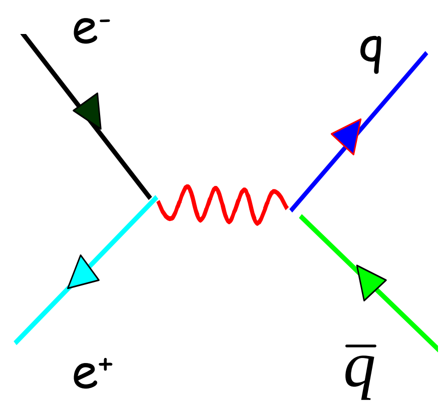
QCD in e^+e^- and hadronic collisions

Lecture 18 Physics 152A/252A

Lance Dixon

(thanks again to Colin Jessop)

QCD in e^+e^- Annihilation

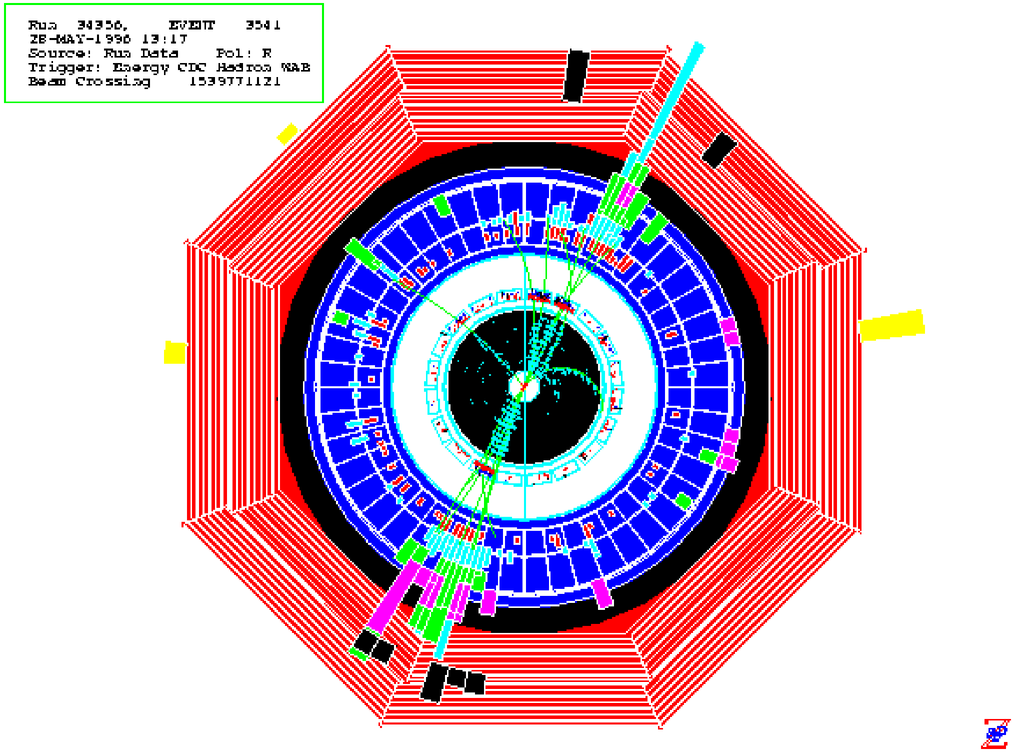


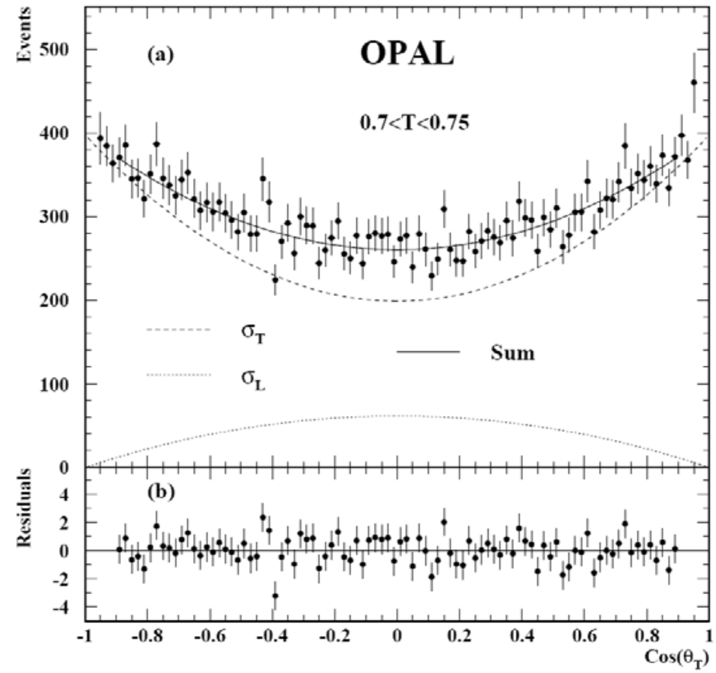
Cleanest laboratory for studying perturbative QCD:
- No hadrons in initial state

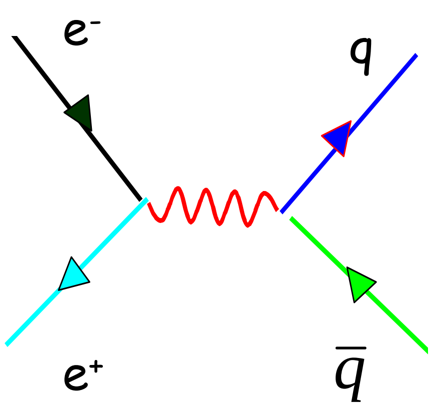
→ no need to use parton distribution functions
- No "remnants" of initial proton(s)
- Quarks and gluons produced at short distances and emerge as jets
- As $s = E_{CM}^2$ increases, $\alpha_s(E_{CM})$ decreases;

