

Particle Accelerators

Lecture 17 Physics 152A/252A

Lance Dixon

(thanks again to Colin Jessop)

Two basic types of accelerator

linear



circular



Two types of beam

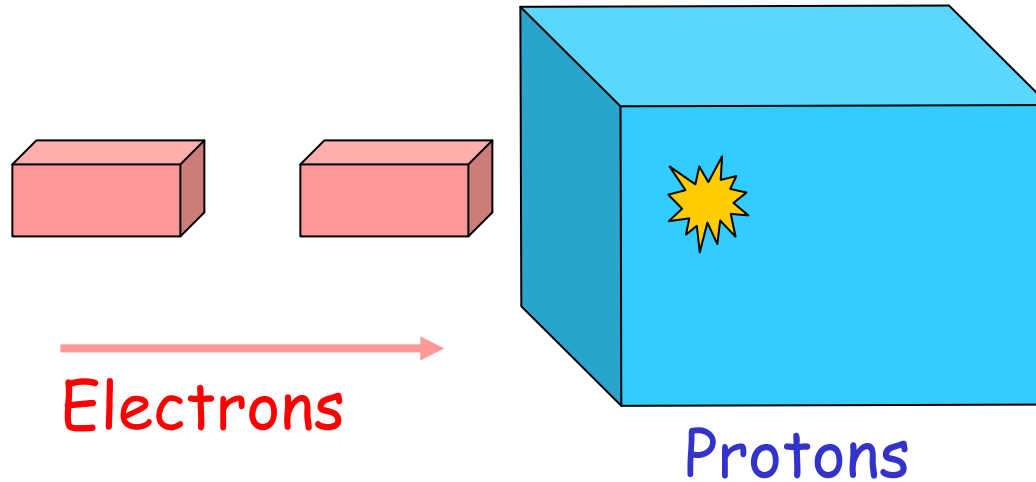
electrons

protons

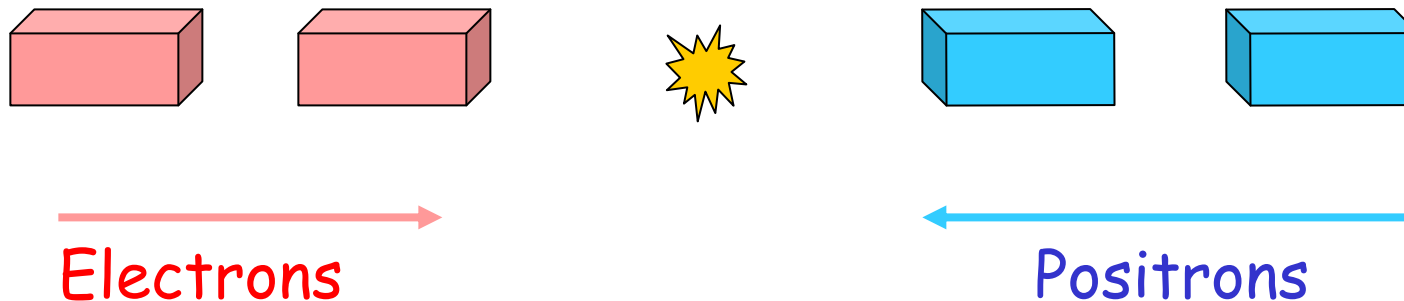
Two types of collisions available:
fixed target & colliding beam

The two most important accelerator properties are Luminosity (L) & Center-of-Mass Energy (E_{CM})

Fixed target: L large, E_{CM} small

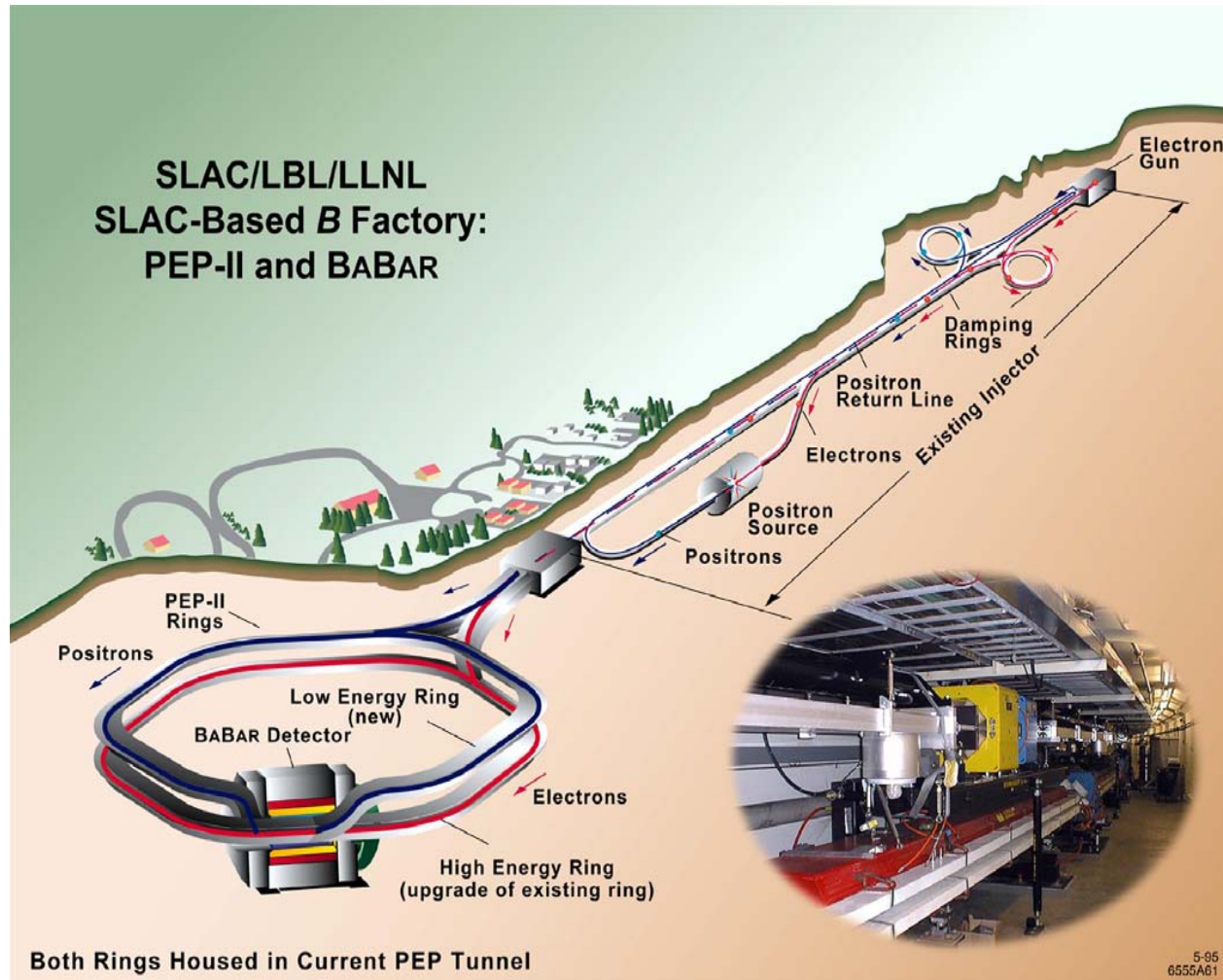


Colliding beams: L small, E_{CM} large



Example: PEP-II @ SLAC

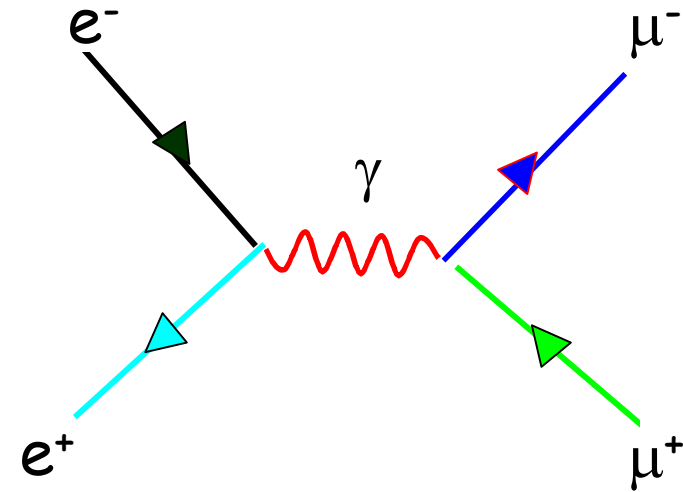
Produced about a billion $B - \bar{B}$ meson pairs in e^+e^- collisions at the PEP-II storage ring



How probable are the various physics processes?

$e^+e^- \rightarrow \mu^+\mu^-$ at PEP II

$$1/\text{GeV}^2 = (0.197 \text{ fm})^2 = 388 \mu\text{b}$$



$$\sigma = \frac{4\pi\alpha^2}{3s} = 87 \text{ nb} / s (\text{GeV}^2)$$

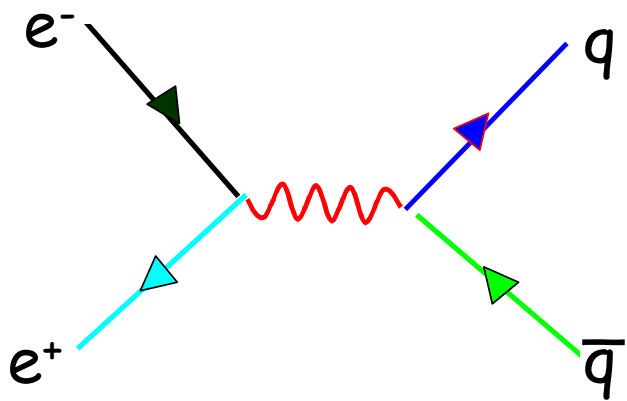
$$\alpha = 1/137$$

$$s = (E_{CM})^2$$

At $s = (10.58 \text{ GeV})^2$ (PEP II): $\sigma(e^+e^- \rightarrow \mu^+\mu^-) = 0.78 \text{ nb}$

$e^+e^- \rightarrow b\bar{b}$ at PEP II

At SLAC we want to make as many $b\bar{b}$ pairs as possible. For light quarks (u,d,s,c), similar process to $e^+e^- \rightarrow \mu^+\mu^-$



$$\sigma = 3Q_q^2 \frac{4\pi\alpha^2}{3s} \quad q = u, d, s, c$$

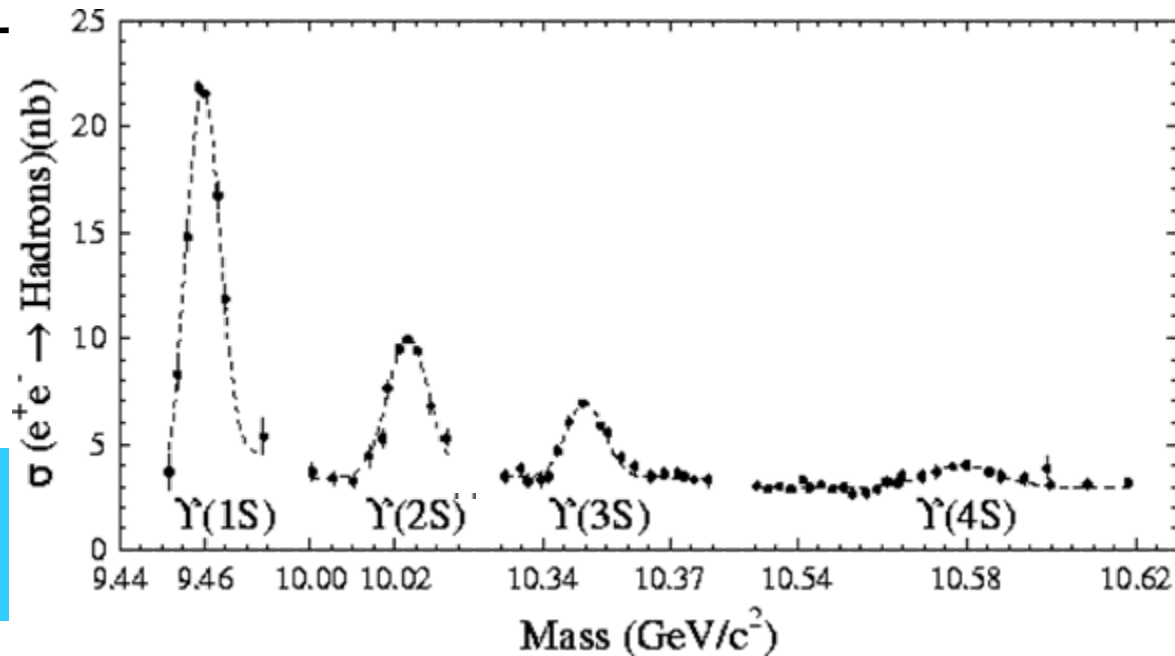
accounts for **off-resonance** σ

$$3/3^2 \cdot 0.78 \text{ nb} = 0.26 \text{ nb}$$

But for b in B meson there is a **resonance**, the $\Upsilon(4S)$.

