Lecture 6:
A NNLO Distribution: Rapidity of Drell-Yan Lepton Pairs

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Motivation

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- But, state-of-the-art is not quite there yet. Require:
  - building an intricate set of subtraction terms
  - evaluating their phase-space integrals analytically
  - stable numerical integration of subtracted cross section
- In the meantime, some NNLO distributions of experimental interest can be attacked more directly, and analytically, using traditional multi-loop techniques to do the phase-space integrals:
  - integration-by-parts (IBP)
  - reduction to master integrals

Motivation (cont.)

- One such quantity is the rapidity distribution $d\sigma/dY$ for inclusive hadronic production of a massive color-singlet object, *e.g.* $V = \gamma^*, W, Z$, or H (where $\gamma^* \to \ell^+\ell^-$). Here $Y = \frac{1}{2}\ln\left(\frac{E+p_z}{E-p_z}\right)$.

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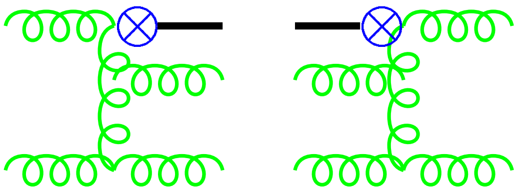
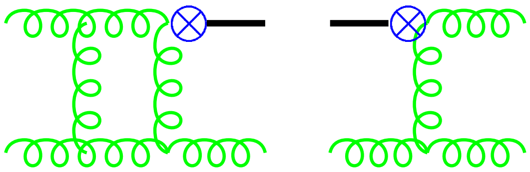
- First application of “IBP method” to hadron collider cross sections was to total inclusive cross section for
  $\mathrm{pp} \to H + X$. Anastasiou, Melnikov
  Here extend the method to a distribution.

Anatomy of NNLO $gg \to H + X$

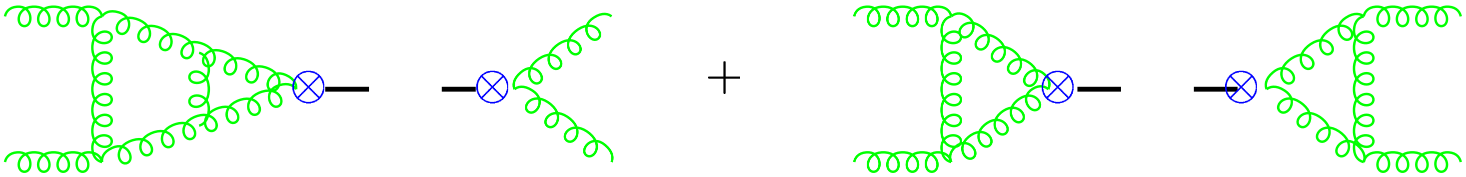
- Topologically exactly the same as $q\bar{q} \to V + X$.
- 3 types of terms:
  - real $\times$ real



