Lecture 1:
Multi-loop Techniques in Field Theory
and Collider Applications

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Lecture Series Outline

- Lecture 1 (*Antipasti*): Motivation
- Lecture 2 (*Prima Piatti*): NNLO Computations and Two-loop Amplitudes
- Lecture 3 (*Carpaccio*): Unitarity Techniques and Two-loop Integrals
- Lecture 4 (*Insalata*): A Toy Example: The Energy-Energy Correlation at Leading Order
- Lecture 5 (*Secondi Piatti*): The Higgs $\to \gamma\gamma$ signal at the LHC
- Lecture 6 (*Dolci*): NNLO Rapidity Distribution of Drell-Yan Lepton Pairs

Precision Collider Physics

- High energy collider processes provide our most direct look at short-distance physics and — we hope — new physics beyond the Standard Model:
  - Hadron colliders, Tevatron and LHC, over next decade
  - e^+e^- linear collider thereafter (?)
- If new physics is subtle, we will need the best possible predictions of SM backgrounds.
- Most collider physics involves partonic interactions — even at an e^+e^- collider, most events are hadronic.
- Advances in perturbative QCD pave way for more precise predictions of SM backgrounds to new physics.

Precision physics at the LHC

- Enormous amount of high quality data will become available
- One year at low luminosity $\Rightarrow \int \mathcal{L} dt = 10$ fb^{-1} $\Rightarrow$
  - $W \to \mathrm{e}\nu$: 10^8 events
  - $Z \to \mathrm{e}^+\mathrm{e}^-$: 10^7 events
  - $t\bar{t}$: 10^7 events
  - Higgs ($m_H = 700$ GeV): 10^4 events
  - Jets with $p_T > 200$ GeV: 10^9 events

Gianotti, Altarelli

- Statistical errors $< 1\%$. Measurements limited by experimental systematics, theoretical uncertainties.

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- Few collider cross sections are available yet at NNLO:
  - pp$\rightarrow (W, Z, \gamma^*, H) + X$ total cross sections
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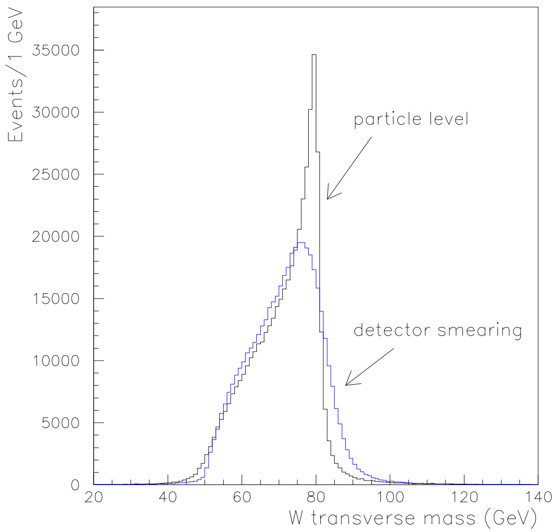
  - deep inelastic scattering (DIS) & sum rules
- What do they have in common?
  1. "totally inclusive" — simple final state definition
  2. few scales involved

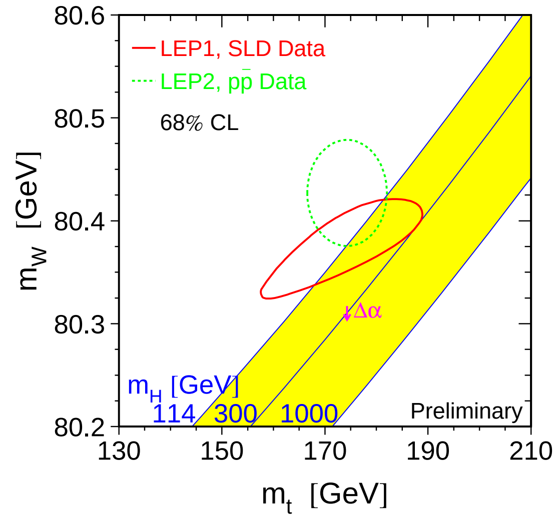
Status of pQCD for colliders (cont.)

- Experiments require NNLO precision for more differential, less inclusive distributions.
- This is true for a variety of processes, as we'll see in the following examples.

Example: W mass measurement

- LHC goal:
  $\Delta M_W = 15$ MeV, so ΔM_W isn't dominant in EW fits.





LEPEWWG

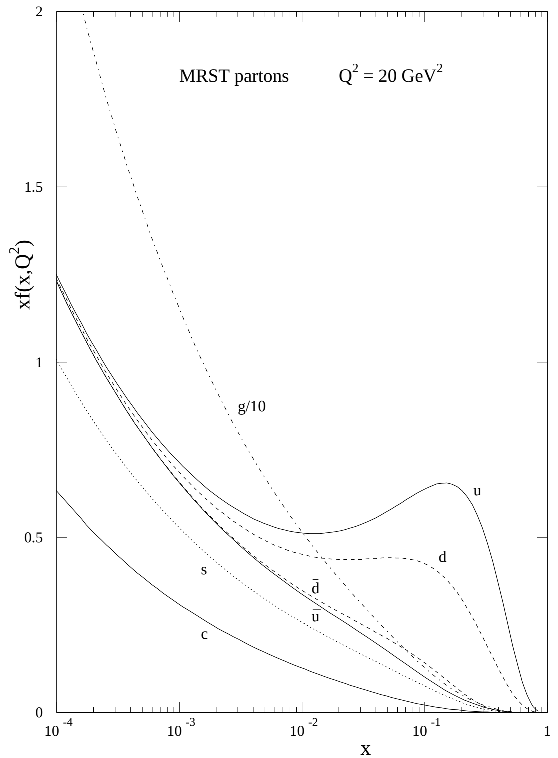
- Requires excellent understanding of W transverse mass distribution, $M_T = \sqrt{2 p_T^{\ell} p_T^{\nu} (1 - \cos\phi_{\ell\nu})}$