At $s = (10.58 \text{ GeV})^2$:

$$\sigma(e^+e^- \rightarrow b\bar{b}) = 1.05 \text{ nb}$$



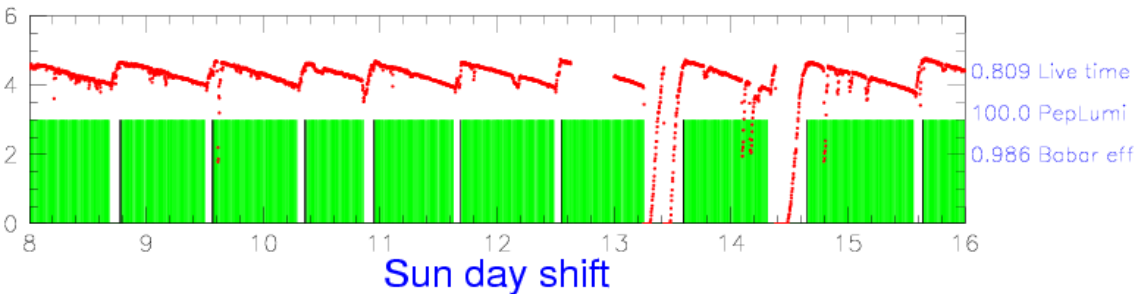
Luminosity (Flux)

Number of events/sec = σ (cm²) × Luminosity (cm⁻² s⁻¹)

$$\sigma(e+e^- \rightarrow b\bar{b}) = 1.05 \text{ nb}$$

$$(1 \text{ nb} = 10^{-9} \times 10^{-24} \text{ cm}^2)$$

PEP II-BABAR: Apr 6-7



PEP II Luminosity
about $4 \times 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$
 $= 4 \text{ nb}^{-1} \text{ s}^{-1}$

Produced about 5 $b\bar{b}$ pairs per second

In the form of $B^0 \bar{B}^0$ pairs ($B^0 = B_d = \bar{b}d$ meson)
and $B^+ B^-$ pairs ($B^+ = B_u = \bar{b}u$ meson)
50% of each.

Integrated Luminosity

$$L_{\text{int}} = \int L dt \quad (\text{cm}^{-2})$$

$$1 \text{ nb} = 10^{-33} \text{ cm}^2$$

$$1 \text{ nb}^{-1} = 10^{33} \text{ cm}^{-2}$$

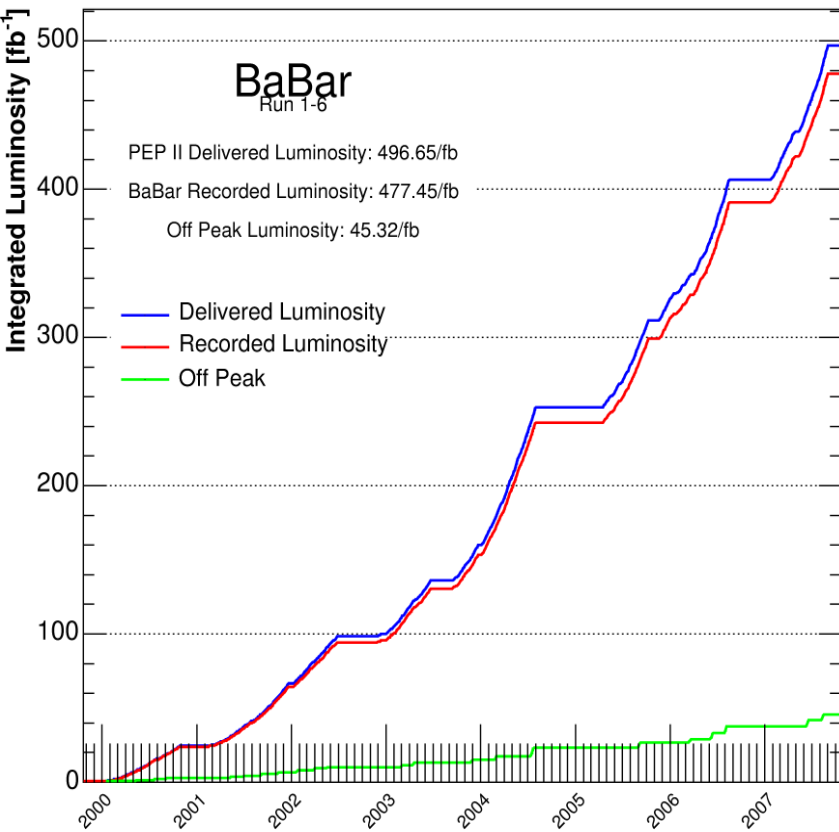
$$1 \text{ pb} = 10^{-36} \text{ cm}^2$$

$$1 \text{ pb}^{-1} = 10^{36} \text{ cm}^{-2}$$

$$1 \text{ fb} = 10^{-39} \text{ cm}^2$$

$$1 \text{ fb}^{-1} = 10^{39} \text{ cm}^{-2}$$

As of 2007/10/23 00:00



$$\text{Total BaBar events} = L_{\text{int}} \cdot \sigma$$

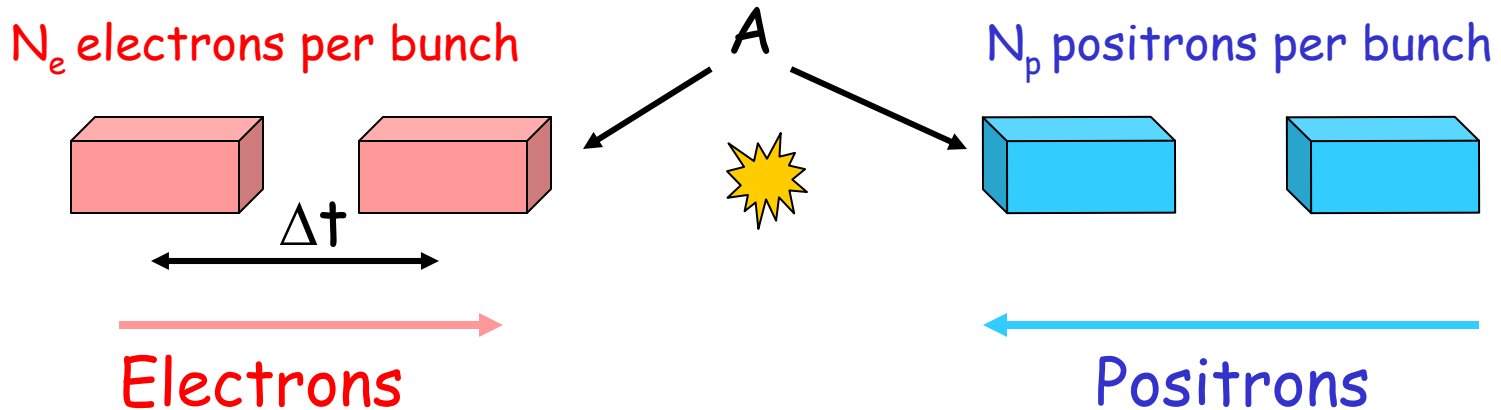
$$= 477 \times 10^6 \text{ nb}^{-1} \times 1.05 \text{ nb}$$

$$= 500 \text{ million } \bar{b}b \text{ pairs}$$

So **many** are required because "interesting" decays like $B_d \rightarrow (J/\psi) K_S \leftarrow \bar{B}_d$ are **rare**, $\text{Br}(B \rightarrow \psi K_S) < 10^{-3}$, cannot "find" all decays, etc.

plus 900 fb^{-1} at KEK (Japan)

Luminosity

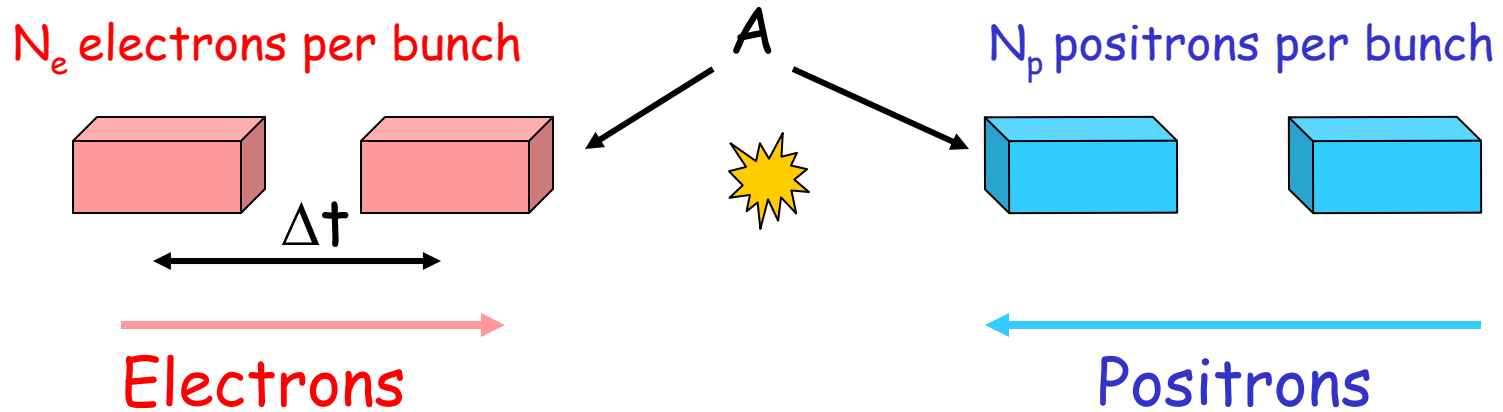


$$L = \frac{f N_e N_p}{A}$$

Frequency of collisions: $f = 1/\Delta t$
Area of beam = A

For large L need large N , f and small A (focusing)

