PS#1 Solns

1.) (1.3)

$\Delta x \approx 10^{-13}\ cm$

$p \approx \Delta p \gtrsim \frac{\hbar}{\Delta x} \approx 10^{-19}$, SI units

$E = \sqrt{p^2c^2 + m^2c^4}$

$\approx 3 \cdot 10^{-11}\ J \approx 2 \cdot 10^{8}\ eV \approx 200\ MeV$

or in natural units to start w/

$\Delta x \approx 10^{-13}\ cm \ \frac{1}{197\ MeV\ fm} \ \frac{10^{13}\ fm}{cm} = .005\ MeV^{-1}$

$\Delta p \gtrsim \frac{1}{\Delta x} \Rightarrow p \approx 200\ MeV$

$E = \sqrt{p^2 + m^2} = \sqrt{(200 MeV)^2 + (.511 MeV)^2} \approx 200\ MeV$

$200\ MeV \gg 15\ KeV$

2.) (3.4)

a.) $d = vt \approx 660\ m$

No, they don't make it.

b.) $d = v t \gamma = \frac{vt}{\sqrt{1-v^2}} = 10{,}000\ m$. Yes they make it

c.) μ sees distance contracted to $\frac{8000 m}{\gamma} = 505\ m$

so the 660 m that it can travel is far enough

d.) $d = v\tilde{t}\gamma = \frac{vt/100}{\sqrt{1-v^2}} = 100 m$ & π's don't make it.

3.) (3.16)

$A \Rightarrow B + C$ use 4-vectors

a.) $p_A = (m_A, 0, 0, 0)$ $p_B = (E_B, \vec{p}_B)$ $p_C = (E_C, \vec{p}_C)$

$$p^2 = m^2 \ \& \ p_A = p_B + p_C$$

$$p_A^2 = m_A^2 \, , \quad p_B^2 = E_B^2 - \vec{p}_B^{\,2} = m_B^2 \, , \quad p_C^2 = E_C^2 - \vec{p}_C^{\,2} = m_C^2$$

$$m_A = E_B + E_C \, , \quad 0 = \vec{p}_B + \vec{p}_C$$

$$-(E_B - m_A) = E_C = \sqrt{m_C^2 + \vec{p}_C^{\,2}} = \sqrt{m_C^2 + \vec{p}_B^{\,2}}$$

$$= \sqrt{m_C^2 + E_B^2 - m_B^2}$$

$$(E_B - m_A)^2 = m_C^2 + E_B^2 - m_B^2$$

$$-2E_B m_A + m_A^2 = m_C^2 - m_B^2$$

$$E_B = \frac{m_A^2 + m_B^2 - m_C^2}{2m_A}$$

b.) $$|\vec{p}_B| = |\vec{p}_C| = \sqrt{E_B^2 - m_B^2}$$

$$= \left(\frac{(m_A^2 + m_B^2 - m_C^2)^2 - m_B^2(4m_A^2)}{4m_A^2} \right)^{1/2}$$

$$= \frac{1}{2m_A} \sqrt{(m_A^2)^2 + (m_B^2)^2 + (m_C^2)^2 - 2m_A^2 m_B^2 - 2m_A^2 m_C^2 - 2m_B^2 m_C^2}$$

$$= \frac{1}{2m_A} \sqrt{\lambda(m_A^2, m_B^2, m_C^2)}$$

c.) conservation of energy.
A light particle cannot decay into two particles whose combined rest energy is greater than that of the initial particle.

4.) $B^0 \to K^+ \pi^-$

$m_{B^0} = 5280$ meV

$m_{K^+} = 494$ meV

$m_{\pi^-} = 140$ meV

$|\vec{p}_1| = 2615$ meV $= |\vec{p}_K|$

$B^0 \Rightarrow \pi^+ \pi^-$

$m_{\pi^+} = m_{\pi^-} = 140$ meV

$|\vec{p}_2| = 2636$ meV $= |\vec{p}_\pi|$

$$\frac{|\vec{p}_2| - |\vec{p}_1|}{\frac{1}{2}(|\vec{p}_2| + |\vec{p}_1|)} = ,0081 = ,81\%$$

$$p = m v \gamma = m v (1 - v^2)^{-1/2}$$

$$\frac{p}{m} = \frac{1}{\sqrt{v^{-2} - 1}} \Rightarrow v^{-2} = 1 + \left(\frac{m}{p}\right)^2 \Rightarrow v = \frac{1}{\sqrt{1 + \left(\frac{m}{p}\right)^2}}$$

$v_{K^+} = .983$

$v_{\pi^+} = ,998$

$,983 < \frac{1}{n} < .998$

$1,001 < n < 1,017$

5.) Let $E = 45\ GeV$

Let $m = .511\ MeV$ = electron mass

I'll first do the easy case when $E \gg m$, which applies here. On the next page, I'll do the full calculation for arbitrary E. ($E > m$ for consistency)

$P_{e^-_1} = P_1 = (E, 0, 0, E)$ (e^-_1 is effectively massless initially — at least wrt E)

$P_{e^-_2} = P_2 = (m, 0, 0, 0)$

$P = P_1 + P_2 = (E+m, 0, 0, E)$

$P^2 = (E+m)^2 - E^2 = 2Em$

$v_{cm} = \dfrac{|\vec{P}_{TOT}|}{E_{TOT}} = \dfrac{E}{E+m}$ (Griffiths, 3.52 - useful to just know)

