1.) Griffiths 9.8

$\sqrt{2}(rb+br)$

$$f = \frac{1}{4}(c_3^\dagger \lambda^\alpha c_1)(c_4^\dagger \lambda^\alpha c_2)$$

$$= \frac{1}{4}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\Bigg(\left((100)\lambda^\alpha \begin{pmatrix}1\\0\\0\end{pmatrix}\right)\left((010)\lambda^\alpha\begin{pmatrix}0\\1\\6\end{pmatrix}\right)$$
$$+ \left((010)\lambda^\alpha\begin{pmatrix}1\\0\\0\end{pmatrix}\right)\left((100)\lambda^\alpha\begin{pmatrix}0\\1\\0\end{pmatrix}\right)$$
$$+ \left((100)\lambda^\alpha\begin{pmatrix}0\\1\\0\end{pmatrix}\right)\left((010)\lambda^\alpha\begin{pmatrix}1\\0\\0\end{pmatrix}\right)$$
$$+ \left((010)\lambda^\alpha\begin{pmatrix}0\\1\\0\end{pmatrix}\right)\left((100)\lambda^\alpha\begin{pmatrix}1\\0\\0\end{pmatrix}\right) \Bigg)$$

$$= \frac{1}{8}\left(\lambda_{11}^\alpha \lambda_{22}^\alpha + \lambda_{21}^\alpha \lambda_{12}^\alpha + \lambda_{12}^\alpha \lambda_{21}^\alpha + \lambda_{22}^\alpha \lambda_{11}^\alpha\right)$$

$$= \frac{1}{4}\left(\lambda_{11}^\alpha \lambda_{22}^\alpha + \lambda_{12}^\alpha \lambda_{21}^\alpha\right)$$

$$= \frac{1}{4}\left(\lambda_{11}^3 \lambda_{22}^3 + \lambda_{11}^8 \lambda_{22}^8 + \lambda_{12}^1 \lambda_{12}^1 + \lambda_{12}^2 \lambda_{21}^2\right)$$

$$= \frac{1}{4}\left(-1 + \frac{1}{3} + 1 + 1\right)$$

$$= \frac{1}{3}$$

2.) Griffiths 9.17

$$\alpha_s(q^2) = \frac{\alpha_s(\mu^2)}{1 + \frac{\alpha_s(\mu^2)}{12\pi}(11n - 2f)\log\frac{q^2}{\mu^2}}$$

$$\log\Lambda^2 = \log\mu^2 - \frac{12\pi}{(11n-2f)\alpha_s(\mu^2)}$$

$$\alpha_s(q^2) = \frac{12\pi}{\frac{12\pi}{\alpha_s(\mu^2)} - (11n-2f)\log\mu^2 + (11n-2f)\log q^2}$$

$$= \frac{12\pi}{-(11n-2f)\log\Lambda^2 + (11n-2f)\log q^2}$$

$$\alpha_s(q^2) = \frac{12\pi}{(11n-2f)\log\frac{q^2}{\Lambda^2}}$$

3.) Griffiths 10.20

a.) $Z^0 \to f + \bar{f}$

z-polarization

(see 10.93)

$$iM = \epsilon_\mu \, \bar{u}^{s_f}\left(\frac{p}{2}\right) \frac{-ig_z}{2} \gamma^\mu (c_V^f - c_A^f \gamma^5) v^{s_{\bar f}}\left(\frac{\tilde{p}}{2}\right)$$

average over incoming

(Note: $\frac{1}{3} \sum_{\text{spins}} \epsilon^\mu \epsilon^{\nu *} \to \frac{-g^{\mu\nu}}{3}$)

$p = (m_Z, 0, 0, m_Z)$, $\tilde{p} = (m_Z, 0, 0, -m_Z)$

$$\langle |M|^2 \rangle = \frac{-1}{3}\frac{g_z^2}{4} \, tr\left(\frac{\not p}{2} \gamma^\mu (c_V^f - c_A^f \gamma^5)\left(\frac{\tilde{\not p}}{2}\right) \gamma_0 (c_V^{f*} - c_A^{f*}\gamma^5) \gamma_\mu^\dagger \gamma_0\right)$$

