1.)

counting error $= \sqrt{N}$

fractional error $= \frac{1}{\sqrt{N}}$

$60 e^-$'s per MeV $\Rightarrow$ $N = E \cdot (60/\mathrm{MeV})$

$$\frac{\Delta E}{E} = \frac{1}{\sqrt{N}} = \frac{1}{\sqrt{60\ E/\mathrm{MeV}}}$$

if E is measured in GeV

$$\frac{\Delta E}{E} = (60{,}000\ E)^{-1/2} \simeq .004\ E^{-1/2}$$

if only 1% of E is deposited,

$$N \approx .6\ E/\mathrm{MeV} \approx 600\ E$$

$$\frac{\Delta E}{E} \simeq .04\ E^{-1/2}$$

2.) Griffiths 6.9

$1+2 \to 3+4$ w/ $|M|^2$

lab frame, $\vec{p}_2 = 0$

$m_3 = m_4 = 0$

$$d\sigma = |M|^2 \frac{S}{4}\left((p_1 \cdot p_2)^2 - (m_1 m_2)^2\right)^{-1/2} \frac{d^3p_3}{(2\pi)^3}\frac{1}{2E_3}\frac{d^3p_4}{(2\pi)^3}\frac{1}{2E_4}(2\pi)^4 \delta^{(4)}(p_1+p_2-p_3-p_4)$$

$$= |M|^2 \frac{S}{4}\frac{1}{m_2}\frac{1}{\sqrt{E_1 - m_1}}\frac{1}{(2\pi)^2}\frac{1}{4}\frac{d^3p_3\, d^3p_4}{E_3E_4}\delta^{(4)}(p_1+p_2-p_3-p_4)$$

$$= \frac{1}{(8\pi)^2}\frac{S|M|^2}{m_2|\vec{p}_1|}\frac{d^3p_3\, d^3p_4}{E_3E_4}\delta^{(3)}(\vec{p}_1 - \vec{p}_3 - \vec{p}_4)\delta(E_1+m_2-E_3-E_4)$$

$$= \frac{1}{(8\pi)^2}\frac{S|M|^2}{m_2|\vec{p}_1|} d^3p_3 \frac{\delta(E_1+m_2-E_3-E_4)}{E_3E_4}$$

w/ $\vec{p}_4 = \vec{p}_1 - \vec{p}_3$, $|\vec{p}_4|^2 = |\vec{p}_1|^2 + |\vec{p}_3|^2 - 2|\vec{p}_1||\vec{p}_3|\cos\theta$

$E_3 = \sqrt{\vec{p}_3^{\,2} - m_3^2} = |\vec{p}_3|$, $E_4 = |\vec{p}_4|$

b/c divided by $d\Omega$: $d^3p_3 \Rightarrow |p_3|^2 d|p_3|$

$$\frac{d\sigma}{d\Omega} = \frac{1}{(8\pi)^2}\frac{S|M|^2}{m_2|\vec{p}_1|}\int |p_3|^2 d|p_3| \frac{\delta(E_1+m_2-E_3-E_4)}{E_3E_4}$$

let $\rho = |p_3|^2$

$$\frac{d\sigma}{d\Omega} = \frac{1}{(8\pi)^2}\frac{S|M|^2}{m_2|p_1|}\int \rho^2 d\rho \frac{\delta(E_1+m_2-\rho-\sqrt{p_1^2+\rho^2+2|p|\rho\cos\theta})}{\rho(p_1^2+\rho^2-2|p_1|\rho\cos\theta)^{1/2}}$$

let $E = \rho + \sqrt{p_1^2+\rho^2-2\rho|\vec{p}_1|\cos\theta}$

$$dE = d\rho\left(1+\frac{1}{2}\frac{2\rho-2|\vec{p}_1|\cos\theta}{(p_1^2+\rho^2-2\rho|p_1|\cos\theta)^{1/2}}\right) = d\rho\left(\frac{E-|\vec{p}_1|\cos\theta}{(p_1^2+\rho-2|\vec{p}_1|\cos\theta)^{1/2}}\right)$$