PEP II Luminosity



$$2\pi R = 2200\text{m}, 1658 \text{ bunches} \rightarrow 0.0075 \text{ bunches/cm}$$

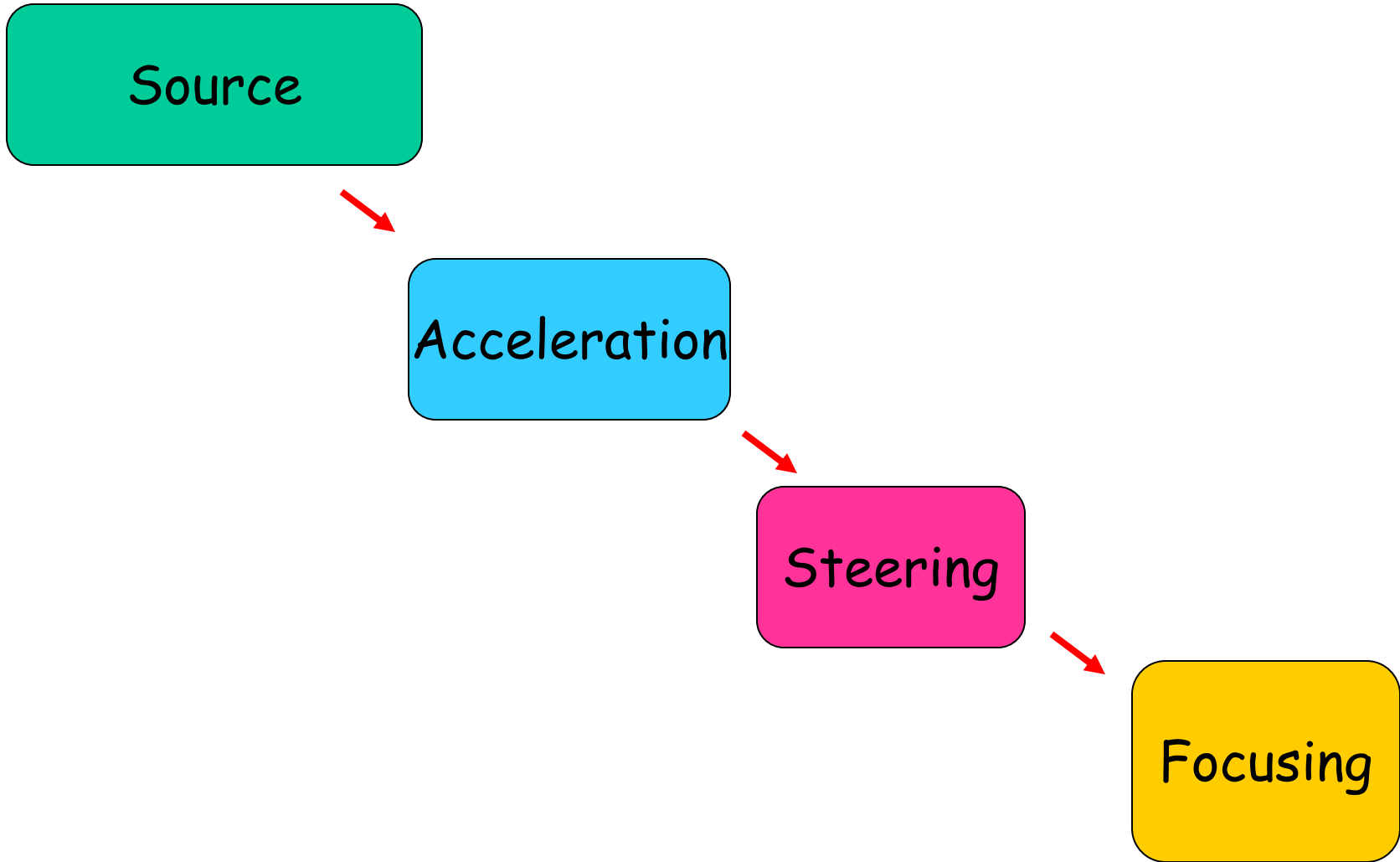
$$\rightarrow f = 2.3 \times 10^8 \text{ bunches/sec}$$

$$L = k \frac{f N_e N_p}{A} = k \frac{(2.3 \times 10^8)(2.7 \times 10^{10})(5.9 \times 10^{10})}{150 \mu\text{m} \times 15 \mu\text{m}}$$

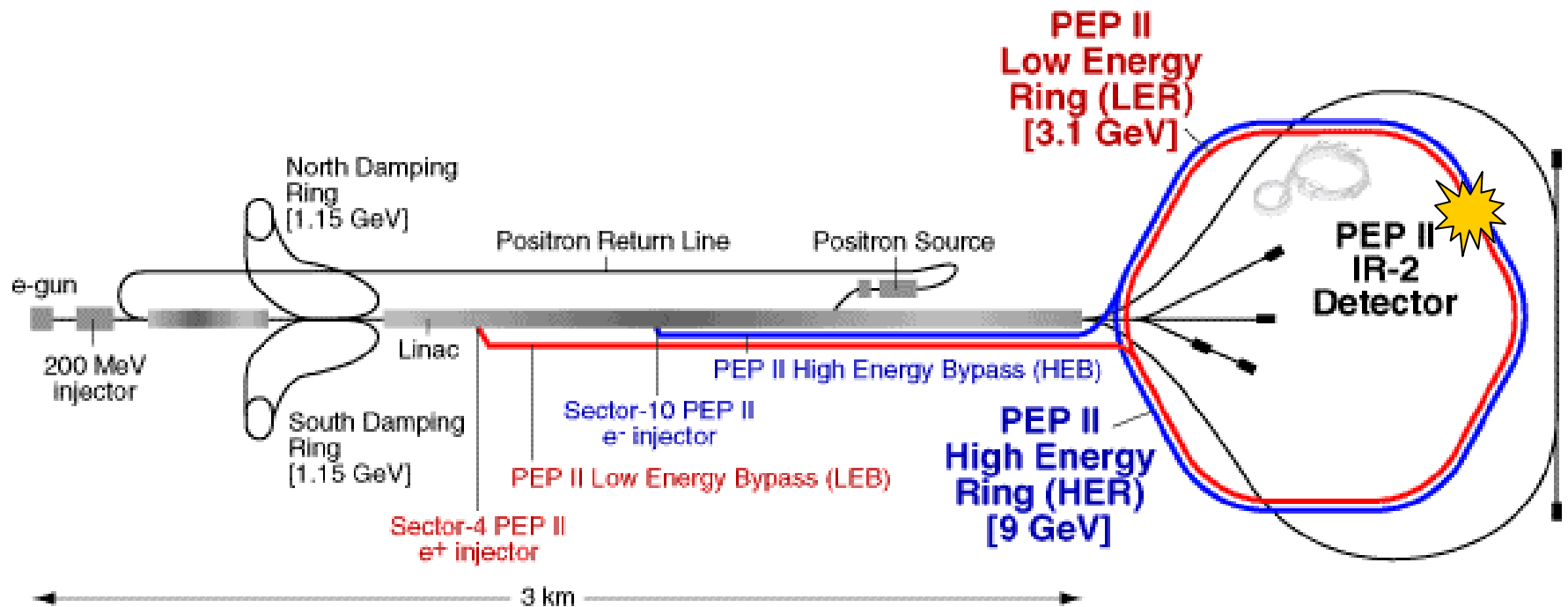
$$L = k \times 1.6 \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$$

