Particle Accelerators

Lecture 5 Physics 152

Lance Dixon

(thanks again due to Colin Jessop)

Two basic types of accelerator

linear

circular

Two types of beam

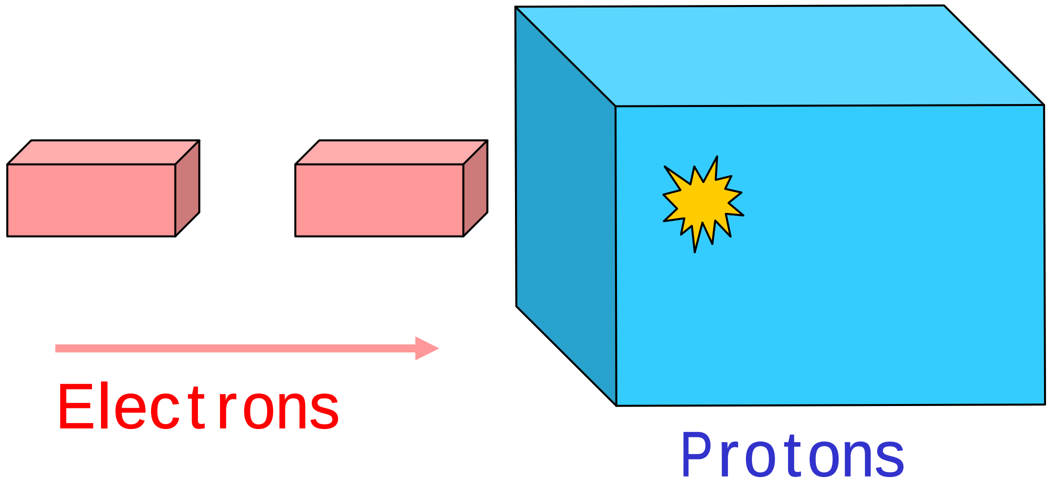
electrons

protons

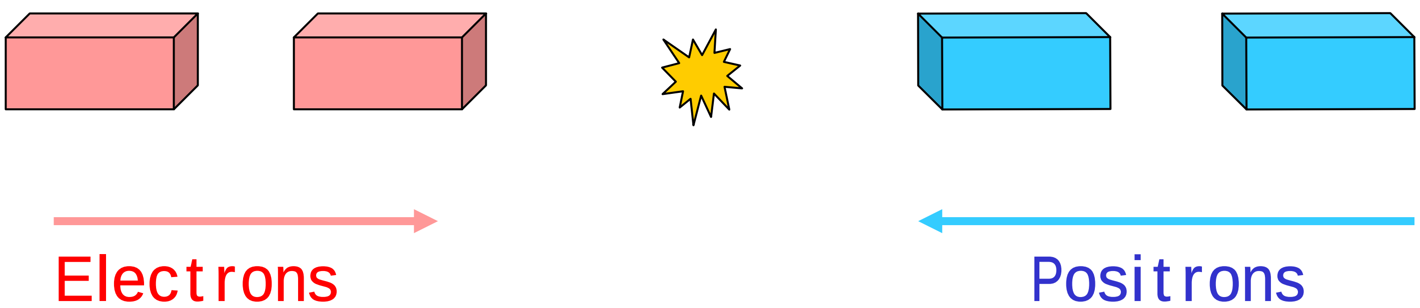
Two types of collisions available:
fixed target & colliding beam

The two most important accelerator properties are Luminosity (L) & Center-of-Mass Energy (E_{CM})

Fixed target: L large, E_{CM} small

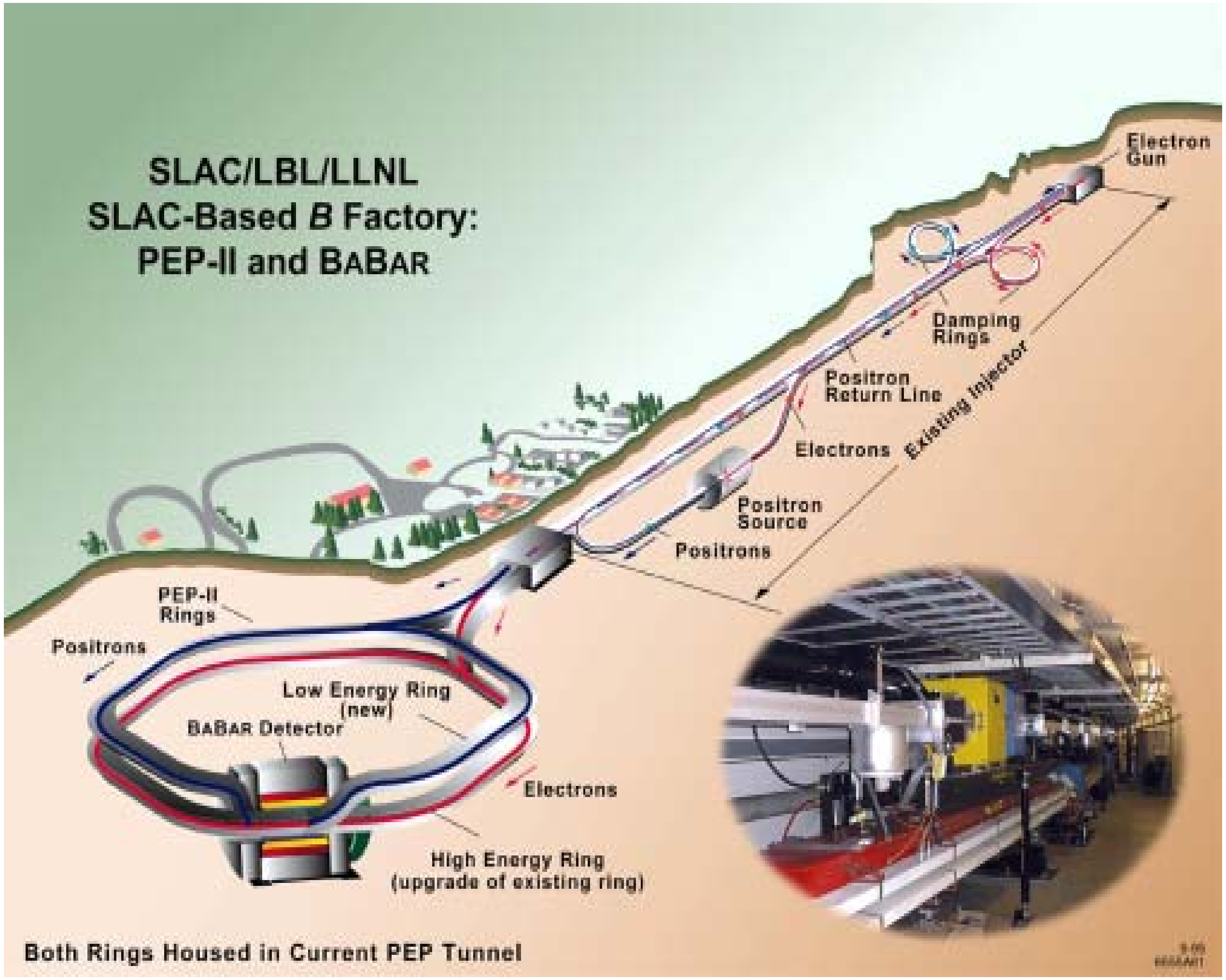


Colliding beams: L small, E_{CM} large



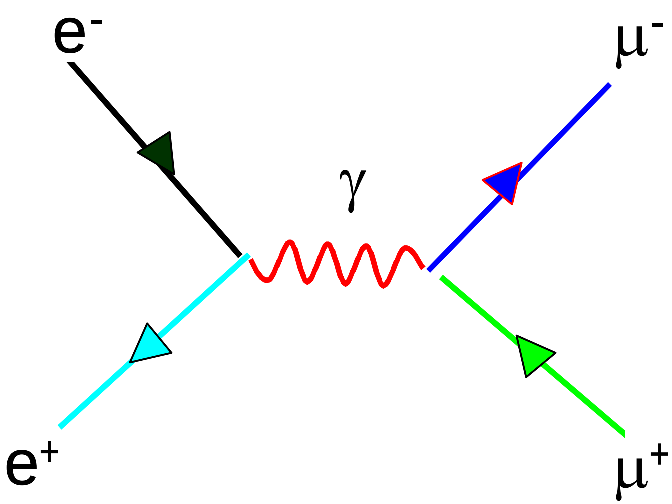
Example: PEPII @ SLAC

Currently producing millions of beauty -antibeauty quarks in e+e- collisions at the PEPII storage ring



How probable are the various physics processes?

