Problem Set 3 — due Feb. 11

1. (10 pts) Problem 6.11 of Griffiths:
(a) Is $A \to B + B$ a possible process in the ABC theory?
(b) Suppose a diagram has n_A external A lines, n_B external B lines, and n_C external C lines. Develop a simple criterion for determining whether it is an allowed reaction.
(c) Assuming A is heavy enough, what are the next most likely decay modes, after $A \to B + C$? Draw a Feynman diagram for each decay.

2. (15 pts) Problem 7.9 of Griffiths (plus):
(a) The charge conjugation operator (C) acts on a Dirac spinor ψ by $\psi \to i\gamma^2\,\psi^*$. Find the charge conjugates of $u^{(1,2)}$ and compare them to $v^{(1,2)}$.
(b) Also, work out how the fermion bilinears $\bar\psi\psi$, $\bar\psi\gamma^\mu\psi$ and $\bar\psi\gamma^\mu\gamma^5\psi$ transform under C. (Hint: rewrite the bilinear after applying C by setting it equal to its transpose, and moving the various extra γ matrices past $(\gamma^\mu)^T$.)
(c) Show that $A_\mu \to -A_\mu$ under C, by considering how (moving) charges give rise to **E** and **B** fields, and how they change when the sign of all charges are reversed. How does the QED interaction $e\bar\psi\gamma^\mu\psi\,A_\mu$ transform under C?
(d) How does the electron-Z axial coupling $g_A\bar\psi\gamma^\mu\gamma^5\psi\,Z_\mu$ transform under C? Under the combination of C and P, also known as CP?

3. (5 pts) Problem 7.15 of Griffiths:
Show that the adjoint spinors $\bar u^{(1,2)}$ and $\bar v^{(1,2)}$ satisfy $\bar u(\not p - m) = 0$ and $\bar v(\not p + m) = 0$, given that $(\not p - m)u = 0$ and $(\not p + m)v = 0$.

4. (10 pts) Problem 7.46 of Griffiths:
Why can't the photon "decay" by the process $\gamma \to \gamma\gamma$ mediated by a closed fermion loop (triangle diagram)? (Don't use energy-momentum conservation for this.) This is an example of *Furry's theorem*, which says that any closed electron loop with an odd number of photons emerging from it gives a vanishing contribution to the amplitude.

5. (15 pts) Work out the matrix-element-squared, and differential cross section $d\sigma/d\cos\theta$ for unpolarized Compton scattering, $\gamma e \to \gamma e$, in the CM frame, in the high energy limit when you may neglect $m_e \to 0$. For the sum over polarizations of a photon with momentum q, you may use

$$\sum_{s=1,2} \epsilon^\mu_{(s)}(q)\epsilon^{\nu\,*}_{(s)}(q) = -\eta^{\mu\nu}$$

Is the total cross section finite in the high energy limit? Now use crossing symmetry to get the same results for $e^+e^- \to \gamma\gamma$ and $\gamma\gamma \to e^+e^-$.