierse can then tunnel to a very large expectation value of φ, however, with a very long lifetime of about 10^{600} years. It is not clear that this is a problem. Maybe the ultimate stability of the universe is too much to hope for. In any case, this instability is a prediction of the SM.

Before concluding this lecture, I would like to discuss one more topic that emphasizes the essential role that the Higgs boson plays in the structure of the SM. As I discussed in relation to vector boson mass generation, a massive vector boson is composed of chirality ± 1 states, which come from the original vector field that satisfies Maxwell's equations. That chirality 0 field must be a field that is physical if there is no spontaneous symmetry breaking and disappears when the gauge symmetry is broken. This is the property of the fields in the Higgs multiplet that become Goldstone bosons and are gauged away in the calculation of vector boson masses. The degree of freedom counting in the SM is

$SU(2)\times U(1)$ bosons	4×2 d.o.f.	EWSB $\rightarrow$	$W^+ W^- Z$	3×3 d.o.f.
φ	4 d.o.f.		A	2 d.o.f.
			h	1 d.o.f.

When we boost a massive vector boson to high energy, its states of definite helicity should become the pure states of definite chirality. The $h = \pm 1$ states should have properties of the original massless gauge bosons. But, then, the $h = 0$ state should have properties of the Goldstone bosons from the Higgs multiplet. This idea is encoded in the *Goldstone Boson Equivalence Theorem* of Cornwall, Tiktopoulos, and Vayonakis. For example

$$\mathcal{M}\left((A+B) \to C + W_0^+(p)\right) = \mathcal{M}\left(A+B \to C + \pi^+(p)\right)\left(1 + \mathcal{O}\left(\frac{m_W}{E_W}\right)\right)$$

This important principle governs many phenomena involving W and Z bosons at high energy.

There is some oddity in the calcuation of the amplitude for producing a longitudinally polarized W or Z boson. In the rest frame, the three polarization vectors of a massive vector boson are

$$\epsilon^m_- = \frac{1}{\sqrt{2}}(0\ 1\ -i\ 0)$$
$$\epsilon^m_+ = \frac{1}{\sqrt{2}}(0\ 1\ i\ 0) \qquad\qquad p = (m_W, 0, 0, 0)$$
$$\epsilon^m_0 = (0\ 0\ 0\ 1)$$

These satisfy $\epsilon^2 = -1$, $\epsilon \cdot p = 0$. Now boost p to high velocity

$$p = (E, 0, 0, p)$$

The vectors $\epsilon^m_\pm$ are not changed by the boost. But the vector ϵ^m_0 becomes

$$\epsilon^m_0 = \left(\frac{p}{m_W}, 0, 0, \frac{E}{m_W}\right)$$

so that

$$\epsilon_0 \cdot p = 0 \quad \text{but} \quad \epsilon^m_0 \rightarrow \frac{p^m}{m_W} \quad \text{as } p \rightarrow \infty$$

This means that certain amplitudes for the production of boosted W bosons are enhanced by unexpected factors of E_W/m_W.

An example of this is seen in top quark decay. The top quark decays to a b quark and an on-shell W boson. At leading order, the decay amplitudes for decay to transverse W bosons are

$$\mathcal{M}(t \to b W^+_L) = -1g\sqrt{2pm_t}\ \sin\theta/2$$
$$\mathcal{M}(t \to b W^+_R) = 0$$

where θ is the angle between the t spin and the W momentum. The amplitude for W_R^+ can be shown to vanish by angular momentum considerations. For longitudinally polarized W, we find

$$\mathcal{M}(t \to bW_0^+) = -i \frac{g}{\sqrt{2}} \sqrt{2pm_t} \; \frac{m_t}{m_W} \cos \theta/2$$

So this amplitude is enhanced by a factor

$$\frac{m_t}{\sqrt{2}\, m_W}$$

This reflects the E_W/m_W enhancement of the longitudinal polarization vector. The Higgs field does not appear directly in this calculation, which only involves the weak interaction gauge vertex. But we can also understand the enhancement factor as

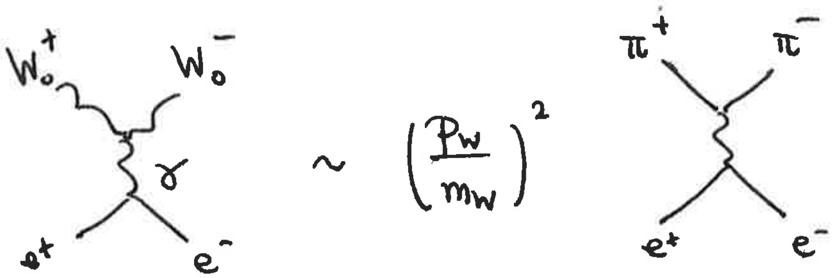
$$\frac{m_t}{\sqrt{2}\, m_W} = \frac{y_t v/\sqrt{2}}{\sqrt{2}\, g v/2} = \frac{y_t}{g}$$

This is, the W_0 couples to the top quark with the strength of a Higgs boson coupling rather than a gauge coupling. These amplitudes predict that

$$\frac{\Gamma(t \to bW_0^+)}{\Gamma(t \to bW^+)} = \frac{m_t^2/m_W^2}{2 + m_t^2/m_W^2} \cong 70\%$$

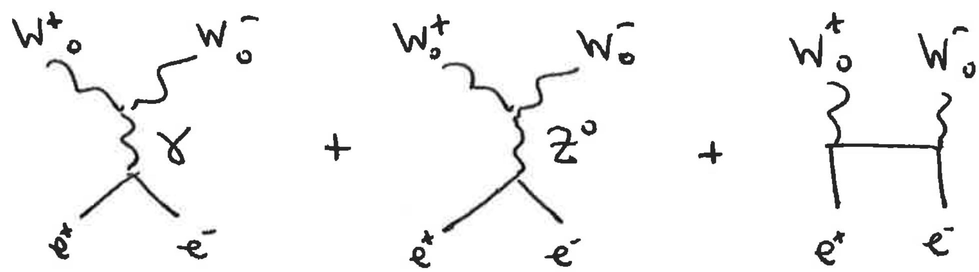
and this predictions is in good agreement with experiments at the Tevatron and the LHC.