$e^+e^- \to \mu^+\mu^-$ at PEP II



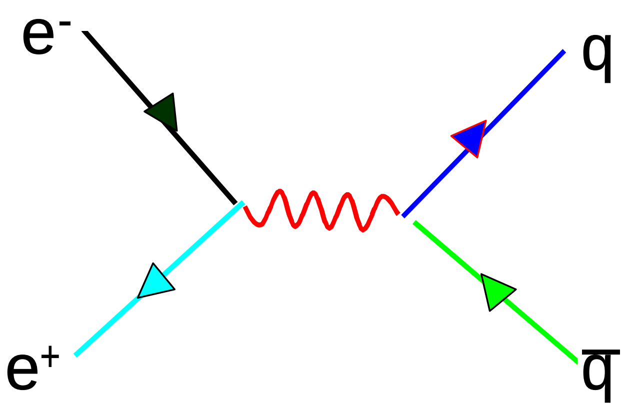
$$\sigma = \frac{4\pi\alpha^2}{3s} = 87 \text{ nb} / s\ (\text{GeV}^2)$$

$\alpha = 1/137$ $s = (E_{CM})^2$

At $s = (10.58 \text{ GeV})^2$ (PEP II): $\sigma(e^+e^- \to \mu^+\mu^-) = 0.78$ nb

$e^+e^- \to b\bar{b}$ at PEP II

At SLAC we want to make as many $b\bar{b}$ pairs as possible.
For light quarks (u,d,s,c), similar process to $e^+e^- \to \mu^+\mu^-$

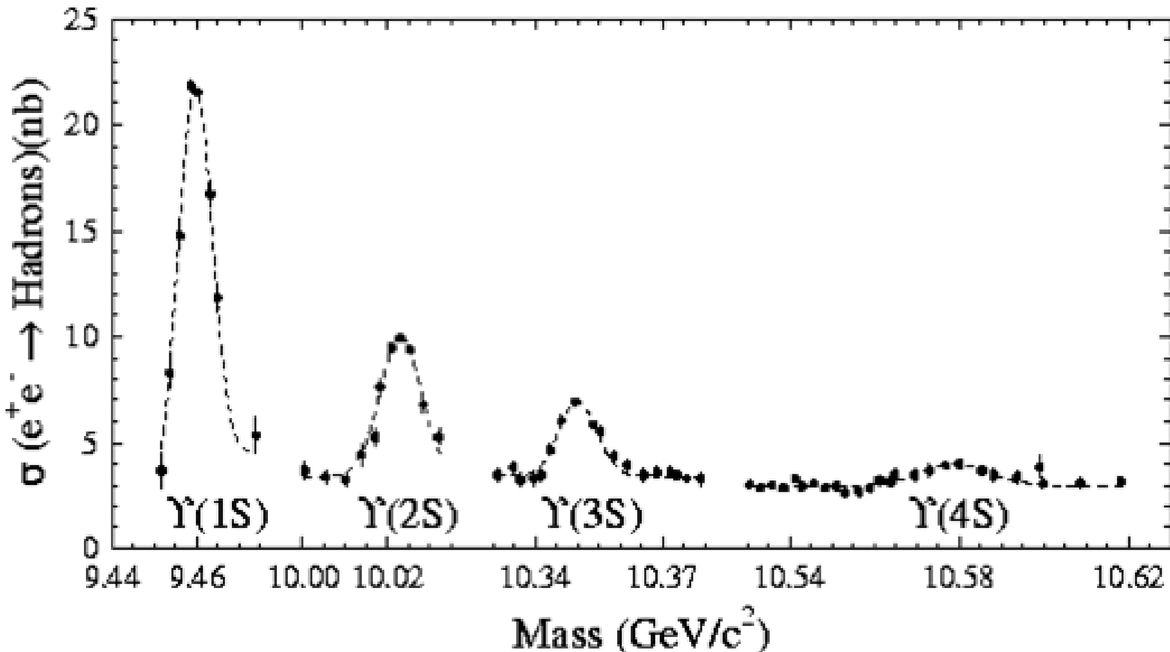


$$\sigma = 3Q_q^2 \frac{4\pi\alpha^2}{3s}$$

q = u,d,s,c

accounts for off-resonance σ

But for b in B meson there is a resonance, the $\Upsilon(4S)$.

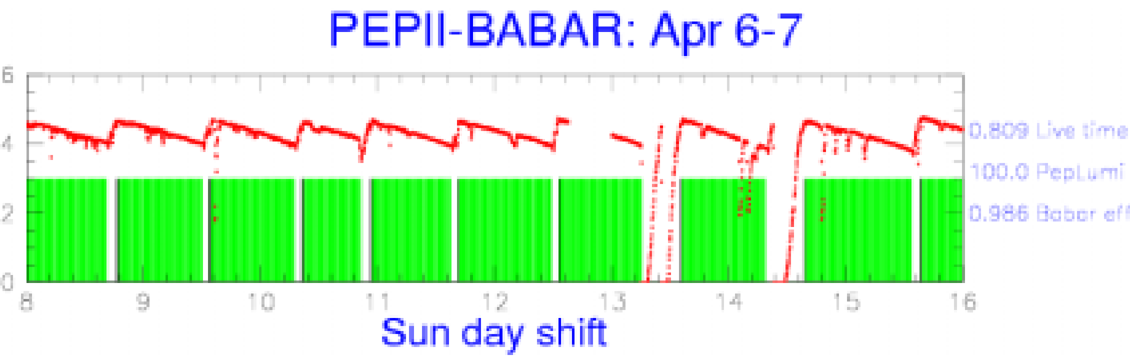


At $s = (10.58\ \text{GeV})^2$:
$\sigma(e^+e^- \to b\bar{b}) = 1.05$ nb

Luminosity (Flux)

Number of events/sec = σ (cm^2) x Luminosity (cm^{-2} s^{-1})

$\sigma(e^+e^- \to b\bar{b})$ = 1.05 nb (1 nb = $10^{-9} \times 10^{-24}$ cm^2)



PEP II Luminosity
currently = 4×10^{33} cm^{-2} s^{-1}
= 4 nb^{-1} s^{-1}

Currently producing 4.2 $b\bar{b}$ pairs per second

In the form of $B^0 \bar{B}^0$ pairs ($B^0 = B_d = \bar{b}d$ meson)
and $B^+ B^-$ pairs ($B^+ = B_u = \bar{b}u$ meson)
50% of each.

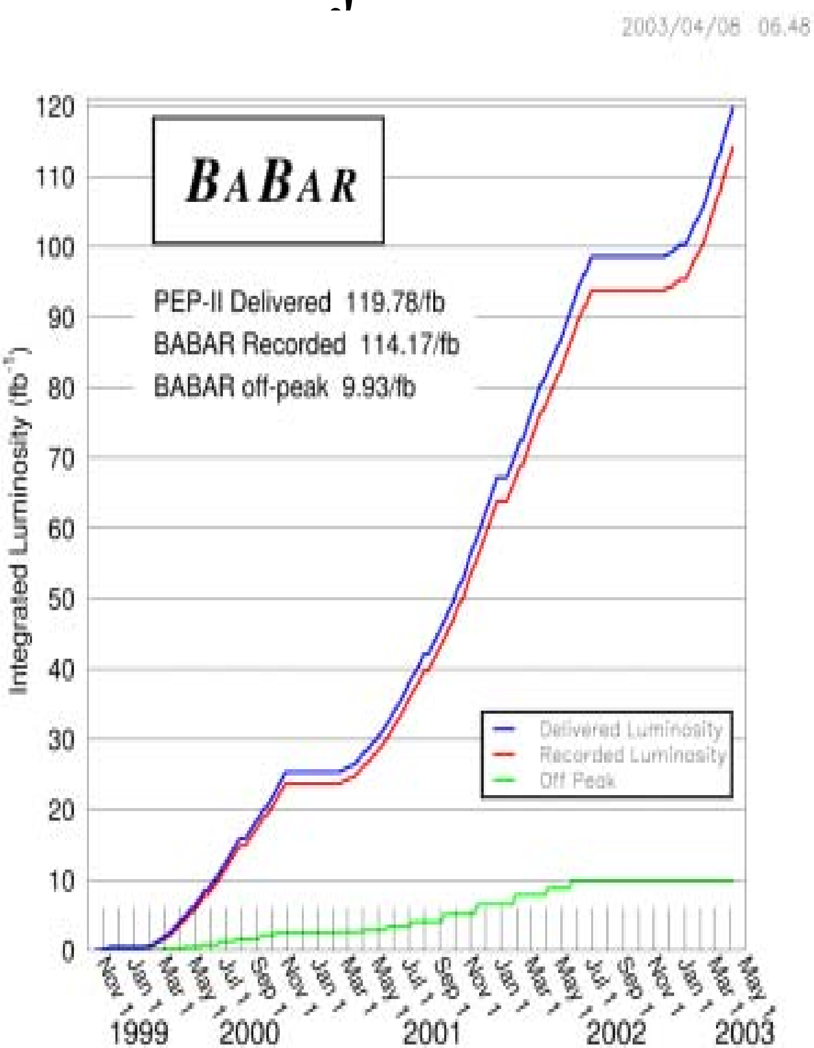
Integrated Luminosity

$$L_{int} = \int L dt \quad (cm^{-2})$$

$1\ nb = 10^{-33}\ cm^2$ $\quad$ $1\ nb^{-1} = 10^{33}\ cm^{-2}$

$1\ pb = 10^{-36}\ cm^2$ $\quad$ $1\ pb^{-1} = 10^{36}\ cm^{-2}$

$1\ fb = 10^{-39}\ cm^2$ $\quad$ $1\ fb^{-1} = 10^{39}\ cm^{-2}$



Total events = $L_{int} \cdot \sigma$

= $120 \times 10^6\ nb^{-1} \times 1.05$

= 126 million $b\bar{b}$ pairs

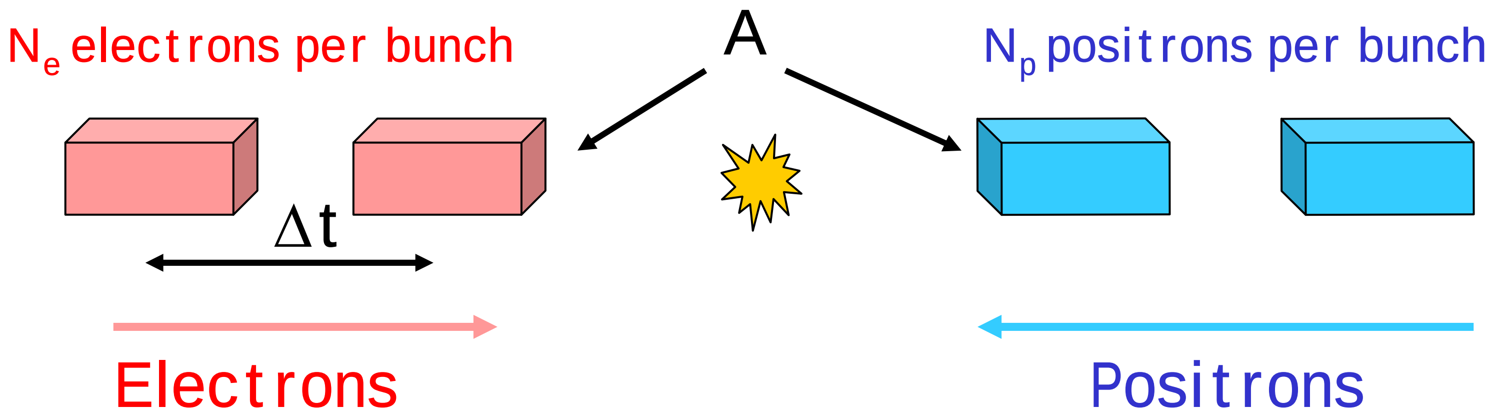
So many are required because "interesting" decays like

$B_d \to (J/\psi) K_S \leftarrow \bar{B}_d$

are rare, $Br(B \to \psi K_S) < 10^{-3}$,

cannot "find" all decays, etc.