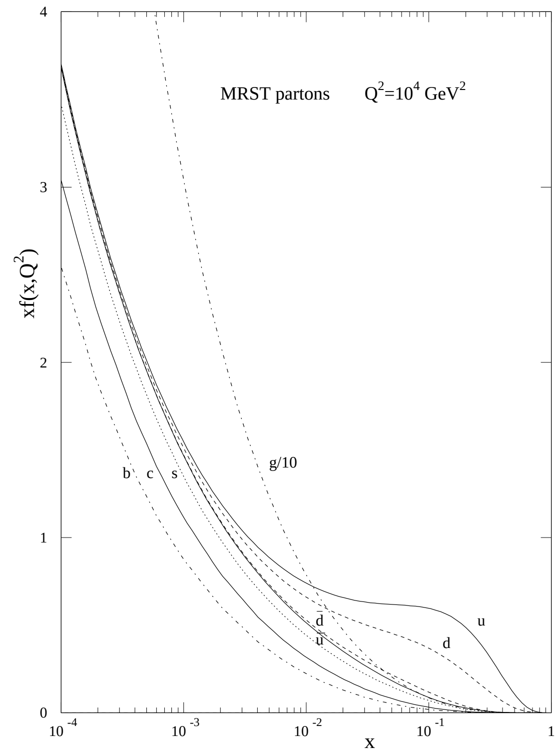
W mass measurement (cont.)

- Understanding detector M_T distribution in turn requires understanding transverse momentum p_T^W and rapidity $Y^W = \frac{1}{2}\ln(\frac{E+p_z}{E-p_z})$ distributions:
  - p_T^W distribution needed to predict p_T^ℓ, p_T^ν
  - Y^W distribution affects lepton acceptance
- To bring error from each source below 10 MeV, few % precision needed for each distribution $\Rightarrow$ NNLO QCD required.

Example: Constraining parton distributions

- All hadron collider predictions depend on parton distribution functions inside the proton (pdfs).





MRST

$Q^2 = 20$ GeV^2 $\Rightarrow$ $Q^2 = 10^4$ GeV^2

Parton distributions (cont.)

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- Small x gluons are huge at large $Q^2 \Rightarrow$ LHC is a gluon factory. Important for Higgs production, $gg \to H$.
- Ideally, perform global fit to data with NNLO cross sections and NNLO evolution in Q^2.

Parton distributions (cont.)

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- Besides DIS — Zijlstra, van Neerven
  and very recent Drell-Yan rapidity distribution — Anastasiou et al.
  no other hard cross sections known at NNLO yet either.

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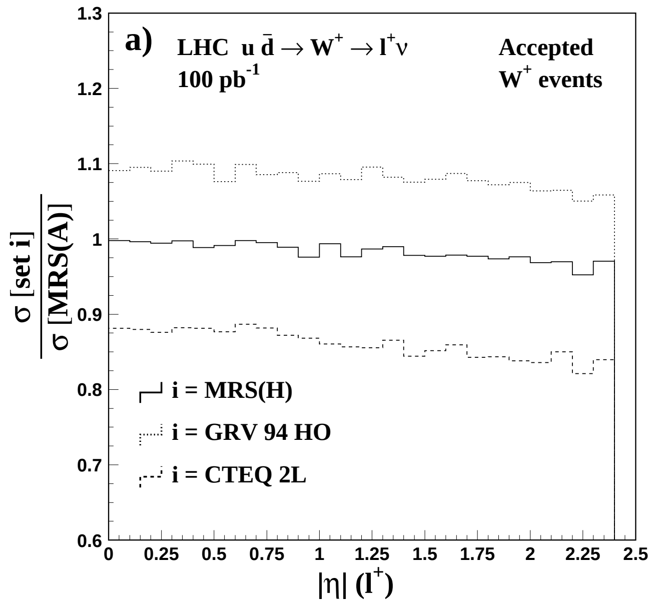
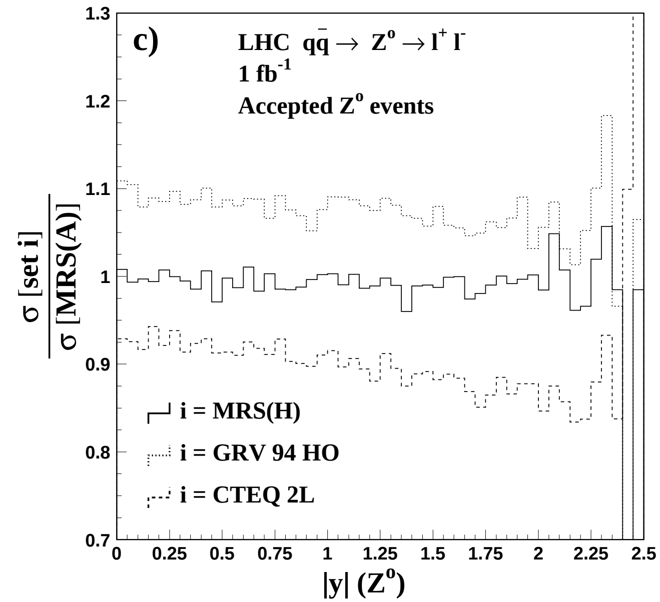
- But an approximate NNLO form exists — van Neerven, Vogt
- Besides DIS (Zijlstra, van Neerven) and very recent Drell-Yan rapidity distribution (Anastasiou et al.) no other hard cross sections known at NNLO yet either.
- When LHC W, Z rapidity distributions are known at NNLO, data should allow a few % measurement of pdfs — "partonic luminosity monitor" — Dittmar, Pauss, Zuercher