$\rho = |\vec{p}_3|$

$$\frac{d\sigma}{d\Omega} = \frac{1}{(8\pi)^2}\frac{S|M|^2}{m_2|\vec{p}_1|}\int \frac{\rho\,\delta(E_1+m_2-E)}{E-|\vec{p}_1|\cos\theta}dE$$

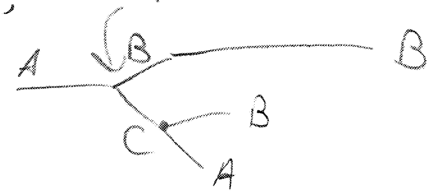
$$\frac{d\sigma}{d\Omega} = \frac{1}{(8\pi)^2}\frac{S|M|^2}{m_2|\vec{p}_1|}\frac{|\vec{p}_3|}{E_1+m_2-|\vec{p}_1|\cos\theta}$$

3.) Griffiths 6.11

a.) $A \to B + B$ is not allowed (at least perturbatively :))

try it:

only allowed vertex



^ but now what do we do w/ him?

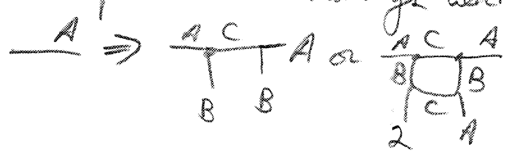
& spinning off a B into another vertex

(i.e. 

) gets us back to where we were before

b.) (n_A, n_B, n_C)

2 basic diagrams  $= (1,1,1)$ & — $= (1,0,0)$ or $(0,1,0)$ or $(0,0,1)$

starting w/ a triplet we can always add 2 more of any particle : 

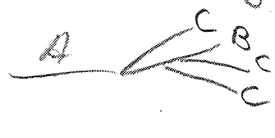
Notice, that we start out (even, even, even) or (odd, odd, odd) & that by adding $(2,0,0)$ or $(0,2,0)$ or $(0,0,2)$ we can get anything of this form.

Finally, notice we cannot get anything else, b/c we can always subtract $(2,0,0)$ or perms (assuming $n \geq 2$ to start w/) by joining two ext. lines into a loop, & $(1,1,0)$ & $(1,0,0)$ are both impossible.

c.) other possible decays

$A \to 3B + C$

$A \to B + 3C$



4.) Griffiths 7.1

$\frac{\partial \varphi}{\partial x^\mu}$ boost in 1-direction: $x^{0\prime} = \gamma(x^0 - \beta x^1)$ $\quad x_0' = \gamma(x_0 + \beta x_1)$

$x^{1\prime} = \gamma(x^1 - \beta x^0)$ $\quad x_1' = \gamma(x_1 + \beta x_0)$

or $x^0 = \gamma(x^{0\prime} + \beta x^{1\prime})$, $x_0 = \gamma(x_0' - \beta x_1')$

$x^1 = \gamma(x^{1\prime} + \beta x^{0\prime})$, $x_1 = \gamma(x_1' - \beta x_0')$

$$\frac{\partial \varphi}{\partial x^{0\prime}} = \frac{\partial \varphi}{\partial x^\mu}\frac{\partial x^\mu}{\partial x^{0\prime}} = \gamma \frac{\partial \varphi}{\partial x^0} + \beta\gamma \frac{\partial \varphi}{\partial x^1}$$

$$\frac{\partial \varphi}{\partial x^{1\prime}} = \frac{\partial \varphi}{\partial x^\mu}\frac{\partial x^\mu}{\partial x^{1\prime}} = \beta\gamma \frac{\partial \varphi}{\partial x^0} + \gamma \frac{\partial \varphi}{\partial x^1}$$

$$\Rightarrow \frac{\partial \varphi}{\partial x^\mu} = v_\mu$$

real, lower index

5.) Griffiths 7.7

$$(\not{p}-m)u=0$$

$$(\hat{p}\cdot\vec{\Sigma})u^{\pm}=\pm u^{\pm}$$

$$u^{\dagger}u=2|E|$$

let $u=\begin{pmatrix}u_A\\u_B\end{pmatrix}$ w/ u_A, u_B 2-component spinors

$$(\not{p}-m)u=\begin{pmatrix}E-m & -\vec{p}\cdot\vec{\sigma}\\ \vec{p}\cdot\vec{\sigma} & -E-m\end{pmatrix}\begin{pmatrix}u_A\\u_B\end{pmatrix}=0$$