Luminosity



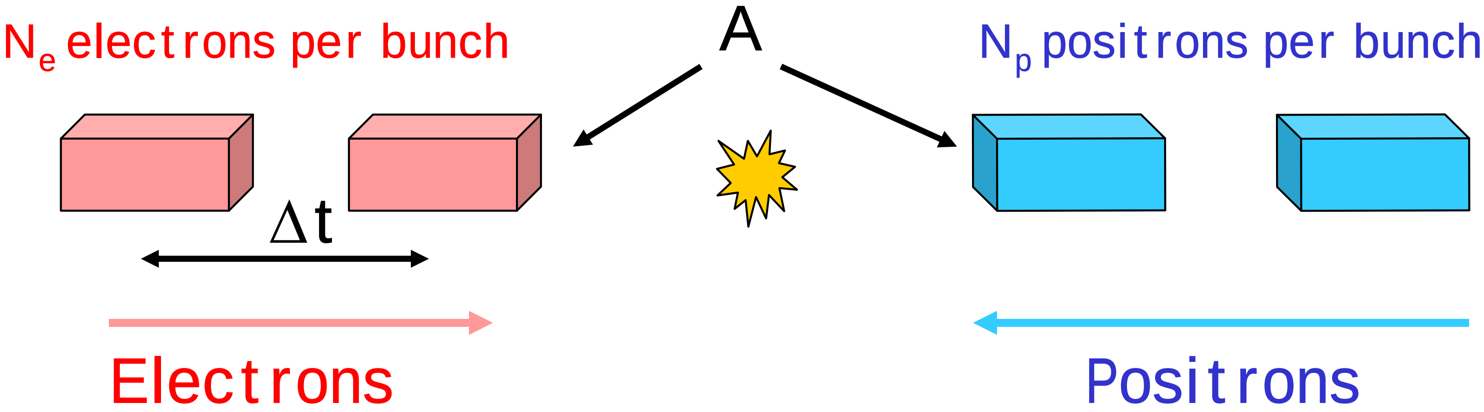
$$L = \frac{f N_e N_p}{A}$$

Frequency of collisions: $f = 1/\Delta t$

Area of beam = A

For large L need large N, f and small A (focusing)

PEP II Luminosity



$2\pi R$ = 2200m, 1658 bunches -> 0.0075 bunches/cm

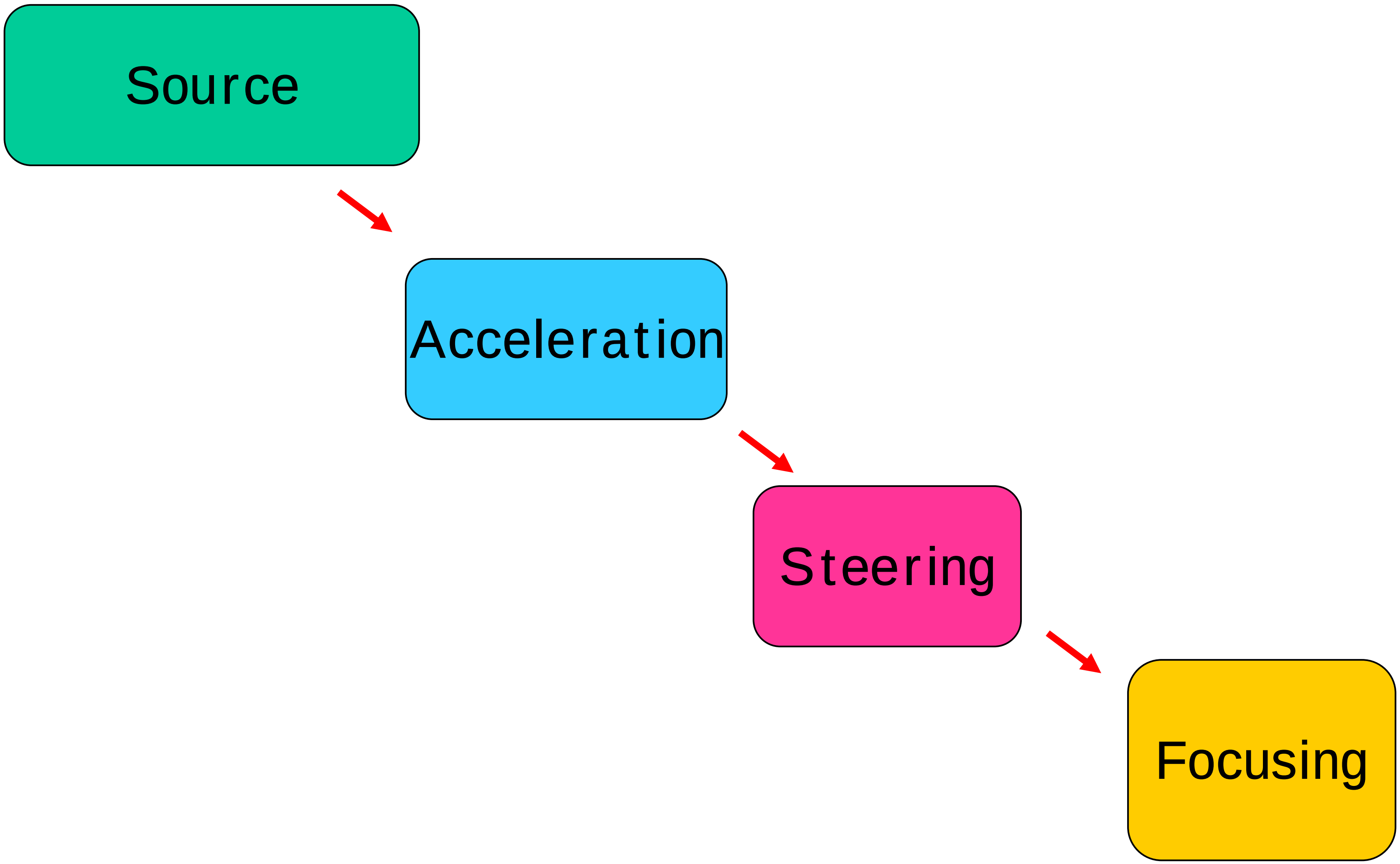
-> $f = 2.3 \times 10^8$ bunches/sec

$$L = k\frac{fN_eN_p}{A} = k\frac{(2.3\times10^8)(2.7\times10^{10})(5.9\times10^{10})}{150\mu m\times15\mu m}$$

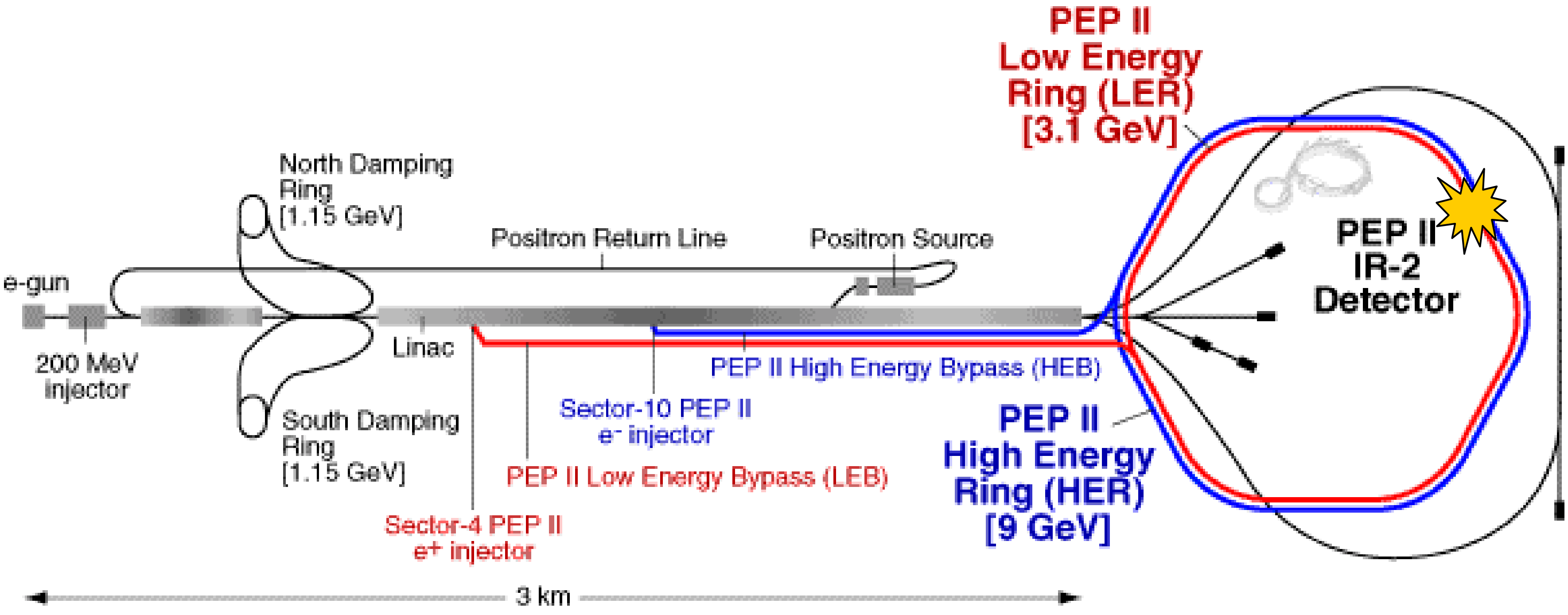
$L = k \times 1.6 \times 10^{34}\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}$