jets become more distinct

Two Jet Events at $E_{CM} = M_Z = 91$ GeV







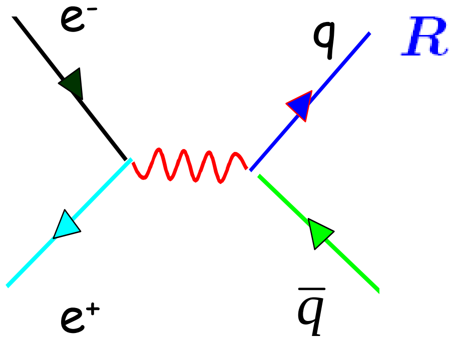
Angular distribution follows

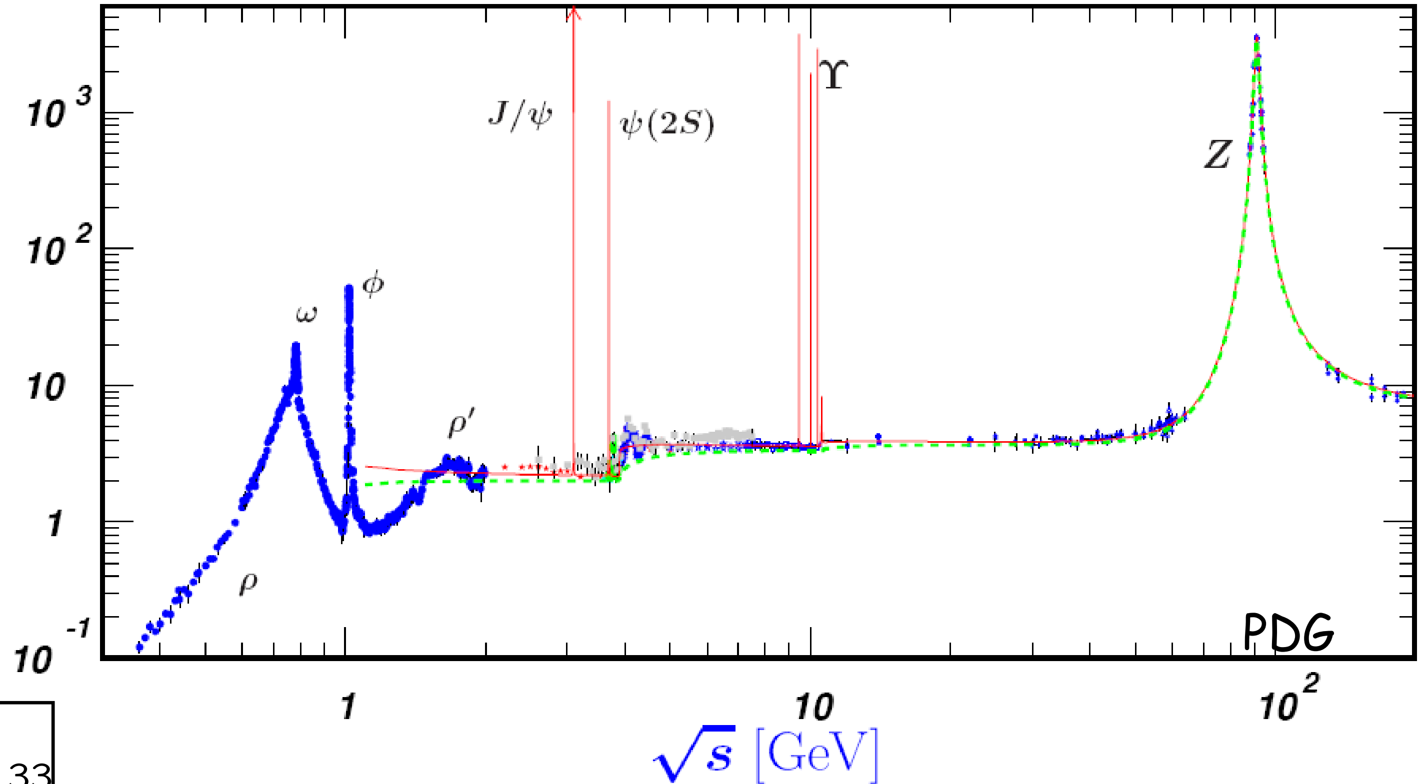
$$\frac{d\sigma}{d(\cos\vartheta)_{cm}} \propto (1+\cos^2\vartheta)$$

Just like $e^+ e^- \rightarrow \mu^+ \mu^-$

-> quarks have spin 1/2

Overall Rate: $R(s) \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$

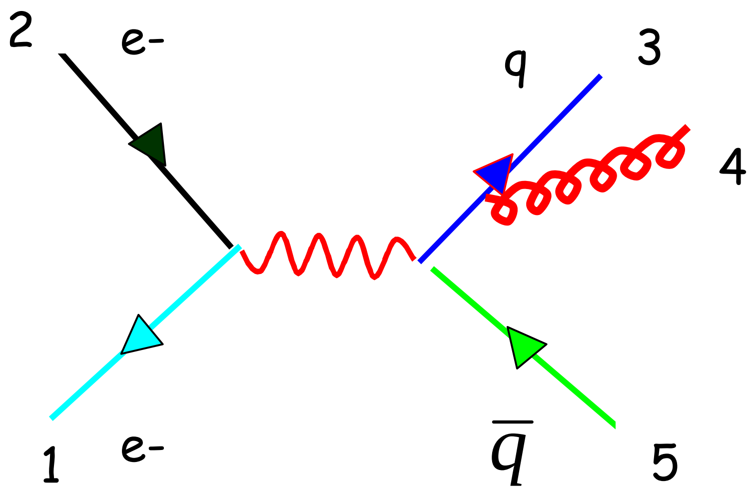


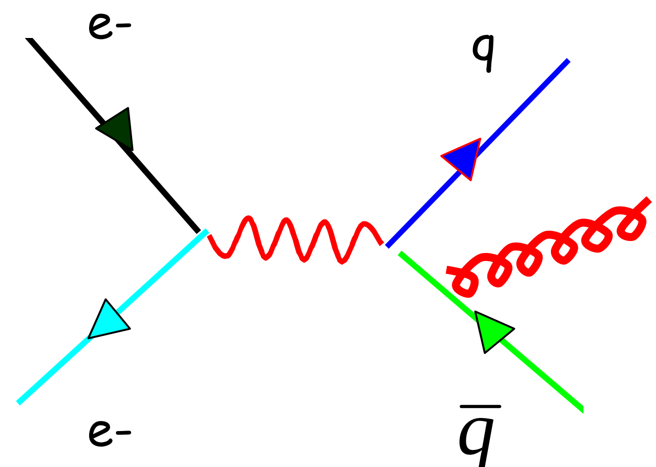


u, d, s	$\to$	2
u, d, s, c	$\to$	$10/3 = 3.33$
u, d, s, c, b	$\to$	$11/3 = 3.67$

$$R(s) = 3\sum_q Q_q^2 \left[1 + \frac{\alpha_s}{\pi} + \mathcal{O}(\alpha_s^2)\right]$$

How to See Gluons (See also talk by M.S. Shaw)





Gluons fragment into jets similarly to quarks so expect 3 jet events

$$|A_5|^2 \propto \frac{s_{13}^2 + s_{15}^2 + s_{23}^2 + s_{25}^2}{s_{12} s_{34} s_{45}}$$

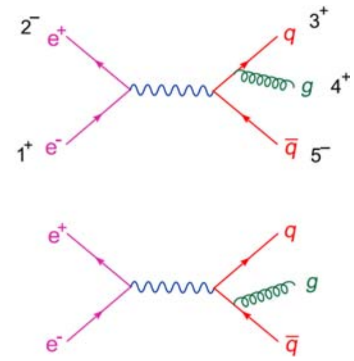
S-matrix diverges in two regions:

1) soft gluon ($k_4 \to 0$; $s_{34} \to 0$ and $s_{45} \to 0$)

2) gluon collinear with quark or anti-quark
($k_4 \,||\, k_3$, $s_{34} \to 0$ or $k_4 \,||\, k_5$, $s_{45} \to 0$)

Can only use QCD Feynman diagrams to predict "infrared safe" quantities which are insensitive to these "long-distance" regions.

