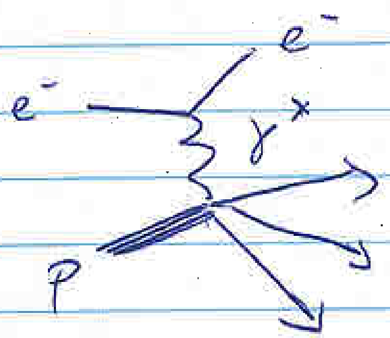
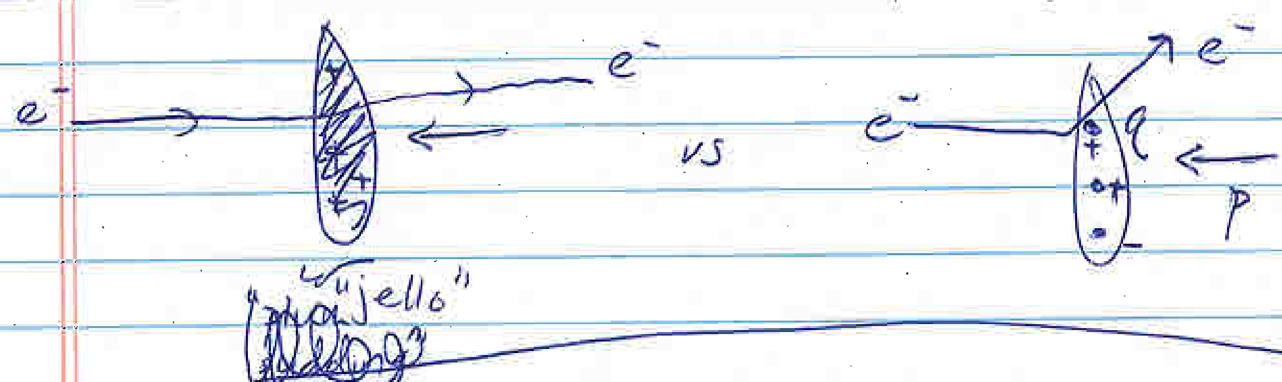
Lecture 15: QCD and Parton Model (I)

- First (dynamical) direct evidence for quark substructure of proton came from deep inelastic scattering (DIS) at SLAC ~ 1969.

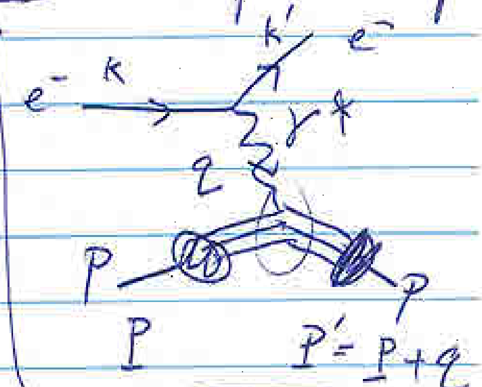
$$e^- + p \to e^- + \underbrace{X}_{\text{anything (inclusive)}}$$



Large cross sections at large angles indicated hard (pointlike) electrically charged constituents



Kinematics of elastic scattering $e^- p \to e^- p$



$$q = k - k'$$

$$q^2 = (k-k')^2$$

$$= 2m_e^2 - 2(EE' - pp'\cos\theta)$$

$$q^2 \simeq -2EE'(1-\cos\theta) < 0$$

$\Rightarrow$ Define $Q^2 \equiv -q^2 > 0$

Elastic Scattering Kinematics (CONT.)

$$P'^2 = m_p^2 = (P+q)^2 = P^2 + 2P\cdot q + q^2$$
$$= m_p^2 + 2P\cdot q - Q^2$$

$$\Rightarrow 0 = 2P\cdot q - Q^2,$$

or $$\boxed{\frac{Q^2}{2P\cdot q} = 1}$$

* Only 1 kinematic variable in elastic scattering, given $\sqrt{s}$ = C.M. energy — the C.M. scattering angle.

* But in inelastic scattering, there is a second variable, the amount of energy loss.