k a factor specific to machine design

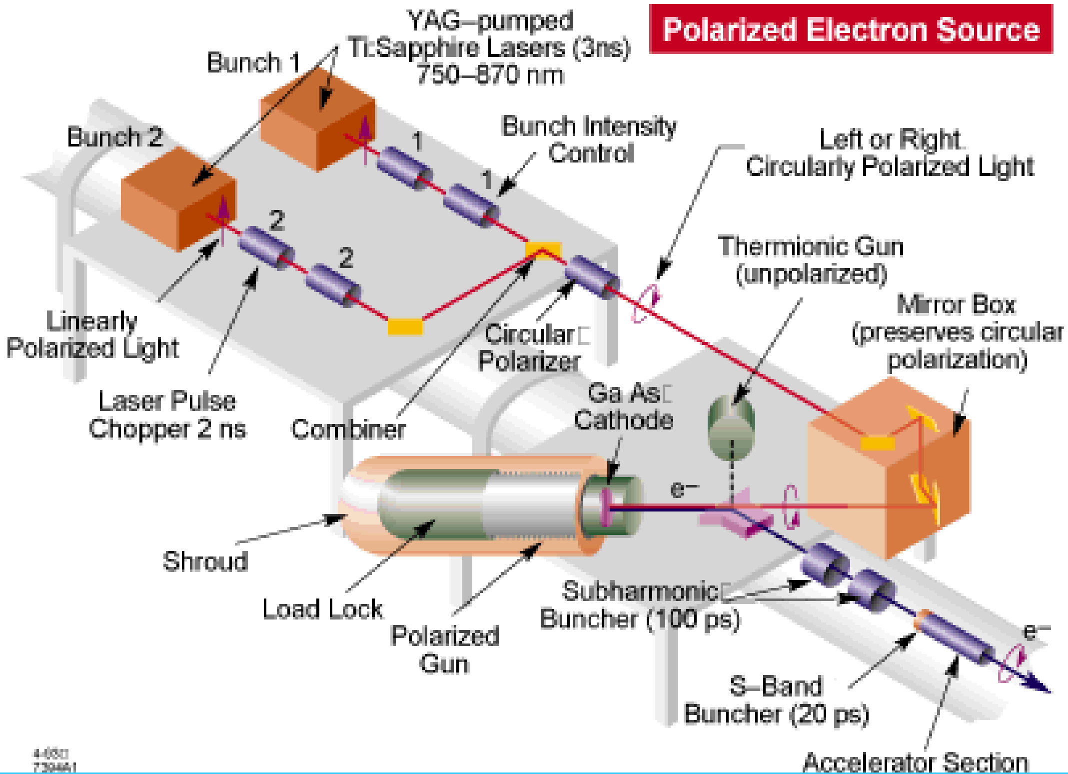
Basic Accelerator Components



The SLAC accelerator complex

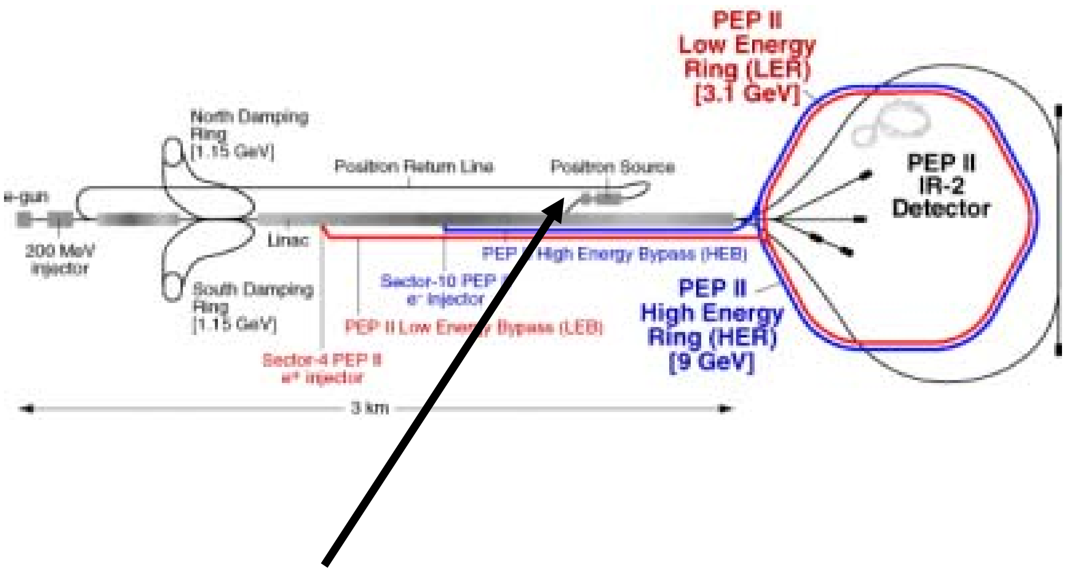


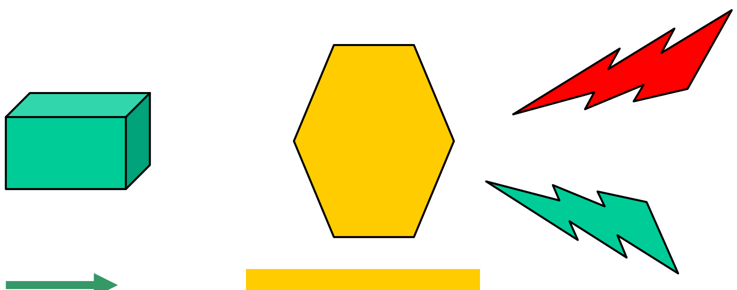
Electron Source



Thermionic (hot high voltage cathode)
or photo-electric emission to create electrons.
With polarized laser light can create polarized electrons.
Pulse the high voltage
-> bunches created for insertion into linear accelerator.

Positron Source

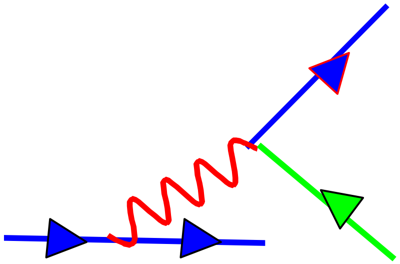




electrons

tungsten target

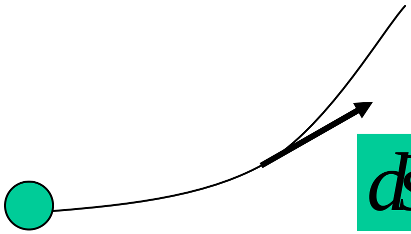
Electrons+ positrons



Particle Acceleration

$$\vec{F} = e\vec{E} + e(\vec{v} \times \vec{B})$$

$$\Delta\vec{p} = \int \vec{F}\,dt \qquad \Delta E = \int \vec{F}\cdot d\vec{s}$$



$$d\vec{s} = \vec{v}\,dt$$

$$\Delta E = \int e\vec{E}\cdot\vec{v} + e(\vec{v}\times\vec{B})\cdot\vec{v}\,dt \qquad (\vec{v}\times\vec{B})\cdot\vec{v} = 0$$

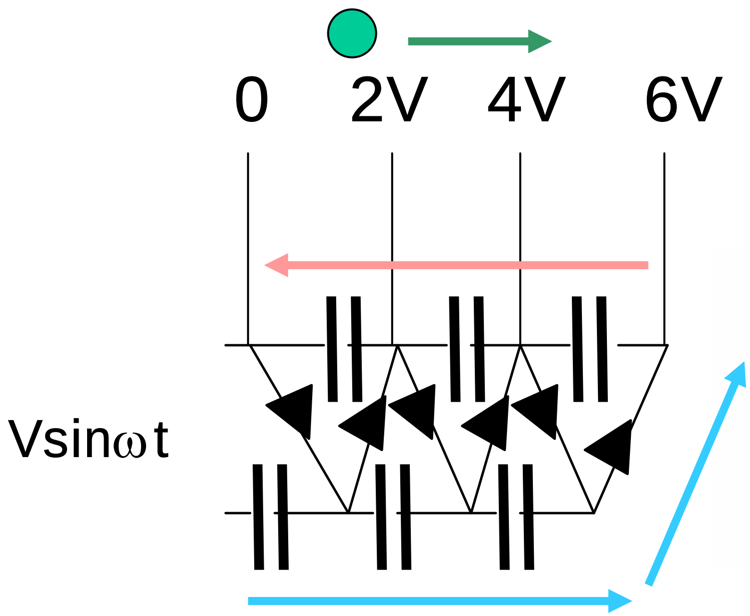
$$\Delta E = \int e\vec{E}\cdot\vec{v}\,dt$$

$\vec{E}$ accelerate

$\vec{B}$ steer / focus

Earliest and Simplest Accelerator

Cockcroft-Walton Accelerator - Voltage Doubler

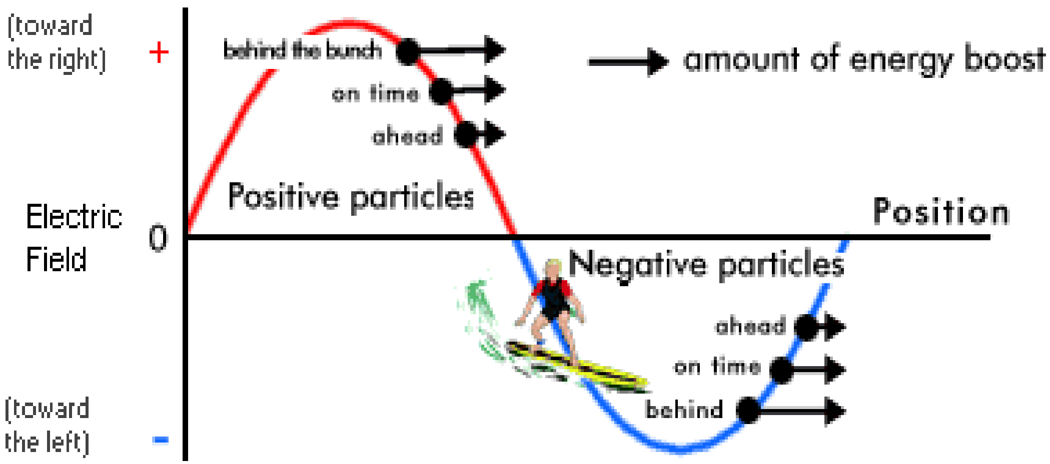


Diodes -> capacitors charge in parallel
discharge in series

Still used as first stage accelerator in proton machines.
Up to ~ 1 MV.   Watch out for arcing!