Three-body phase space for $q\bar{q}g$

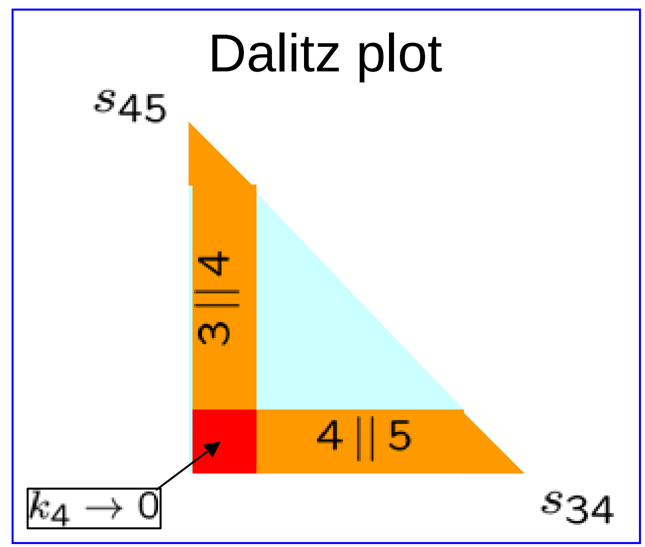


Dalitz plot: Can show that the phase space is constant density when plotted in terms of 2 of the three possible invariant masses:

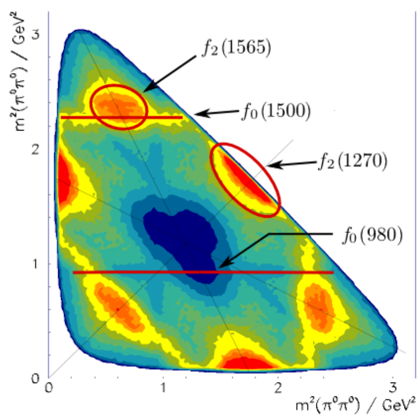
$$s_{ij} \equiv (k_i + k_j)^2$$

For massless q,g it is simple

$$s_{34} + s_{45} + s_{35} \;=\; s_{12}$$

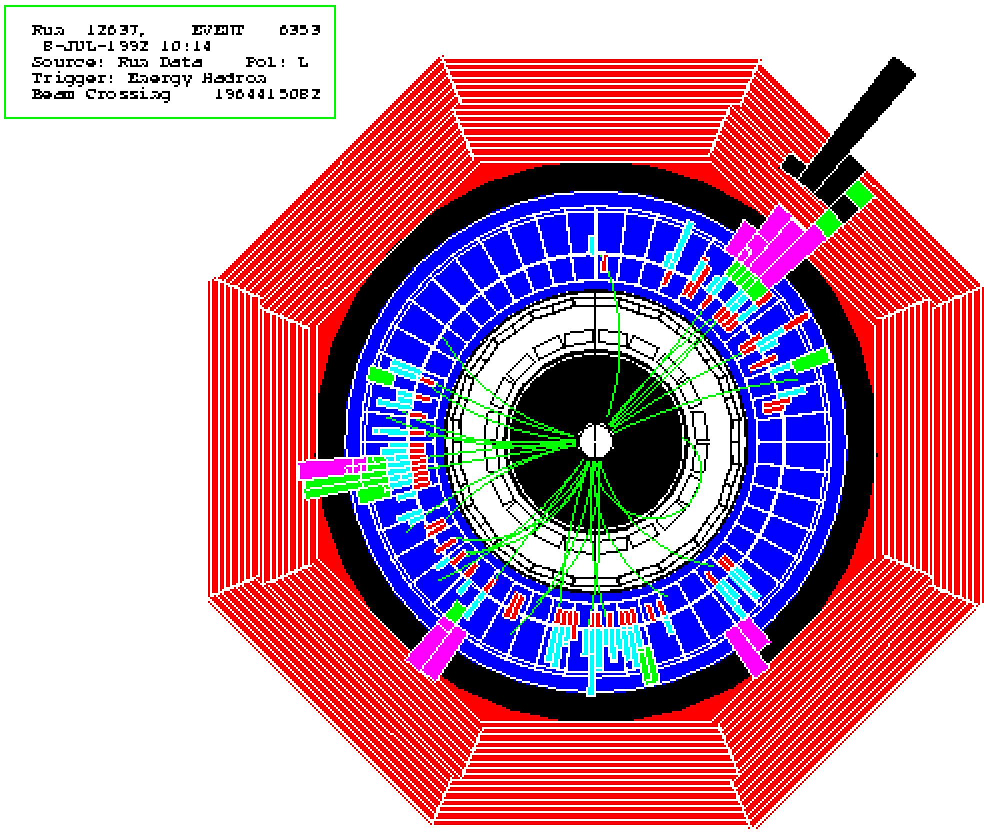


For massive particles the boundary of the Dalitz plot is complicated.



$p\bar{p} \to \pi^0\pi^0\pi^0$ at rest

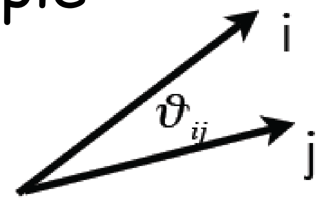
Proof that Gluons Exist



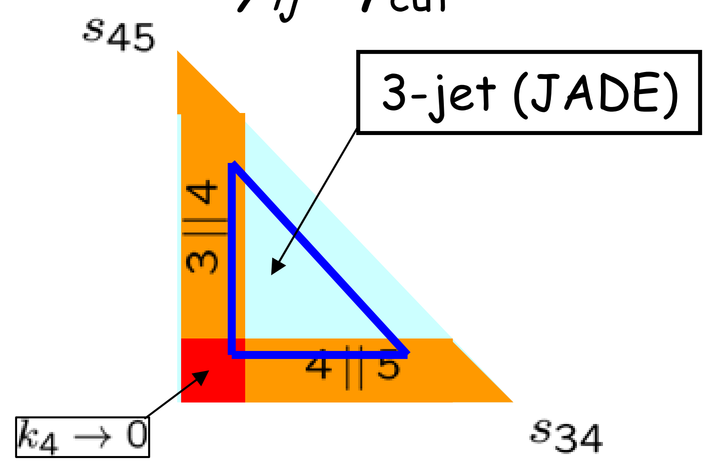
3 jet events observed at rate consistent with expectations

e^+e^- Jet Algorithms

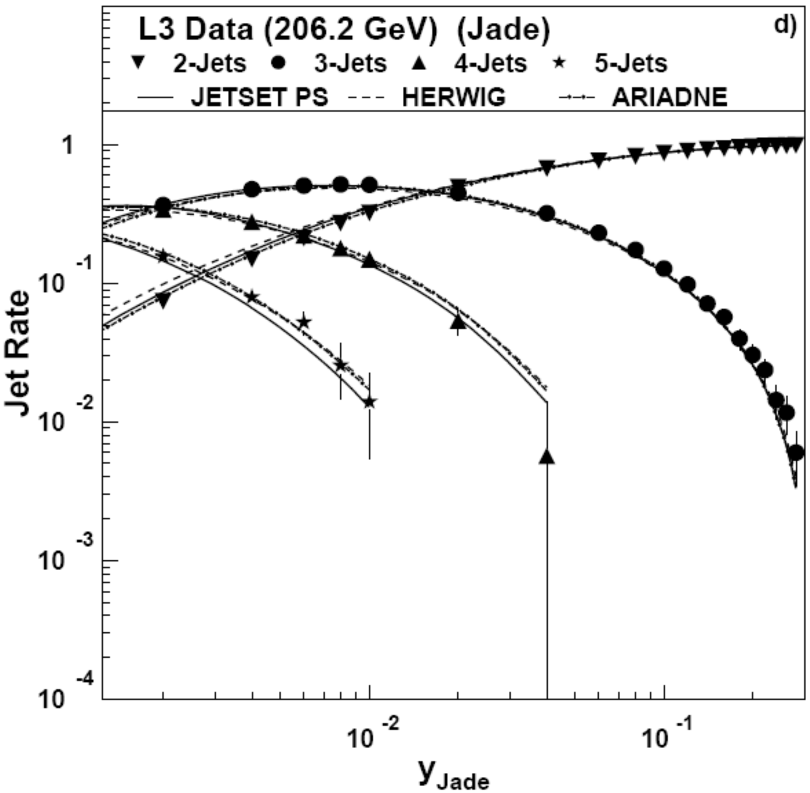
- Start with list of all n hadrons and 4-momenta
- Cluster the two "closest" with a distance d_{ij} that vanishes in soft and collinear limits, for example



