1.) i) $\nabla \cdot E = 0$

ii) $\nabla \cdot B = 0$

iii) $\nabla \times E = -\frac{\partial B}{\partial t}$

iv) $\nabla \times B = \frac{\partial E}{\partial t}$

$\frac{\partial}{\partial t}$ of iv. $\Rightarrow \frac{\partial^2 E}{\partial t^2} = \nabla \times \frac{\partial B}{\partial t}$

use iii. $\Rightarrow \frac{\partial^2 E}{\partial t^2} = -\nabla \times (\nabla \times E)$

$$\frac{\partial^2 E}{\partial t^2} = -\nabla \times \nabla \times E = -(\nabla(\nabla \cdot E) - \nabla^2 E)$$

use i $\Rightarrow$ $\nabla^2 E = \frac{\partial^2 E}{\partial t^2}$

$$\nabla^2 E = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \vec{E}}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \vec{E}}{\partial \varphi^2} + \frac{\partial^2 \vec{E}}{\partial z^2} = \frac{\partial^2 \vec{E}}{\partial t^2}$$

take z-component & use ansatz $E_z = E_0 f(r) e^{i(\omega t - kz)}$

$$\frac{1}{r}\frac{\partial}{\partial r} r f' - k^2 f = -\omega^2 f$$

$$f'' + \frac{1}{r} f' + (\omega^2 - k^2) f = 0$$

This ~~satisfies~~ Bessel's Eqn, for J_0 $(n=0)$ if $\omega^2 - k^2 = 1$

We ~~thus~~ get $f(r) = J_0(r)$ w/ our dispersion relation

$$\omega = \pm\sqrt{1+k^2},$$

But we need $E_z = 0$ at the boundary so we need to scale our soln st we find $f(R) = 0$. We will use the fact that $J_0(2.405\ldots) = 0$

Let $u = \alpha r$, Let $g(u) = g(\alpha r) = f(r)$

$$g' = \frac{dg}{du} = \frac{dg(u)}{du} = \frac{d\,g(\alpha r)}{du} = \frac{dg(\alpha r)}{dr}\frac{du}{dr} = \frac{df}{dr}\frac{du}{dr} = \alpha f'$$

Similarly, $g'' = \alpha^2 f''$

$$\alpha^2 g'' + \frac{\alpha^2}{u} g' + (\omega^2 - k^2) g = 0$$

$$g'' + \frac{1}{u} g' + \left(\frac{\omega^2 - k^2}{\alpha^2}\right) g = 0$$

$\Rightarrow g(u) = J_0(u)$, $f(r) = J_0(\alpha r)$, $f(R) = 0 \Rightarrow \alpha R \approx 2.405\ldots \Rightarrow \alpha \approx \frac{2.405\ldots}{R}$

$f(r) = J_0(2.405 r/R)$ & $\frac{\omega^2 - k^2}{\alpha^2} = 1 \Rightarrow \omega = \pm\sqrt{\left(\frac{2.405}{R}\right)^2 + k^2}$

$$f(r) = J_0\left(\frac{u_{01} r}{R}\right)$$

$$\omega = \pm\sqrt{\left(\frac{u_{01}}{R}\right)^2 + k^2}$$

$$u_{01} \approx 2.405\ldots, \quad J_0(u_{01}) = 0$$

$$\omega^2 - k^2 = \frac{u_{01}^2}{R^2}, \quad \frac{\omega^2}{k^2} - 1 = \frac{u_{01}^2}{k^2 R^2} = \frac{u_{01}^2}{R^2} \frac{1}{\omega^2 - \frac{u_{01}^2}{R^2}} = \frac{u_{01}^2}{R^2\omega^2 - u_{01}^2}$$

$$\frac{\omega^2}{k^2} = 1 + \frac{u_{01}^2}{R^2\omega^2 - u_{01}^2} = \frac{R^2\omega^2}{R^2\omega^2 - u_{01}^2}$$

$$v_p(\omega) = \frac{\omega}{k} = \left(\frac{R^2\omega^2}{R^2\omega^2 - u_{01}^2}\right)^{1/2}$$

$$R^2\omega^2 > R^2\omega^2 - u_{01}^2 \Rightarrow v_p(\omega) > 1 \quad \forall \omega$$ (provided v_p is real, or $R^2\omega^2 - u_{01}^2 > 0$) $\left(\omega > \frac{u_{01}}{R}\right)$

minimum ω_c is given as $k \to 0$ (or as $v_p \to$ imaginary)

$$\omega_c = \frac{u_{01}}{R} \approx \frac{2.405}{R}$$

if $R = 5\text{cm}$, $\nu_c = \frac{\omega_c}{2\pi} = \frac{u_{01}}{2\pi R} = \frac{2.405}{2\pi \cdot 5\,\text{cm}} \approx .077\ \text{cm}^{-1}$

$$\approx 2.30\ \text{GHz}$$

2.)

