FE.1

Ph 152A/252A Final Exam Solutions

(1) • The quark momentum fraction is given (at lowest order in QCD) by

$$x = \frac{Q^2}{2p_P \cdot q} = \frac{Q^2}{2\underbrace{M_p}_{\text{proton mass}}\underbrace{(E_e - E_e')}_{\text{electron energy loss in lab frame}}}$$

$q^\mu = p_e^\mu - p_{e'}^\mu$

• And Q^2 is given by

$$Q^2 = -(p_e - p_{e'})^2 = +2E_e E_e'(1-\cos\theta)$$

So, $$E_e - E_e' = \frac{Q^2}{2M_p x}$$

or $$E_e' = E_e - \frac{Q^2}{2M_p x} = \left[30 - \frac{4}{2(0.938)(0.3)}\right] \text{GeV}$$

$$\Rightarrow \boxed{E_e' = 22.89\,\text{GeV}}$$

$$1-\cos\theta = \frac{Q^2}{2E_e E_e'} = \frac{4}{2\cdot 30 \cdot 22.89}$$

$$\Rightarrow \boxed{\begin{aligned}\theta &= 0.07633 \text{ radians}\\ &= 4.374^\circ\end{aligned}}$$

• For the minimum value of x, we solve the Q^2 equation for E_e', $E_e' = \dfrac{Q^2}{2E_e(1-\cos\theta)}$

$$\Rightarrow x = \frac{Q^2}{2M_p\underbrace{\left(E_e - \frac{Q^2}{2E_e(1-\cos\theta)}\right)}_{\text{want as big as possible}}}$$

(want as small as possible $\Rightarrow \cos\theta = -1$

[Although it's hard to put a detector here!]

$$\Rightarrow \quad x_{min} = \frac{Q^2}{2M_p\left(E_e - \frac{Q^2}{4E_e}\right)}$$

FE.2

1 (CONT.) For $Q^2 = 4\ \text{GeV}^2$, $E_e = 30\ \text{GeV}$,

$$\boxed{x_{min} = \frac{4}{2 \cdot 0.938\left(30 - \frac{4}{4 \cdot 30}\right)} = 0.0712}$$

(Assuming you could put the e^- detector right in the beamline!)

- Now for HERA, $\underset{e^- \rightarrow}{30\ \text{GeV}}$ $\underset{\longleftarrow p}{820\ \text{GeV}}$

we also go to the frame in which the p is at rest, e^- has energy E_e

In this frame,

$$\Rightarrow s = (p_e + p_p)^2 = \underbrace{m_p^2}_{\text{also} \sim 0} + 2m_p E_e + \underbrace{m_e^2}_{0}$$

In the HERA frame

$$s = (p_e + p_p)^2 = m_p^2 + m_e^2 + 2(30,0,0,30)\cdot(820,0,0,-820)$$

$$= 4 \cdot 30 \cdot 820\ \text{GeV}^2$$

$$\Rightarrow E_e = \frac{4 \cdot 30 \cdot 820}{2\ 0.938}\ \text{GeV} = \underline{\underline{52{,}450\ \text{GeV}}}$$

Using this value (instead of 30 GeV) ~~for~~ in the formula for x_{min} gives

$$\boxed{x_{min} = 4.07 \times 10^{-5}}$$

So HERA can measure ~ 4 orders of magnitude lower in x than SLAC can. ($E_e = 30$ GeV is typical SLAC electron energy.)

(2)

$k_1^\nu + k_2^\mu = (M, 0, 0, 0) \qquad k_1^2 = k_2^2 = 0$

$k_1^\mu = (\frac{M}{2}, 0, 0, \frac{M}{2}) \quad k_2^\mu = (\frac{M}{2}, 0, 0, -\frac{M}{2})$

Answering the last question first

· As we rescale the meson mass,

$k_i^\mu \propto M$, so $\mathcal{M} \propto k^2 \propto M^2$

$$\Gamma(\pi^0/\eta \to \gamma\gamma) \propto |\mathcal{M}|^2 \cdot \frac{(2\text{-body phase space})}{2M} \quad \leftarrow \propto \frac{P}{E} = 1$$

(flux factor)

$$\propto c_\alpha^2 (M^2)^2 / M$$

$$\propto c_\alpha^2 M^3$$

$$\therefore \quad \frac{\Gamma(\eta \to \gamma\gamma)}{\Gamma(\pi^0 \to \gamma\gamma)} = \left|\frac{c_\eta}{c_\pi}\right|^2 \cdot \frac{M_\eta^3}{M_\pi^3}$$

$$\text{or} \quad \left|\frac{c_\eta}{c_\pi}\right| = \sqrt{\frac{\Gamma(\eta \to \gamma\gamma)}{\Gamma(\pi^0 \to \gamma\gamma)} \cdot \frac{M_\pi^3}{M_\eta^3}}$$

$$= \sqrt{\frac{\tau_{\pi^0}\, \Gamma_\eta\, Br(\eta \to \gamma\gamma)}{Br(\pi^0 \to \gamma\gamma)} \frac{M_\pi^3}{M_\eta^3}}$$