$$\Rightarrow u_A=\frac{\vec{p}\cdot\vec{\sigma}}{E-m}u_B,\quad u_B=\frac{\vec{p}\cdot\vec{\sigma}}{E+m}u_A,\ (\vec{p}\cdot\vec{\sigma})^2=|\vec{p}|^2\Rightarrow E^2-\vec{p}^2=m^2$$

$$(\hat{p}\cdot\vec{\Sigma})\begin{pmatrix}u_A\\u_B\end{pmatrix}=\pm\begin{pmatrix}u_A\\u_B\end{pmatrix}$$

$$\begin{pmatrix}\hat{p}\cdot\vec{\sigma}\mp1 & \\ & \hat{p}\cdot\vec{\sigma}\mp1\end{pmatrix}\begin{pmatrix}u_A\\u_B\end{pmatrix}=0 \text{ multiplying by } |\vec{p}|: \begin{pmatrix}\vec{p}\cdot\vec{\sigma}\mp p & \\ & \vec{p}\cdot\vec{\sigma}\mp p\end{pmatrix}\begin{pmatrix}u_A\\u_B\end{pmatrix}=0$$

$$(\vec{p}\cdot\vec{\sigma}\mp p)u_A=\begin{pmatrix}p_z\mp p & p_x-ip_y\\ p_x+ip_y & -p_z\mp p\end{pmatrix}\begin{pmatrix}u_{A_1}\\u_{A_2}\end{pmatrix}=0$$

$$u_A=\begin{pmatrix}p_z\pm p\\ p_x+ip_y\end{pmatrix}\qquad u=N\begin{pmatrix}u_A\\u_B\end{pmatrix}$$

$$u_B=\frac{\pm p}{E+m}u_A$$

& finally we need $u^{\dagger}u=2E$

$$u^{\dagger}u=N^2(|u_{A_1}|^2+|u_{A_2}|^2+|u_{B_1}|^2+|u_{B_2}|^2)$$

5) $$u^\dagger u = N^2 |u_{A_1}|^2 \left(\left(1 + \left|\frac{-p_z \pm p}{p_x - i p_y}\right|^2 + \frac{p^2}{(E+m)^2}\left(1 + \left|\frac{-p_z \pm p}{p_x - i p_y}\right|^2\right)\right)\right)$$

$$= N^2 |u_{A_1}|^2 \left(\frac{(E+m)^2 + p^2}{(E+m)^2}\right)\left(\frac{p_x^2 + p_y^2 + p_z^2 + p^2 \mp 2pp_z}{p_x^2 + p_y^2}\right)$$

$$= N^2 |u_{A_1}|^2 \frac{2E^2 + 2Em}{(E+m)^2} \frac{2p^2 \mp 2pp_z}{p^2 - p_z^2}$$

$$= N^2 |u_{A_1}|^2 \left(\frac{2E}{E+m}\right)\left(\frac{2p(p \mp p_z)}{(p+p_z)(p-p_z)}\right)$$

$$= N^2 |u_{A_1}|^2 \frac{4Ep}{(E+m)(p \pm p_z)}$$

$$= N^2 (p \pm p_z)^2 \frac{4Ep}{(E+m)(p \pm p_z)}$$

$$= N^2 \frac{4Ep(p \pm p_z)}{E+m} = 2E$$

$$\Rightarrow N^2 = \frac{E+m}{2p(p \pm p_z)}$$

6.) Griffiths 7.34

$$\mathcal{M} = -\frac{g^2}{(p_1-p_3)^2}(\bar{u}(3)\gamma^\mu u(1))(\bar{u}(4)\gamma_\mu u(2)) + \frac{g^2}{(p_1-p_4)^2}(\bar{u}(4)\gamma^\mu u(1)) \times (\bar{u}(3)\gamma_\mu u(2))$$