RF acceleration

Particle "surfs" an electromagnetic wave

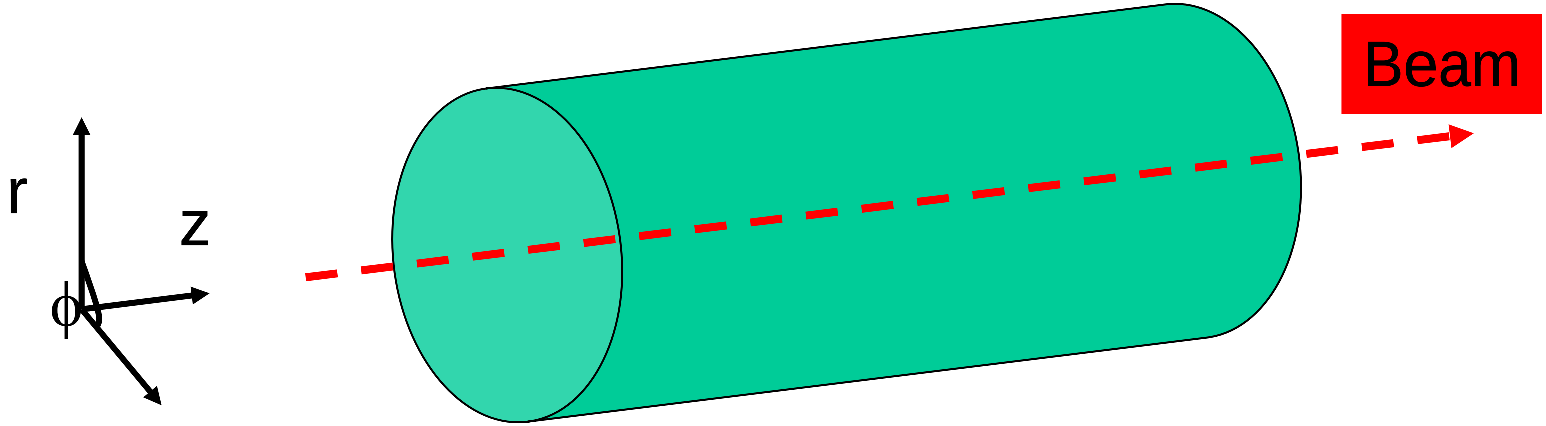


1. Need E field in direction of motion
   (i.e. longitudinal E-field - not free space EM wave)

2. Need particles locked in phase with the wave

Creating longitudinal E for acceleration

Use waveguides and resonant cavities.
Geometry of waveguide can create the correct field.
(Free space EM waves are transverse; need walls.)



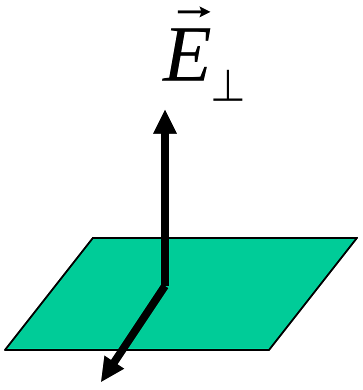
Solve Maxwell's equations with appropriate boundary conditions for a cylindrical cavity.

Maxwell’s Equations and boundary conditions

$$\vec{\nabla}\cdot\vec{E}=0 \qquad \vec{\nabla}\cdot\vec{B}=0 \qquad \vec{\nabla}\times\vec{E}=\frac{-1}{c}\frac{\partial\vec{B}}{\partial t} \qquad \vec{\nabla}\times\vec{B}=\frac{\mu\varepsilon}{c}\frac{\partial\vec{E}}{\partial t}$$

From these equations can derive:

Solve $\nabla^2\vec{E}=\frac{\mu\varepsilon}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}$ In cylindrical coordinates



With boundary conditions
for surface of conductor

$$\vec{E}_\parallel=0 \qquad \vec{B}_\perp=0$$

Creating longitudinal E for acceleration

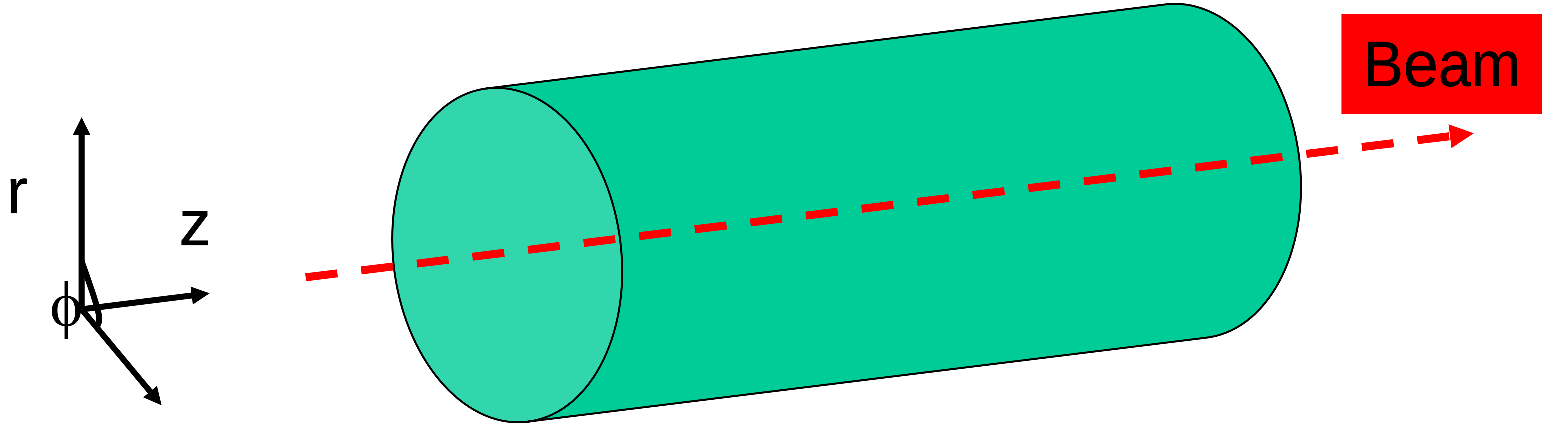
Use separation of variables to solve.

Two sets of solutions:

TE_{klm} Transverse E (longitudinal B) modes

TM_{klm} Transverse B (longitudinal E) modes

klm is the periodicity of the solution in ϕrz

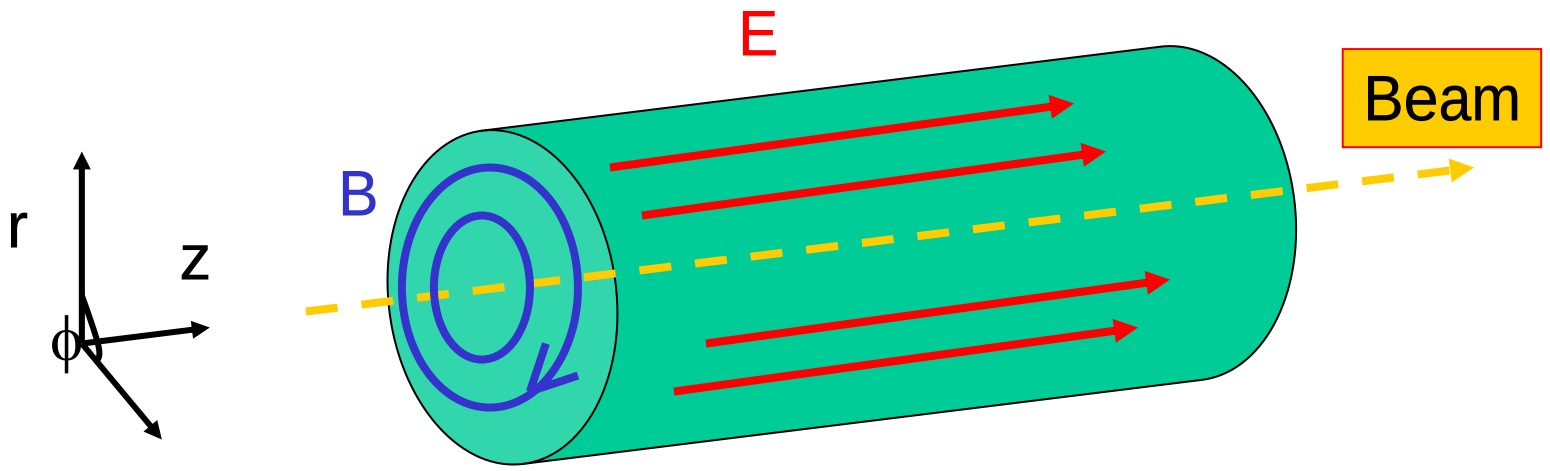


The TM modes are useful for particle acceleration

The TM$_{010}$ mode

$$E_z = E_0 f(r) e^{i(wt-kz)}$$

$$B_\phi = E_0 g(r) e^{i(wt-kz)}$$

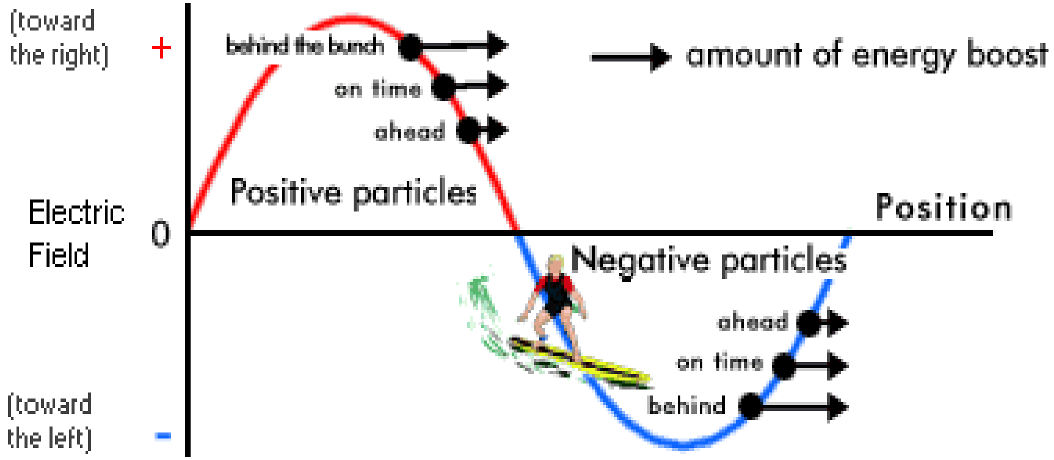


f(r) is a Bessel function, with *f(R)* = 0.

$f(r) = J_0(2.405\, r/R)$.

g(r) obtained from *f(r)*.

Phase Synchronization

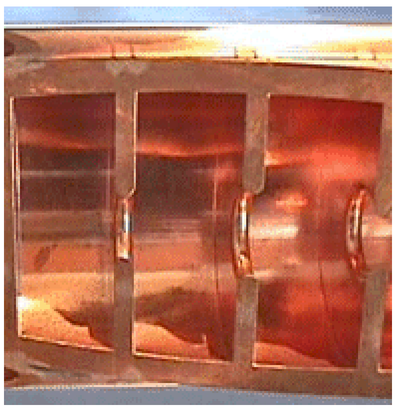


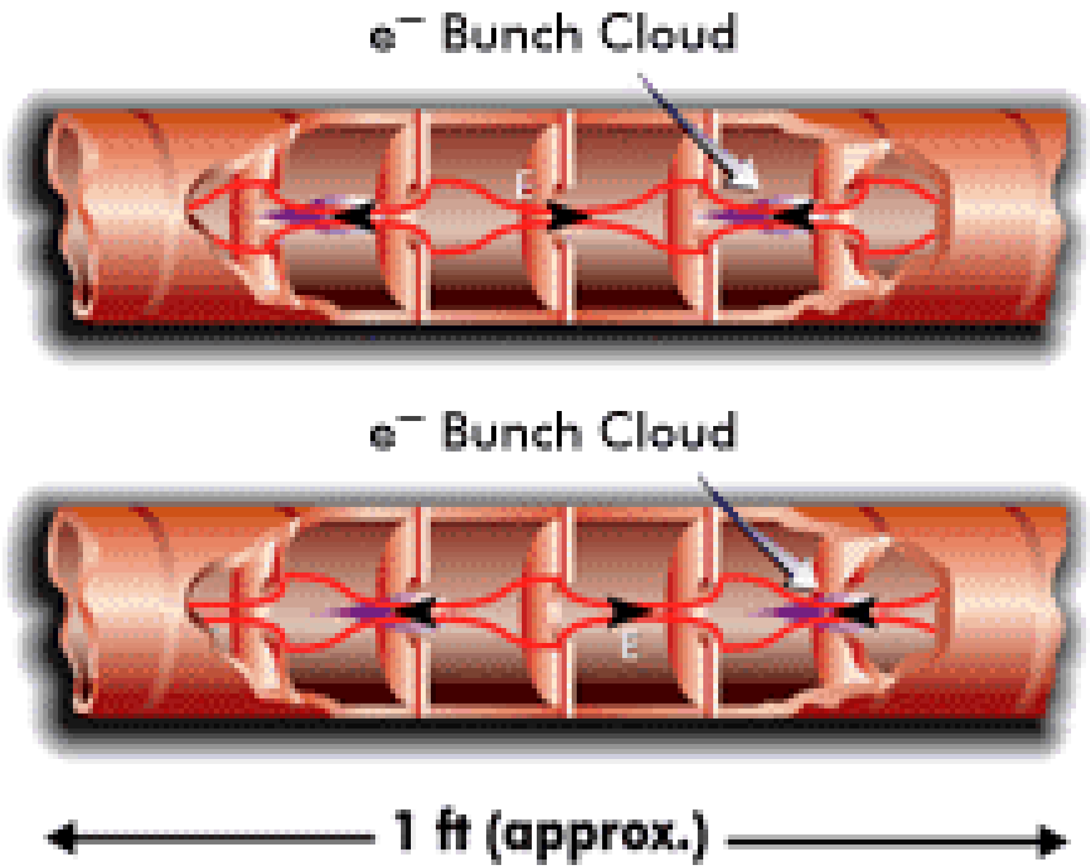
1. Arrange phase velocity (ω/k) = particle velocity

2. Stable for small perturbations about synchronized point

Disk Loaded Waveguide

Insertion of disks controls phase velocity
(interference of reflected waves)