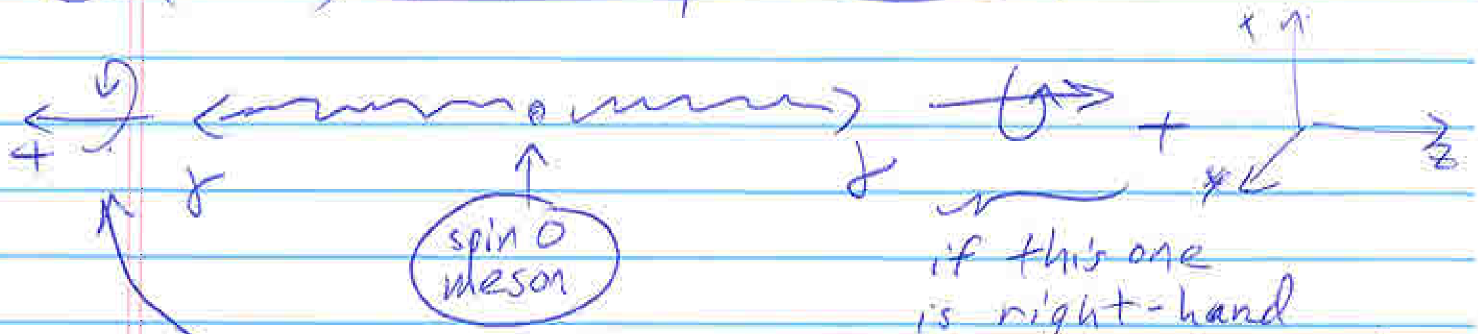
$$= \sqrt{\frac{(8.4 \times 10^{-17}\,s)(3.0 \times 10^{23}\,fm\,s^{-1})(1.30 \times 10^{-6}\,GeV)(0.39\ldots)}{0.988} \frac{(135)^3}{(548)^3} \Big/ (0.197\,GeV\,fm)}$$

$$\Rightarrow \boxed{\left|\frac{c_\eta}{c_\pi}\right| = 0.994}$$

· Pretty close to 1 [Not surprisingly, because they are $\frac{1}{\sqrt{2}}(u\bar{u} \pm d\bar{d})$ states

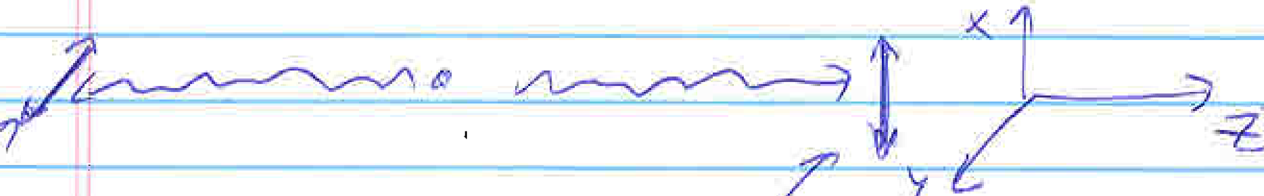
FE. 4

(2) (CONT.) Photon polarization:



if this one is right-hand circular polarized = helicity +1

then this one must be also RH circularly polarized using J_z conservation.



If this one is x-polarized,

then

$$\mathcal{M} \propto \varepsilon_{\mu_1 \mu_2 \nu_1 \nu_2} \underbrace{k_1^{\mu_1} k_2^{\mu_2}}_{\text{along } t,\, z} \underbrace{\epsilon_1^{\nu_1}}_{\text{along } x} \epsilon_2^{\nu_2}$$

along y

This one must be linearly polarized in the y direction to make the contraction with the Levi-Civita tensor nonzero.

Parenthetical Remark) Note: A scalar (not pseudoscalar) decay to $\gamma\gamma$ is $\propto \varepsilon_1 \cdot k_2\, \varepsilon_2 \cdot k_1 - \varepsilon_1 \cdot \varepsilon_2\, k_1 \cdot k_2$ (also $(+,+)$, $(-,-)$ but (x,x) not (x,y) (y,y))

(3) η has $I=0$

π^0 has $I=1$

· Let's group 2 of the π^0's:

$$\underbrace{(\pi^0\ \pi^0)}_{\tilde{L},\ \tilde{I}}\ \pi^0 \quad I=1$$

· We could only get total $I=0$ if $\tilde{I}=1$ (to conserve isospin in the decay)

· But ~~that would~~ the product of two $(I=1)$'s to give $I=1$ only happens in the antisymmetric product, which violates Bose statistics.

