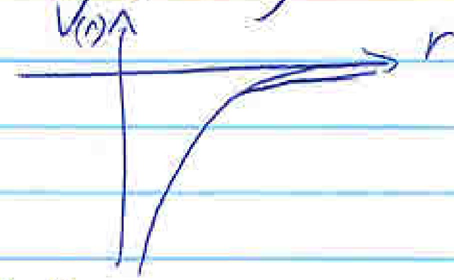
Lecture 13

Heavy Quarkonia ($c\bar{c}$, $b\bar{b}$) as "Hydrogen Atoms" of Strong Interactions

Recall for hydrogen, e^- is nonrel, electrostatic potential is

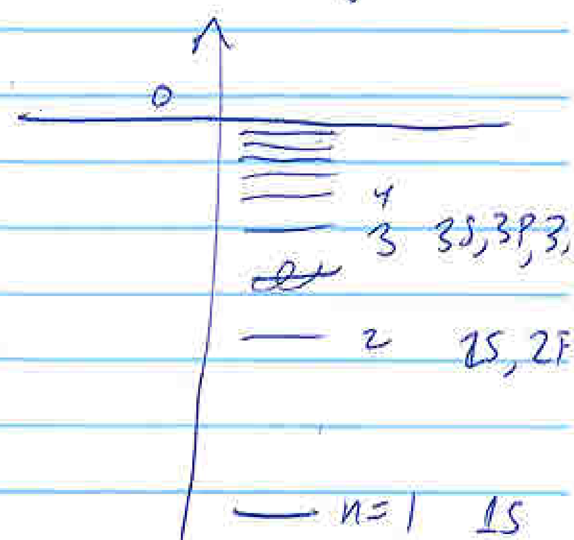


$$V(r) = -\frac{\alpha}{r}$$

$\Rightarrow$ spectrum $E_n = -\frac{1}{2}\frac{\alpha^2 m_r}{n^2} = -\frac{13.6\,eV}{n^2}$

Reduced mass

$$m_r = \frac{m_e m_p}{m_e + m_p} \approx m_e$$



Note
Virial theorem
$\Rightarrow \langle KE \rangle = \frac{1}{2} m_r \langle v^2 \rangle \approx \langle E \rangle = \frac{1}{2}\alpha^2 m_r$
$\Rightarrow v \simeq \alpha \simeq \frac{1}{137}$
slow, nonrel for hydrogen
$\alpha \langle \frac{1}{r} \rangle \approx \alpha^2 m$

2S, 2P (degen accidental) split by "fine structure"
– 2 distinct effects:

1) rel. corrections
$$H = \frac{p^2}{2m} + V - \frac{p^4}{8m^3}$$

2) spin-orbit $\vec{L}\cdot\vec{S}_e$ $\quad \vec{\mu}_e = g_e \frac{e}{m_e} \vec{S}_e$ $\leftarrow$ (electron spin indep)

Both $\propto \alpha^4 m\, f(n, j)$ $\quad j = l \pm \frac{1}{2}$ $\quad (\vec{J} = \vec{L} + \vec{S})$

◦ Leaves $2S_{1/2} = 2P_{1/2}$

split by Lamb shift – the loop diagram in QED

$\mathcal{O}(\alpha^5 m$

◦ Finally, hyperfine from $\vec{S}_p \cdot \vec{S}_e$ $\Rightarrow 1S_{1/2} \nearrow$ triplet, $\searrow$ singlet

$\vec{\mu}_p = g_p \frac{e}{m_p} \vec{S}_p$ $\Rightarrow$ hfi splitting $\sim \frac{m_e}{m_p}\alpha^4 m_e$

In QCD with heavy enough quarks, should get something similar.

~~ϕ~~ $m_t \simeq 170$ GeV $\xrightarrow{\text{top decays too fast}}$ $t \to W b$ ~~1 GeV~~

$t \to b + W$

$\Gamma_t \sim \alpha_w m_t \sim 1\text{-}2$ GeV

Compare with $(\text{time})^{-1}$ for one orbit:

$(2\pi r v)^{-1} \simeq \left(\frac{2\pi}{\alpha_s m_t}\alpha_s\right)^{-1}$

$= \frac{m_t}{2\pi} \simeq \frac{m_t}{(2\pi)} \simeq 30$ GeV

$\Rightarrow$ Not very different

$\Rightarrow$ top decays too fast to form bound state

(best available) $m_b = 5$ GeV

(a little "too light") $m_c = 1.3$ GeV

$m_r = \frac{m_q m_q}{m_q + m_q} = \frac{m_q}{2}$

But $\frac{\alpha}{r} \to \frac{\alpha_s(r)}{r}$

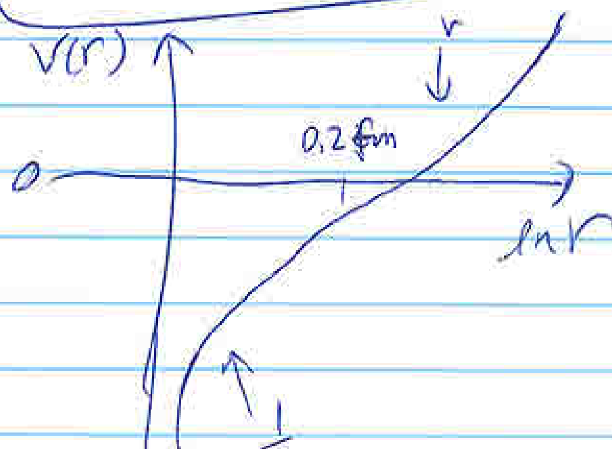
$\alpha_s(r)$ changes slowly with r for small r

$\frac{1}{r} \gg 1$ GeV

But at $\frac{1}{r} \simeq 1$ GeV

$r \simeq 0.2$ fm,

$V(r) = \frac{\alpha_s(r)}{r} \to V(r) \simeq r$



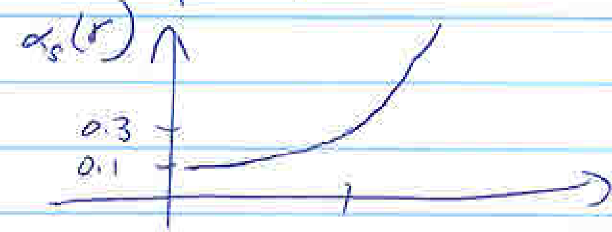
- Now $\alpha_s(r) \gg \alpha$

at $r \simeq \frac{1}{\alpha_s m_q}$

- So $c\bar{c}$, $b\bar{b}$ are not really in $\frac{1}{r}$ region.

$\therefore$ Don't expect $2S \simeq 2P$

Also ~~because~~ large spin-orbit splitting



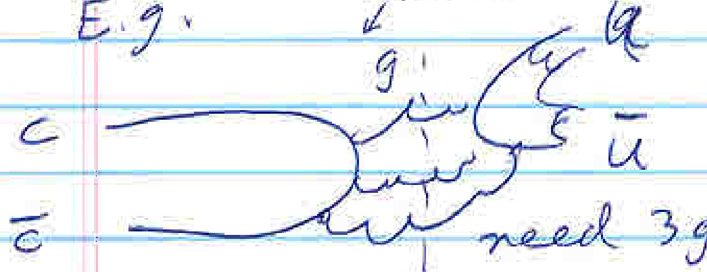
- Finally, hyperfine splitting not mass-suppressed: $\dfrac{\frac{m_q}{m_q}\,\alpha_s^4\, m_q}{\alpha_s^2 m_q} \simeq \alpha_s^2$

$\Rightarrow \simeq$ same size as fine structure.

- Why are "onia" long-lived at all?

E.g.

has color $\Rightarrow$ need at least 2 g's



?

need 3 g's

colorless

1^{--} cannot decay to 2 g's

sym. in color

"OZI rule" if you have to go through gluons, it's small.

- Reason: Momentum transfers q at these vertices are large

$\Rightarrow \alpha_s(q \simeq \frac{1}{r})$ relatively small

rate $\sim \alpha_s^6$

$\Gamma_{J/\psi} = .93$ keV very narrow.

$\Gamma_{\Upsilon(1S)} = 54$ keV 〃 〃

The excited states can decay to ground states (or other excited states) <u>hadronically</u>, but splittings are small, so not a lot of phase space.

$\Gamma(\chi_{c0}(1P)) = 10.0$ MeV

$\chi_{c1} \quad = 0.9$ MeV.