Parton distributions (cont.)

- At LO, $\mathrm{pp} \to W^+ X$ directly measures $u(x_1)\bar{d}(x_2)$,
  where $M_W^2 = s x_1 x_2$, $Y^W = \ln(x_1/x_2)/2$.





Dittmar

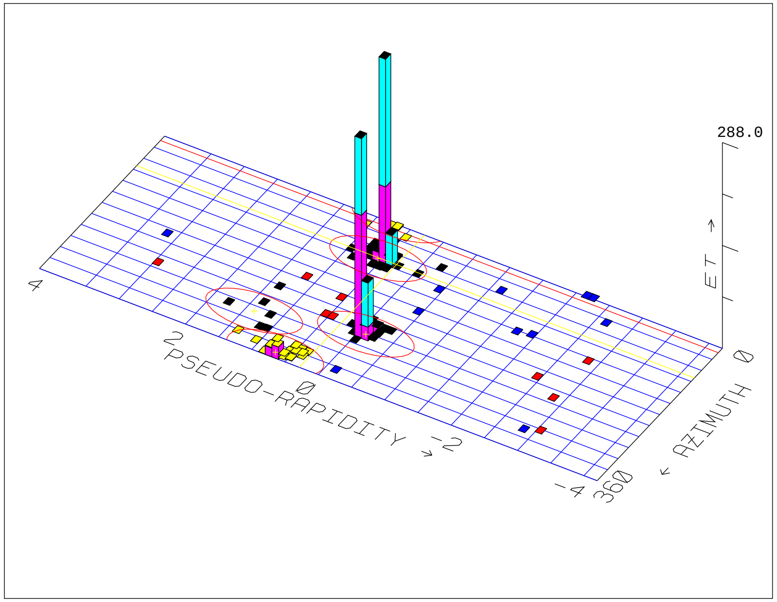
- Experimental error for each bin is 1%,
  for a mere $0.1,\ 1$ fb^{-1}!

Example: Jets at hadron colliders

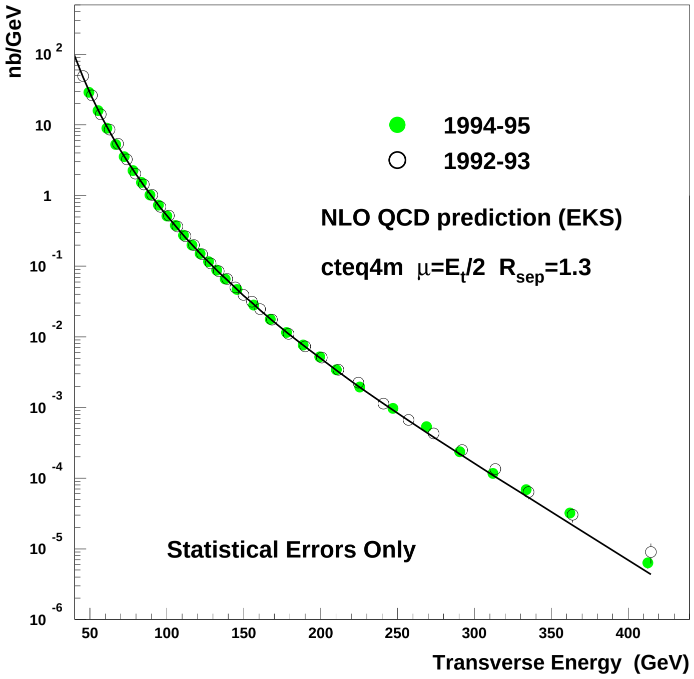
- Parton-parton scattering, $q\bar{q} \to q\bar{q}$, $q\bar{q} \to gg$, $gg \to gg$, etc., produces high-E_T jets at hadron colliders.
- Because $\alpha_s > \alpha_{EW}$, jet rates are large, and give access to the shortest distance scales.
- However:
  - a jet is a complicated object, compared with a lepton $\Rightarrow$ jet energy (p_T) scale hard to determine precisely (3% at Tevatron $\to$ 1% at LHC)
  - jet p_T distributions are steeply falling, so a small energy error can produce a big cross section error
  - cross sections also depend on large x pdfs, especially $g(x)$.