* Was traditional to use energy loss in lab frame $\nu = E - E' = \frac{m_p(E-E')}{m_p} = \frac{P\cdot q}{m_p}$

As well as Q^2. $\Rightarrow \frac{d\sigma(\nu, Q^2)}{d\nu\, dQ^2}$

~~[illegible]~~

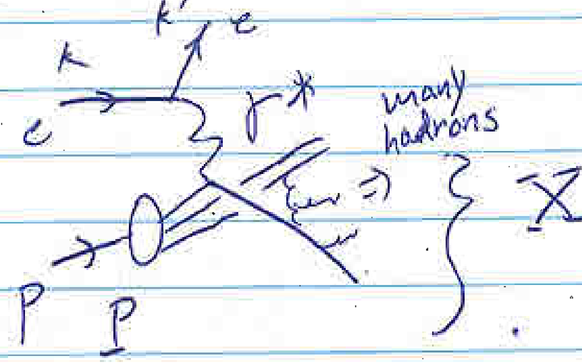
$$\cancel{\frac{d\sigma}{d\Omega\, dE'} = \left(\frac{d\sigma}{d\Omega}\right)_{point} W_2(q^2,\nu)}$$

Bjorken suggested trading ν for

$$\boxed{x = \frac{Q^2}{2P\cdot q} = \frac{Q^2}{2m_p\nu}}$$

$x = 1$ for elastic scattering

Inelastic Scattering Kinematics:



- suppose $Q^2 \gg m_p^2$ (short-distances probed) and $m_X^2 \gg m_p^2$

$$m_X^2 \equiv P'^2 = (P+q)^2 = \cancel{m_p^2} + 2P\cdot q - Q^2$$
$$= \frac{Q^2}{x} - Q^2$$

$$m_X^2 = Q^2\left(\frac{1-x}{x}\right)$$

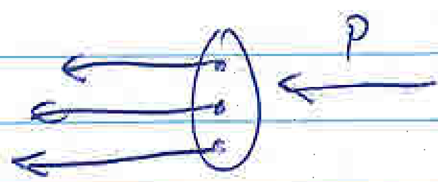
$$\Downarrow$$

$$0 < x < 1$$

$m_X^2 \to \infty$ $\qquad$ $m_X^2 \to m_p^2$

- Now think of boosting a proton containing quarks to very high momentum, $\gg$ few hundred MeV

characteristic momenta of quarks in proton

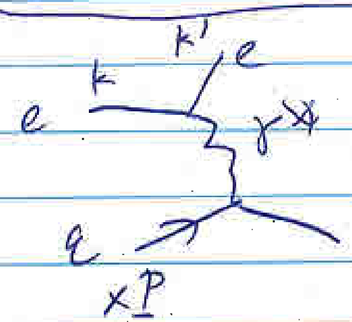


$\Rightarrow$ quark momenta are $\simeq$ parallel to proton momentum, but could have smaller fraction x of proton momentum

$p_q^\mu \simeq x P^\mu$ $\qquad 0 < x < 1$ with probability

(quark distribution function) $\rightarrow$ $\boxed{f_q(x) \equiv q(x)}$

Quark collision kinematics — same as elastic scattering kinematics except $P \to xP$



$$\Rightarrow \frac{Q^2}{2xP\cdot q} = 1$$

or $$\boxed{x = \frac{Q^2}{2P\cdot q}}$$

charged quarks + neutral gluons $\equiv$ partons

• So x defined by Bjorken is the quark longitudinal momentum fraction (up to QCD corrections)

Dynamics:

* Now let's ~~change~~ adapt our $e\mu \to e\mu$ formula (for $m_e, m_\mu \to 0$) to $eq \to eq$

$$\Rightarrow \boxed{\frac{1}{4}\sum_{\text{spins}} |\mathcal{M}|^2 = \frac{8e^4 Q_q^2}{\hat{t}^2}\,\frac{\hat{s}^2+\hat{u}^2}{4}}$$

(Q_q = charge of (anti-)quark q, $\pm 2/3$ or $\mp 1/3$)

