Physics 152A/252A Final Exam

Take Home, due at noon Wednesday March 18, in Tomo Miyasita's Varian mailbox

This exam is governed by the Stanford Honor Code. It is open book: You may use Bettini, *Introduction to Elementary Particle Physics*, Griffiths, *Introduction to Elementary Particles*, the particle data book, your personal class lecture notes, and any of the lecture notes at `http://www.slac.stanford.edu/~lance/phys152A`; but no other sources, and of course no aid from other individuals.

Problem 1 (20 pts): An experimentalist wishes to measure the quark distribution $q(x)$ in the proton, at $x = 0.3$ and $Q^2 = 4$ GeV^2, using deep inelastic scattering and a 30 GeV electron beam striking a fixed hydrogen target. At what angle from the beamline should the electron spectrometer be placed? What momentum (or energy) outgoing electrons should be measured? What is the minimum value of x that can possibly be measured for $Q^2 \geq 4$ GeV^2, with this beam and target? At HERA, a 30 GeV electron beam colliding with an 820 GeV proton beam, what is the minimum value of x that could possibly be measured for $Q^2 \geq 4$ GeV^2. Hint: Since x is Lorentz invariant, you can work this part out by first finding the electron energy in the frame in which the proton is at rest, and then using the previous formula.

Problem 2 (25 pts): From the particle data book, the η and the π^0 are both neutral, $J^{PC} = 0^{-+}$ mesons that decay mainly to two photons. The lifetime of the π^0 is given as $8.4 \times 10^{-17} s$, and the branching fraction is $\mathrm{Br}(\pi^0 \to \gamma\gamma) = 98.8\%$. The width of the η is given as $\Gamma = 1.30$ keV, and the branching fraction is $\mathrm{Br}(\eta \to \gamma\gamma) = 39.3\%$. Their masses are $m_\eta = 548$ MeV and $m_\pi = 135$ MeV. Parity implies that the amplitude for their decay to two photons must involve the Levi-Civita tensor $\varepsilon_{\mu\nu\sigma\rho}$, and have the form $\mathcal{M} = c_\alpha \, \varepsilon_{\mu_1\mu_2\nu_1\nu_2} k_1^{\mu_1} k_2^{\mu_2} \epsilon_1^{\nu_1} \epsilon_2^{\nu_2}$, where $\alpha = \pi, \eta$. Here k_i are the photon 4-momenta and ϵ_i are their polarization vectors; c_α is a constant. If one photon emerging from the decay is right-hand-circularly polarized, what is the polarization of the other photon? If one photon emerging from the decay in the z direction is linearly polarized in the x direction, what is the polarization of the other photon? Now use the experimental data to determine the ratio of the two coupling strengths, $|c_\eta / c_\pi|$.

Problem 3 (10 pts): The η also decays to 3 π^0's with a similar branching fraction, 32.5%. Use isospin to explain why this decay does not strongly dominate over the electromagnetic decay to $\gamma\gamma$. Also, explain why the η has never been seen to decay to $\pi^+\pi^-$, or to 2 π^0's.

Problem 4 (25 pts): In the process $e_1^+ e_2^- \to q_3 g_4 \bar{q}_5$, the spin-averaged squared matrix element given in class is proportional to

$$\langle |\mathcal{M}_5|^2 \rangle \propto \frac{s_{13}^2 + s_{15}^2 + s_{23}^2 + s_{25}^2}{s_{12} s_{34} s_{45}} \,.$$

Show that in the limit that the quark and gluon momenta become parallel, or *collinear*, $k_3 || k_4$, the matrix element *factorizes* as follows,

$$\langle |\mathcal{M}_5|^2 \rangle \to \langle |\mathcal{M}_4|^2 \rangle \times \frac{1}{s_{34}} \times P_{qq}(z),$$

where $\langle |\mathcal{M}_4|^2 \rangle$ is the spin-averaged squared matrix element for $e_1^+ e_2^- \to q_P \bar{q}_5$ with quark momentum $k_P = k_3 + k_4$, and z is the fraction of this momentum carried by k_3, $k_3 \approx z k_P$, and $k_4 \approx (1-z) k_P$,

$$\langle |\mathcal{M}_4|^2 \rangle \propto \frac{s_{15}^2 + s_{25}^2}{s_{12}^2} \,.$$

Determine the z-dependence of the Altarelli-Parisi *splitting function* for $q \to q$, $P_{qq}(z)$.

Problem 5 (10 pts): The same z dependence, $P_{ee}(z) = P_{qq}(z)$, also holds for QED initial-state radiation (ISR), the process by which a high-energy electron radiates a collinear photon, retaining a fraction z of its momentum, just before it annihilates with a positron at a lower center of mass energy-squared, given by $s' = zs$. Suppose an e^+e^- collider is operating at the $\Upsilon(3S)$ resonance, with $m_{\Upsilon(3S)} = 10.355$ GeV, and e^+e^- partial width $\Gamma_{ee}(\Upsilon(3S)) = 0.43$ keV. Work out the ratio of $\Upsilon(2S)$ to $\Upsilon(1S)$ states that are produced via ISR,

$$\frac{\sigma(e^+e^- \to \gamma\Upsilon(2S))}{\sigma(e^+e^- \to \gamma\Upsilon(1S))}.$$

Use $m_{\Upsilon(1S)} = 9.460$ GeV, $\Gamma_{ee}(\Upsilon(1S)) = 1.34$ keV, and $m_{\Upsilon(2S)} = 10.023$ GeV, $\Gamma_{ee}(\Upsilon(2S)) = 0.612$ keV.

Problem 6 (10 pts): The LHC will produce millions of Z bosons through the Drell-Yan process. These data can also be used to measure the parton distributions. At lowest order in QCD, we saw in class that $M_Z^2 = sx_1x_2$, where s is the pp center-of-mass energy squared, and x_1 and x_2 are the quark/anti-quark momentum fractions. The detectors can also measure the Z boson *rapidity* Y, defined in terms of the Z boson energy and z (beam) component of momentum p_z,

$$Y \equiv \frac{1}{2}\ln\left(\frac{E + p_z}{E - p_z}\right).$$

Determine x_1 and x_2 in terms of s, M_Z and Y, at lowest order in QCD. What are the numerical values of x_1 and x_2 for $s = (14 \text{ TeV})^2$ and $Y = 0$? $Y = 3$?