Jets at hadron colliders (cont.)

- Jets at Tevatron fit NLO theory pretty well over many orders of magnitude in cross section:

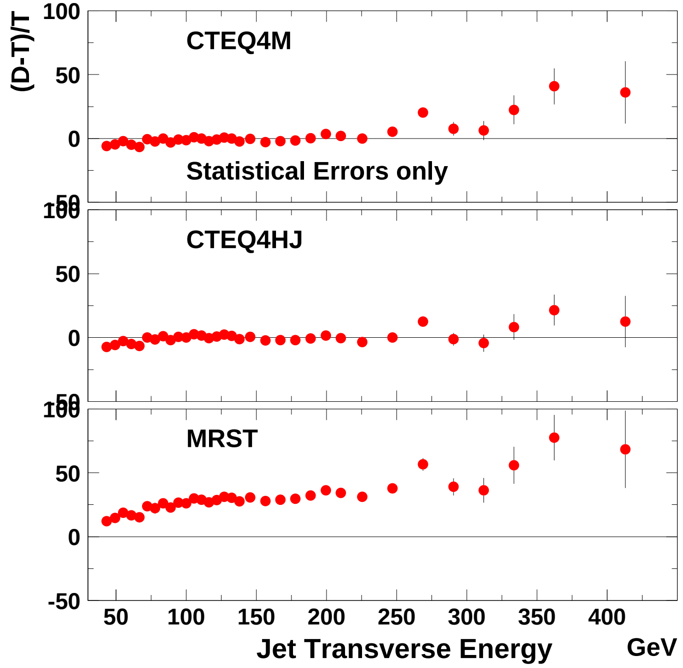


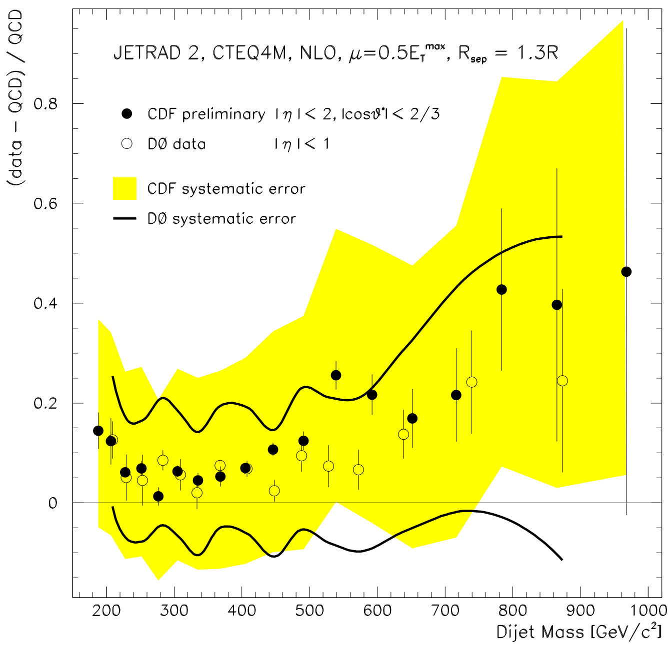
CDF



Jets at hadron colliders (cont.)

- Reported excess at high E_T





- Can be removed by increasing large x gluons.

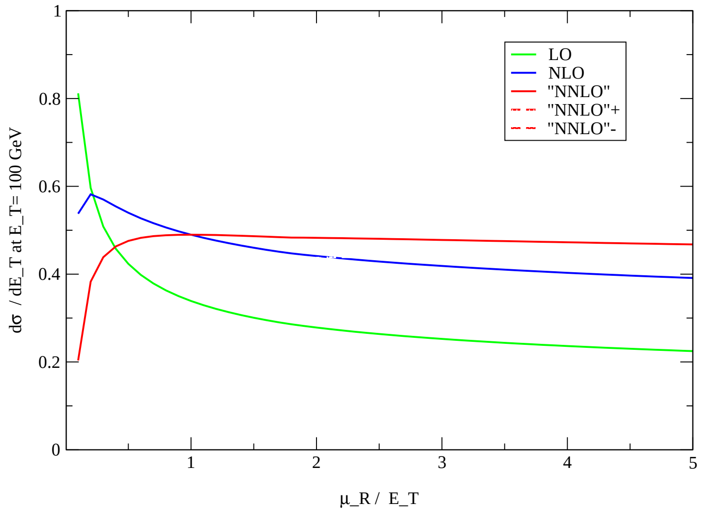
Jets at hadron colliders (cont.)