$\{\gamma^5, \gamma^\nu\} = 0$

$$= \frac{-g_z^2}{48} tr\left(\not p \gamma^\mu (c_V^f - c_A^f \gamma^5) \tilde{\not p} \gamma_\mu (c_V^{f*} - c_A^{f*} \gamma^5)\right)$$

any term w/ exactly 1 γ^5 cancels b/c $tr(\gamma^5 \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma) \propto \epsilon^{\mu\nu\rho\sigma}$ but $p^\mu \tilde{p}^\rho$ is symmetric in $\mu\rho$.

$$= \frac{-g_z^2}{48} tr(\not p \gamma^\mu \tilde{\not p} \gamma_\mu)(|c_V^f|^2 + |c_A^f|^2)$$

$(\gamma^5\gamma^5 = 1)$

$$= + \frac{g_z^2}{6} p \cdot \tilde{p} \left(|c_V^f|^2 + |c_A^f|^2\right)$$

$$= + \frac{g_z^2}{3} m_Z^2 \left(|c_V^f|^2 + |c_A^f|^2\right)$$

$$\Gamma = + \frac{g_z^2 m_Z}{48\pi}\left(|c_V^f|^2 + |c_A^f|^2\right) \quad \text{(see 6.22)}$$

b.)

f	c_V	c_A	$\lvert c_V\rvert^2 + \lvert c_A\rvert^2$	$(\lvert c_A\rvert^2 + \lvert c_V\rvert^2) \times \{1$ for leptons, 3 for quarks$\}$ (color) $\times$ # particles	branching ratio (per particle)
ν_e, ν_μ, ν_τ	1/2	1/2	.5	1.5	6.8%
e^-, μ^-, τ^-	$(-\frac{1}{2} + 2\sin^2\theta)$	-1/2	.2514	.7542	3.4%
u, c	$(\frac{1}{2} - \frac{4}{3}\sin^2\theta)$	1/2	.2868	1.7208	11.8%
d, s, b	$(-\frac{1}{2} + \frac{2}{3}\sin^2\theta)$	-1/2	.3696	3.3264	15.2%
				7.3014	

$g_Z^2 = \frac{g_e^2}{\sin^2\theta \cos^2\theta} = .5160$

3.) c.) $\Gamma_{tot} = \frac{g_Z^2 M_Z}{48\pi} \sum_f (|c_V^f|^2 + |c_A^f|^2)$

$= (.3120\,GeV)(1.5 + .7542 + 1.7208 + 3.3264)$

$= 2.278\,GeV$

$\tau = 1/\Gamma = \frac{1}{2.278\,GeV} = .446\,GeV^{-1} = 3 \cdot 10^{-25}\,s$

Actually, b/c Z is so short lived, instead of talking about its lifetime, one talks about its width, or uncertainty in its mass. (rough reason: $\Delta E \Delta t \gtrsim \hbar \Rightarrow$ (Δt small $\Rightarrow \Delta E$ large) & ΔE is like an uncertainty in mass.)

A fourth light neutrino would increase Γ by

$\Gamma_{\nu_4} = \frac{g_Z^2 M_Z}{48\pi} (|c_V|^2 + |c_A|^2) = (.3120\,GeV)(.5) = .156\,GeV$

For this to be off by 3σ, we need $\sigma = 52\,MeV$, or $\sigma = (2\%)\Gamma$

$\Rightarrow$ measure Γ to within 2% at 1σ.

4.) Griffiths 11.21

$$\mathcal{L} = \bar{\psi}(i\not{\partial} - m_1)\psi + \frac{1}{2}(\partial_\mu \varphi)^2 - \frac{1}{2} m_2^2 \varphi^2 + \alpha_Y \bar{\psi}\psi\varphi$$

ψ = spin-½ Dirac fermion w/ mass m_1

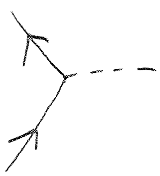
φ = spin-1 scalar boson w/ mass m_2

propagators

ψ: $\xrightarrow{p \to}$ $= \dfrac{i}{\not{p} - m_1} = \dfrac{i(\not{p} + m_1)}{p^2 - m^2}$

φ: $\overset{p \to}{- - - -}$ $= \dfrac{i}{p^2 - m^2}$

Vertex

 $= i\alpha_Y$

physical interpretation: electron+positron interacting w/ a scalar.