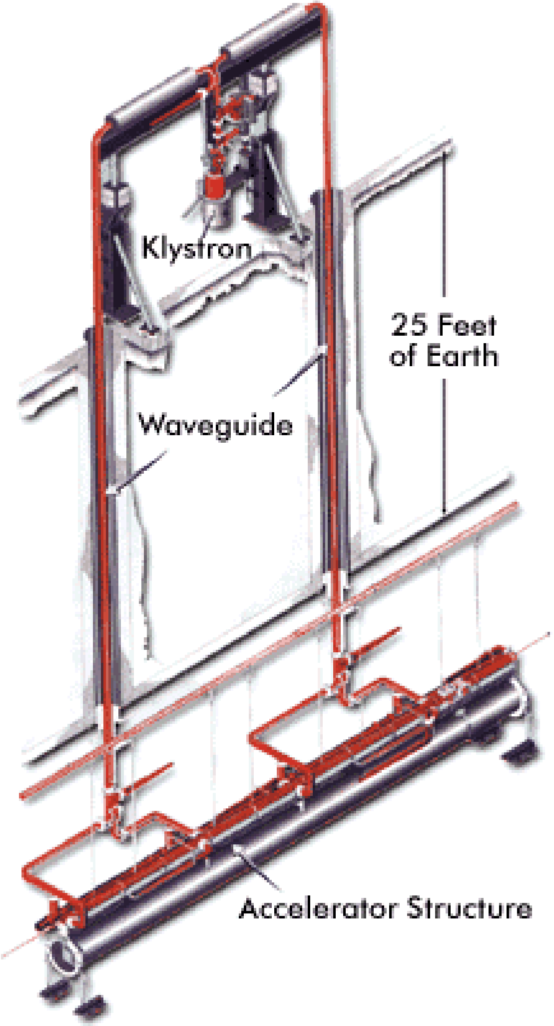




Stanford Linear Accelerator

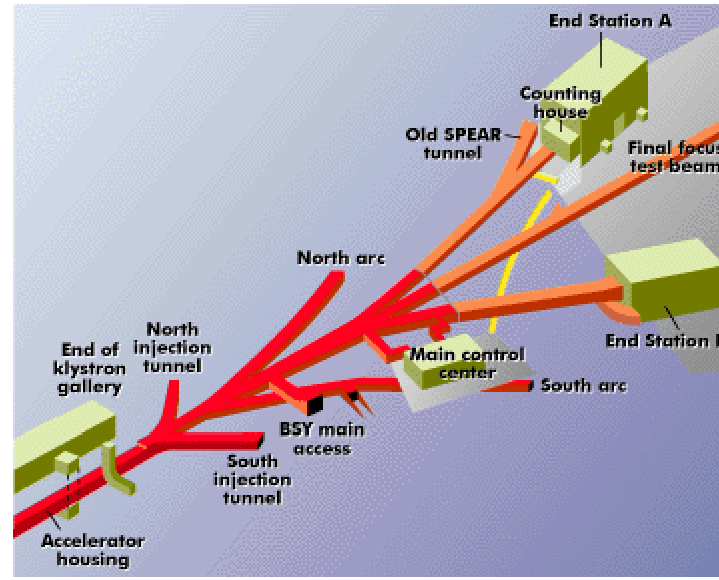
240 x 8 MW Klystrons
provide the power

ACC_01 PEP-II Accelerator at Slac 8-19-97

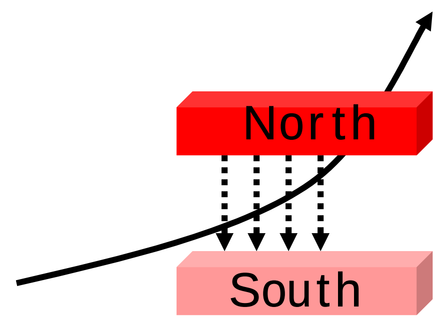


Beam Steering

Use Magnetic Dipoles
for Steering

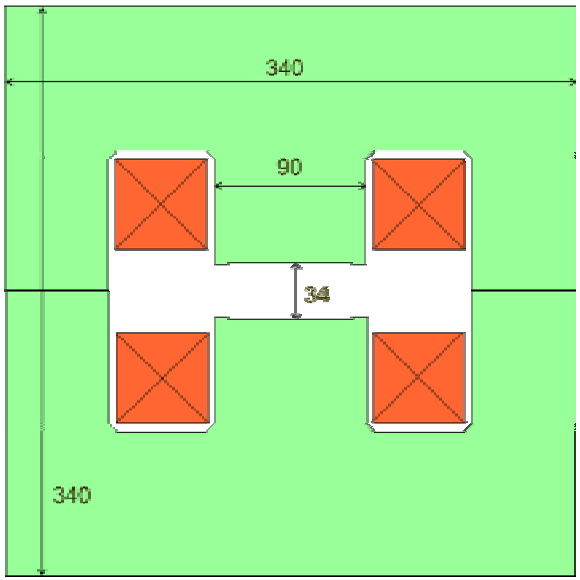


PD_001 PEP-II Dedication 10/26/98



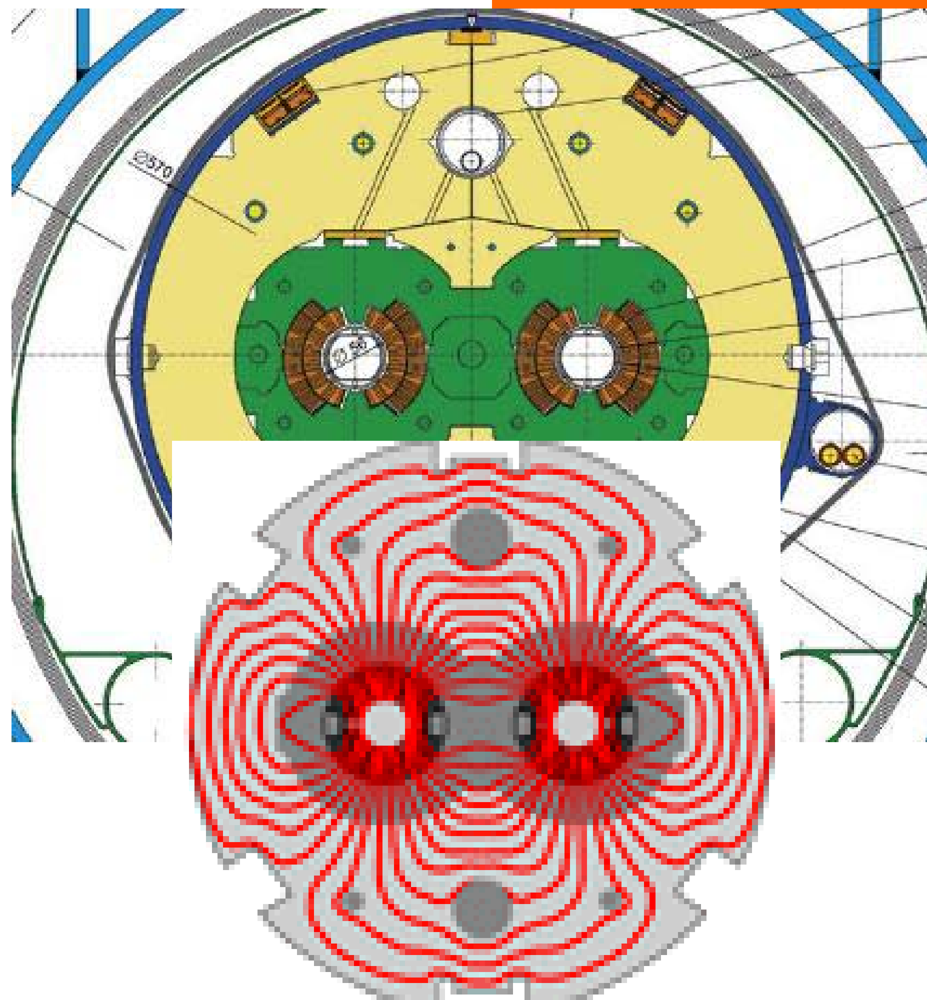
Two examples of Dipole Magnets

Synchroton light source:
one e^- beam, 1 Tesla,
room temperature

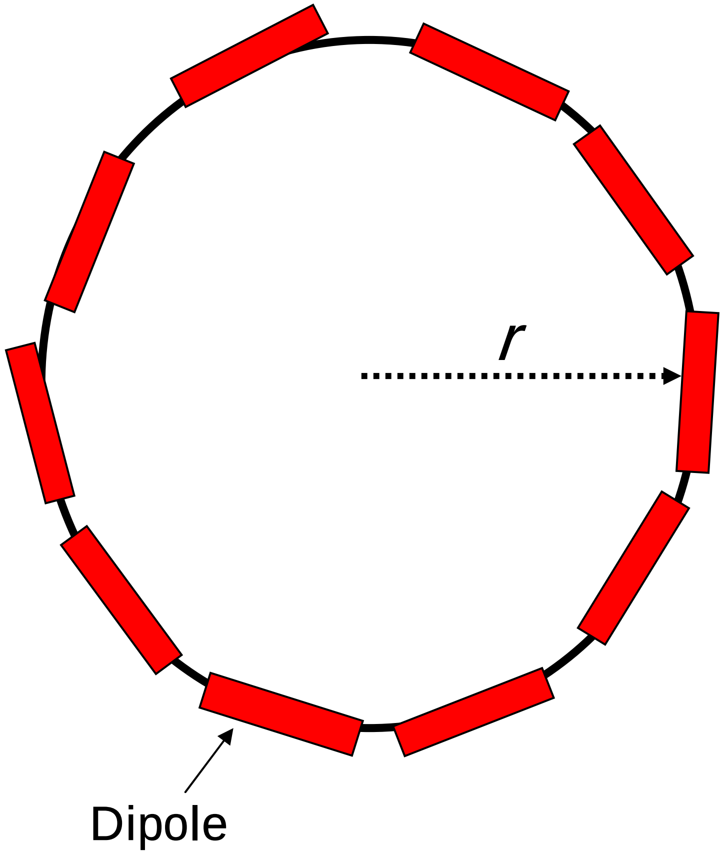


LHC: two p beams, 8.4 Tesla,
superconducting NbTi @ 1.9K

1,296 of these!



Steering Particle in Circle

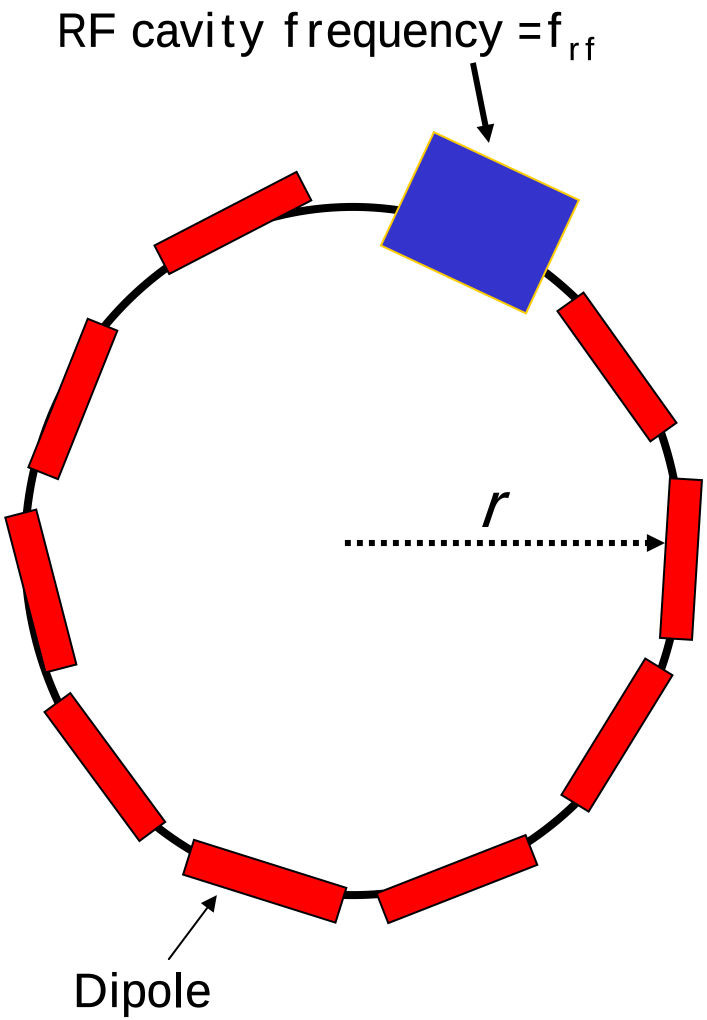


$$p = mv\gamma = eBr$$

$$\gamma = \frac{1}{\sqrt{1-v^2}}$$

$$f_{rev} = \frac{1}{t_{rev}} = \frac{v}{2\pi r} = \frac{eB}{2\pi\gamma m}$$

The Synchrotron



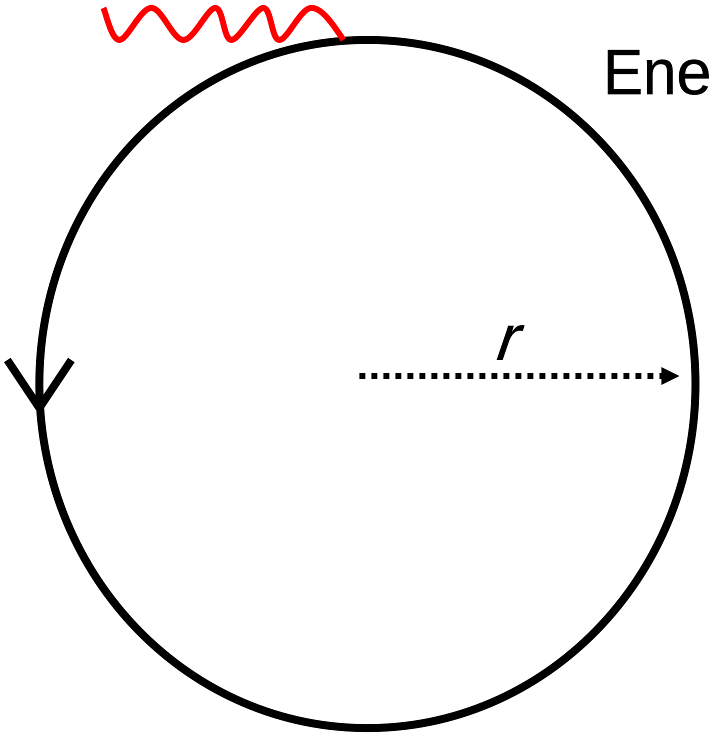
$$f_{rf} = f_{rev} = \frac{eB}{2\pi\gamma m}$$

$$\gamma = \frac{1}{\sqrt{1-v^2}}$$

Circular particle accelerator (1 rf cavity) with rf changing as *v* increases to keep in phase

Synchrotron Radiation

As particle circles, it loses energy through EM radiation



Energy loss per turn: $$\Delta E_{loss} = \frac{4\pi\alpha}{3}\frac{E^4}{m^4 r}$$

Limits maximum E
for circular electron accelerator
because m_e is small

LEP (Large Electron-Positron Collider)

C = 26.7 km

At 100 GeV per beam, Energy loss per turn = 2 GeV
Operates at 11 kHz revolution frequency.