Another example of the GBET is seen in the process $e^+e^- \to W^+W^-$. The enhancement factor in the polarization vectors implies that the diagrams

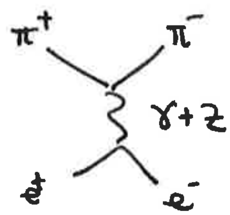


This enhancement is a factor s/m_w^2 in the amplitude or $(s/m_w^2)^2$ in the cross section. This would make the cross section grow rapidly, in fact, so rapidly that it violates unitarity.

The GBET tells us that this cannot happen. In fact, by explicit calculation, there is a massive cancellation among the three diagrams



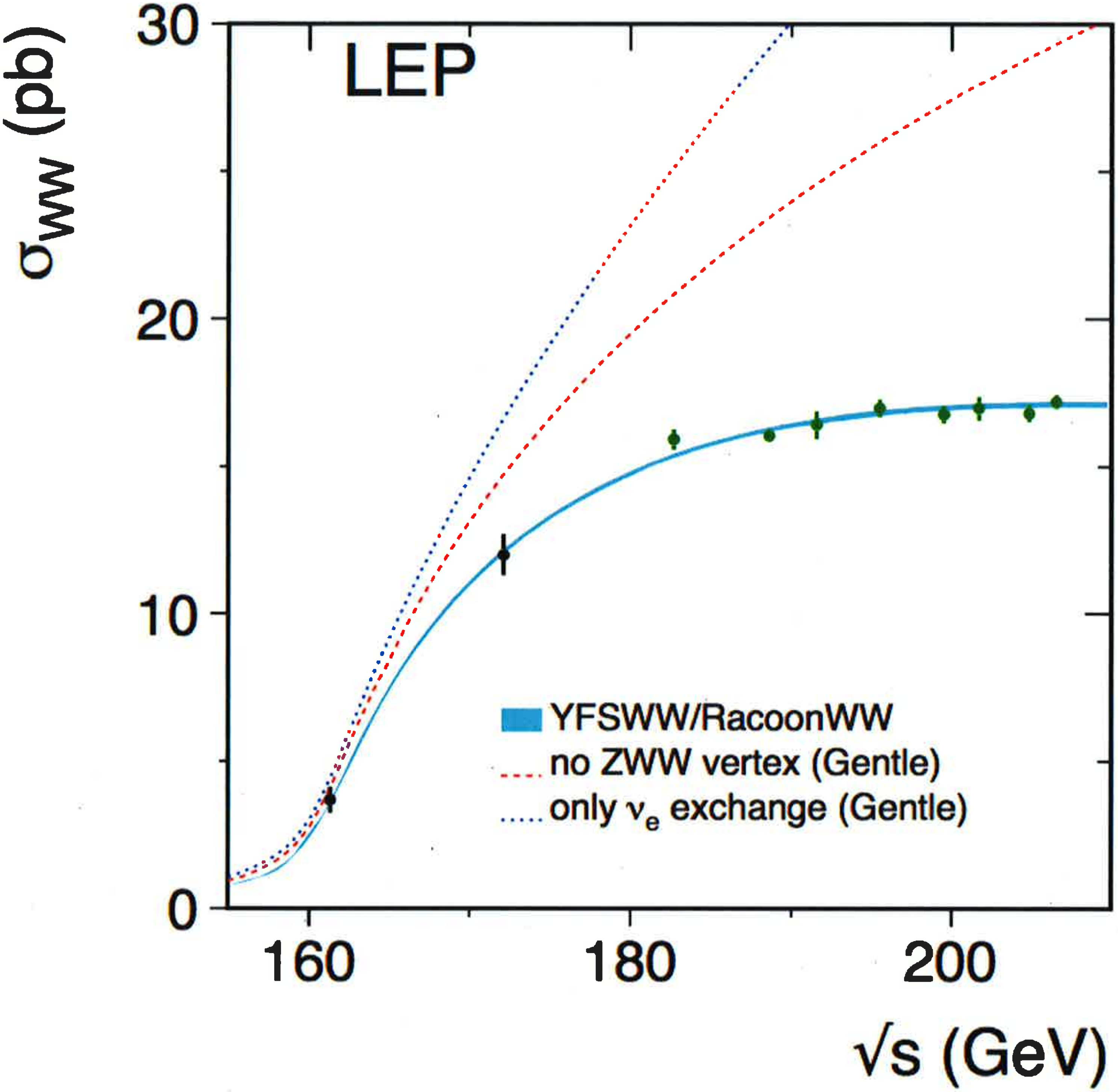
leading to a result that equals the value of the diagram



at high energy. The cross section for W pair production has been measured at LEP (Figure 4), and the cancellation is necessary for agreement with the measurements.

These are some of the most surprising features of the SM predictions for W production at high energies. The Higgs boson does not appear in the calculations I have just described. But the answers are what they are only because the W boson secretly knows that its mass comes from spontaneous symmetry breaking.

$\sigma(e^+e^- \to W^+W^-)$



LEP Electroweak Working Group.