k a factor specific to machine design

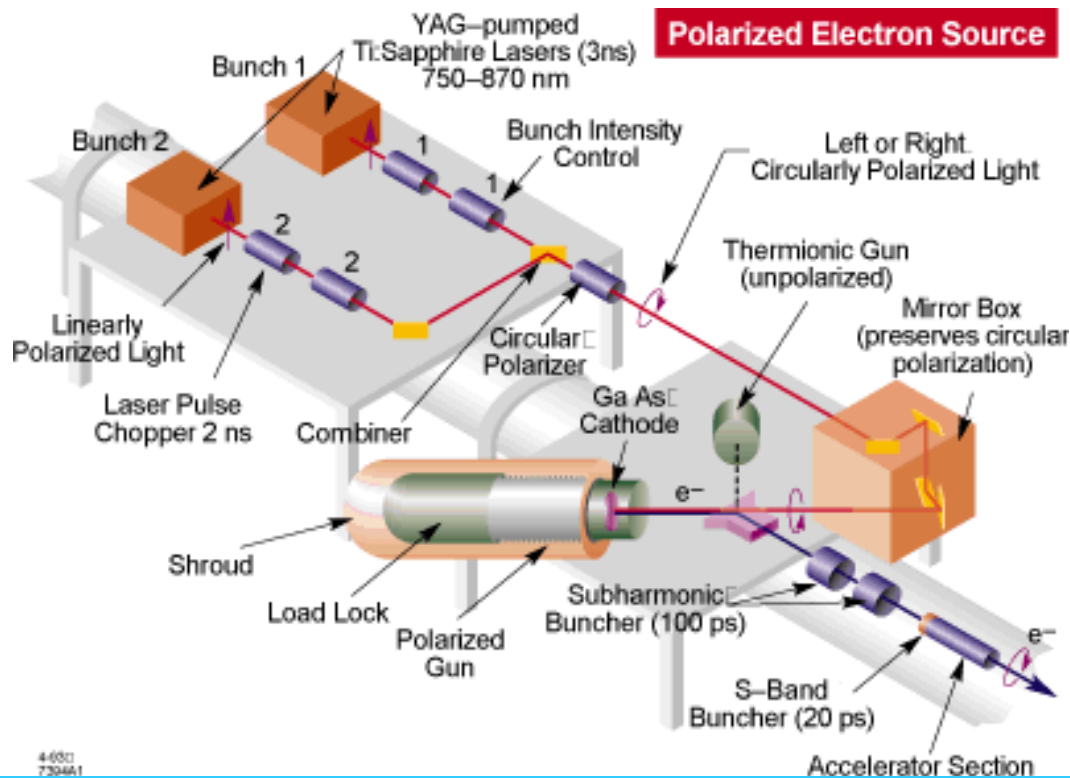
Basic Accelerator Components



The SLAC accelerator complex

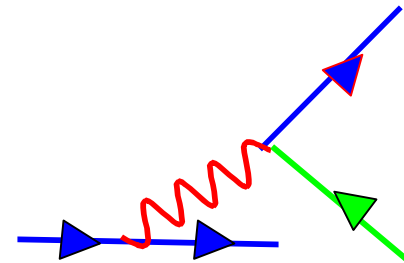
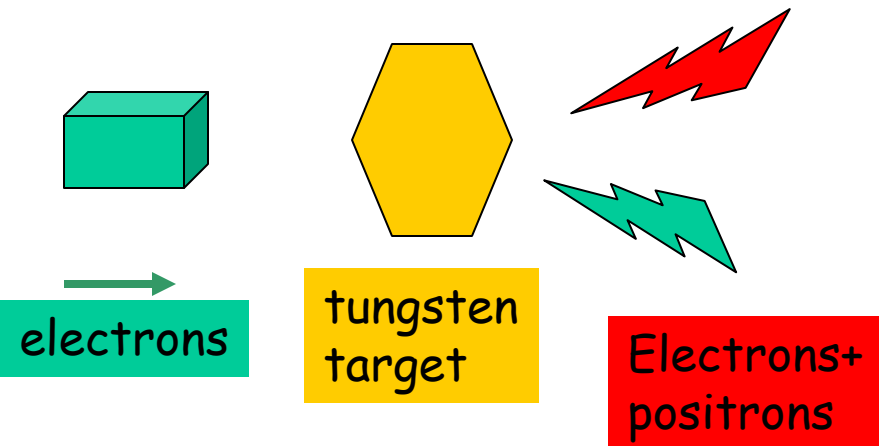
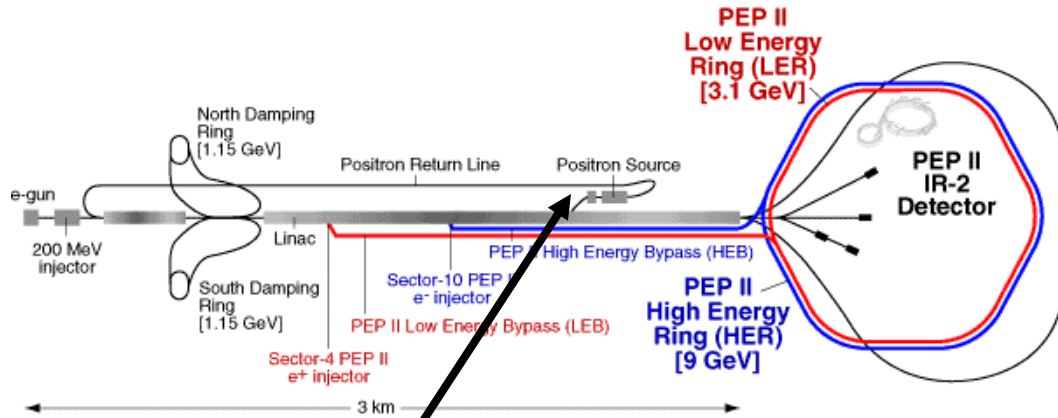


Electron Source



Thermionic (hot high voltage cathode)
or **photo-electric** emission to create electrons.
With **polarized** laser light can create **polarized** electrons.
Pulse the high voltage
→ bunches created for insertion into linear accelerator.

Positron Source

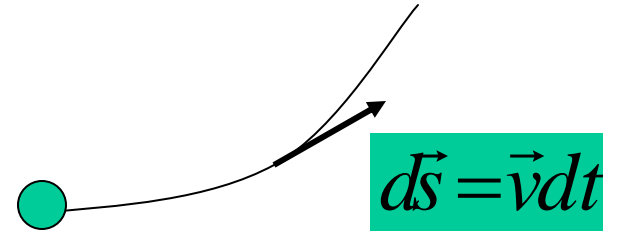


bremsstrahlung in target followed by pair production

Particle Acceleration

$$\vec{F} = e\vec{E} + e(\vec{v} \times \vec{B})$$

$$\Delta\vec{p} = \int \vec{F} dt \quad \Delta E = \int \vec{F} \cdot d\vec{s}$$



$$\Delta E = \int e\vec{E} \cdot \vec{v} + e(\vec{v} \times \vec{B}) \cdot \vec{v} dt$$

$$(\vec{v} \times \vec{B}) \cdot \vec{v} = 0$$

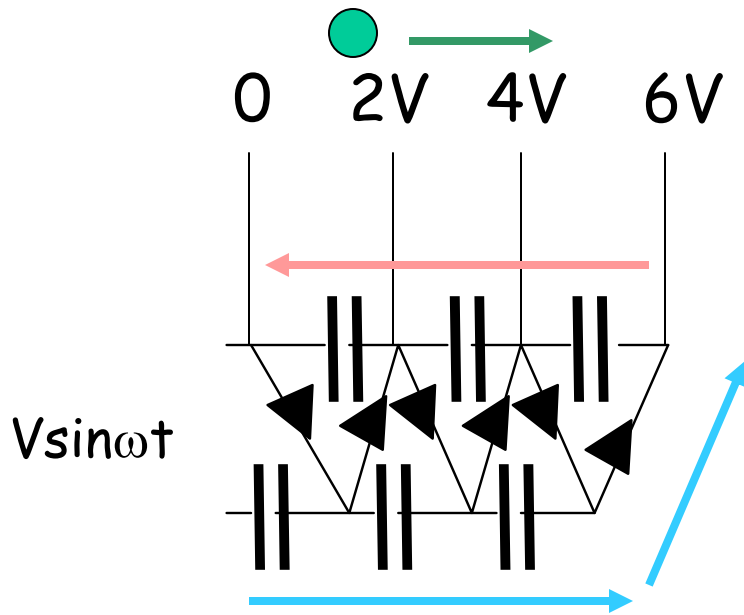
$$\Delta E = \int e\vec{E} \cdot \vec{v} dt$$

\vec{E} accelerate

\vec{B} steer/focus

Earliest and Simplest Accelerator

Cockcroft-Walton Accelerator - Voltage Doubler



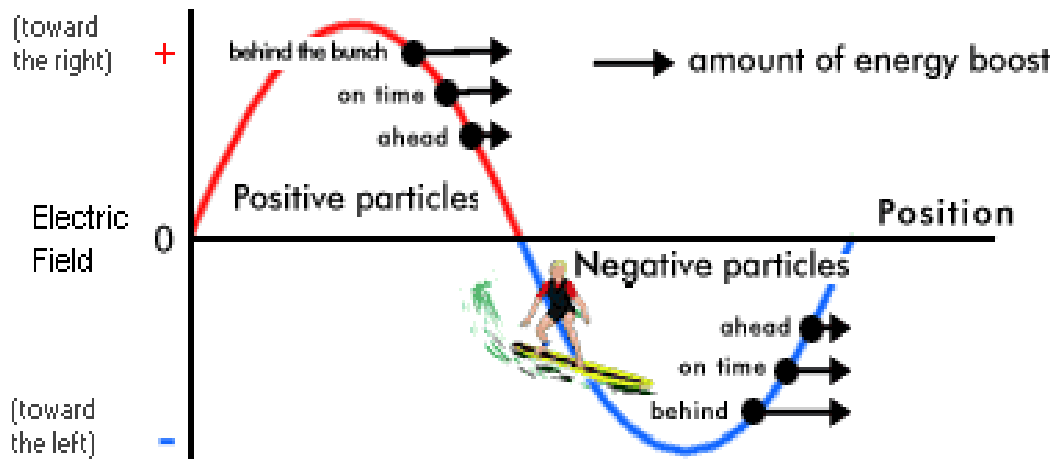
Diodes -> capacitors **charge** in parallel
discharge in series



Still used as first stage accelerator in proton machines.
Up to ~ 1 MV. Watch out for arcing!

RF acceleration

Particle "surfs" an electromagnetic wave



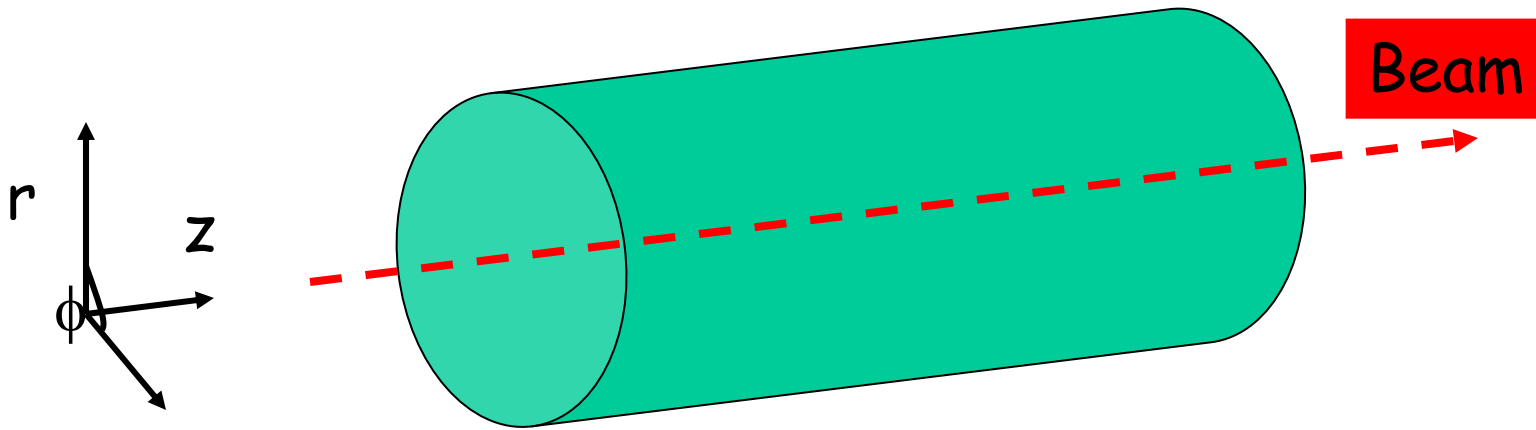
1. Need E field in direction of motion (i.e. longitudinal E-field - not free space EM wave)
2. Need particles locked in phase with the wave

Creating longitudinal E for acceleration

Use waveguides and resonant cavities.

Geometry of waveguide can create the correct field.

(Free space EM waves are **transverse**; need walls.)



Solve Maxwell's equations with appropriate boundary conditions for a cylindrical cavity.

Maxwell's Equations and boundary conditions

$$\vec{\nabla} \cdot \vec{E} = 0$$

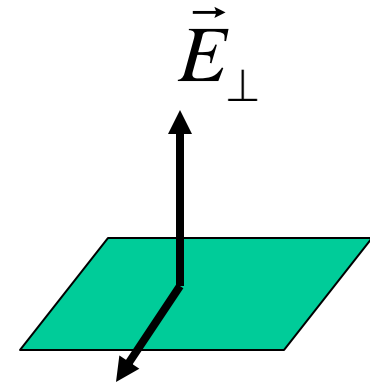
$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \frac{\mu\epsilon}{c} \frac{\partial \vec{E}}{\partial t}$$

From these equations can derive:

Solve $\nabla^2 \vec{E} = \frac{\mu\epsilon}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2}$ In cylindrical coordinates



With boundary conditions
for surface of conductor

$$\vec{E}_\parallel = 0$$

$$\vec{B}_\perp = 0$$

Creating longitudinal E for acceleration

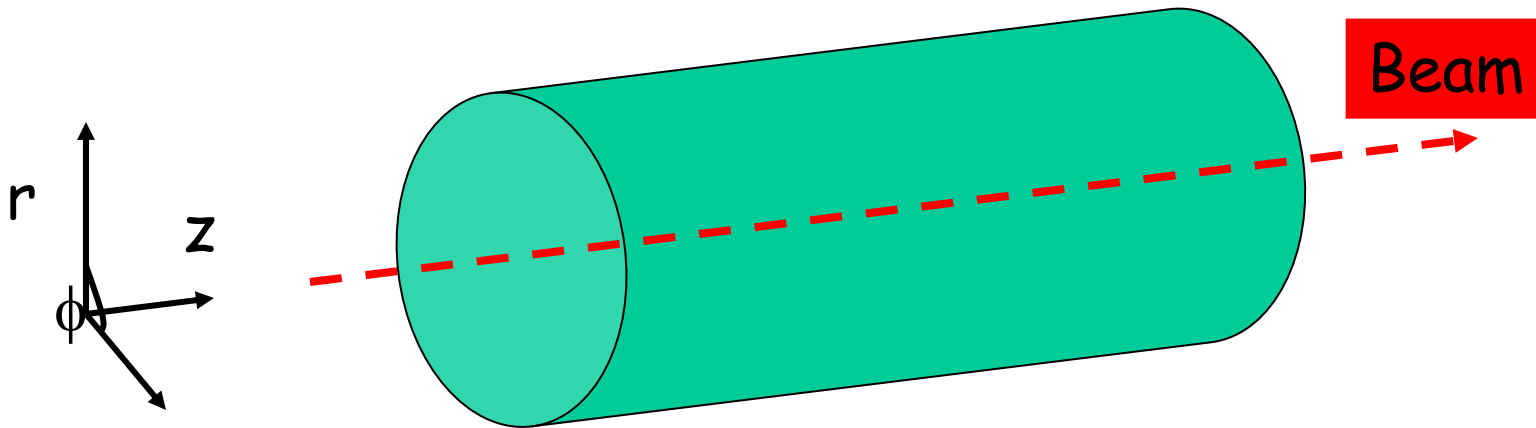
Use separation of variables to solve.