$p_1 = (E_1, \vec{p}_1) = E_1(1,1)$

$p_2 = E_2(1,-1)$

$p_{TOT} = (E_1+E_2, E_1-E_2)$

$p_{TOT}^2 = (E_1+E_2)^2 - (E_1-E_2)^2 = 4E_1E_2 = E_{cm}^2$

$E_1 = 9\,GeV$

$E_{cm} = 10.56\,GeV$

$E_2 = \frac{E_{cm}^2}{4E_1} = 3.098\,GeV$

$\text{current} = \frac{\#e^-}{\text{dist}} \cdot \text{speed} \cdot \frac{C}{e^-}$

speed = c for both beams (b/c 9, 3 GeV ≫ .5 meV)

HER: $I = \frac{1658 \cdot 2.7 \cdot 10^{10} e^-}{2200\,m} \, 2.998 \cdot 10^8 \frac{m}{s} \, \frac{1.602 \cdot 10^{-19} C}{e^-} \approx .9773\,C/s \approx .98\,amps$

LER: $I = \frac{1658 \cdot 5.9 \cdot 10^{10} e^-}{2200\,m} \, 2.998 \cdot 10^8 \frac{m}{s} \, \frac{1.6 \cdot 10^{-19} C}{e^-} \approx 2.1\,amps$

$\Delta E = \frac{4\pi\alpha}{3} \frac{E^4}{m^4 r}$

HER: $\Delta E = \frac{4\pi}{3} \frac{1}{137} \frac{(9\,GeV)^4}{(5.11 \cdot 10^{-4}\,GeV)^4} \frac{1}{\left(\frac{2200\,m}{2\pi}\right)} \approx 8.4 \cdot 10^{2}\,m^{-1} \cdot (197\,MeV \cdot fm)\left(\frac{m}{10^{15}\,fm}\right) \approx \boxed{1.7\,MeV}$

LER: $\Delta E \approx .023\,MeV$

$P = \frac{\Delta E}{\Delta t} = (\Delta E)(\#e^-/s) \; \frac{2.998 \cdot 10^8\,m/s}{2200\,m}$

HER: $P = (1.7\,MeV)(1658 \cdot 2.7 \cdot 10^{10})\left(\frac{2.998 \cdot 10^8\,m/s}{2200\,m}\right)\left(\frac{1.6 \cdot 10^{-13} J}{MeV}\right) \approx 1.7 \cdot 10^6\,W$

LER: $P \approx 4.9 \cdot 10^4\,W$ * see note on next page about bending

Tevatron

p^+ ring: $P \approx \frac{4\pi}{3} \frac{1}{137} \left(\frac{960\,GeV}{.938\,GeV}\right)^4 \left(\frac{1}{\frac{6300\,m}{2\pi}}\right)\left(\frac{197}{10^{15}}\,MeV\,m\right)\left(36 \cdot 2.7 \cdot 10^{11}\right)\left(\frac{3 \cdot 10^8\,m/s}{6300\,m}\right)\left(\frac{1.6 \cdot 10^{-13}}{MeV}\right)$

$\approx .49\,W$

if e^-: $P \approx 5.5 \cdot 10^{12}\,W$

*

Why would we want to lump the bending into short segments to increase the radiation?

Well, the LER is actually "too big" for its energy. Some synchrotron radiation is good in that it acts a little like focusing — it reduces the energy of the higher energy particles more than the lower energy.

Plus we can then also control where the radiation strikes the walls of the vacuum chamber. We can then make a cheaper ring b/c we need less of the special surface designed to absorb the radiation.

3.) $$M_x = \begin{pmatrix} \cos kl & k^{-1}\sin kl \\ -k\sin kl & \cos kl \end{pmatrix}$$

$$\approx \begin{pmatrix} 1-\frac{(kl)^2}{2} & l \\ -k^2 l & 1-\frac{(kl)^2}{2} \end{pmatrix} \approx \begin{pmatrix} 1 & l \\ -k^2 l & 1 \end{pmatrix}$$

$$M_c = \begin{pmatrix} 1 & \\ -1/f & 1 \end{pmatrix} . \quad M_z = \begin{pmatrix} 1 & z \\ & 1 \end{pmatrix}$$