  - virtual $\times$ real

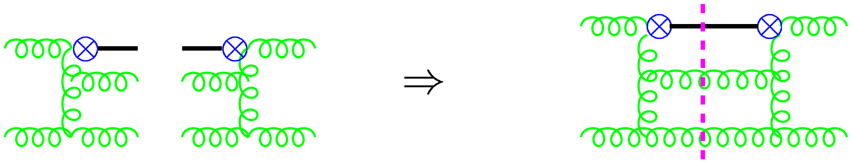


  - doubly virtual $+$ virtual $\times$ virtual

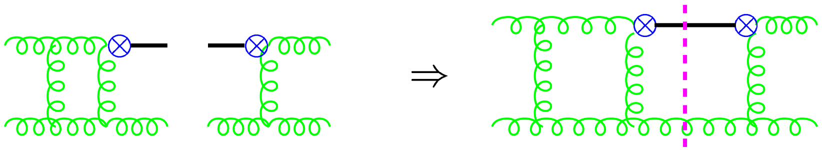


Use optical theorem

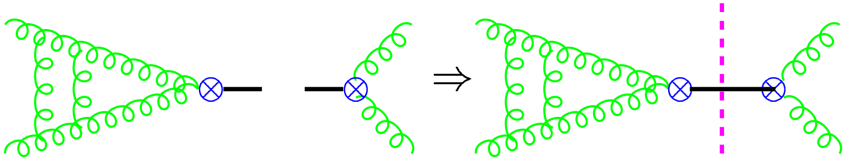
- All 3 terms can be treated uniformly as different cuts contributing to imaginary part of $gg \to gg$ forward scattering amplitude:
- real $\times$ real $\Rightarrow$ 3-particle cut



- virtual $\times$ real $\Rightarrow$ 2-particle cut



- doubly virtual $+$ virtual $\times$ virtual $\Rightarrow$ 1-particle cut

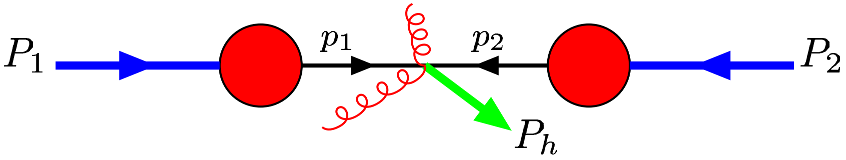


One more δ-function

- Optical theorem corresponds to Cutkosky rules:

$$\delta(p_i^2 - m_i^2) \Rightarrow \frac{1}{2\pi i}\left(\frac{1}{p_i^2 - m_i^2 - i\varepsilon} - \frac{1}{p_i^2 - m_i^2 + i\varepsilon}\right)$$

- As in EEC case (lecture 4), can insert another δ-function in the same way.



$P_1 = \frac{\sqrt{s}}{2}(1, \vec{0}, 1)$ $\qquad p_1 = x_1 P_1$

$P_2 = \frac{\sqrt{s}}{2}(1, \vec{0}, -1)$ $\qquad p_2 = x_2 P_2$ $\qquad P_h = (E, \vec{p_T}, p_z)$

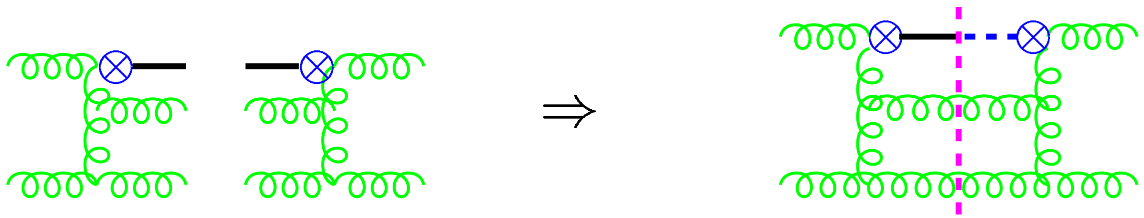
$$e^{-2Y} = \frac{E - p_z}{E + p_z} = \frac{p_1 \cdot P_h / x_1}{p_2 \cdot P_h / x_2} \quad \Rightarrow \quad u \equiv \frac{x_1}{x_2} e^{-2Y} = \frac{p_1 \cdot P_h}{p_2 \cdot P_h}$$

The new δ-function

- To get $d\sigma/dY$, just insert extra factor of $\delta\left(u - \frac{p_1 \cdot P_h}{p_2 \cdot P_h}\right)$ inside all integrals for total cross section σ.
- To treat as cut "funny" propagator, replace

$$\delta\left(u - \frac{p_1 \cdot P_h}{p_2 \cdot P_h}\right) \quad \Rightarrow \quad \frac{1}{2\pi i}\left(\frac{p_2 \cdot P_h}{(p_1 - up_2) \cdot P_h - i\varepsilon} - (+i\varepsilon)\right)$$

as indicated by dashed blue line in



Apply multi-loop/IBP technology

- Reduce integrals to master integrals.
  Same basic procedure discussed for EEC in lecture 4

  Laporta

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- Extract soft/collinear limits $u \to z$ and $u \to 1/z$
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  interior region: $z < u < 1/z$; $z < 1$.