Proton Synchrotrons

Energy loss per turn much more favorable for protons, because m_p large (= 1837 m_e)

$$\Delta E_{loss} = \frac{4\pi\alpha}{3}\frac{E^4}{m^4 r}$$

Tevatron is currently highest energy collider:
960 GeV protons collide with 960 GeV anti-protons

Proton synchrotrons limited in E by strength of bending magnets:

$$E = p = eBr$$

Focusing

To decrease beam area at a collision point, we need to focus the beam.

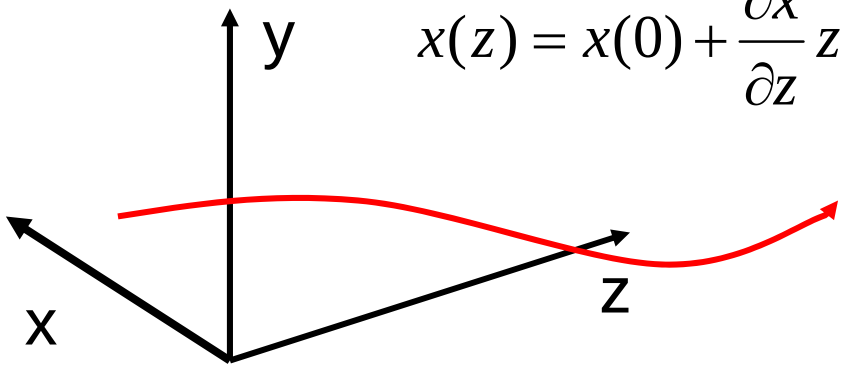
Also a number of effects in the accelerator cause the beam to defocus (space charge repulsion, residual transverse E fields, etc.)

All accelerators need focusing throughout the accelerator to maintain a stable beam.

To study the beam behavior we use beam optics.

Beam Optics

Consider a particle moving in the z direction



$$x(z) = x(0) + \frac{\partial x}{\partial z} z$$

$$x' = \frac{\partial x}{\partial z}$$

If the x and y motions are decoupled then we can describe the motion by a transfer matrix M

$$\begin{pmatrix} x \\ x' \end{pmatrix}_z = M \begin{pmatrix} x \\ x' \end{pmatrix}_0 = \begin{pmatrix} 1 & z \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_0$$

The M shown here describes a drift region of empty space.

Combining Transfer Matrices

Successive optical components can be described by multiplying the matrices.
For example, for two successive drift regions:

$$M_1 = \begin{pmatrix} 1 & z_1 \\ 0 & 1 \end{pmatrix} \qquad M_2 = \begin{pmatrix} 1 & z_2 \\ 0 & 1 \end{pmatrix}$$

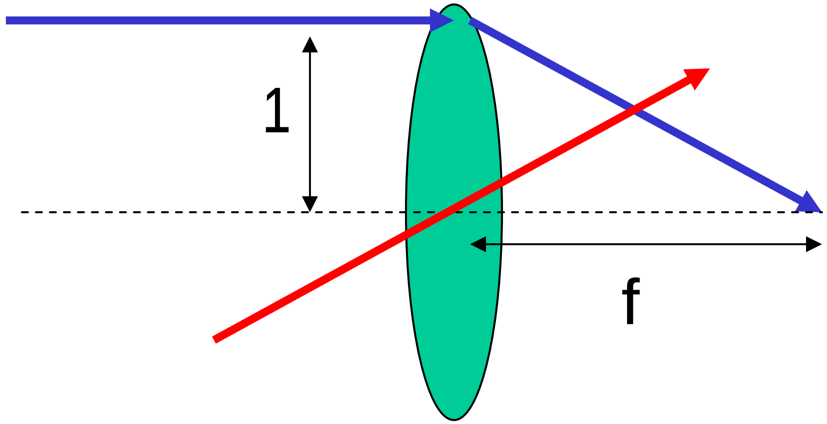
$$M = M_2 M_1 = \begin{pmatrix} 1 & z_1 + z_2 \\ 0 & 1 \end{pmatrix}$$

In this case the transfer matrices commute but this is not always the case

Transfer matrix of a thin lens

An off-axis parallel ray is focused

$$\begin{pmatrix} x \\ x' \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ -1/f \end{pmatrix}$$



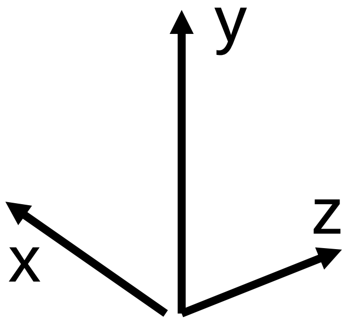
Ray through center is unaffected

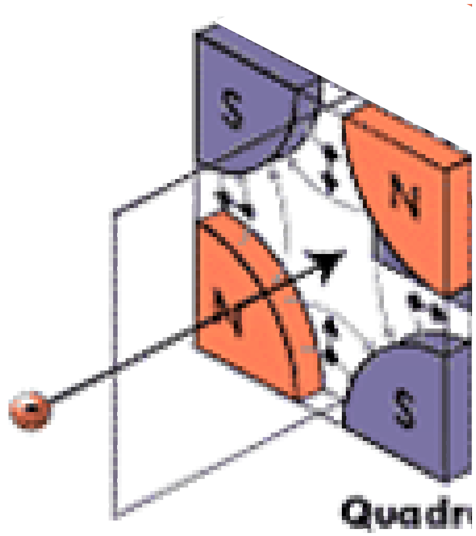
$$\begin{pmatrix} x \\ x' \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

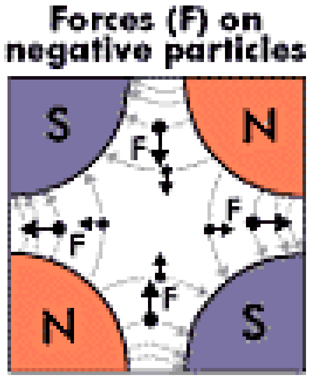
$$M_{convergent} = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$$

$$M_{divergent} = \begin{pmatrix} 1 & 0 \\ 1/f & 1 \end{pmatrix}$$

Quadrupole Focusing







$\vec{\nabla} \times \vec{B} = 0$ and $B_z = 0$ so $\dfrac{\partial B_x}{\partial y} = \dfrac{\partial B_y}{\partial x}$

then $B_x = by$ $B_y = bx$

Quadrupole focusing

In the *x* dimension the particle executes simple harmonic motion since

$$m\gamma^2 \frac{d^2x}{dt^2} = e\gamma(\vec{v}\times\vec{B})_x = -e\frac{p}{m}bx \qquad dz = \frac{p}{m\gamma}dt$$

$$\frac{d^2x}{dz^2} = -k^2 x \qquad k = \sqrt{\frac{eb}{p}}$$

The solution for *x* is then $x = A\cos kz + B\sin kz$

so

$$x(z) = x(0)\cos kz + x'(0)k^{-1}\sin kz$$
$$x'(z) = -x(0)k\sin kz + x'(0)\cos kz$$

If magnet length is *l*, then

$$M_x = \begin{pmatrix} \cos kl & k^{-1}\sin kl \\ -k\sin kl & \cos kl \end{pmatrix}$$

Quadrupole focusing

Can show that M_x is equivalent to $M_{convergent}$

$$M_x = \begin{pmatrix} \cos kl & k^{-1}\sin kl \\ -k\sin kl & \cos kl \end{pmatrix}$$

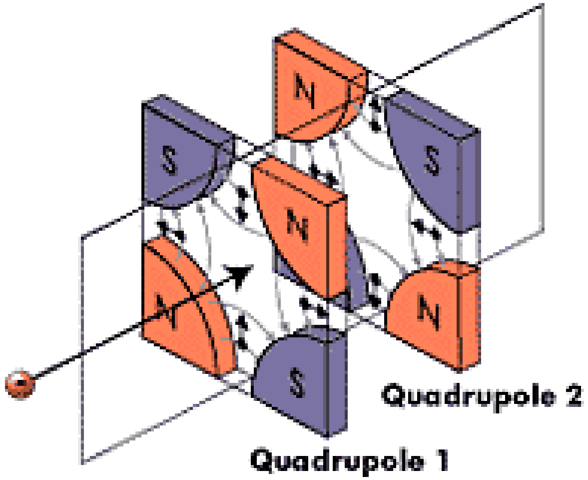
Similarly, can show that M_y is equivalent to $M_{divergent}$

$$M_y = \begin{pmatrix} \cosh kl & k^{-1}\sinh kl \\ k\sinh kl & \cosh kl \end{pmatrix}$$

If focus in *x* then defocus in *y* !

Quadrupole focusing

However, you can prove the surprising result that that two opposite polarity quads separated by a drift distance cause focusing in both *x* and *y*



Elements of an Accelerator