• "Hats" refer to "partonic" kinematic variables ($eq \to eq$), as opposed to hadronic variables ($ep \to e X$)

• We should also { sum over final colors / average over initial colors } (since we do not observe them)

• In $eq \to eq$ this gives $\frac{1}{3}\cdot 3 = 1$

Now, $\hat{s} = (p_c + k)^2 = (xP + k)^2 \cong 2xP\cdot k = xs$

$$\boxed{\hat{s} = xs} \qquad \boxed{\hat{u} = -\hat{s} - \hat{t}} \qquad \frac{\hat{s}+\hat{t}}{\hat{s}} = 1 - \frac{Q^2}{xs}$$

$$\boxed{\hat{t} = (k-k')^2 \equiv -Q^2}$$

$$\hat{t} = -\frac{\hat{s}}{2}(1-\cos\hat{\theta}_{cm})$$

Putting in the flux factors and 2-body final state phase space,

$$\frac{d\sigma}{d\cos\hat{\theta}_{CM}} = \frac{1}{\cancel{2}\hat{s}} \cdot \frac{1}{16\pi} \cdot \frac{\cancel{8} e^4 Q_q^2}{\hat{t}^2} \, \frac{\hat{s}^2 + (\hat{s}+\hat{t})^2}{\cancel{4}}$$

$$\frac{d\sigma}{dQ^2} = \frac{d\sigma}{d\hat{t}} = \frac{2}{\hat{s}} \frac{d\sigma}{d\cos\hat{\theta}_{cM}} = \frac{2}{\hat{s}^2} \frac{(4\pi\alpha)^2 Q_q^2}{16\pi \, \hat{t}^2} \, \hat{s}^2 \left[1 + \left(1 - \frac{Q^2}{xs}\right)^2\right]$$

$$\boxed{\frac{d\sigma(eq \to eq)}{dQ^2} = Q_q^2 \, \frac{2\pi\alpha^2}{Q^4}\left[1 + \left(1 - \frac{Q^2}{xs}\right)^2\right]} \leftarrow \boxed{y \equiv \frac{P\cdot q}{P\cdot k} \equiv \frac{Q^2}{sx} \quad (0<y<1)}$$

Now we invoke the quark distribution function

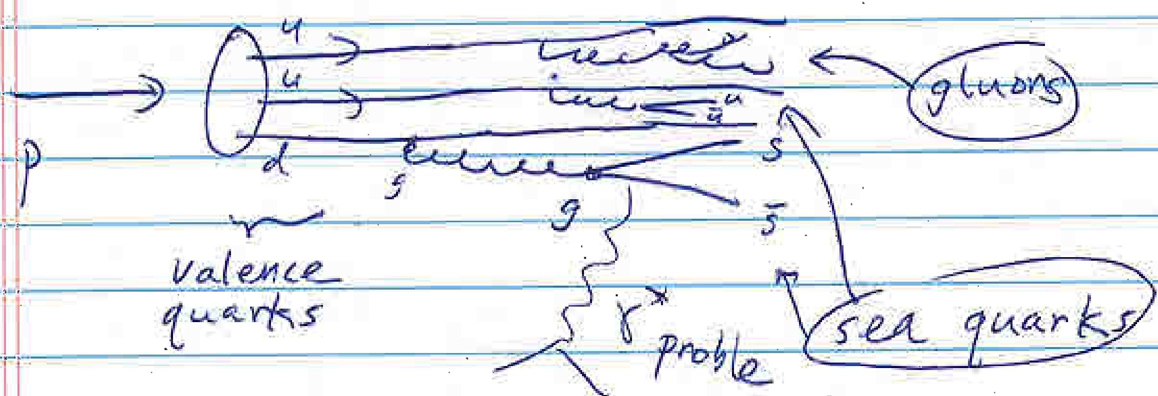
$f_{\overset{(-)}{q}}(x) = \overset{(-)}{q}(x) \Rightarrow$ probability ~~of~~ (anti)quark mom. fraction being between $(x, x+dx)$

$$\Rightarrow \boxed{\frac{d^2\sigma(ep \to eX)}{dx\, dQ^2} = \sum_{q=u,d,s} \left[f_q(x) + f_{\bar{q}}(x)\right] \times Q_q^2 \cdot \frac{2\pi\alpha^2}{Q^4}\left[1 + (1-y)^2\right]}$$