For the future (LHC):

$$\delta\sigma^{\text{exp}}_{\text{CDF}} = 2\%(\text{stat}) + 10\%(\text{syst}) \Rightarrow \delta\sigma^{\text{exp}}_{\text{ATLAS}} = 1\%(\text{stat}) + 5\%(\text{syst})$$

NNLO required to match this precision theoretically

$$\delta\sigma^{\text{th}}_{\text{NLO}} = 15\%(\text{intrinsic}) \quad \Rightarrow \quad \delta\sigma^{\text{exp}}_{\text{NNLO}} = (3-4)\%(\text{intrinsic})$$

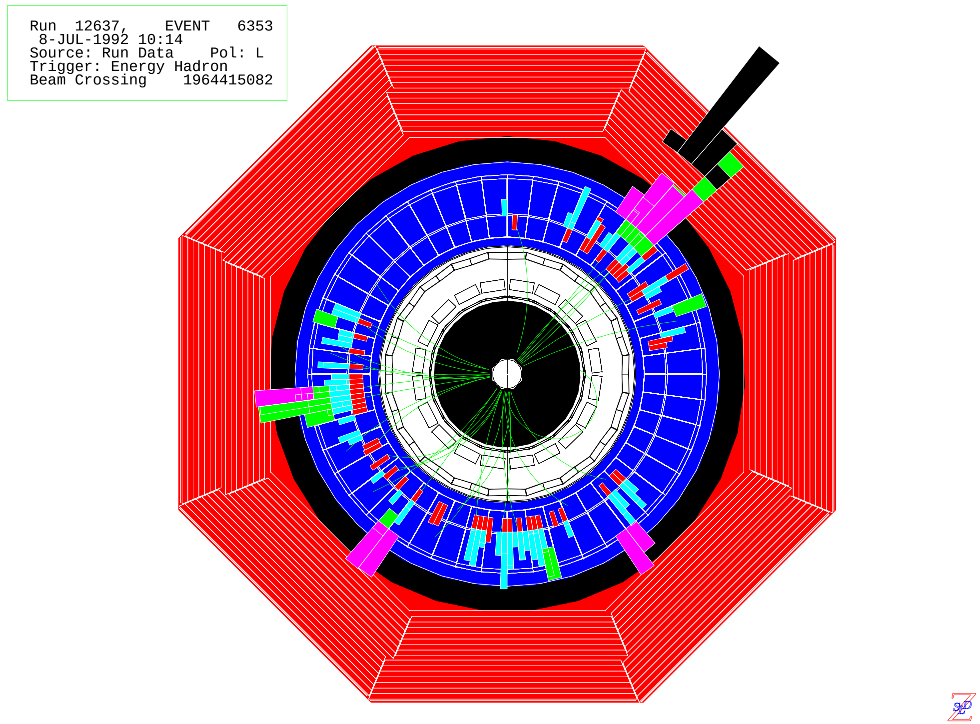


Glover

Example: Jets at e^+e^- colliders

- Experimentally clean; well-defined initial state (no pdfs)

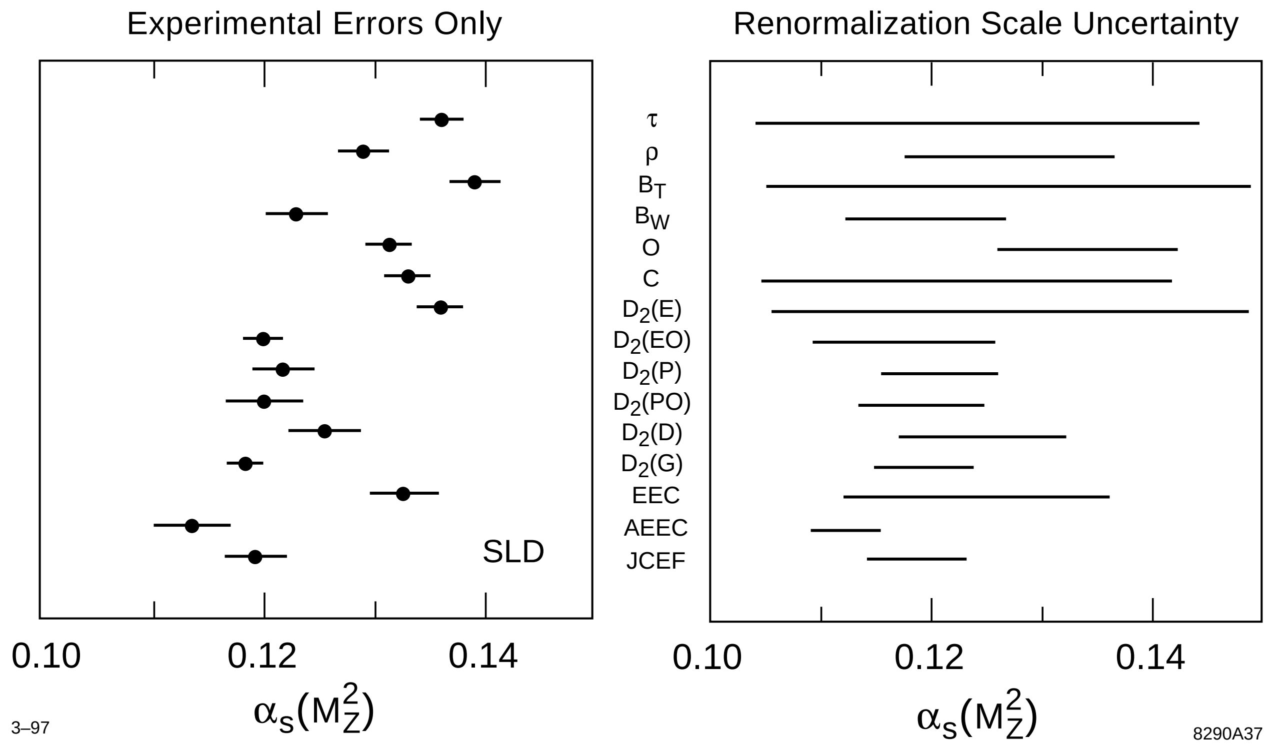
SLD



- every particle “belongs to a jet” (no underlying event)
- $\Rightarrow R_{3-\mathrm{jet}} \equiv \sigma_{3-\mathrm{jet}}/\sigma_{\mathrm{had}}$ is ideal for measuring α_s

Jets at e^+e^- colliders (cont.)

