Ph 152/252a Problem Set I Solutions

January 29, 2009

1.

$$r = 1\,\text{fm} = 10^{-13}\,\text{cm}$$

$$\Delta x \cdot \Delta p \geq \hbar/2$$

$$\implies \Delta p \geq \hbar/2r = 98.7\,\text{MeV/c}$$

$$\boxed{(\Delta p)_{min} = 98.7\,\text{MeV/c}}$$

$$m_e \ll (\Delta p)_{min}$$

$$\implies \boxed{(\Delta E)_{min} = 98.7\,\text{MeV} \gg 18\,\text{keV}}$$

Note: Since Δx and Δp are meant to be standard deviations in the formal definition of the uncertainty principle, this is really just a rough order of magnitude estimate. That is, the radius is not the standard deviation of position and σ_x cannot really be calculated without knowing the wavefunction of the electron. Because of differences in convention, the uncertainty principle is also sometimes expressed as $\Delta x \cdot \Delta p \geq \hbar$, though this means that Δx and Δp are no longer equal to the standard deviation of position and momentum. Considering the approximate nature of this calculation, either solution was accepted for full credit.

2. Bettini 1.4

$$\tau = \hbar/\Gamma$$

$$\hbar = 6.58 \cdot 10^{-22}\,\text{MeV s}$$

$$\Gamma_\rho = 149\,\text{MeV} \implies \tau_\rho = 4.41 \cdot 10^{-24}\text{s}$$

$$\Gamma_\omega = 8.5\,\text{MeV} \implies \tau_\omega = 7.74 \cdot 10^{-23}\text{s}$$

$$\Gamma_\phi = 4.3\,\text{MeV} \implies \tau_\phi = 1.53 \cdot 10^{-22}\text{s}$$

$$\Gamma_{K^*} = 51\,\text{MeV} \implies \tau_{K^*} = 1.29 \cdot 10^{-23}\text{s}$$

$$\Gamma_{J/\psi} = 93\,\text{keV} \implies \tau_{J/\psi} = 7.08 \cdot 10^{-21}\text{s}$$

$$\Gamma_\Delta = 118\,\text{MeV} \implies \tau_\Delta = 5.58 \cdot 10^{-24}\text{s}$$

3. Bettini 1.8

$$\text{CMB} \implies T = 3\,\text{K}$$

$$E_{\gamma,3\text{K}} \approx 1\,\text{meV} = 10^{-3}\,\text{eV}$$

$$\text{EBL} \implies \lambda \approx 1\mu\text{m} \implies E = h\nu = hc/\lambda = 1.24\,\text{eV}$$

We obtain the maximum energy in the CM if we have a head-on collision in the lab frame so before the collision:

$$s_{\text{LAB}} = (E_\gamma + E)^2 - (|p_\gamma| - |p|)^2 = (E_\gamma + E)^2 - (E_\gamma - E)^2 = 4E_\gamma E$$

After the collision in the CM, we have:

$$s'_{\text{CM}} = (2m_e)^2 - 0^2$$

Setting these invariant masses equal gives:

$$E = m_e^2/E_\gamma$$

so plugging in the photon energies for the CMB and EBL yields:

$$\boxed{E_{\text{thresh}}^{\text{CMB}} = 261\,\text{TeV}}$$

$$\boxed{E_{\text{thresh}}^{\text{EBL}} = 211\,\text{GeV}}$$

4. Bettini 1.10

$$E_p = 7\,\text{TeV}$$

Since we have a head on collision of two equal energy protons, the total CM/LAB energy is (obviously) just:

$$E^\star_{\text{TOT}} = 14\,\text{TeV}$$

For a fixed hydrogen target before the collision in the LAB frame,

$$s_{\text{LAB}} = (E_p + m_p)^2 - p_p^2 = m_p^2 + m_p^2 + 2E_p m_p$$

We want the CM energy to be 14 TeV as above, so

$$s_{CM} = (E^\star_{\text{TOT}})^2$$

$$s_{\text{CM}} = s_{\text{LAB}} \implies (E^\star_{\text{TOT}})^2 = 2m_p^2 + 2E_p m_p$$

or:

$$E_p = \frac{(E^\star_{\text{TOT}})^2 - 2m_p^2}{2m_p}$$

So we have that

$$\boxed{E_p = 1.04 \cdot 10^{17}\,\text{eV}}$$

if we want the CM energy of the fixed target experiment to be the same as that of the head-on collision experiment at LHC. This is well below the maximum energy of cosmic rays which is on the order of 10^{20} eV.