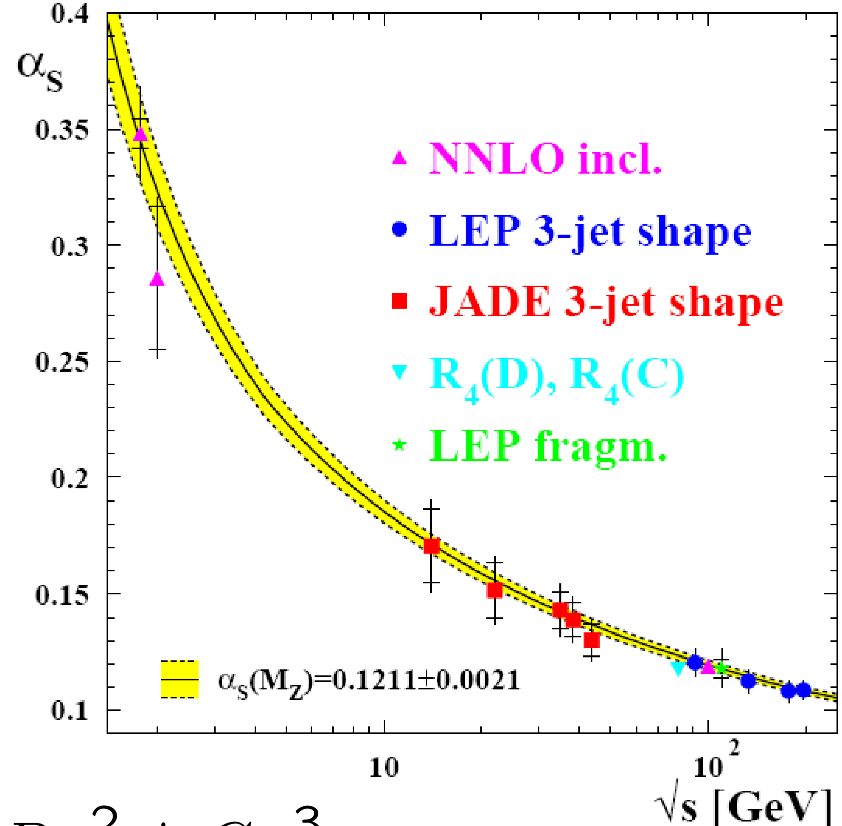
- $y_{ij} = s_{ij}/s$ (JADE)
  or
  $y_{ij} = 2\min(E_i^2, E_j^2)(1-\cos\theta_{ij})/s$ (DURHAM)
  then $p_i + p_j = p_k$ = momentum of "proto-jet"
- Now have (n-1) proto-jets.
- Repeat above steps on proto-jets until all $y_{ij} > y_{cut}$
- Left with N jets.
- N-jet rate depends on jet resolution parameter, y_{cut}.



Jet Rates, α_s



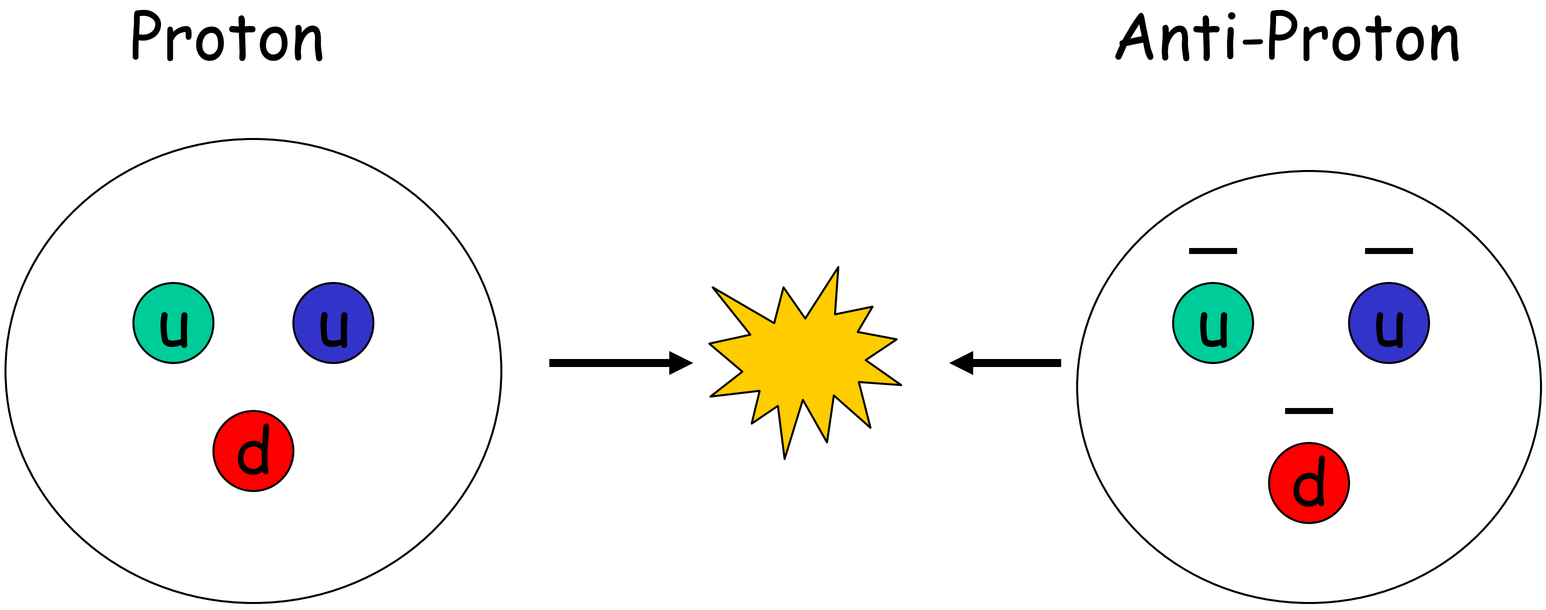
→



$$R_{3-\mathrm{jet}} = \underbrace{A\alpha_s}_{\text{LO}} + \underbrace{B\alpha_s^2}_{\text{NLO}} + \underbrace{C\alpha_s^3}_{\text{NNLO (very recent)}}$$

Hadron Colliders

It is possible to accelerate protons and antiprotons to much greater energies than electrons and positrons. This effectively makes a quark-antiquark collider



QCD factorization & parton model

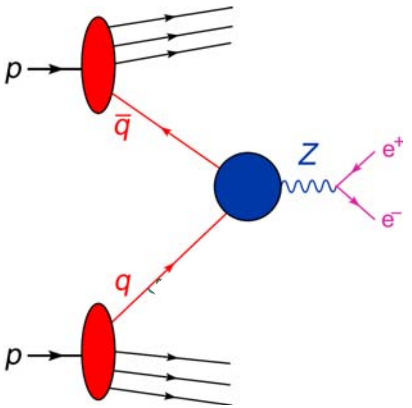
- Asymptotic freedom guarantees that at short distances (large transverse momenta), partons in the proton are almost free.
- Only one parton at a time participates in a "hard" collision.
- Leads to QCD-improved parton model:

"suitable" final state

Parton distribution function: prob. of finding parton *a* in proton 1, carrying fraction x_1 of its momentum

factorization scale ("arbitrary")

$$\sigma^{pp\to X}(s;\alpha_s,\mu_R,\mu_F) = \sum_{a,b}\int_0^1 dx_1 \int_0^1 dx_2\; f_a(x_1,\alpha_s,\mu_F)\, f_b(x_2,\alpha_s,\mu_F) \times \hat{\sigma}^{ab\to X}(sx_1x_2;\alpha_s,\mu_R,\mu_F)$$



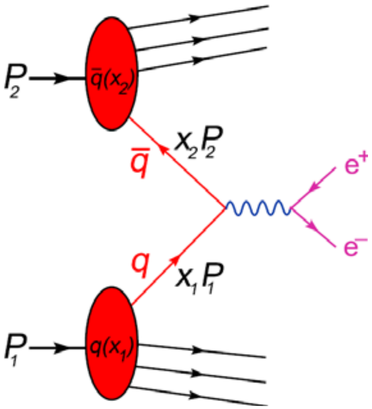
Partonic cross section, computable in perturbative QCD

partonic CM energy^2

renormalization scale ("arbitrary")

Simplest Case: Drell-Yan process

$$pp \longrightarrow e^+e^-$$



LO partonic cross section:

$$\hat{s} = x_1 x_2 s = M^2_{e^+e^-}$$

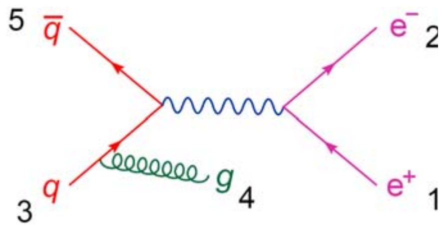
$$\hat{\sigma}(q\bar{q} \to e^+e^-) = \frac{1}{2\hat{s}}\frac{1}{4N_c^2}\sum_{h,c}|\mathcal{A}_4|^2 = \frac{4\pi\alpha^2}{3\hat{s}}\frac{1}{N_c}Q_q^2$$

$$\frac{d\hat{\sigma}}{dM^2} = \frac{\sigma_0}{N_c}Q_q^2\delta(\hat{s}-M^2), \qquad \sigma_0 \equiv \frac{4\pi\alpha^2}{3M^2}$$

LO hadronic cross section:

$$\begin{aligned}\frac{d\sigma}{dM^2} &= \int_0^1 dx_1 dx_2 \sum_q [q(x_1)\bar{q}(x_2) + \bar{q}(x_1)q(x_2)]\frac{d\hat{\sigma}}{dM^2} \\ &= \frac{\sigma_0}{N_c}\int_0^1 dx_1 dx_2 \delta(x_1x_2 s - M^2)\sum_q Q_q^2[q(x_1)\bar{q}(x_2) + \bar{q}(x_1)q(x_2)] \\ &= \frac{\sigma_0\, s}{N_c}\int_0^1 dx_1 dx_2 \delta(x_1x_2 - \tau)\sum_q Q_q^2[q(x_1)\bar{q}(x_2) + \bar{q}(x_1)q(x_2)], \qquad \tau \equiv \frac{M^2}{s}\end{aligned}$$

NLO QCD corrections to Drell-Yan process



$$|A_5|^2 = \frac{s_{13}^2 + s_{15}^2 + s_{23}^2 + s_{25}^2}{s_{12}s_{34}s_{45}}$$

As at LO, average over decay direction of e^+ and e^-:

$$\langle k_1^\mu k_1^\nu \rangle_\Omega \equiv \int \frac{d\Omega_{e^+e^-}}{4\pi} k_1^\mu k_1^\nu = -\frac{s_{12}}{12}\eta^{\mu\nu} + \frac{1}{3}(k_1+k_2)^\mu (k_1+k_2)^\nu = \langle k_2^\mu k_2^\nu \rangle_\Omega$$

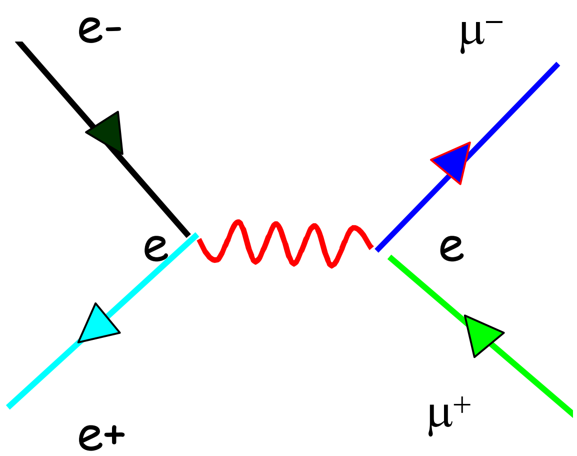
$$\langle s_{13}^2 \rangle_\Omega = \langle s_{23}^2 \rangle_\Omega = \frac{1}{3}(s_{13}+s_{23})^2 = \frac{1}{3}(s_{34}+s_{35})^2$$

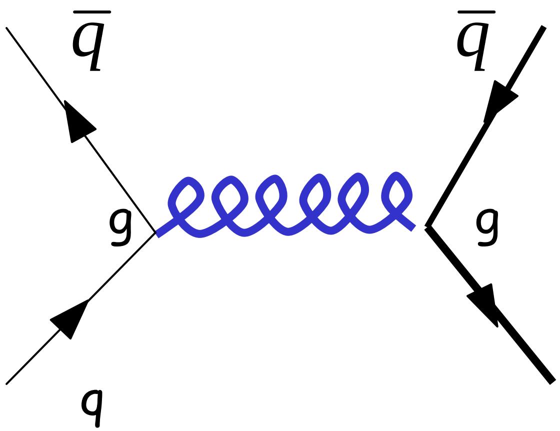
$$\Rightarrow \quad \langle |A_5|^2 \rangle_\Omega = \frac{2}{3}\frac{(s_{34}+s_{35})^2 + (s_{35}+s_{45})^2}{s_{12}s_{34}s_{45}}$$

NLO corrections: cross section increased by ~ 30%

collinear singularities: have to be absorbed into a "renormalization" of the parton distribution functions