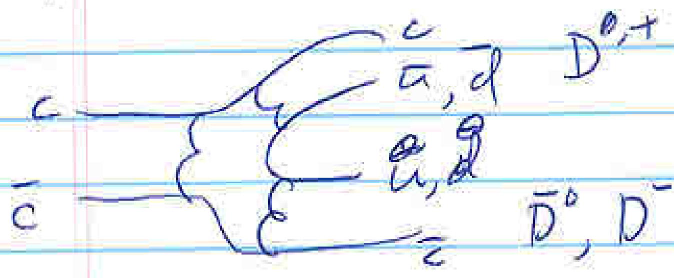
$\chi_{c2} \quad = 2$ MeV

$\Gamma(\eta_c) = 25$ MeV $\gg \Gamma_{J/\psi}$

because only 2 gluons required $\Rightarrow \ \sim \alpha_s^4$ not α_s^6

0^{-+}

* Above D-$\bar{D}$ or B-$\bar{B}$ thresholds, widths get larger



$\Gamma(\psi(3770)) = 23$ MeV

$\Gamma(\Upsilon(4S)) = 20$ MeV

$\hookrightarrow B\bar{B} \simeq$ all the time

K. Königsmann, Phys. Rep'ts 139 243 (1986)

(13.5)

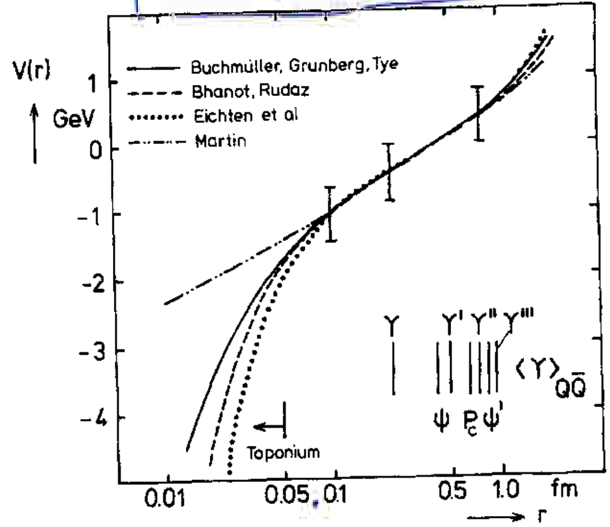


Fig. 4. The radial dependence of some typical quark–antiquark potentials for heavy quarkonia systems (from ref. [38]). The potentials have been shifted to agree at a radius of $r = 0.5$ fm. Average radii ($\sqrt{\langle r^2 \rangle}$) of the observed $c\bar{c}$ and $b\bar{b}$ states are indicated. The potential models used are by Bhanot and Rudaz (ref. [35]), Buchmüller, Grunberg and Tye (ref. [38]), Eichten et al. (ref. [33]) and Martin (ref. [32]).

3.1. *Theoretical introduction*

Radiative transitions offer the possibility to produce charmonium states with quantum numbers

h.f.

fine structure

hyperfine splitting

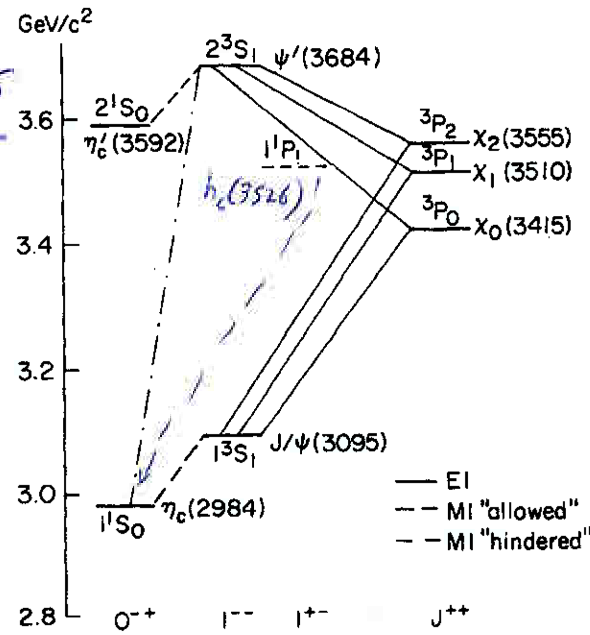


Fig. 3. The observed charmonium levels below charm threshold. The $^1P_1$ state has not yet been found. Measured radiative transitions are indicated as solid lines (electric dipole), dashed lines (magnetic dipole) and dashed-dotted lines (hindered magnetic dipole).