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- Same integrals for all processes: γ^*, W, Z, H.

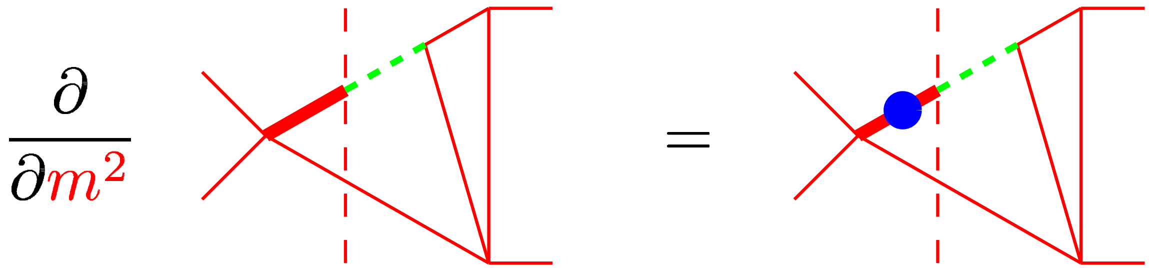
Calculation of master integrals

- Use differential equations to integrate difficult master integrals.
- For example,

$$\frac{\partial}{\partial m^2}\left\{\int d^Dk \frac{1}{(k^2-m^2)(k+p)^2\cdots}\right\} =$$

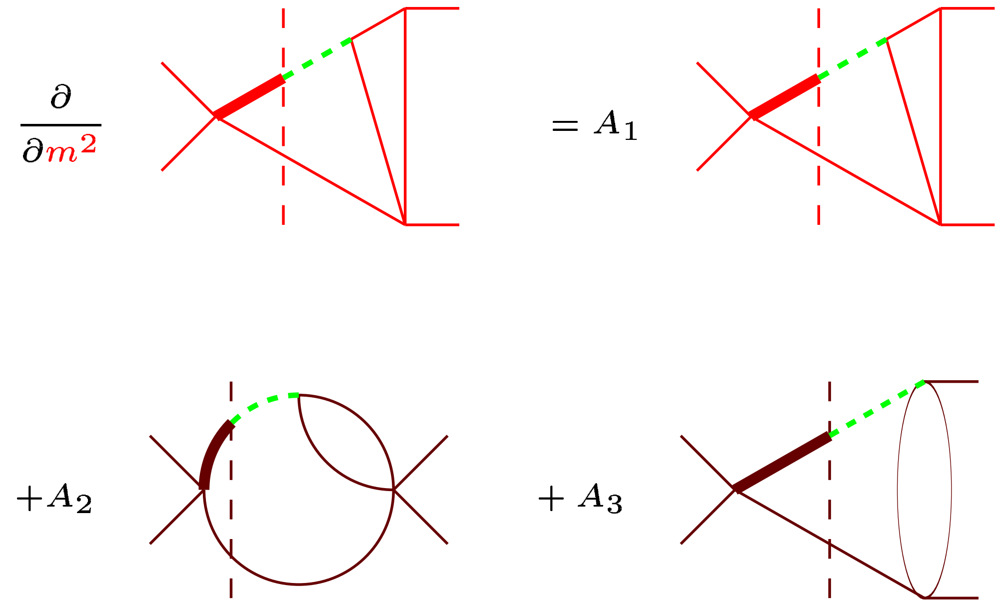
$$\int d^Dk \frac{1}{(k^2-m^2)^2(k+p)^2\cdots}$$

- Diagrammatically,



Differential equations

- Reduce "dotted" integral to master integrals $\Rightarrow$



- Similar differential eqn. for u
- Solve as Laurent series in ϵ from $1/\epsilon^2$ in interior; $1/\epsilon^3$ and $1/\epsilon^4$ on soft/collinear boundaries.
- Solve for simpler "boundary integrals" on RHS first.