Two sets of solutions:

TE_{klm} Transverse E (longitudinal B) modes

TM_{klm} Transverse B (longitudinal E) modes

klm is the periodicity of the solution in $\phi r z$

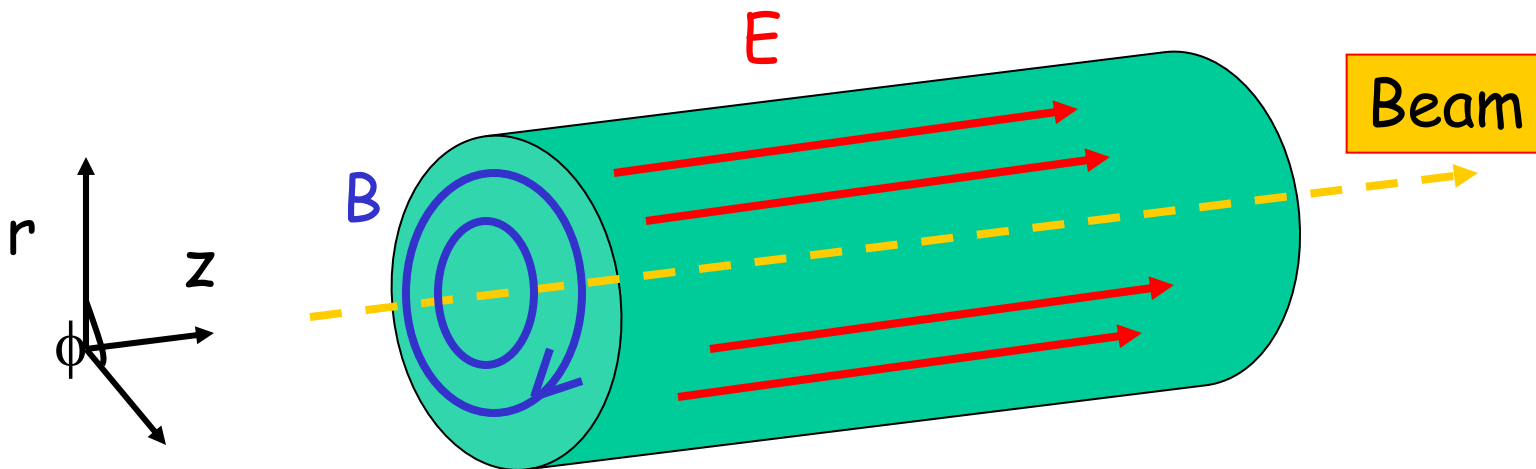


The TM modes are useful for particle acceleration

The TM_{010} mode

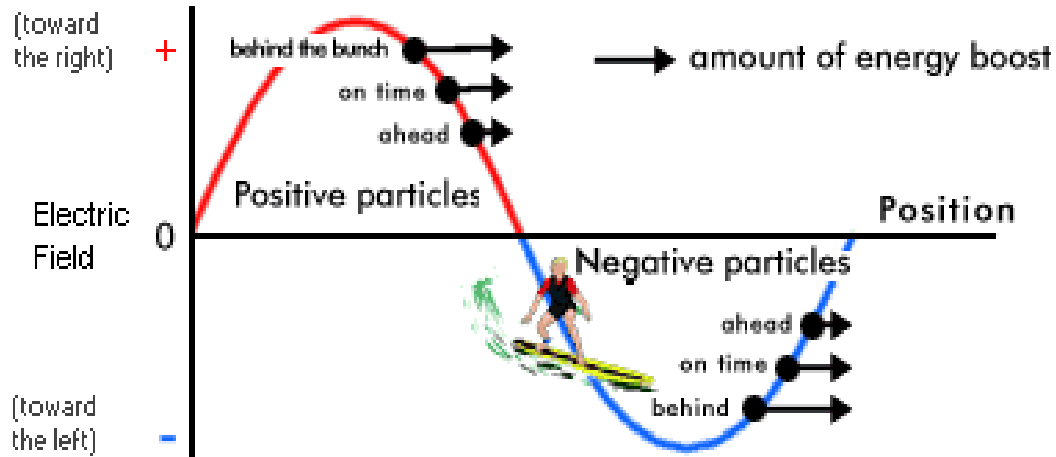
$$E_z = E_0 f(r) e^{i(\omega t - kz)}$$

$$B_\phi = E_0 g(r) e^{i(\omega t - kz)}$$



$f(r)$ is a Bessel function, with $f(R) = 0$.
 $f(r) = J_0(2.405 r/R)$.
 $g(r)$ obtained from $f(r)$.

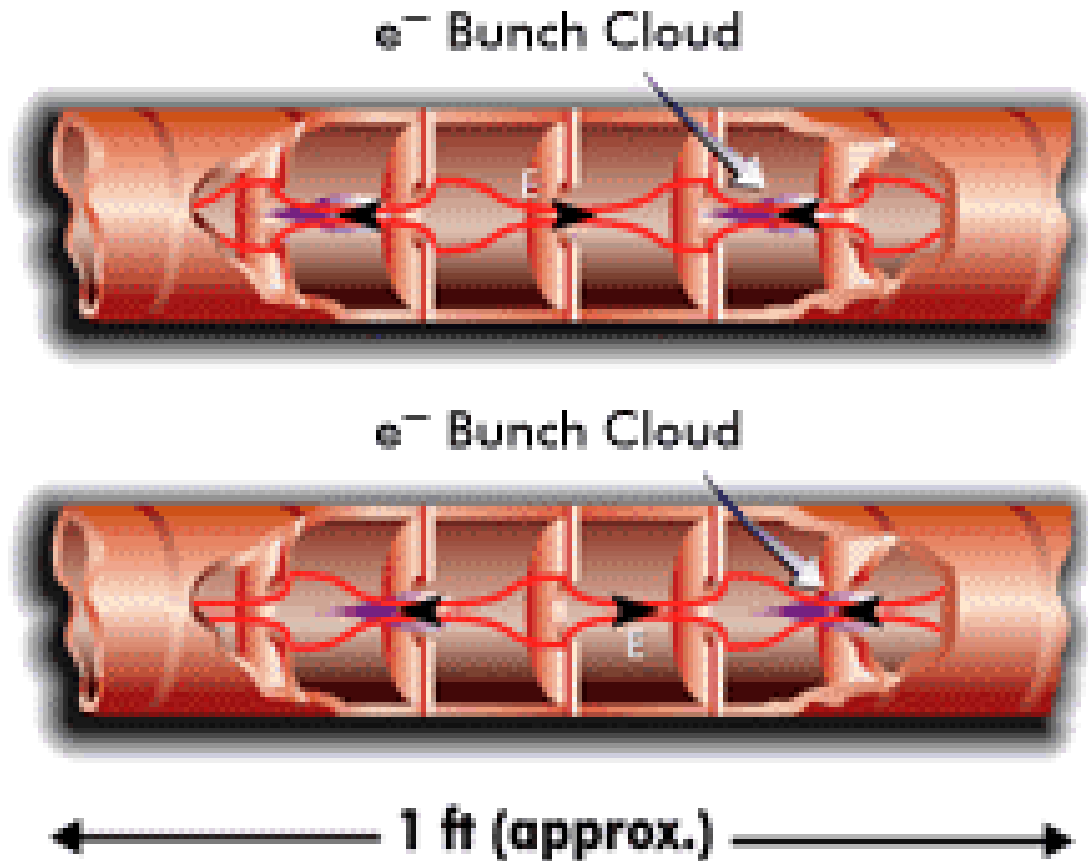
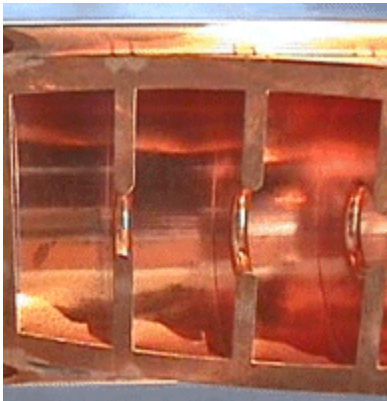
Phase Synchronization



1. Arrange phase velocity $(\omega/k) =$ particle velocity
2. Stable for small perturbations about synchronized point

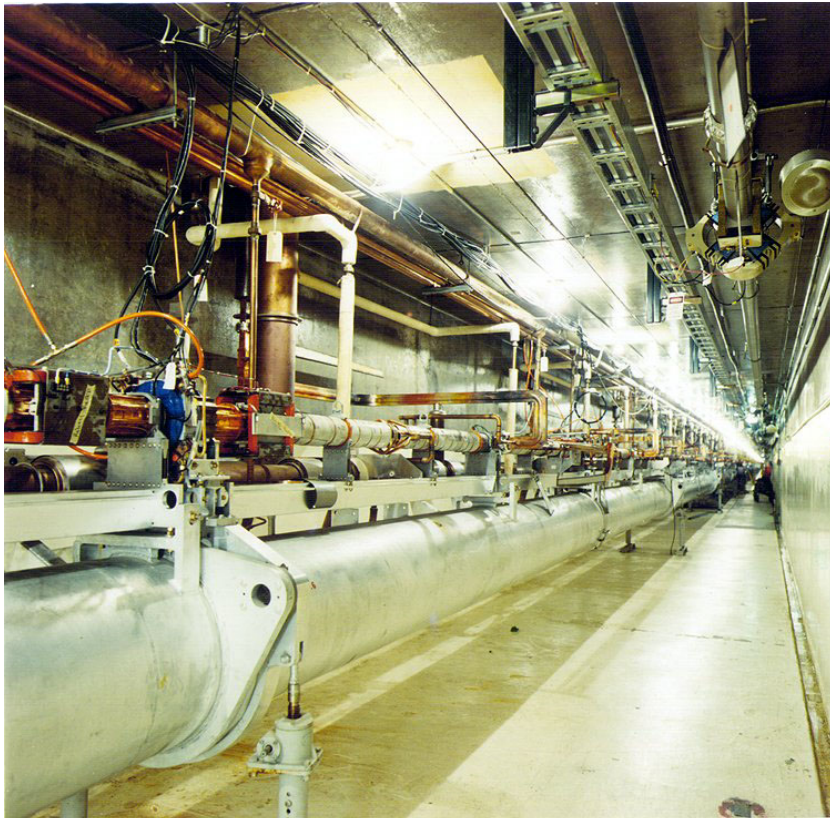
Disk Loaded Waveguide

Insertion of disks controls phase velocity
(interference of reflected waves)



