Ph 152/252a Problem Set II Solutions

1)
$$F^{\mu\nu} = \begin{bmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{bmatrix}$$

Or, in our case where $\vec{E} = 0$ and $\vec{B} = 0\hat{i} + 0\hat{j} + B\hat{k}$,

$$F^{\mu\nu} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -B & 0 \\ 0 & B & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Lowering an index with η gives:

$$F^{\mu}{}_{\nu} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & B & 0 \\ 0 & -B & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$m \frac{dp^{\mu}}{d\tau} = q p^{\nu} F^{\mu}{}_{\nu}$$

$$\Rightarrow \quad m \frac{dp^2}{d\tau} = q p^{\nu} F^{2}{}_{\nu} = q p^1 F^2{}_1 = q p_x (-B)$$

and

$$m \frac{dp^1}{d\tau} = q p^{\nu} F^{1}{}_{\nu} = q p^2 F^1{}_2 = q p_y (+B)$$

and $m \frac{dp^3}{d\tau} = 0 \Rightarrow$ There is no force in the z direction so v_z is constant

Or: $\quad \gamma m \frac{dp_y}{dt} = -q p_x B \quad \Big| \quad \gamma m \frac{dp_x}{dt} = +q p_y B$

Note: $\frac{d}{dt}(p_x^2 + p_y^2) = 2p_x \frac{dp_x}{dt} + 2p_y \frac{dp_y}{dt} = 0$, so $\dot{\gamma} = 0$

$$\Rightarrow \quad \gamma^2 m^2 \ddot{y} = -q \gamma m \dot{x} B$$
$$\gamma^2 m^2 \ddot{x} = +q \gamma m \dot{y} B$$

where the dot denotes differentiation with respect to t

$$\Rightarrow \left.\begin{aligned} \dot{x} &= -\frac{\gamma m \ddot{y}}{qB} \\ \dot{y} &= +\frac{\gamma m \ddot{x}}{qB} \end{aligned}\right\} \Rightarrow \left.\begin{aligned} x &= -\frac{\gamma m \dot{y}}{qB} + x_0 \\ y &= +\frac{\gamma m \dot{x}}{qB} + y_0 \end{aligned}\right\} \Rightarrow \begin{aligned} (x - x_0) &= -\frac{p_y}{qB} \\ (y - y_0) &= +\frac{p_x}{qB} \end{aligned}$$

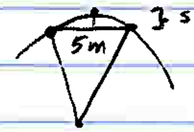
$$\therefore \quad r = \sqrt{(x-x_0)^2 + (y-y_0)^2} = \sqrt{(p_y^2 + p_x^2)/(qB)^2}$$

$\Rightarrow r = p/(qB)$ where $p = \sqrt{p_x^2 + p_y^2}$ So, because v_z is constant and the particle moves in a circle of radius $r = p/(qB)$ when projected onto the x-y plane

2) $B = 1\,T$

$1\ TeV\ \mu$

$m_\mu = 105.7\ MeV$



From Bettini, $s \doteq \frac{L^2}{8r}$ where r is the radius of curvature

$$r = \frac{p}{qB} = \frac{\sqrt{E^2 - m^2}}{qB} = 3.34 \cdot 10^5\ cm$$

$$\Rightarrow \boxed{s \approx 0.937\ mm}$$

3) For large N, statistical fluctuations which are Poisson become approximately gaussian with $\sigma = \sqrt{N}$

We get 60 photoelectrons / MeV so $N = 6 \cdot 10^4 \cdot E(GeV)$

$$\Rightarrow \quad \sigma_N = \sqrt{N} = \sqrt{6 \cdot 10^4} \cdot \sqrt{E(GeV)}$$

$$\Rightarrow \sigma_E = \frac{\sigma_N}{6 \cdot 10^4} = \frac{\sqrt{E(GeV)}}{\sqrt{6 \cdot 10^4}} \Rightarrow \frac{\sigma_E}{E} = \frac{1}{\sqrt{6 \cdot 10^4 \cdot E(GeV)}}$$

$\Rightarrow$ $\boxed{\text{In percent, the resolution is } \frac{\Delta E}{E} \doteq \frac{0.408\%}{\sqrt{E(GeV)}}}$

If we only actually detect 1% of the energy deposited, then we have a resolution of

$$\frac{\Delta E}{E} = \frac{1}{\sqrt{6 \cdot 10^4 \cdot 0.01 \cdot E(GeV)}} = \frac{4.08\%}{\sqrt{E(GeV)}}$$

which is 10 times worse.

4) Bettini 6.19

$\ell = 20 m$

$I = 90\% I_0$

$\tau_{\pi^-} = 2.60 \cdot 10^{-8} s$ is the mean lifetime

The distance travelled in the lab frame in terms of the time elapsed in the π frame (t_0) is:

$$d = (\beta c)(\gamma t_0)$$

$$p = \gamma \beta c m \Rightarrow \gamma \beta c = \frac{p}{m}$$

$$\Rightarrow d = \frac{p t_0}{m} \Rightarrow p_\pi = \frac{\ell \cdot m_\pi}{t_0}$$

The decay of the πs means that:

$$N(t) = N_0 e^{-t/\tau_\pi}$$

where t is measured in the π rest frame

$\therefore$ In our case,

$$\frac{N(t)}{N_0} = 0.90 = e^{-t_0/\tau}$$

$$\Rightarrow -\ln(0.90) \cdot \tau_\pi = t_0$$

$$\Rightarrow p_{\pi^-} = \frac{\ell m_{\pi^-}}{-\tau_{\pi^-} \ln(0.90)}$$

$$\Rightarrow \boxed{p_\pi = 3.40 \text{ GeV}/c}$$

$$\Rightarrow \boxed{E_\pi = \sqrt{p_\pi^2 + m_\pi^2} \doteq 3.40 \text{ GeV}}$$

5 From the PDG, $X_0 \doteq \dfrac{716.4\,g\,cm^{-2}\,A}{Z(Z+1)\ln(287/\sqrt{Z})}$

For Al, we find in the periodic table that

$Z = 13$ and $A = 26.98$

$\Rightarrow X_0 = 24.26\ g/cm^2$

The density of Al is $2.7 g/cm^3$ so in terms of length,

$$\boxed{X_0 \doteq 8.99 cm}$$

Also, $R_M = 0.0265\, X_0\, (Z+1.2)$ so,

$$\boxed{R_M \approx 3.38 cm}$$

As everyone found, these values roughly agree with the simulation.