- Unfortunately, limited by NLO theoretical uncertainty:
  $\delta\alpha_s^{\rm th} \approx 0.007 \quad \gg \quad \delta\alpha_s^{\rm exp}$ at $\sqrt{s} = M_Z$.
- In fact, many different "event shape" measures of $\alpha_s(M_Z)$ give inconsistent results when fit to NLO predictions:



Burrows

Jets at $\mathrm{e}^+\mathrm{e}^-$ colliders (cont.)

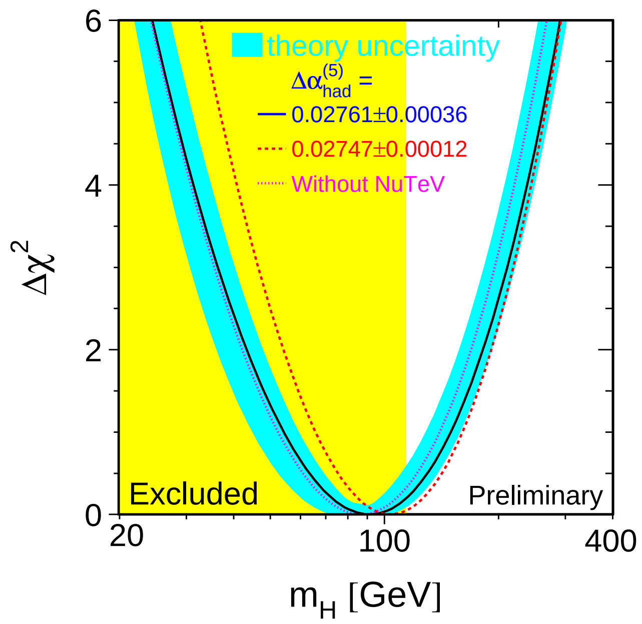
For the future (linear collider):

- High luminosity $\Rightarrow$ plenty of statistics
- Larger $\sqrt{s}$ $\Rightarrow$ nonperturbative corrections smaller ($\propto \Lambda_{\mathrm{QCD}}/\sqrt{s}$)
- Events are not quite as clean as at M_Z, but can:
  - use e^-_R to suppress W^+W^- background — Schumm
  - anti-b tag to suppress $t\bar{t}$ background — Burrows
- A high precision α_s measurement would
  - improve tests of coupling constant unification in GUTs
  - provide important input into other physics, e.g. precise extraction of m_t from $\mathrm{e}^+\mathrm{e}^- \to t\bar{t}$ threshold scan

Example: Higgs production at LHC

The Higgs boson is probably light

- Precison electroweak data favor $m_H < 195$ GeV   LEP



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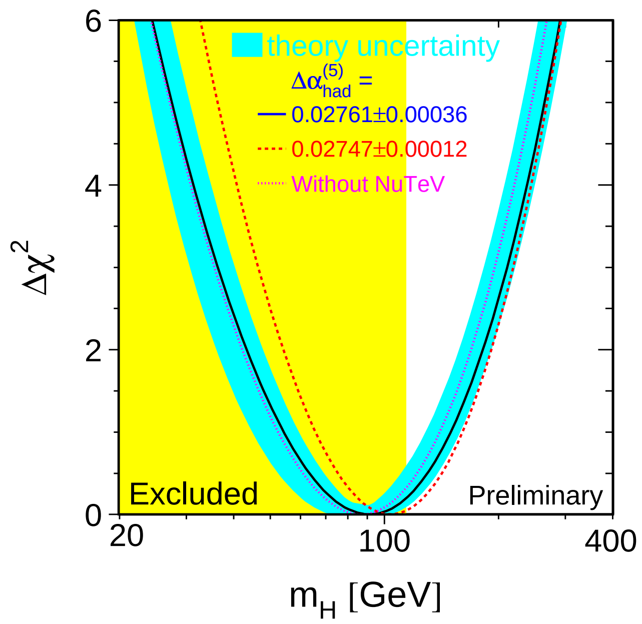