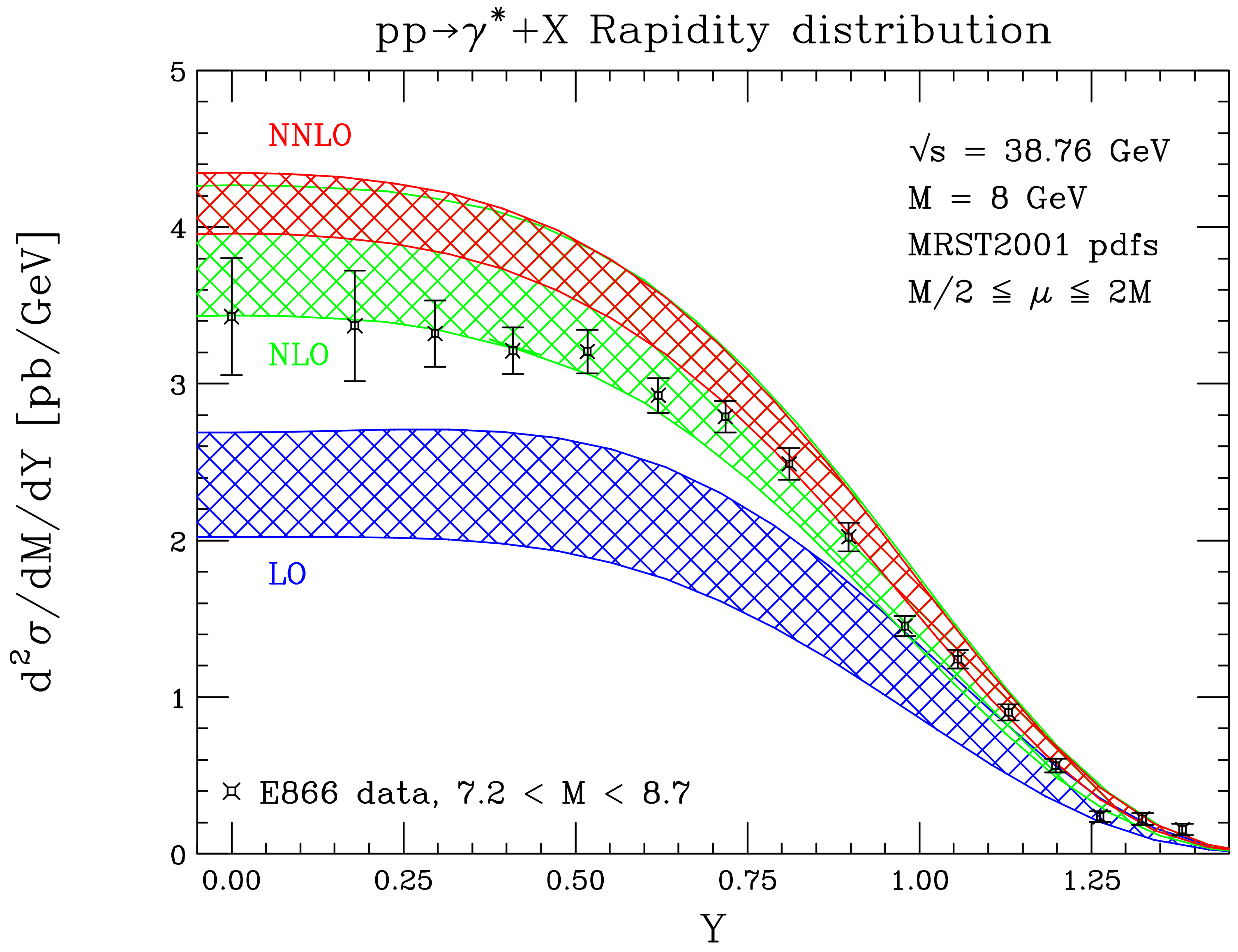
Checks & Phenomenology

- After renormalizing virtual terms, and factorizing initial-state collinear singularities, result is finite.
- Integral of $d\sigma/dY$ over Y reproduces NNLO Drell-Yan total cross section σ