13.6

inclusive radiative branching ratios. In the fall of 1978, a dedicated neutral particle detector covering a large solid angle, the Crystal Ball, started operating. Its excellent photon measurement capabilities yielded precise branching ratios [55]. Figure 5 shows the inclusive photon spectrum obtained by the Crystal Ball experiment. Prominent peaks (labeled 2, 3 and 4) correspond to the transitions to the χ states. The inclusive radiative branching ratios obtained by all experiments are summarized in table 3.

The third method to determine the χ states consists of measuring the cascade process $\psi' \to \gamma\chi$, $\chi \to \gamma J/\psi$. Here the J/ψ is identified by its decay to $\mu^+\mu^-$ or e^+e^-. The magnetic detector experiments Mark I, Mark II, and DASP as well as the non-magnetic detectors DESY–Heidelberg and Crystal Ball identified this process. Results are included in table 3. Both detector types yield very similar values for the product branching ratios. The branching ratios $BR(\chi \to \gamma J/\psi)$ have been calculated from the average values of the inclusive and exclusive measurements, and the results are shown in the last row of table 3. In addition, the data from the Crystal Ball experiment yielded high confidence levels for the spins [58] of $\chi_{1,2}$ as preferred in the charmonium model, i.e., $J = 1$ and 2 for χ_1 and χ_2, respectively. Note that the spin

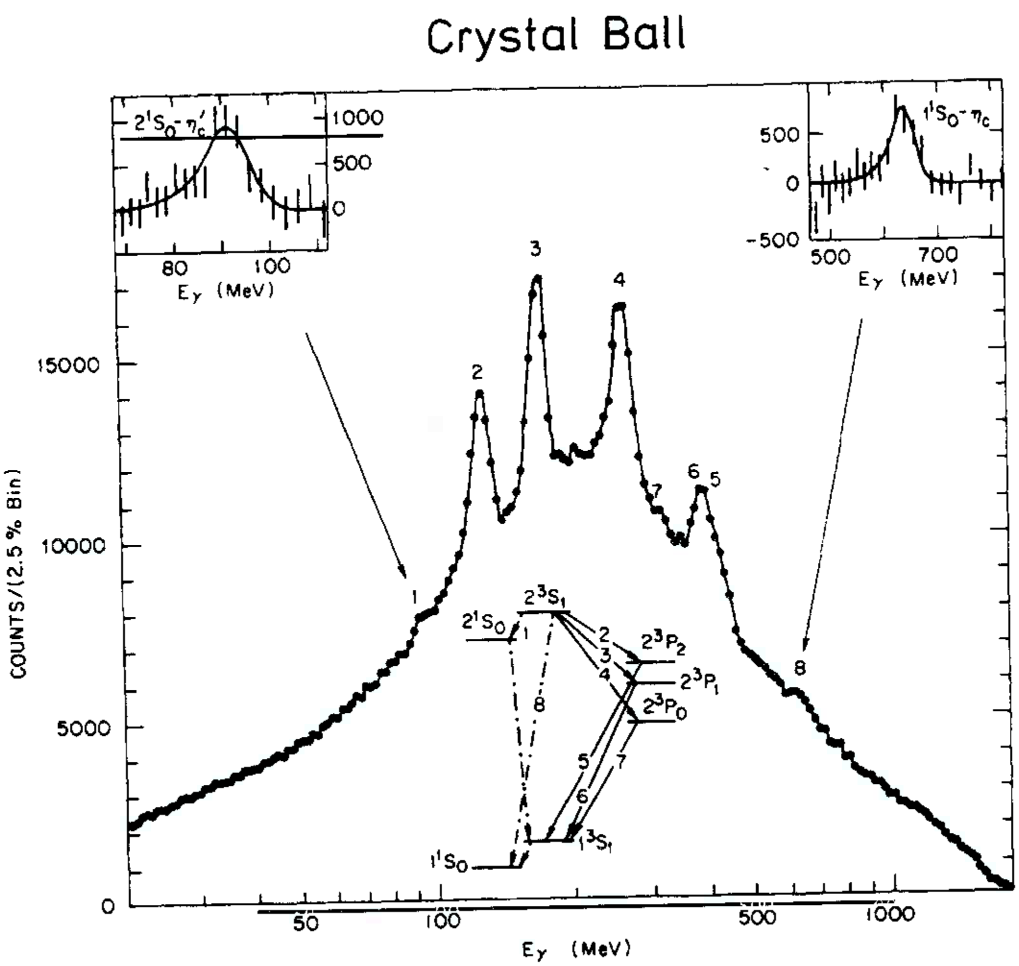


Fig. 5. Inclusive photon spectrum at the ψ' obtained by the Crystal Ball experiment. Note that the logarithmic energy scale yields bin sizes approximately proportional to photon energy resolution. The numbers over the spectrum correspond to the expected radiative transitions shown in the spectrum inset.