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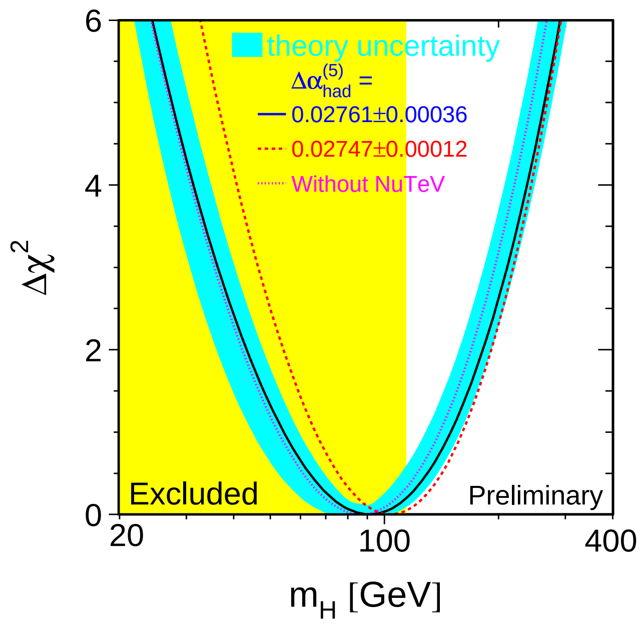
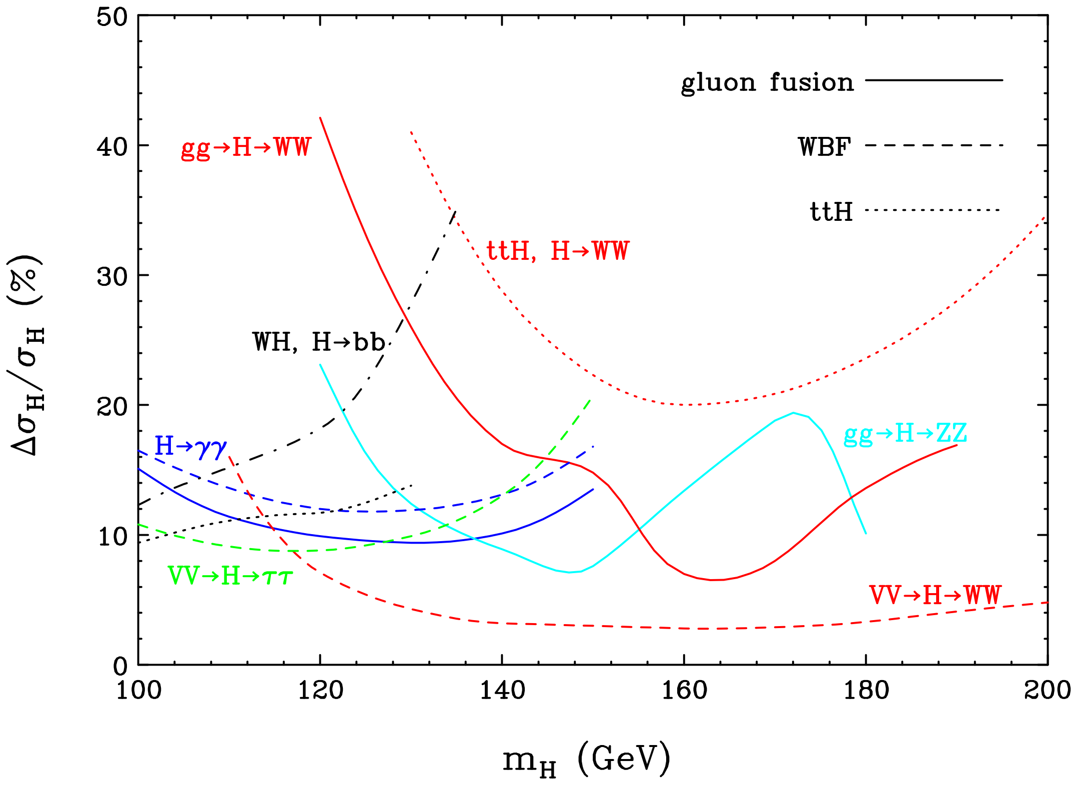


LEPEWWG

- Supersymmetry favors $m_H < 140$ GeV — Carena et al.
- LEP hints at $m_H = 115$ GeV — ALEPH

Higgs production (cont.)

- At the LHC, a light Higgs boson can be produced and detected in a number of channels.
- Typical anticipated experimental accuracies are $\sim 10\%$.



Zeppenfeld

Higgs production (cont.)

- $\Rightarrow$ Need theoretical prediction for production cross section to $\sim 10\%$ also.
- For largest production channel, gluon fusion, $gg \to H$, NLO corrections are huge, $\sim 80\%$ Dawson; Djouadi, Spira, Zerwas
- This necessitated the recent computation of the NNLO corrections to the total cross section; Catani, De Florian, Grazzini now understood at $\sim 15-20\%$ level Harlander, Kilgore Anastasiou, Melnikov
- NNLO corrections to Y^H distribution would be useful.
- Higher-order corrections to certain background processes, e.g. $\mathrm{pp} \to \gamma\gamma X$, of interest too.

A few more examples

- Heavy quark production ($b\bar{b}$ and $t\bar{t}$) at hadron colliders.
  - reported $b\bar{b}$ "excess" at Tevatron — CDF; D0
    — but see also Cacciari & Nason
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- Hadronic production of $W+$jets, WZ and WW pairs.
  - Important backgrounds to Tevatron Higgs search, supersymmetry.
- NNLO QED corrections to Bhabha scattering, $\mathrm{e}^+\mathrm{e}^- \to \mathrm{e}^+\mathrm{e}^-$
  - Precision luminosity monitor at a linear collider

Uncertainty estimation

- An observable $A(\alpha_s)$ in QCD should not depend on the renormalization scale μ_R, but in fact it does, due to truncation error. Suppose

$$A(\alpha_s) = \sum_{i=0}^{N} A_i(\mu_R) \left(\frac{\alpha_s}{2\pi}\right)^{n+i} \quad \Rightarrow \quad \frac{\partial A(\alpha_s)}{\partial \ln(\mu_R^2)} = \mathcal{O}(\alpha_s^{n+N+1})$$

- Using the β function,

$$\frac{\partial}{\partial \ln(\mu_R^2)} \frac{\alpha_s(\mu_R)}{2\pi} = \frac{\beta(\alpha_s)}{2\pi} = -b_0 \left(\frac{\alpha_s}{2\pi}\right)^2 - b_1 \left(\frac{\alpha_s}{2\pi}\right)^3 - \cdots$$

can predict the μ_R dependence of $A_i(\mu_R)$.