$\gamma = (1-v^2)^{-1/2} = \left(\dfrac{2Em + m^2}{(E+m)^2}\right)^{-1/2} = \dfrac{E+m}{\sqrt{2Em}}$

in CM frame, $P' = (m, 0, 0, 0)$, but $P'^2 = P^2$ (Lorentz invariant)

so, $P' = (\sqrt{2Em}, 0, 0, 0)$

$P_1' = (\tfrac{1}{2}\sqrt{2Em}, 0, \tfrac{1}{2}\sqrt{2Em}, 0)$ Want $90°$ scattering & $P_1' + P_2' = P'$

$P_2' = (\tfrac{1}{2}\sqrt{2Em}, 0, -\tfrac{1}{2}\sqrt{2Em}, 0)$ & $P_1'^2 = P_2'^2 = m^2$

Boost P_1' back to lab frame

$$\sqrt{\frac{Em}{2}}\begin{pmatrix} \gamma & & & \beta\gamma \\ & 1 & & \\ & & 1 & \\ \beta\gamma & & & \gamma \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \sqrt{\frac{Em}{2}}\begin{pmatrix} \gamma \\ 0 \\ 1 \\ \beta\gamma \end{pmatrix}$$

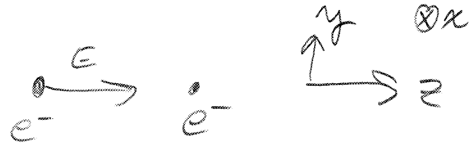
Angle of scattering $= \tan^{-1}\left(\dfrac{1}{\beta\gamma}\right) = \tan^{-1}\left(\dfrac{\sqrt{2Em}}{E}\right) = \tan^{-1}\left(\sqrt{\dfrac{2m}{E}}\right)$

$= \tan^{-1}\left(\sqrt{\dfrac{2 \cdot .000511\ GeV}{45\ GeV}}\right) \approx .00477$

≈ 5 milliradians

5.) Let $E = 45$ GeV (Same method – less rounding works for all $E > m$)

Let $m = .511$ MeV $\approx m_{e^-}$



$p_1 = (E, 0, 0, \sqrt{E^2 - m^2})$, $p_2 = (m, 0, 0, 0)$ (b/c $p_i^2 = E^2 - \vec{p}^{\,2} = p_0^2 - p_3^2 = m^2$)

$p = p_1 + p_2 = (E + m, 0, 0, \sqrt{E^2 - m^2})$

$v_{cm} = \dfrac{|\vec{P}_{TOT}|}{E_{TOT}}$ (Griffiths 3.52 – very useful to know)

$= \sqrt{E^2 - m^2}/(E + m)$

$p^2 = (E+m)^2 - (\sqrt{E^2 - m^2})^2 = 2Em + 2m^2$

In CM frame, $p' = (E, 0, 0, 0)$. But $p^2 = p'^2$ (Lorentz invariant)

$p' = ((2Em + 2m^2)^{1/2}, 0, 0, 0)$

$p_1' = \left(\frac{1}{2}\sqrt{2Em + 2m^2}, 0, \sqrt{\frac{Em}{2} - \frac{m^2}{2}}, 0\right) = \sqrt{\frac{m}{2}}\left(\sqrt{E+m}, 0, \sqrt{E-m}, 0\right)$

$p_2' = \left(\frac{1}{2}\sqrt{2Em + 2m^2}, 0, -\sqrt{\frac{Em}{2} - \frac{m^2}{2}}, 0\right) = \sqrt{\frac{m}{2}}\left(\sqrt{E+m}, 0, -\sqrt{E-m}, 0\right)$

b/c $p_1' + p_2' = p'$ & $p_1'^2 = p_2'^2 = m^2$ & we want $90°$ scattering

Now we take p_1' & boost back to lab frame.

$v_{cm} = \dfrac{\sqrt{E^2 - m^2}}{E + m}$, $\gamma = \dfrac{1}{(1 - v^2)^{1/2}} = \left(\dfrac{(E+m)^2 - (E^2 - m^2)}{(E+m)^2}\right)^{-1/2} = \dfrac{E + m}{\sqrt{2Em + 2m^2}}$

$$\sqrt{\frac{m}{2}}\begin{pmatrix} \gamma & & & v\gamma \\ & 1 & & \\ & & 1 & \\ v\gamma & & & \gamma \end{pmatrix}\begin{pmatrix} \sqrt{E+m} \\ 0 \\ \sqrt{E-m} \\ 0 \end{pmatrix} = \sqrt{\frac{m}{2}}\begin{pmatrix} \gamma\sqrt{E+m} \\ 0 \\ \sqrt{E-m} \\ v\gamma\sqrt{E+m} \end{pmatrix}$$

Scattering angle $= \tan^{-1}\left(\dfrac{\sqrt{E-m}}{v\gamma\sqrt{E+m}}\right) = \tan^{-1}\left(\dfrac{\sqrt{E-m}}{\sqrt{E^2-m^2}\,\frac{1}{\sqrt{2m}\sqrt{E+m}}\sqrt{E+m}}\right)$

$= \tan^{-1}\left(\dfrac{\sqrt{E-m}\,\sqrt{2m}}{\sqrt{E+m}\,\sqrt{E-m}}\right) = \tan^{-1}\left(\sqrt{\dfrac{2m}{E+m}}\right)$

$= \tan^{-1}\left(\sqrt{\dfrac{2 \cdot .000511\text{GeV}}{45\text{GeV} + m}}\right)$

$\approx .00477 \approx 5$ milliradians (as before)