Problem Set 1

(5) In the two-body decay $B^0 \to K^+\pi^-$, with the B^0 at rest, the value of $p_K = |\vec{p}_K| = |\vec{p}_\pi|$ is

$$p_K = \frac{\sqrt{\lambda(m_B^2, m_K^2, m_\pi^2)}}{2m_B}$$

where $\lambda(x,y,z) = x^2 + y^2 + z^2 - 2(xz + yz + zx)$

plugging in $m_B = 5280$ MeV, $m_K = 493.7$ MeV, $m_\pi = 139.6$ MeV

$\Rightarrow$ $p_K = 2615$ MeV

Similarly, for $B^0 \to \pi^+\pi^-$, $p_\pi = |\vec{p}_\pi|$ is given by

$$p_\pi = \frac{\sqrt{\lambda(m_B^2, m_\pi^2, m_\pi^2)}}{2m_B} = 2636 \text{ MeV}$$

The fractional difference in momentum is $\frac{|p_K - p_\pi|}{p_\pi} = 0.8\%$ ← momentum resolution required

Velocity $v_K = \frac{1}{\sqrt{1 - \frac{1}{\gamma_K^2}}}$, where $\gamma_K = \frac{E_K}{m_K} = \sqrt{\frac{p_K^2}{m_K^2} + 1}$

$\Rightarrow$ $v_K = 0.9826$

Similarly, $v_\pi = 0.9986$

We want $v_\pi > \frac{1}{n}$ for Čerenkov radiation with π

and $v_K < \frac{1}{n}$ for no " " " K.

$$\Rightarrow \frac{1}{v_\pi} < n < \frac{1}{v_K}$$

or $1.0014 < n < 1.0177$

Problem Set 1

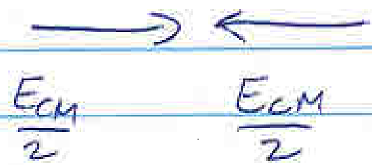
(6) $e^- e^- \to e^- e^-$:

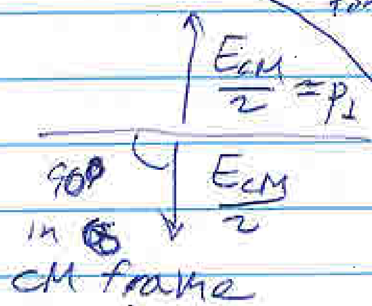
Lab frame:

$p_1^\mu = (E, 0, 0, p)$ $\quad$ $p_2^\mu = (m_e, 0, 0, 0)$

$p_\parallel = 22.5$ GeV for each e^- by symmetry

CM frame:





$$(\text{CM energy})^2 = s = (p_1^\mu + p_2^\mu)^2 = (E + m_e, 0, 0, E)^2$$

$$= (E + m_e)^2 - p^2 = 2 m_e E + 2 m_e^2 \to \text{neglect}$$

with $E = 45$ GeV, $m_e = 0.511$ MeV

$$\Rightarrow \boxed{E_{CM} = \sqrt{s} = \sqrt{2 m_e E} = 211.4 \text{ MeV}}$$

Since $E_{CM} \gg m_e$,

$$p_\perp = \sqrt{\left(\frac{E_{CM}}{2}\right)^2 - m_e^2} \simeq \frac{E_{CM}}{2} = 105.7 \text{ MeV}$$

By symmetry of the 90° scattering, $p_\parallel = \frac{E_{beam}}{2} = 22.5$ GeV

$$\Rightarrow \underline{\theta_{lab}} = \tan^{-1} \frac{p_\perp}{p_\parallel} \simeq \tan^{-1} \frac{105.7}{22{,}500} \simeq \underline{4.7 \text{ milliradians}}$$