Stanford Linear Accelerator

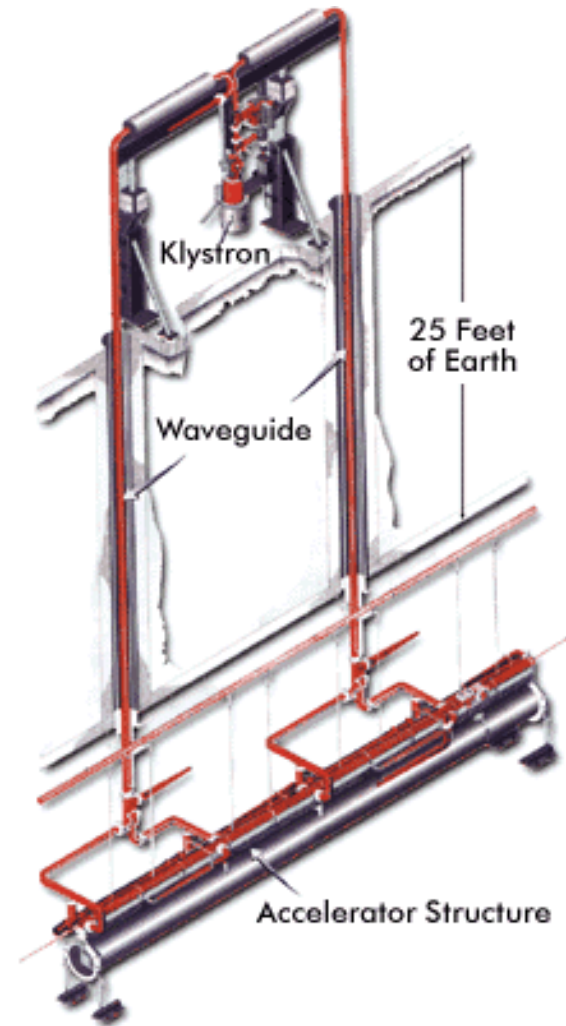
240 x 8 MW Klystrons
provide the power



ACC_01

PEP-II Accelerator at Slac

8-19-97



Beam Steering

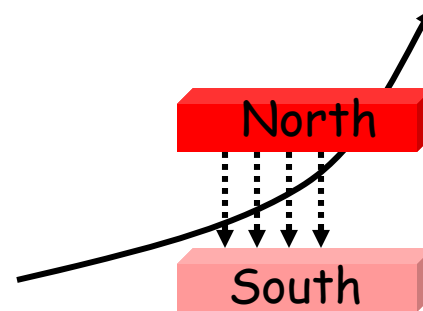
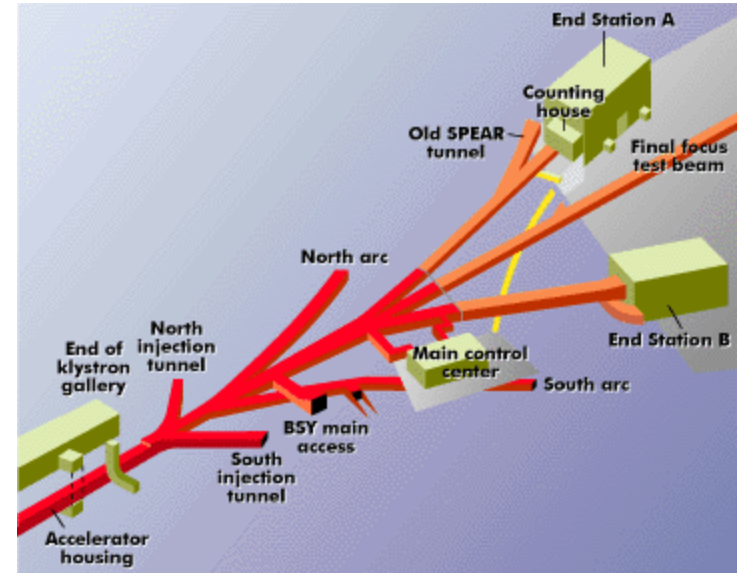
Use Magnetic Dipoles for Steering



PD_001

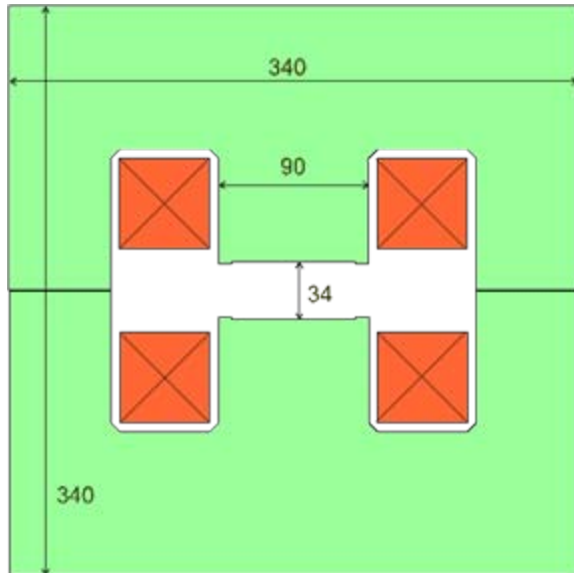
PEP-II Dedication

10/26/98



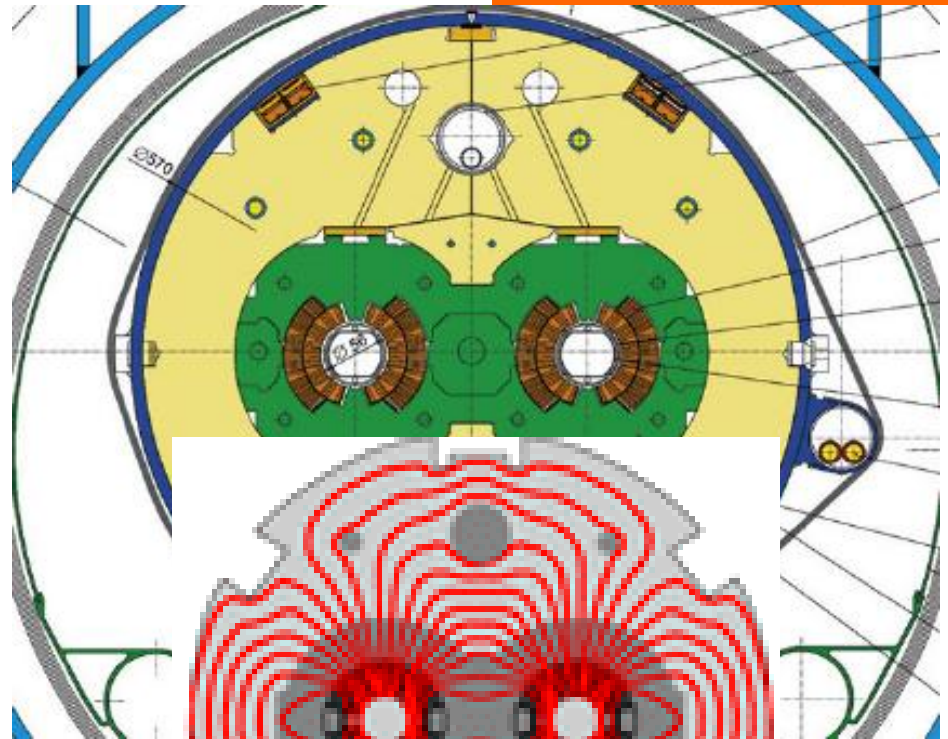
Two examples of Dipole Magnets

Synchrotron light source:
one e^- beam, 1 Tesla,
room temperature



LHC: two p beams, 8.4 Tesla,
superconducting NbTi @ 1.9K

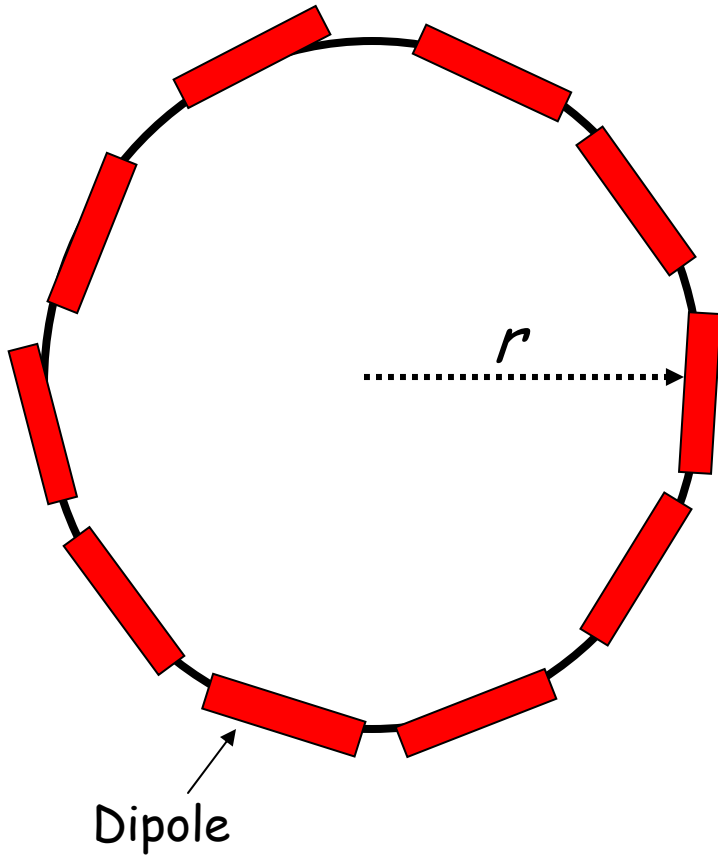
1,296 of these!



Steering Particle in Circle

$$p = mv\gamma = eBr$$

$$\gamma = \frac{1}{\sqrt{1-v^2}}$$

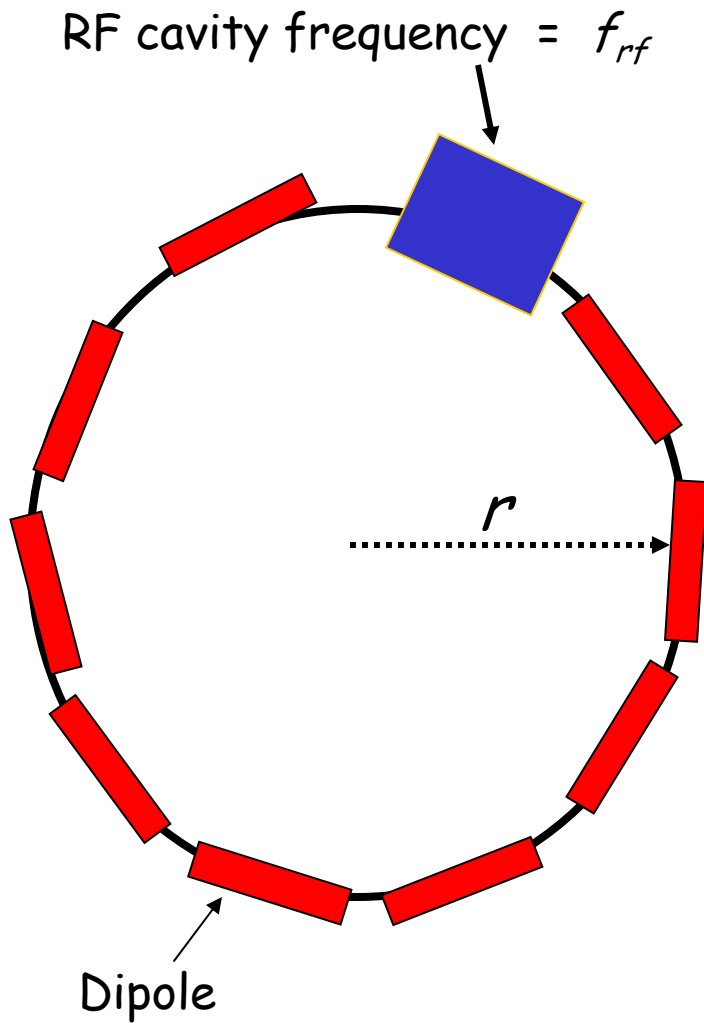


$$f_{rev} = \frac{1}{t_{rev}} = \frac{v}{2\pi r} = \frac{eB}{2\pi\gamma m}$$

assuming the ring is completely filled with dipoles of strength B

If one accelerates a beam in a ring, B must increase in time ("ramp") as p does.

