Problem Set 2 — due Jan. 28

1. (10 pts) From the relativistic force law,

$$m\frac{dp^\mu}{d\tau} = qp^\nu F^\mu_\nu$$

show that a particle of charge q in a uniform magnetic field B in the z direction travels in a helix, which projected into the xy plane is a circle. Show that the radius of the circle is given by

$$r = \frac{p}{qB}$$

where $p = \sqrt{p_x^2 + p_y^2}$ is the magnitude of the momentum in the *transverse* or xy plane, independent of the velocity (*i.e.* show the usual formula is valid relativistically).

2. (5 pts) Muon momenta in the ATLAS detector at the LHC are measured by their bend angle in a toroidal magnetic field of ~ 1 T over a distance of 5 meters. What is the radius of curvature of a 1 TeV muon in a 1 T magnetic field? If three points are measured on this circle, with the outer 2 points separated by 5 meters, what is the *sagitta*, the distance of the middle point from a straight line between the outer 2 points? (This distance controls how well the muon detection system must be aligned.)

3. (10 pts) A CsI crystal scintillator produces light output which (after wavelength shifting) impinges on the photocathode of a photomultiplier tube. This particular device produces 60 photoelectrons per MeV of energy deposited in the crystal. The inferred energy deposition is proportional to the photomultiplier output, which in turn is proportional to the number of photoelectrons. Statistical fluctuations in the number of photoelectrons (sampling fluctuations) control the energy resolution of the device. The fluctuations are Poisson for small average numbers of photons, but become Gaussian for larger numbers of photons. What is the fractional energy resolution, $\Delta E/E$, in percent, as a function of E(GeV) (for E in the GeV range)? Suppose layers of scintillator/photomultiplier tubes, with similar photoelectron yield to the above, were included as the sampling layers of a sampling calorimeter, and suppose they typically capture 1% of the shower energy deposited. Now what would the sampling contributions to the fractional energy resolution be, in percent, as a function of E(GeV)?

4. (5 pts) Problem 1.19 of Bettini.

[turn over for problem 5]

5. (10 pts) What is the radiation length of aluminum in cm? Go to

`http://www2.slac.stanford.edu/vvc/egs/basicsimtool.html`

This program produces a Monte Carlo simulation of an electromagnetic shower. Investigate the showering of a 1 GeV electron in aluminum. Adjust the parameters until you can get a nice picture which lets you estimate the radiation length. Compare with "theory". Estimate the transverse spread, the radius of a cylinder around the initial particle direction that contains 90% of the energy, called the "Molière radius" R_M. The approximate formula is

$$R_M = 0.0265\, X_0\, (Z + 1.2).$$

Compare with theory. Print and attach the picture.