∴ Assuming isospin for the strong interactions, the $\eta \to 3\pi^0$ decay must be electromagnetic

$\Rightarrow$ same rough size as EM $\eta \to \gamma\gamma$ decay.

· $\eta \not\to \pi^+\pi^-$
$\quad \not\to \pi^0\pi^0$ by strong or electromagnetic interactions because of parity:

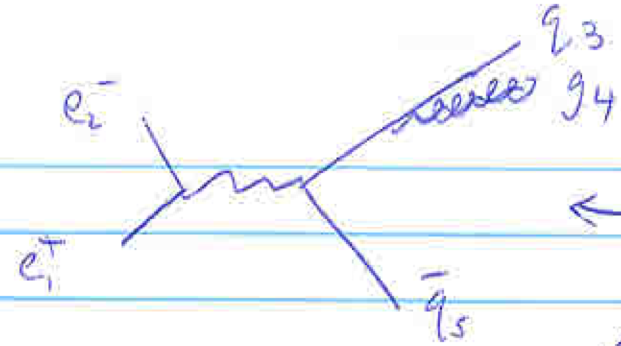
$P_{initial} = -$

both P's the same $(-)$

$\Rightarrow P_{final} = (-1)^\ell$. But angular momentum $(J_\eta = J_\pi = 0)$

$\Rightarrow \ell = 0 \Rightarrow P_{final} = + \neq P_{initial}$

(4)



Collinear configuration with $k_3 \parallel k_4$,

$k_3 \simeq z\, k_p$

$k_4 \simeq (1-z)\, k_p$

with $k_3 + k_4 \equiv k_p$

- Differential cross section is proportional to:

$$\langle |\mathcal{M}_5|^2 \rangle \propto \frac{s_{13}^2 + s_{15}^2 + s_{23}^2 + s_{25}^2}{s_{12}\, s_{34}\, s_{45}}$$

So we need to work out the limiting behavior of:

$$s_{13} = 2k_1 \cdot k_3 \simeq z\,(2k_1 \cdot k_p)$$

$$= z\, s_{1p} = z\, s_{25}$$

(using 4-particle kinematics: $(k_1+k_p)^2 = (k_2+k_5)^2$)

$s_{15} \to s_{15}$

$s_{23} \to z\, s_{2p} = z\, s_{15}$ ($(k_1+k_2)^2 = (k_p+k_5)^2$)

$s_{25} \to s_{25}$, $s_{45} \to (1-z)\, s_{p5} = (1-z)\, s_{12}$

$$\Rightarrow \langle |\mathcal{M}_5|^2 \rangle \to \frac{z^2 s_{25}^2 + s_{15}^2 + z^2 s_{15}^2 + s_{25}^2}{s_{12}\, \underbrace{s_{34}}_{\to 0}\, (1-z)\, s_{12}}$$

$$= \frac{s_{15}^2 + s_{25}^2}{s_{12}^2} \times \frac{1}{s_{34}} \times \underbrace{\frac{1+z^2}{1-z}}_{P_{qq}(z)}$$

$\Rightarrow$ $\boxed{P_{qq}(z) = \frac{1+z^2}{1-z}}$ (up to a z-independent const)

(5) Now we use the fact that

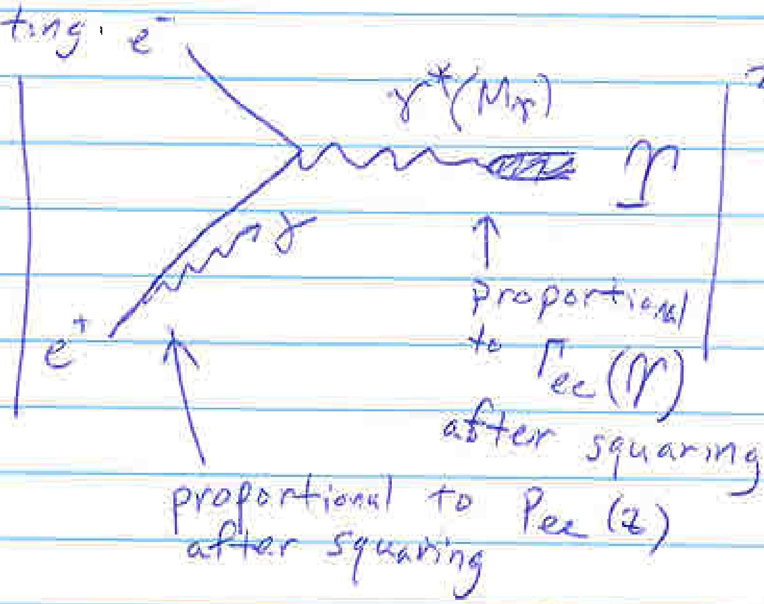
i.e. up to overall constants the collinear splitting is exactly the same. So,

$$P_{ee}(z) \propto P_{qq}(z) \propto \frac{1+z^2}{1-z}$$

The ratio

$$\frac{\sigma(e^+e^- \to \gamma\,\Upsilon(2S))}{\sigma(e^+e^- \to \gamma\,\Upsilon(1S))} = \frac{P_{ee}(z_{2S})}{P_{ee}(z_{1S})} \cdot \frac{\Gamma_{ee}(\Upsilon(2S))}{\Gamma_{ee}(\Upsilon(1S))}$$