  Hamberg, van Neerven & Matsuura (1991); Harlander & Kilgore

- First application: $\mathrm{pp} \to \gamma^* + X$ at $\sqrt{s} = 38.76$ GeV, lepton-pair invariant mass $M = 8$ GeV, for comparison to fixed-target experiment E866/NuSea.
- NLO result dates to 1979: Altarelli, Ellis, Martinelli (1979)

Drell-Yan rapidity distribution vs. data



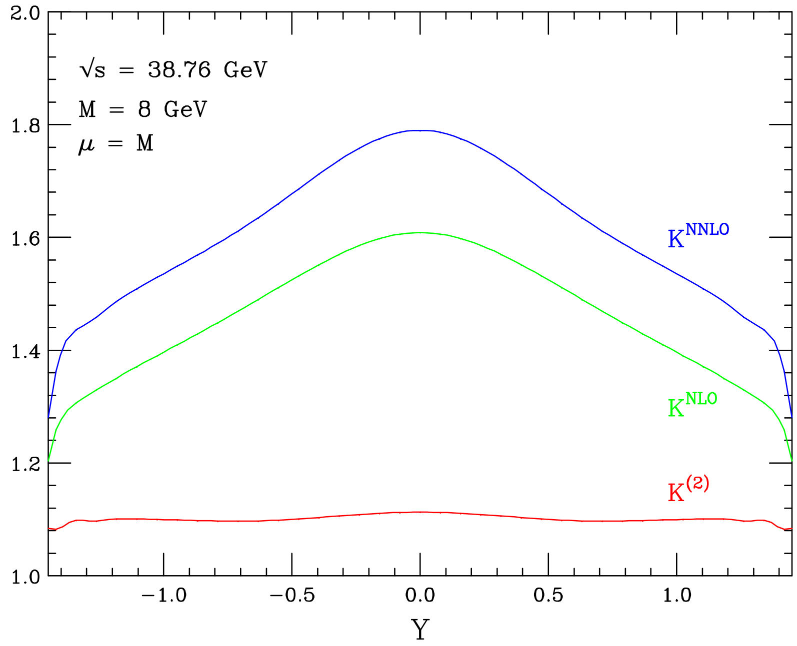
Remarks

- Data is lower than theory for small Y;
  rises up to meet it at larger Y.
- However, input antiquark "NNLO" pdfs from MRST were actually determined in part by fitting to Drell-Yan data using an NLO distribution in $x_F = 2p_z/\sqrt{s}$;
  should probably now refit using NNLO result.

K factors

- Define

$$\mathrm{K}^{\mathrm{NNLO}} \equiv \frac{d\sigma^{\mathrm{NNLO}}/dY}{d\sigma^{\mathrm{LO}}/dY} \qquad \mathrm{K}^{\mathrm{NLO}} \equiv \frac{d\sigma^{\mathrm{NLO}}/dY}{d\sigma^{\mathrm{LO}}/dY} \qquad \mathrm{K}^{(2)} \equiv \frac{d\sigma^{\mathrm{NNLO}}/dY}{d\sigma^{\mathrm{NLO}}/dY}$$



- $\mathrm{K}^{(2)}$ is remarkably independent of Y — suggests using *e.g.* "MC@NLO" and rescaling results by $\mathrm{K}^{(2)}$. Generality?

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- This means that there is still plenty of room for new ideas and techniques, to increase precision, and enlarge the range of processes that can be studied.
- You are more than welcome to join the fun!