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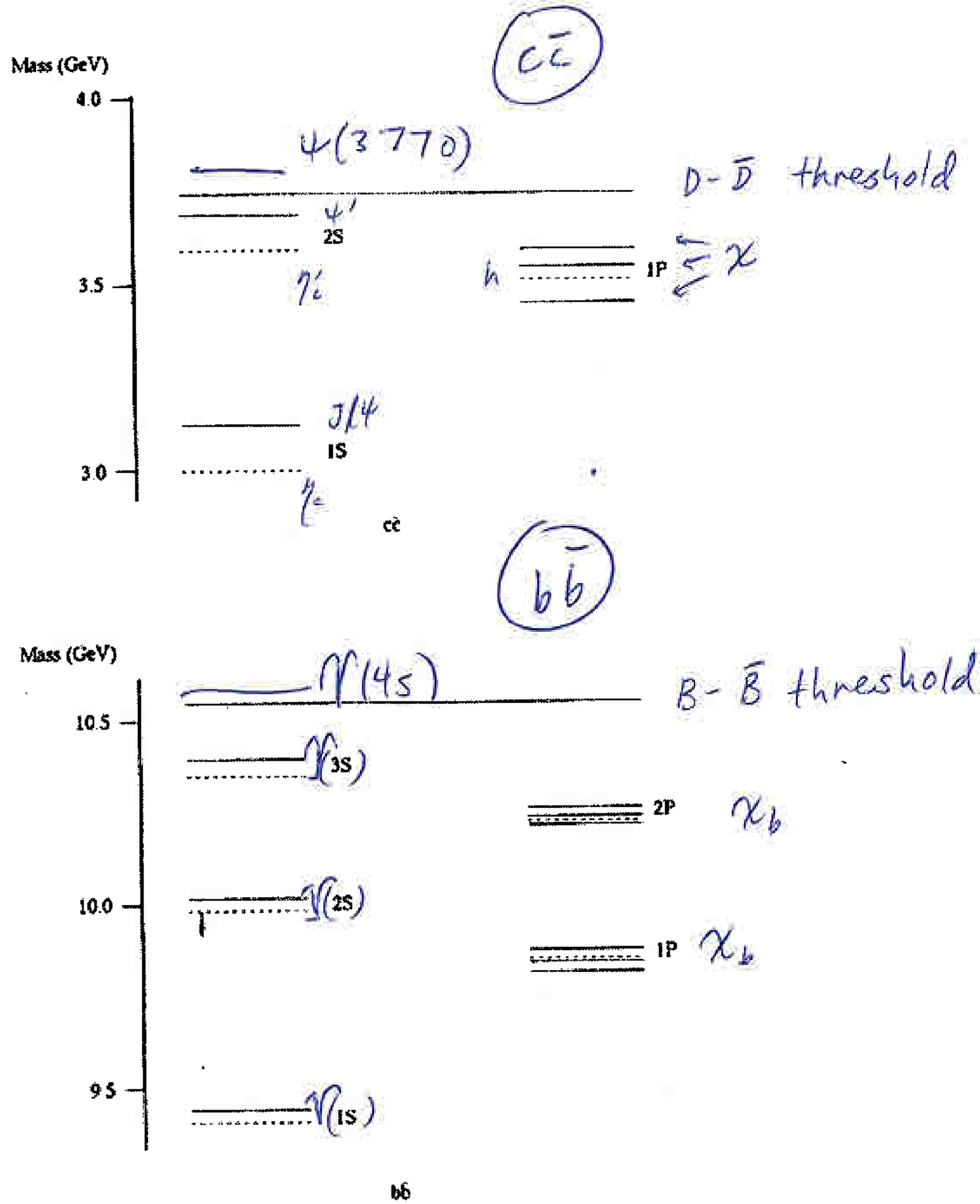
cc̄
Mass (GeV)
4.0
3.5
3.0
ψ(3770)
D - D̄ threshold
ψ'
2S
η_c
h
1P
χ
J/ψ
1S
η_c
cc̄
bb̄
Mass (GeV)
10.5
10.0
9.5
Υ(4S)
B - B̄ threshold
Υ(3S)
2P
χ_b
Υ(2S)
1P
χ_b
Υ(1S)
bb̄