$$\langle|\mathcal{M}|^2\rangle = \underbrace{\frac{1}{4}\sum_{s_1 s_2}}_{\substack{\text{average}\\ \text{over}\\ \text{incoming}}} \underbrace{\sum_{s_3 s_4}}_{\substack{\text{sum}\\ \text{over}\\ \text{outgoing}}} \mathcal{M}\mathcal{M}^*$$

$$= \frac{1}{4}\frac{g^4}{t^2}\sum_{s_i}(\bar{u}_3\gamma^\mu u_1)(\bar{u}_4\gamma_\mu u_2)(\bar{u}_3\gamma^\nu u_1)^*(\bar{u}_4\gamma_\nu u_2)^*$$

$$+ \frac{1}{4}\frac{g^4}{u^2}\sum_{s_i}(\bar{u}_4\gamma^\mu u_1)(\bar{u}_3\gamma_\mu u_2)(\bar{u}_4\gamma^\nu u_1)^*(\bar{u}_3\gamma_\nu u_2)^*$$

$$- \frac{2}{4}\frac{g^4}{tu}\sum_{s_i}(\bar{u}_3\gamma^\mu u_1)(\bar{u}_4\gamma_\mu u_2)(\bar{u}_4\gamma^\nu u_1)^*(\bar{u}_3\gamma_\nu u_2)^*$$

$$= \frac{g^4}{4t^2} tr(\not{p}_3\gamma^\mu \not{p}_1\gamma^\nu) tr(\not{p}_4\gamma_\mu \not{p}_2\gamma_\nu)$$

$$+ \frac{g^4}{4u^2} tr(\not{p}_4\gamma^\mu \not{p}_1\gamma^\nu) tr(\not{p}_3\gamma_\mu \not{p}_2\gamma_\nu)$$

$$- \frac{g^4}{2tu} tr(\not{p}_3\gamma^\mu \not{p}_1\gamma^\nu \not{p}_4\gamma_\mu \not{p}_2\gamma_\nu)$$

$$= \frac{g^4}{4t^2} 32((p_1\cdot p_2)(p_3\cdot p_4) + (p_1\cdot p_4)(p_2\cdot p_3))$$

$$+ \frac{g^4}{4u^2} 32((p_1\cdot p_2)(p_3\cdot p_4) + (p_1\cdot p_3)(p_2\cdot p_4))$$

$$- \frac{g^4}{2tu}(-2) tr(\not{p}_3 \not{p}_4\gamma^\nu \not{p}_1 \not{p}_2\gamma_\nu)$$

6.) $\langle |M|^2 \rangle = \frac{8g^4}{t^2}\left(\left(\frac{s}{2}\right)\left(\frac{s}{2}\right)+\left(-\frac{u}{2}\right)\left(-\frac{u}{2}\right)\right)+\frac{8g^4}{u^2}\left(\left(\frac{s}{2}\right)\left(\frac{s}{2}\right)+\left(-\frac{t}{2}\right)\left(-\frac{t}{2}\right)\right)$

$+\frac{g^4}{tu}\,4\,p_1\cdot p_2 \cancel{\mathrm{tr}}(\cancel{p}_3\cancel{p}_4)$

$= 2g^4\left(\frac{s^2+u^2}{t^2}+\frac{s^2+t^2}{u^2}\right)+16\,\frac{g^4}{tu}\,\frac{s}{2}\,\frac{s}{2}$

$= g^4\left(\frac{2s^2+u^2}{t^2}+\frac{2s^2+2t^2}{u^2}+\frac{4s^2}{tu}\right)$

$=\frac{g^4}{t^2u^2}\left(2s^2u^2+2u^4+2s^2t^2+2t^4 \;\; +4s^2tu\right)$

$=\frac{2g^4}{t^2u^2}\left(s^2(t+u)^2+u^4+t^4\right)$

$s+t+u=\sum_i m_i^2=0$ for $m_{e^-}=0$

$=\frac{2g^4}{t^2u^2}\left(s^4+u^4+t^4\right)$

$=\frac{2g^4}{(p_1\cdot p_3)^2(p_1\cdot p_4)^2}\left((p_1\cdot p_2)^4+(p_1\cdot p_3)^4+(p_1\cdot p_4)^4\right)$