The Synchrotron



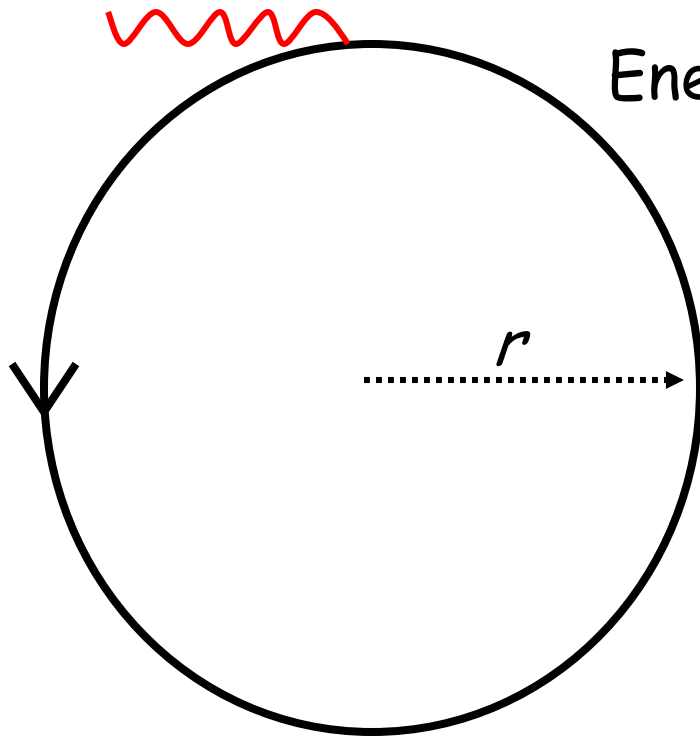
$$f_{rf} = f_{rev} = \frac{v}{2\pi r}$$

$$\gamma = \frac{1}{\sqrt{1-v^2}}$$

Circular particle accelerator
(1 rf cavity) with rf frequency
(and B) changing as v increases
to keep in phase

Synchrotron Radiation

As particle circles, it loses energy through EM radiation



Energy loss per turn:

$$\Delta E_{loss} = \frac{4\pi\alpha}{3} \frac{E^4}{m^4 r}$$

Limits maximum E
for circular electron accelerator
because m_e is small

LEP (Large Electron-Positron Collider)



$$C = 26.7 \text{ km}$$

At 100 GeV per beam, Energy loss per turn = 2 GeV
Operates at 11 kHz revolution frequency.

Proton Synchrotrons

Energy loss per turn much more favorable for protons, because m_p large ($= 1837 m_e$)



$$\Delta E_{loss} = \frac{4\pi\alpha}{3} \frac{E^4}{m^4 r}$$

Tevatron is (still!) the highest energy collider:
980 GeV protons collide with
980 GeV anti-protons

Proton synchrotrons
limited in E by strength
of bending magnets:

$$E = p = eBr$$

Tevatron: 4.4 Tesla, $r = 1$ km

→ $p(\text{GeV}) = 0.3 \text{ B(T)} r(\text{m}) = 1320 \text{ GeV} \times [\text{packing fraction}] = 980 \text{ GeV}$

LHC: 8.4 Tesla, $r = 26.7/(2\pi) \text{ km} = 4.25 \text{ km}$

→ $p(\text{GeV}) = 10.7 \text{ TeV} \times [\text{packing fraction}] = 7 \text{ (or 5?) TeV}$

Focusing

To decrease beam area at a collision point, we need to focus the beam.

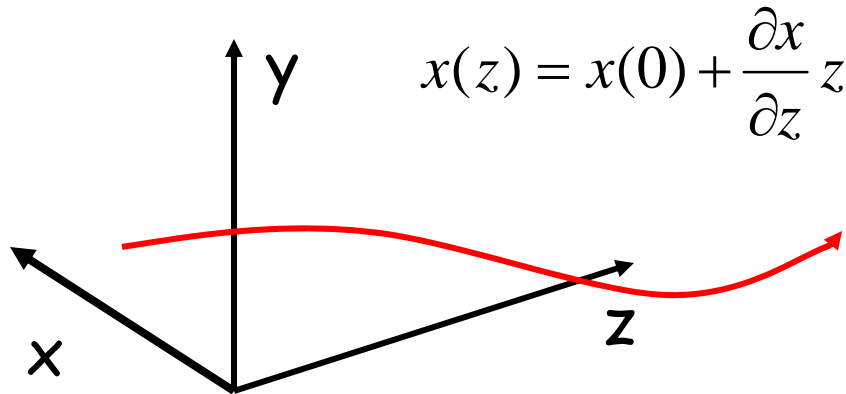
Also a number of effects in the accelerator cause the beam to defocus (space charge repulsion, residual transverse E fields, etc.)

All accelerators need focusing throughout the accelerator to maintain a stable beam.

To study the beam behavior we use beam optics.

Beam Optics

Consider a particle moving in the z direction



$$x' = \frac{\partial x}{\partial z}$$

If the x and y motions are decoupled then we can describe the motion by a **transfer matrix** M which acts in transverse phase space: x and $\sim p_x$.

$$\begin{pmatrix} x \\ x' \end{pmatrix}_z = M \begin{pmatrix} x \\ x' \end{pmatrix}_0 = \begin{pmatrix} 1 & z \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_0$$

The M shown here describes a **drift region** of empty space.

Combining Transfer Matrices

Successive optical components can be described by multiplying the matrices. For example, for two successive drift regions:

$$M_1 = \begin{pmatrix} 1 & z_1 \\ 0 & 1 \end{pmatrix} \quad M_2 = \begin{pmatrix} 1 & z_2 \\ 0 & 1 \end{pmatrix}$$

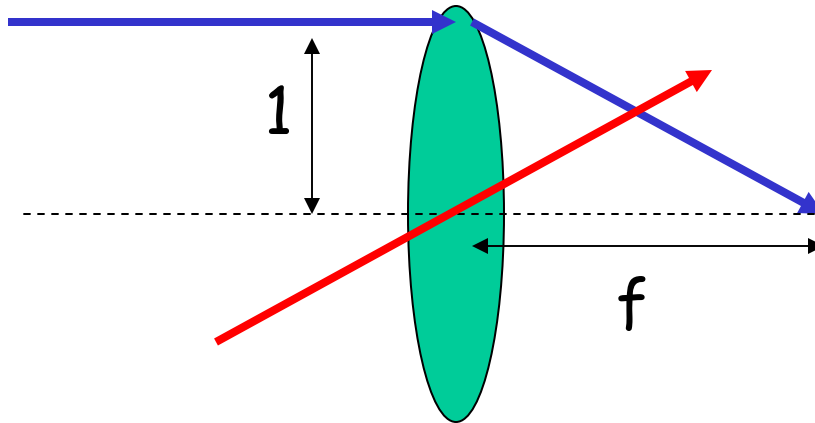
$$M = M_2 M_1 = \begin{pmatrix} 1 & z_1 + z_2 \\ 0 & 1 \end{pmatrix}$$

In this case the transfer matrices commute but this is not always the case

Transfer matrix of a thin lens

An off-axis parallel ray is focused

$$\begin{pmatrix} x \\ x' \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ -1/f \end{pmatrix}$$



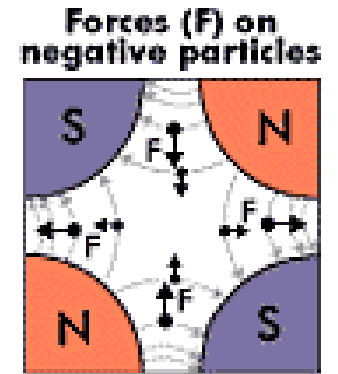
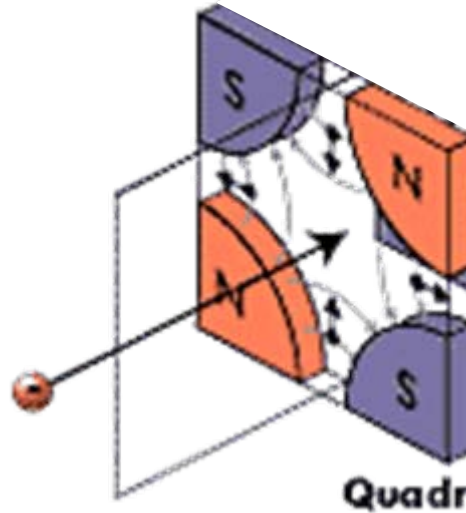
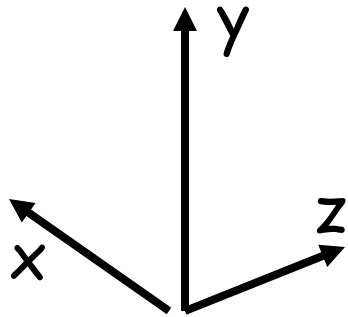
Ray through center is unaffected

$$\begin{pmatrix} x \\ x' \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$M_{convergent} = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$$

$$M_{divergent} = \begin{pmatrix} 1 & 0 \\ 1/f & 1 \end{pmatrix}$$

Quadrupole Focusing



$$\vec{\nabla} \times \vec{B} = 0 \quad \text{and} \quad B_z = 0 \quad \text{so} \quad \frac{\partial B_x}{\partial y} = \frac{\partial B_y}{\partial x}$$

then

$$B_x = by$$

$$B_y = bx$$

Quadrupole focusing

In the x dimension the particle executes simple harmonic motion since

$$m\gamma^2 \frac{d^2 x}{dt^2} = e\gamma(\vec{v} \times \vec{B})_x = -e \frac{p}{m} bx$$

$$dz = \frac{p}{m\gamma} dt$$

$$\frac{d^2 x}{dz^2} = -k^2 x$$

$$k = \sqrt{\frac{eb}{p}}$$

The solution for x is then $x = A \cos kz + B \sin kz$

so $x(z) = x(0) \cos kz + x'(0)k^{-1} \sin kz$

$$x'(z) = -x(0)k \sin kz + x'(0) \cos kz$$

If magnet length is l , then

$$M_x = \begin{pmatrix} \cos kl & k^{-1} \sin kl \\ -k \sin kl & \cos kl \end{pmatrix} \approx \begin{pmatrix} 1 & l \\ -k^2 l & 1 \end{pmatrix}$$

Quadrupole focusing

Can show that M_x is equivalent to $M_{\text{convergent}}$ plus drifts

$$M_x \approx \begin{pmatrix} 1 & l \\ -k^2 l & 1 \end{pmatrix} \approx \begin{pmatrix} 1 & l/2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \begin{pmatrix} 1 & l/2 \\ 0 & 1 \end{pmatrix} = M_{l/2} M_{\text{conv}} M_{l/2}$$

$$f = \frac{1}{k^2 l}$$

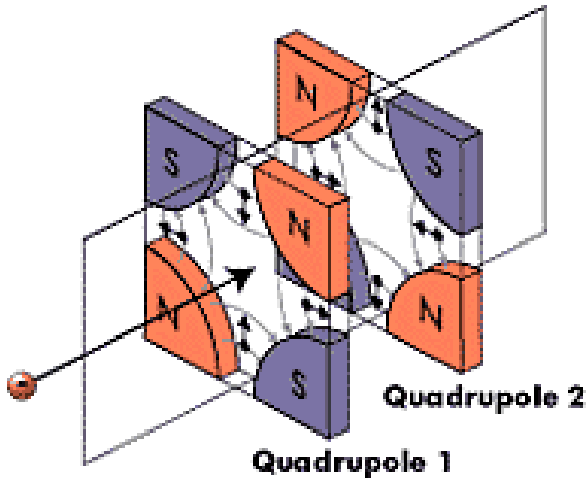
Similarly, can show that M_y is equivalent to $M_{\text{divergent}}$ plus drifts

$$M_y \approx \begin{pmatrix} 1 & l \\ k^2 l & 1 \end{pmatrix} \approx M_{l/2} M_{\text{div}} M_{l/2}$$

If focus in x then defocus in y !