$$M_{z_1} M_c M_{z_2} = \begin{pmatrix} 1 & z_1 \\ & 1 \end{pmatrix}\begin{pmatrix} 1 & \\ -1/f & 1 \end{pmatrix}\begin{pmatrix} 1 & z_2 \\ & 1 \end{pmatrix} \approx \begin{pmatrix} 1-z_1/f & z_1+z_2 \\ -1/f & 1-z_2/f \end{pmatrix}$$

$$f \approx \frac{1}{k^2 l}, \quad z_1 \approx z_2 \approx \frac{l}{2}$$

$$M_y = \begin{pmatrix} \cosh kl & k^{-1}\sinh kl \\ k\sinh kl & \cosh kl \end{pmatrix} \approx \begin{pmatrix} 1+\frac{(kl)^2}{2} & l \\ k^2 l & 1+\frac{(kl)^2}{2} \end{pmatrix}$$

$$M_d = \begin{pmatrix} 1 & \\ 1/f & 1 \end{pmatrix}, \quad M_{z_3} M_d M_{z_4} \approx \begin{pmatrix} 1+z_3/f & z_3+z_4 \\ 1/f & 1+z_4/f \end{pmatrix}$$

$$f \approx \frac{1}{k^2 l}, \quad z_3 \approx z_4 \approx \frac{l}{2}$$

$$M_d M_z M_c = \begin{pmatrix} 1 & \\ 1/f & 1 \end{pmatrix}\begin{pmatrix} 1 & z \\ & 1 \end{pmatrix}\begin{pmatrix} 1 & \\ -1/f & 1 \end{pmatrix} \approx \begin{pmatrix} 1 & z \\ -z/f^2 & 1 \end{pmatrix}$$

$\Rightarrow$ focusing w/ $-\frac{1}{f'} = -\frac{z}{f^2} \Rightarrow f' \approx f^2/z$

$M_c M_z M_d \approx \begin{pmatrix} 1 & z \\ -z/f^2 & 1 \end{pmatrix} \Rightarrow$ again focusing w/ $\frac{1}{f'} \approx \frac{z}{f^2}$

$$f' \approx f^2/z$$

4.) Griffiths 4.11

$\Delta^{++} \to p \pi^+$

init ang mom. = $3/2$ = spin of Δ^{++} = J

spin $p = 1/2$

spin $\pi^+ = 0$

$\Rightarrow$ total final spin ang. mom = $1/2 = S$

$L = J - S = 3/2 - 1/2 = 1$ or 2

5.) Griffiths 4.27

$$I = I_1 + I_2$$

$$I^2 = I_1^2 + I_2^2 + 2 I_1 \cdot I_2$$

$$I_1 \cdot I_2 = \frac{1}{2}(I^2 - I_1^2 - I_2^2)$$

$$I_j = i_j(i_j + 1)$$

$$I_1^2 = \frac{1}{2}\left(\frac{1}{2}+1\right) = \frac{3}{4} = I_2^2$$

in triplet $I^2 = 1(1+1) = 2$

$$I_1 \cdot I_2 = \frac{1}{2}\left(2 - 2 \cdot \frac{3}{4}\right) = \frac{1}{4}$$

in singlet $I^2 = 0(0+1) = 0$

$$I_1 \cdot I_2 = -3/4$$

6.) Griffiths 4.32

Σ^{*0} is isospin $|1,0\rangle$

$\Sigma^+ = |1\ +1\rangle = \pi^+$
$\Sigma^0 = |1\ 0\rangle = \pi^0$
$\Sigma^- = |1\ -1\rangle = \pi^-$

$$\Sigma^+ \pi^- = |1\,1\rangle \otimes |1\,-1\rangle = \tfrac{1}{\sqrt{6}}|2\,0\rangle + \tfrac{1}{\sqrt{2}}|1\,0\rangle + \tfrac{1}{\sqrt{3}}|0\,0\rangle$$
$$\Sigma^0 \pi^0 = |1\,0\rangle \otimes |1\,0\rangle = \sqrt{\tfrac{2}{3}}|2\,0\rangle + 0|1\,0\rangle + -\tfrac{1}{\sqrt{3}}|0\,0\rangle$$
$$\Sigma^- \pi^+ = |1\,-1\rangle \otimes |1\,1\rangle = \tfrac{1}{\sqrt{6}}|2\,0\rangle + -\tfrac{1}{\sqrt{2}}|1\,0\rangle + \tfrac{1}{\sqrt{3}}|0\,0\rangle$$

} from CG tables for 1×1

branching ratio is proportional to square of coefficient of $|1 0\rangle$ (the isospin of Σ^{*0})

So to the extent in which isospin is a good symmetry,
half the Σ^{*0} decays go to $\Sigma^+ \pi^-$ & the other half to $\Sigma^- \pi^+$