Uncertainty estimation (cont.)

Find

$$
\begin{aligned}
A_0(\mu_R) &= A_0 \\
A_1(\mu_R) &= A_1 + n b_0 \ln(\mu_R^2) A_0 \\
A_2(\mu_R) &= A_2 + (n+1) b_0 \ln(\mu_R^2) A_1 \\
&\quad + \left[\frac{n(n+1)}{2} b_0^2 \ln^2(\mu_R^2) + n b_1 \ln(\mu_R^2) \right] A_0
\end{aligned}
$$

with

$$
b_0 = \frac{11 C_A - 4 T_R N_f}{6} \qquad b_1 = \frac{17 C_A^2 - (10 C_A + 6 C_F) T_R N_f}{6}
$$

$$
C_A = N_c = 3, \quad C_F = ({N_c}^2 - 1)/(2N_c) = 4/3, \quad T_R = 1/2
$$

Uncertainty estimation (cont.)

- Thus, can predict μ_R dependence of NNLO term prior to calculation, so can get an idea of how much the scale variation will be reduced. Inclusive jet example:

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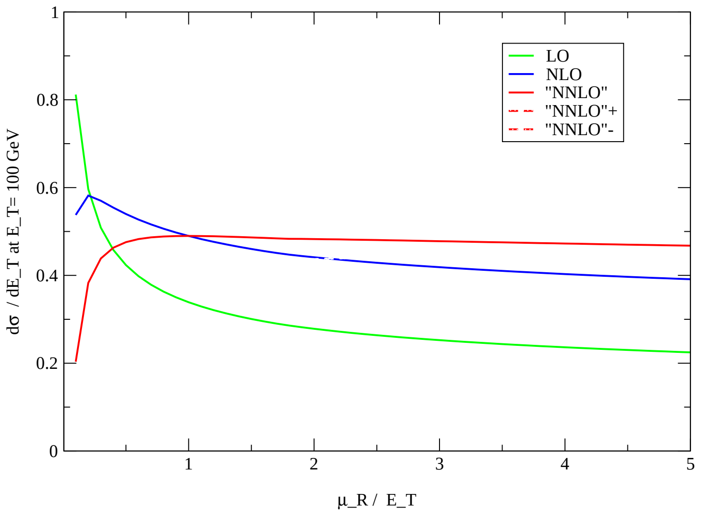

Glover

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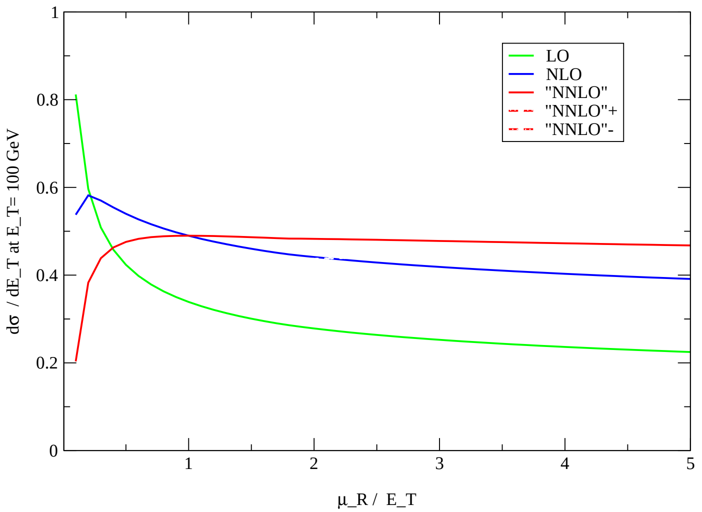

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- Scale-independent term A_2 cannot be predicted.
  Error estimation by scale variation won't capture new physical processes, color structures, etc., opening up at next order.

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- μ_F is arbitrary separation point between scale defining hadron structure and scale defining hard process.
- μ_F dependence controlled by DGLAP equations,

$$
\begin{aligned}
\frac{\partial f_i(x,\mu_F)}{\partial \ln(\mu_F^2)} &= \frac{\alpha_s}{2\pi}\sum_j \int_x^1 \frac{d\xi}{\xi} P_{ij}(x/\xi,\alpha_s) f_j(\xi,\mu_F) \\
P_{ij} &= P_{ij}^{(0)} + \frac{\alpha_s}{2\pi} P_{ij}^{(1)} + \cdots \\
P_{qq}^{(0)} &= C_F\left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\delta(1-x)\right], \cdots
\end{aligned}
$$

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  - some in conjunction with resummation of large logs.
- Ideally, one would like a NNLO program that can compute distributions with flexible cuts to mimic the experimental situation.
- Next time we will start to study the ingredients of a generic NNLO computation.