Quadrupole focusing

However, two **opposite polarity** quads separated by a drift distance cause **focusing in both x and y**



$$M_x M_z M_y = M_{l/2} M_{conv} M_{z+l} M_{div} M_{l/2}$$

find that

$$M_{conv} M_{z+l} M_{div} \approx M_{l/2} M_{conv, f'} M_{l/2}$$

with

$$f' = \frac{f^2}{(z+l)} = \frac{1}{k^4 l^2 (z+l)}$$

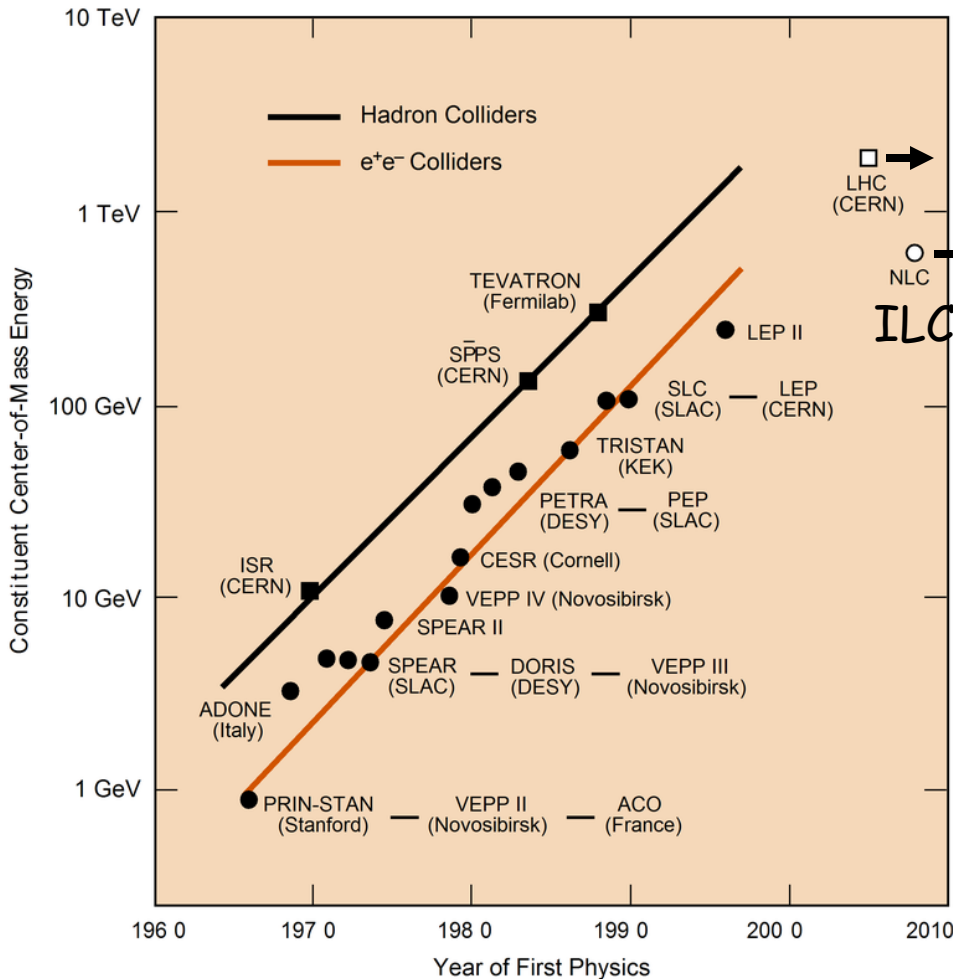
And

$$M_{div} M_{z+l} M_{conv}$$

is same with $f \rightarrow -f$ so it also focuses

Livingston Plot

HEP's version of Moore's Law - but we are falling off it

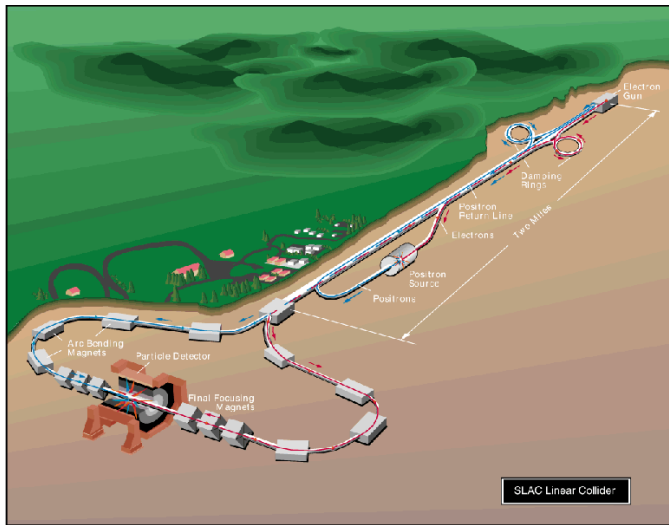


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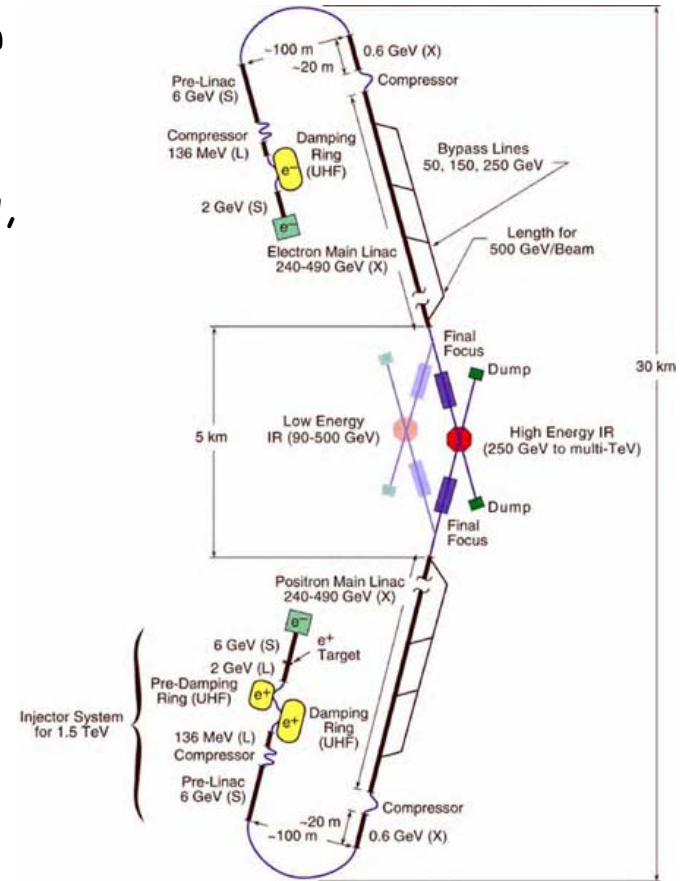
LC = e⁺e⁻ linear collider

e^+e^- Linear Collider

- To avoid synchrotron radiation losses in ring, make "one pass" collider



SLC used same linac twice



Proposed ILC

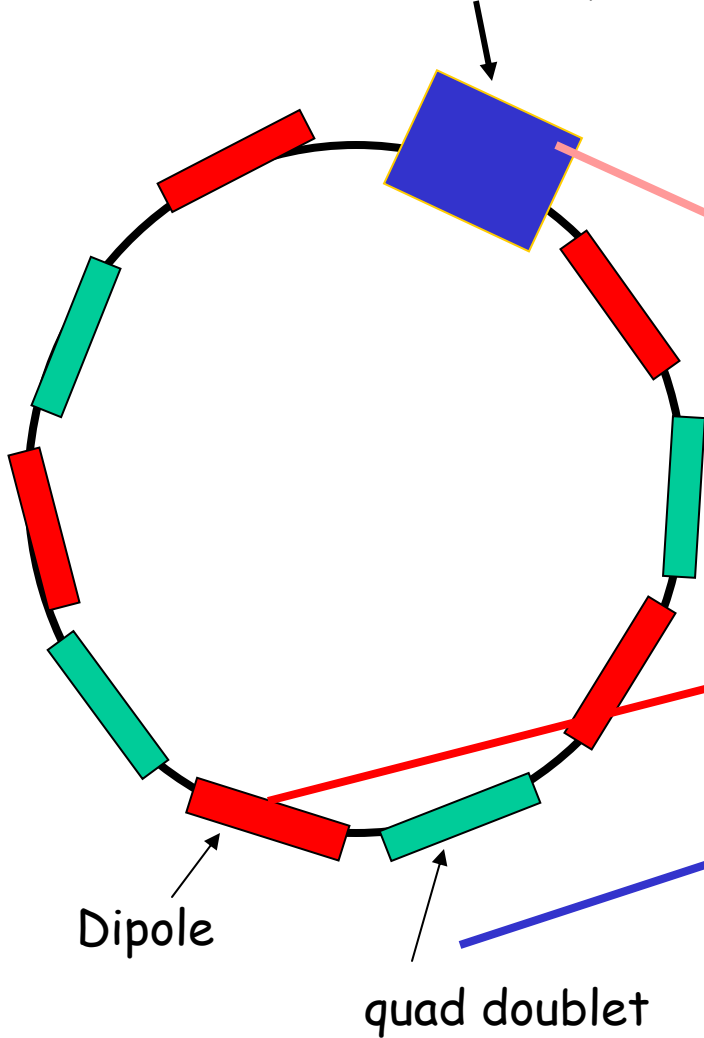
To get enough luminosity L :

$$L = \frac{f N_e N_p}{A}$$

f drops from 10^8 to 10^2 Hz
 $\rightarrow A$ must drop from μm^2 to nm^2

Elements of an Accelerator

RF cavity frequency = f_{rf}



Acceleration - rf cavity
steering - dipoles
focusing - quad doublet



PD_001

PEP-II Dedication

10/26/98