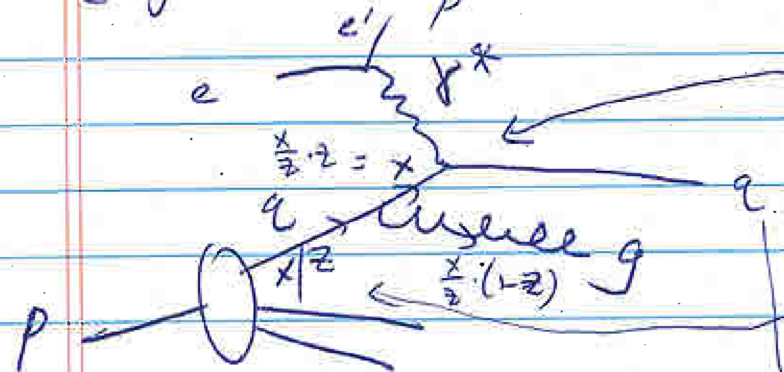
$\boxed{\text{Lowest order (in } \alpha_s \text{) parton model formula for DIS}}$

Why are there antiquarks in p?

· Collinear divergences ⇒ large probability ($f_g(x) \equiv g(x)$) of producing first gluons, then $q\bar{q}$ pairs, inside the high momentum proton:



· These collinear splittings also mean that all the parton distribution functions change ("evolve") slowly (logarithmically) with Q^2



x = momentum fraction of struck quark

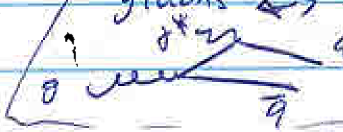
$\frac{x}{z}$ = larger quark momentum fraction before radiating gluon

$$\Rightarrow \boxed{\frac{d f_i(x,Q^2)}{d\ln Q^2} = \frac{\alpha_s(Q^2)}{\pi}\int_x^1 \frac{dz}{z}\sum_j P_{j\leftarrow i}(z)\, f_j\!\left(\frac{x}{z},Q^2\right)}$$

(all partons)

DGLAP eqn. Dokshitzer, Gribov, Lipatov, Altarelli, Parisi

mixes gluons ↔ quarks too



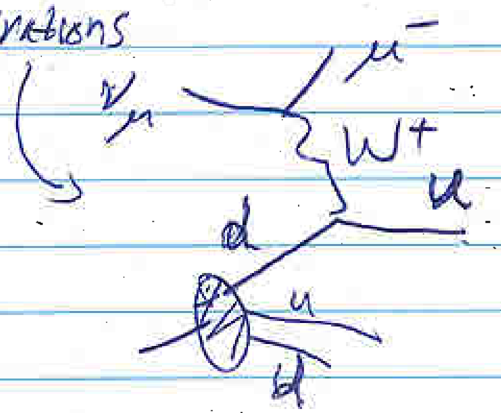
- We can't compute $q(x)$, $g(x)$ theoretically.
- Have to take from experiment.
- Mostly from electron DIS off p, d, heavier targets $ep \to eX$, $ed \to eX$

(Note that $u_n(x) = d_p(x)$ $d_n(x) = u_p(x)$ by isospin)

- But also from neutrino DIS $\nu_\mu p \to \mu X$

Gives access to other combinations

$$\frac{d^2\sigma(\nu_\mu p \to \mu^- X)}{dx\,dQ^2} \propto d(x) \text{ or } \bar{u}(x)$$



$$\frac{d^2\sigma(\bar{\nu}_\mu \to \mu^+ X)}{dx\,dQ^2} \propto u(x) \text{ or } \bar{d}(x)$$

- And some from hadron-hadron collisions