From inspecting



Now

$$s' = z s \Rightarrow \begin{cases} z_{2S} = \dfrac{m^2_{\Upsilon(2S)}}{m^2_{\Upsilon(3S)}} = \left(\dfrac{10.023}{10.355}\right)^2 = 0.9369 \\[2ex] z_{1S} = \dfrac{m^2_{\Upsilon(1S)}}{m^2_{\Upsilon(3S)}} = \left(\dfrac{9.460}{10.355}\right)^2 = 0.8346 \end{cases}$$

$P_{ee}(z_{2S}) = 29.76$

$P_{ee}(z_{1S}) = 10.25$

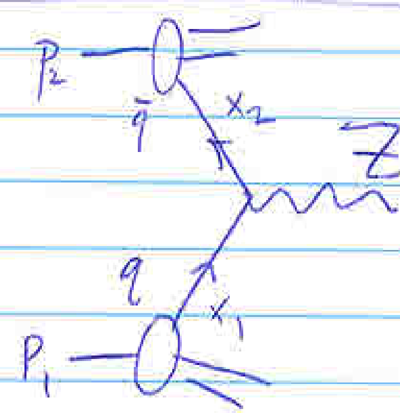
$$\Rightarrow \boxed{\frac{\sigma(e^+e^- \to \gamma\,\Upsilon(2S))}{\sigma(e^+e^- \to \gamma\,\Upsilon(1S))} = \frac{29.76}{10.25}\,\frac{0.612}{1.34} = 1.33}$$

(6) LHC is $p+p$ in center of mass frame

$$\begin{array}{ccccc} & 0 & 1 & 2 & 3=z \end{array}$$

$$P_1 = (P, 0, 0, P)$$

$$P_2 = (P, 0, 0, -P)$$



(anti) quark momenta are proportional, with fractions $x_{1,2}$:

$$k_q = (x_1 P, 0, 0, x_1 P)$$

$$k_{\bar{q}} = (x_2 P, 0, 0, -x_2 P)$$

$$s = (P_1 + P_2)^2 = (2P, 0, 0, 0)^2 = \boxed{4P^2 = s}$$

Now use momentum conservation for

$$q(x_1) + \bar{q}(x_2) \rightarrow Z \qquad \text{[lowest order QCD process]}$$

$$\Rightarrow k_Z = k_q + k_{\bar{q}} = ((x_1+x_2)P, 0, 0, (x_1-x_2)P)$$

First compute the mass-shell condition:

$$M_Z^2 = k_Z^2 = (x_1+x_2)^2 P^2 - (x_1-x_2)^2 P^2$$

$$= (4x_1 x_2)\left(\frac{s}{4}\right) = s\, x_1 x_2$$

$$\Rightarrow \boxed{x_1 x_2 = \frac{M_Z^2}{s}}$$

Now look at the rapidity constraint:

$$Y = \frac{1}{2} \ln\left(\frac{E+p_z}{E-p_z}\right) \Rightarrow e^{2Y} = \frac{E+p_z}{E-p_z}$$

$$= \frac{(x_1+x_2)P + (x_1-x_2)P}{(x_1+x_2)P - (x_1-x_2)P} = \boxed{\frac{x_1}{x_2} = e^{2Y}}$$

6 (CONT.)

- Multiplying the two constraints,

$$(x_1 x_2)\left(\frac{x_1}{x_2}\right) = \frac{M_Z^2}{s} e^{2Y} = x_1^2$$

$$\Rightarrow \boxed{x_1 = \frac{M_Z}{\sqrt{s}} e^{Y}.} \qquad \boxed{x_2 = \frac{M_Z}{\sqrt{s}} e^{-Y}}$$

Numerical Values: $M_Z = 91.187$ GeV
$\sqrt{s} = 14{,}000$ GeV

$$Y = 0 \Rightarrow \boxed{x_1 = x_2 = \frac{M_Z}{\sqrt{s}} = \frac{91.187}{14{,}000} = 6.5 \times 10^{-3}}$$

$$Y = 3 \Rightarrow \boxed{x_1 = (6.5 \times 10^{-3})\, e^{3} = 0.131}$$

$$\boxed{x_2 = (6.5 \times 10^{-3})(e^{-3}) = 3.24 \times 10^{-4}}$$