Can Calculate Simple QCD Processes in same way as QED

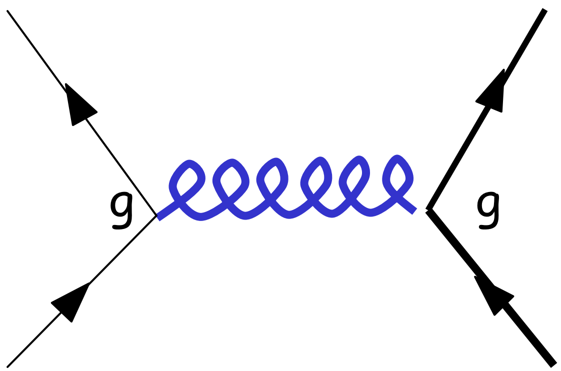


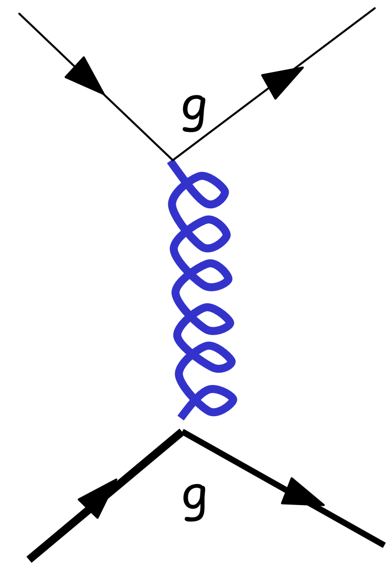


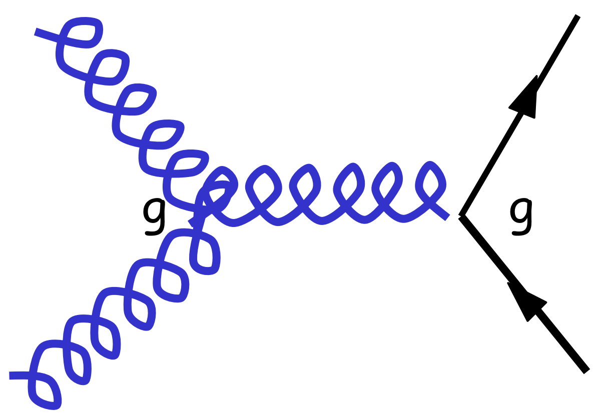
1. e <-> g
2. Average over initial color and sum over final color to get $\hat{\sigma}$
3. Integrate over Parton distribution functions

But this is just one partonic subprocess contributing to pp → 2 jets + X.

Many Different Processes contribute at lowest order to $q\bar{q}$ production



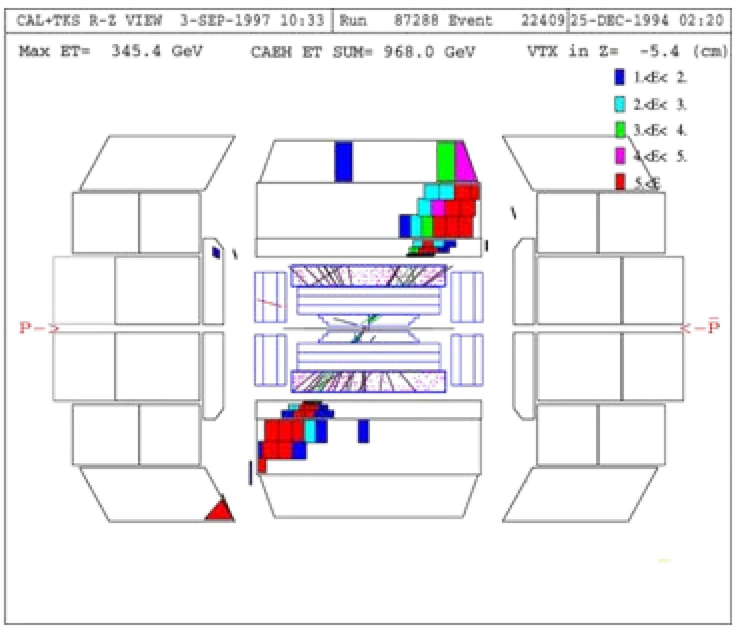


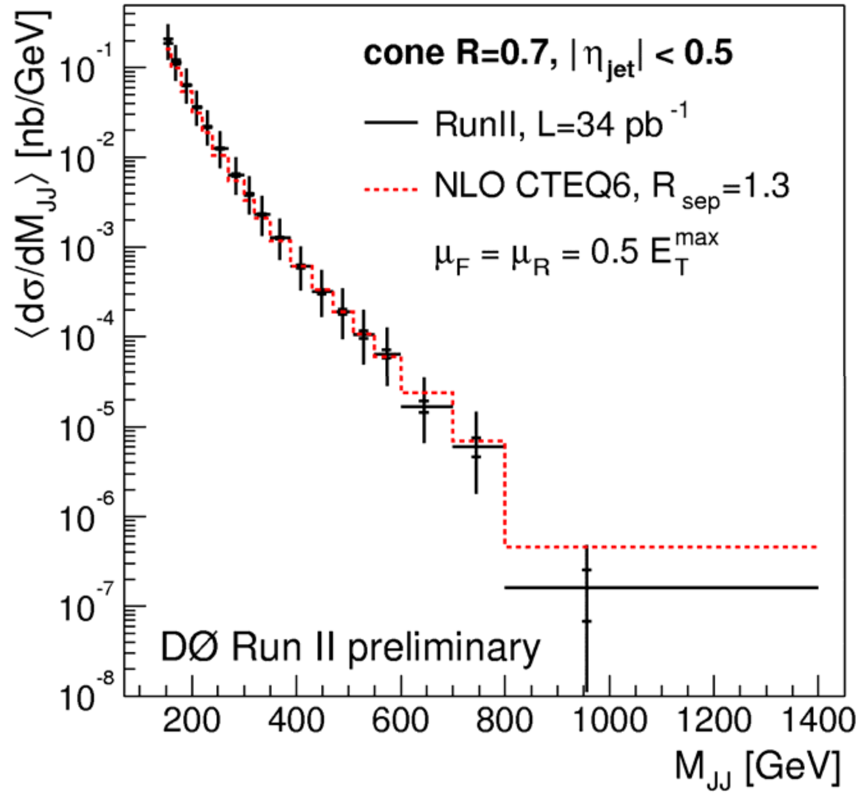


this one involves the non-Abelian self-interaction of gluons

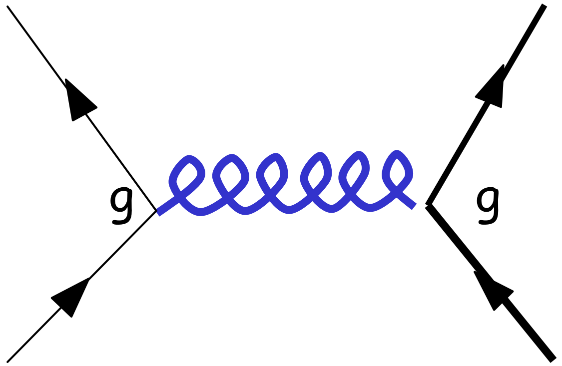
Also, can't tell quark jets from light quark jets. Must add processes with final-state gluons too!

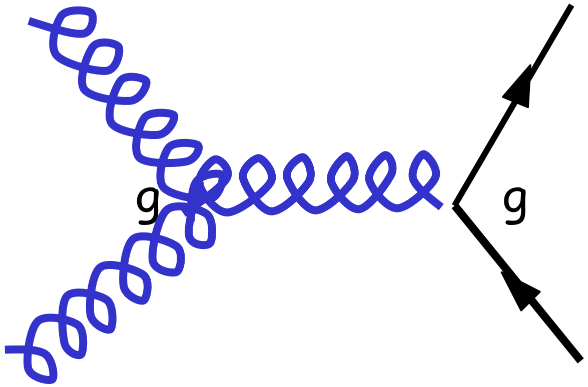
High E_T Jet Events at Tevatron (D0)





Lowest order Processes for $t\,\bar{t}$ production



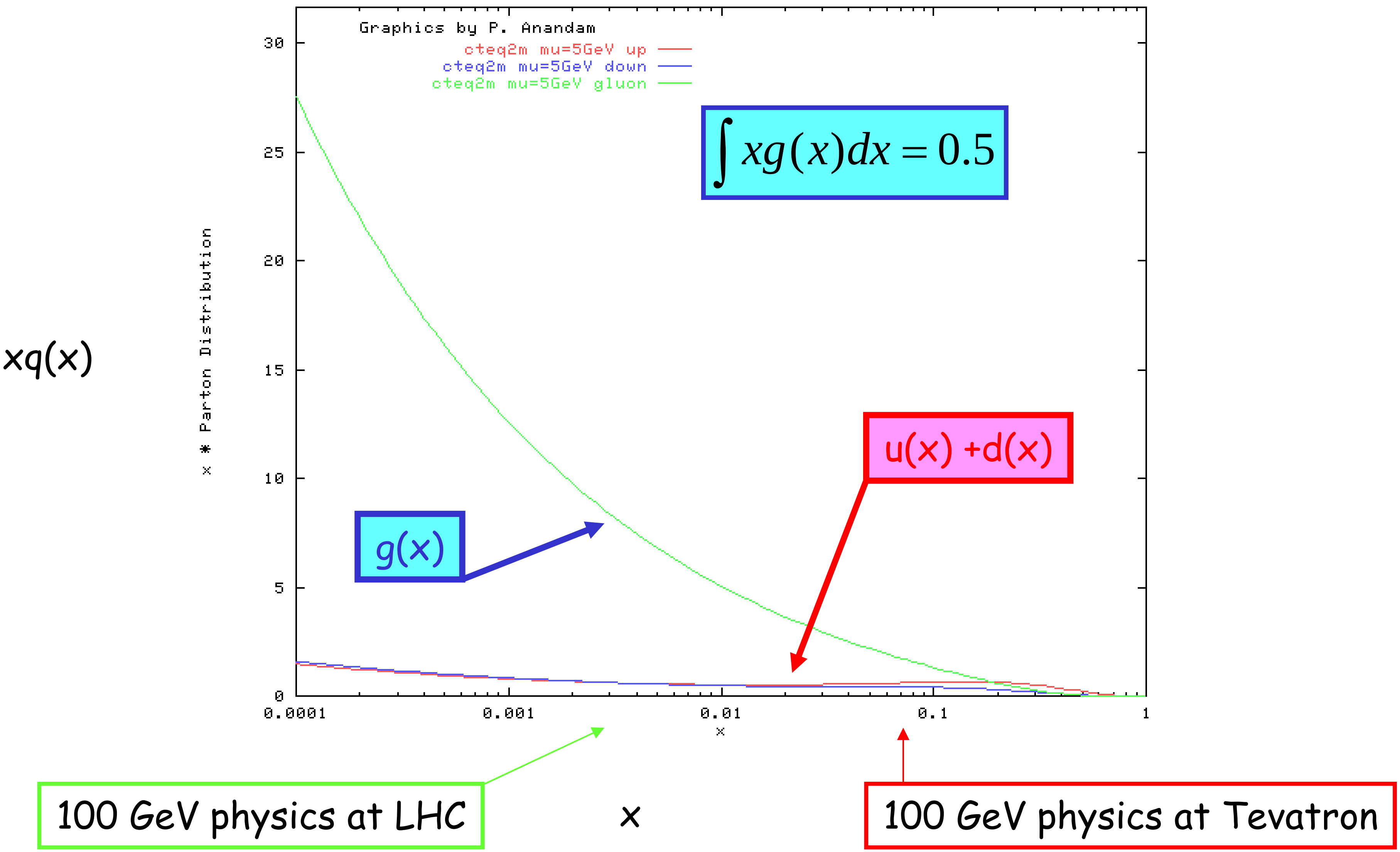


At 2 TeV (Tevatron) top quark production dominated by $\overline{qq}$ initial state.

At 14 TeV (LHC) gg dominates because of smaller x values.

$$\hat{s} = s x_1 x_2 \approx s x_{\text{typical}}^2 \quad \Rightarrow \quad x_{\text{typical}} = \frac{\sqrt{\hat{s}}}{\sqrt{s}}$$

Gluons dominant at small x



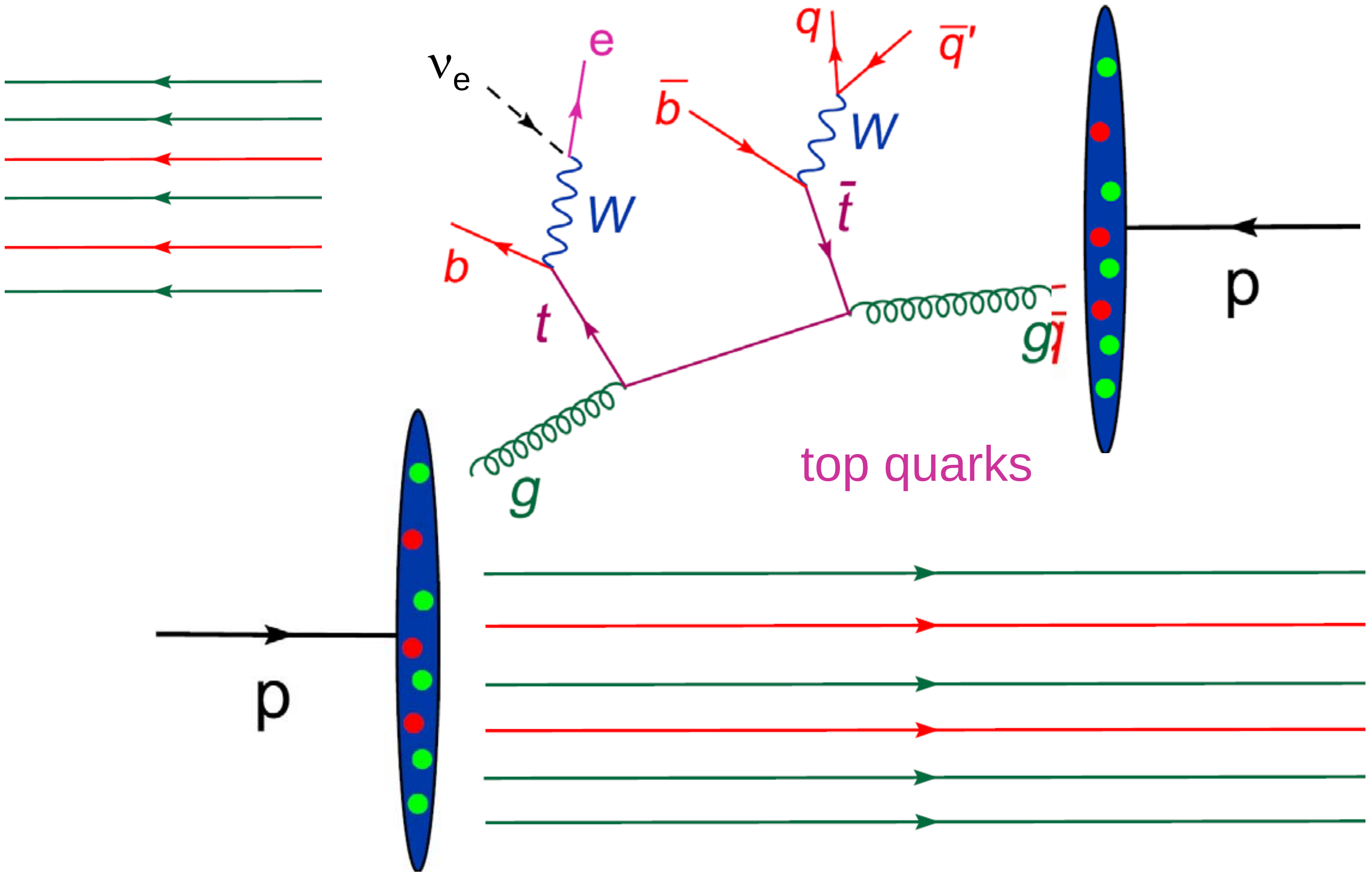
Tevatron 2 GeV

LHC 14 TeV

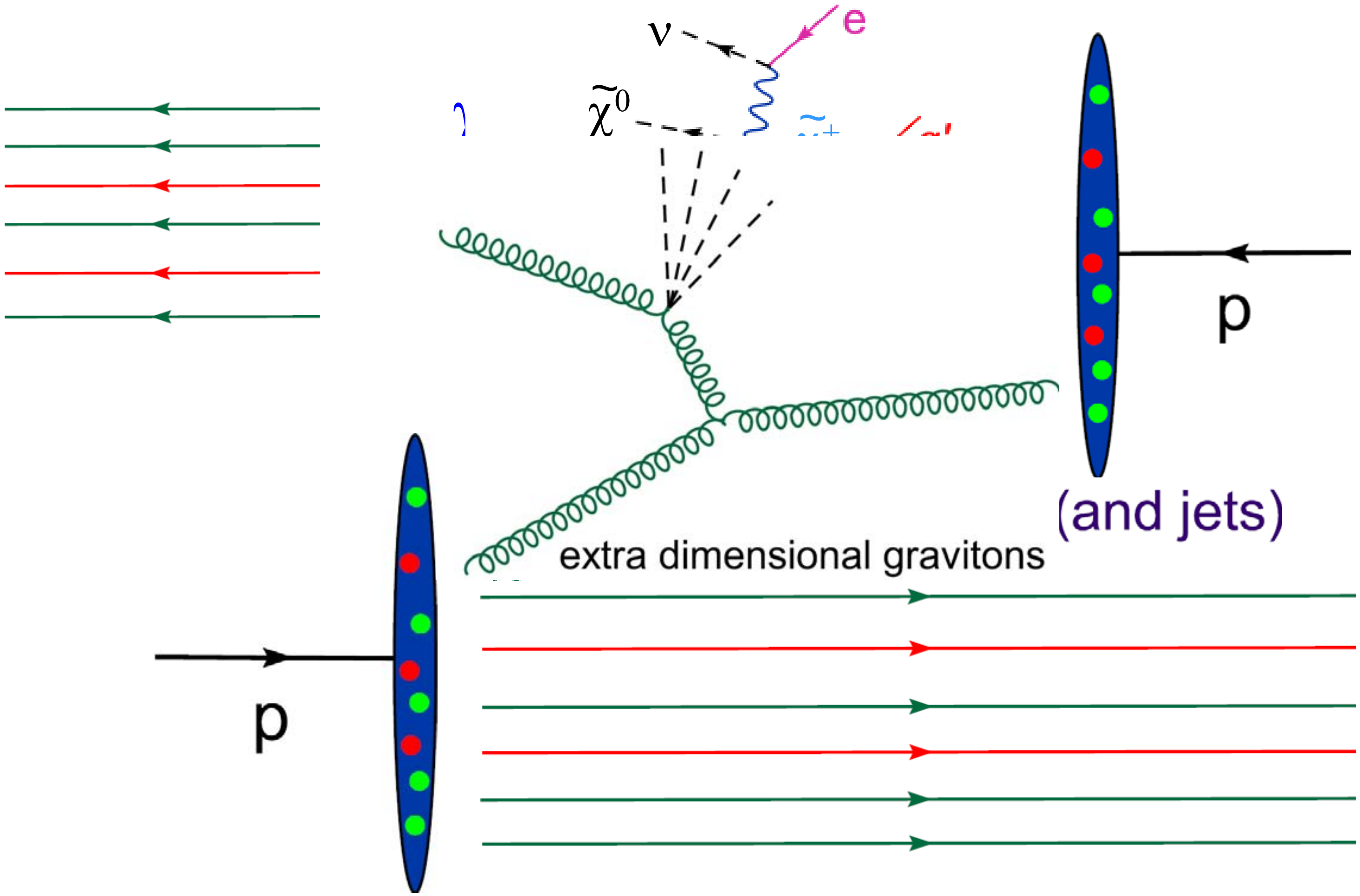
Quark -antiquark collider

Gluon collider

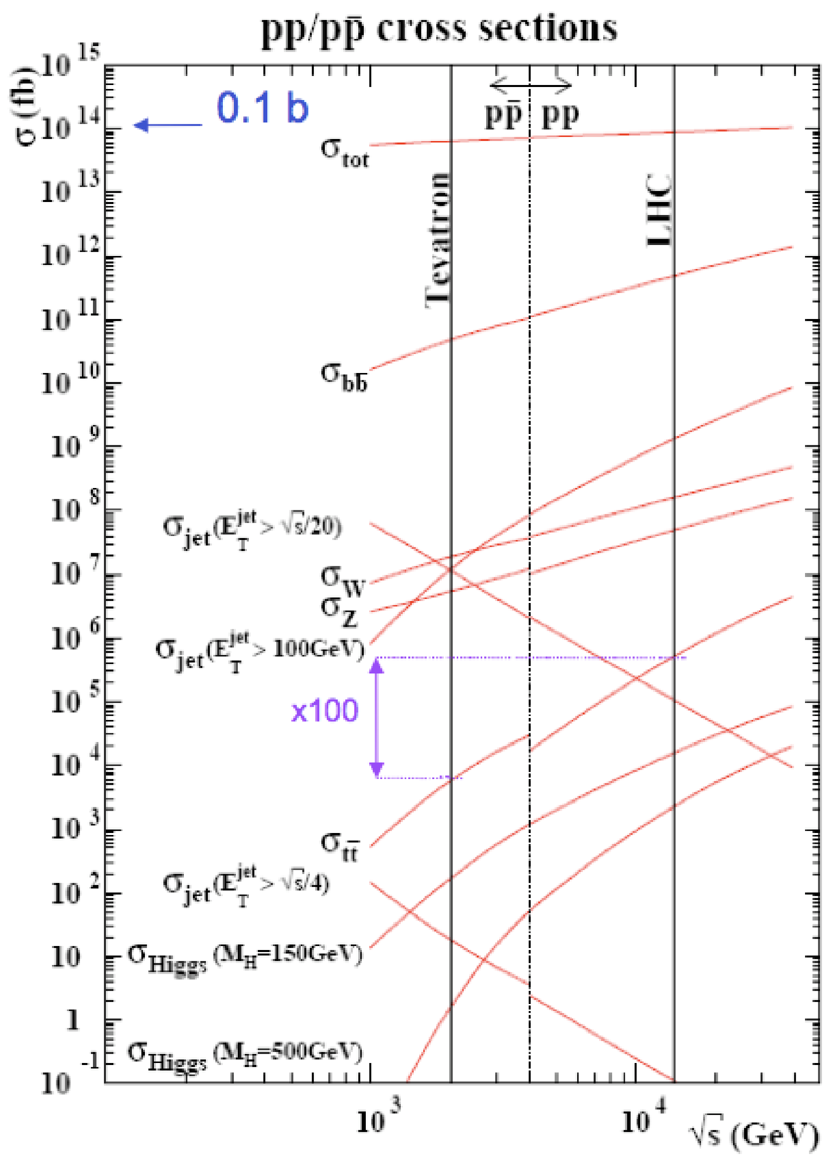
What will the LHC see?



What might the LHC see?



LHC is a QCD machine



But we want to use it to solve puzzles about the **electroweak interactions**, for which the cross sections are often **much smaller**

← new physics?

For more about the electroweak interactions, stay tuned for Ph152B/252B with Helen Quinn

Final Thoughts