Electroweak Interactions (cont.)

Lecture 16   Physics 152/252

Lance Dixon

(thanks again to Colin Jessop)

Precision electroweak physics in $e^+e^- \to Z^0$

In the 1990s, 2 facilities made the most precise tests of the neutral current weak interaction, via $e^+e^- \to Z^0$:
LEP at CERN (4 detectors: ALEPH, DELPHI, L3, OPAL)
SLC at SLAC (1 detector: SLD)

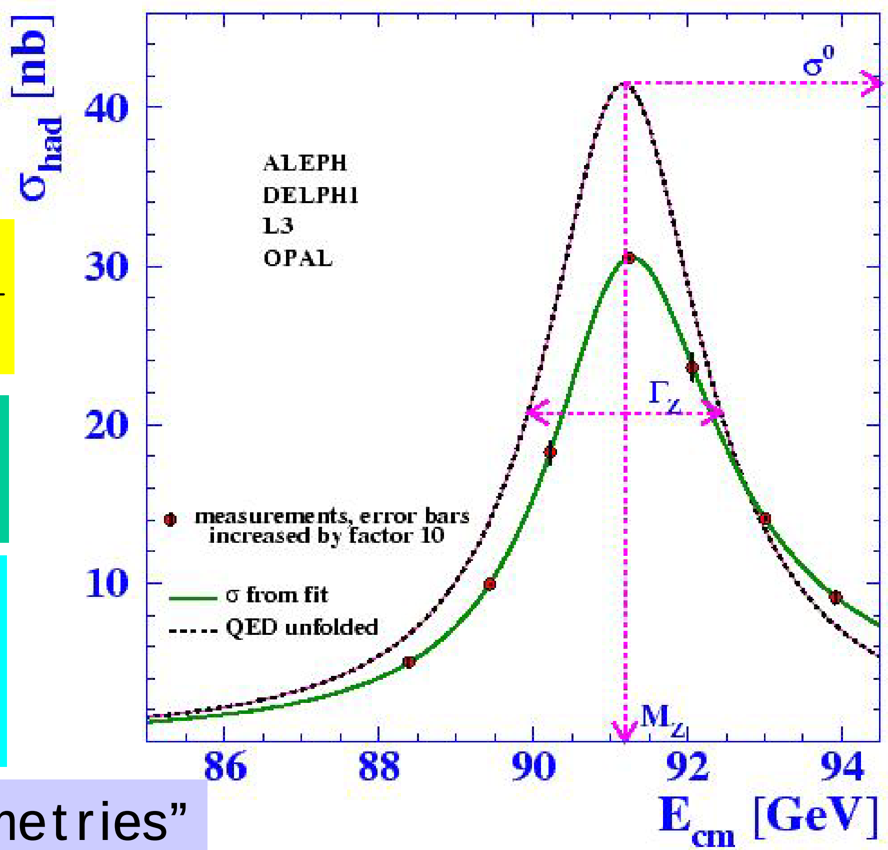
LEP collected 18 million Z^0 decays;
SLD 500,000 Z^0s,
but with polarized electrons.

$$\sigma(e^+e^- \to Z \to \text{hadrons}) = \frac{12\pi\Gamma_{ee}\Gamma_{\text{had}}}{(s - M_Z^2)^2 + M_Z^2\Gamma_Z^2}$$

"giant" resonance (by particle physics Standards)

LEP made very precise measurements of the resonance:
m_Z = 91.187 GeV Γ_Z = 2.495 GeV

LEP/SLD also measured "Z pole asymmetries"



Parameters of the electroweak theory

All W,Z masses, self-couplings, and couplings to leptons, and Z couplings to quarks, depend on just 3 parameters:

g, g', v

Trade these for the 3 most precisely measured quantities:

1) Muon lifetime: $\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3}$ determines $\mathrm{v} = 246\,\mathrm{GeV}$

or $G_F = 1/(\sqrt{2}\mathrm{v}^2) = 1.166\times 10^{-5}\ \mathrm{GeV}^{-2}$

2) Z mass: m_Z = 91.187 GeV determines $\frac{m_Z}{\mathrm{v}} = \frac{1}{2}\sqrt{g^2 + g'^2}$

3) QED coupling: $\alpha(0) = 1/137.036$ determines $e = \frac{gg'}{\sqrt{g^2+g'^2}}$

Predict all other quantities using G_F, m_Z, α as inputs.

Precision electroweak tests

Slight problem regarding α input: Measure $\alpha(0)$, but what appears in electroweak formulas is $\alpha(m_Z) = \alpha(0) + \Delta\alpha$
Part of "running" due to quarks/hadrons cannot be reliably calculated, though experiment can be used.

Net result is that fractional uncertainty is $\delta\alpha(m_Z)/\alpha(m_Z) \sim 2 – 7 \times 10^{-4}$, significantly bigger than
$\delta m_Z/m_Z \sim 2.4 \times 10^{-5}$
$\delta G_F/G_F \sim 9 \times 10^{-6}$

$$\Delta\alpha = \sum_f$$

$$= \frac{\alpha}{3\pi} \sum_f Q_f^2 N_{cf} \left(\ln \frac{M_Z^2}{m_f^2} - \frac{5}{3}\right) = \Delta\alpha_{\mathrm{leptons}} + \Delta\alpha^{(5)}_{\mathrm{quarks}} .$$

Next, test theory with additional quantities that can be predicted/measured accurately:
m_W, Γ_Z, σ_{had}, $Br(Z \to l^+l^-)$, $Br(Z \to b\bar{b})$,
$A_{FB}(Z \to l^+l^-)$, $A_{FB}(Z \to b\bar{b})$,
plus Z pole asymmetries with polarized electrons,
plus a few "low energy" measurements:
atomic parity violation, neutrino scattering.

Precision electroweak tests (cont.)

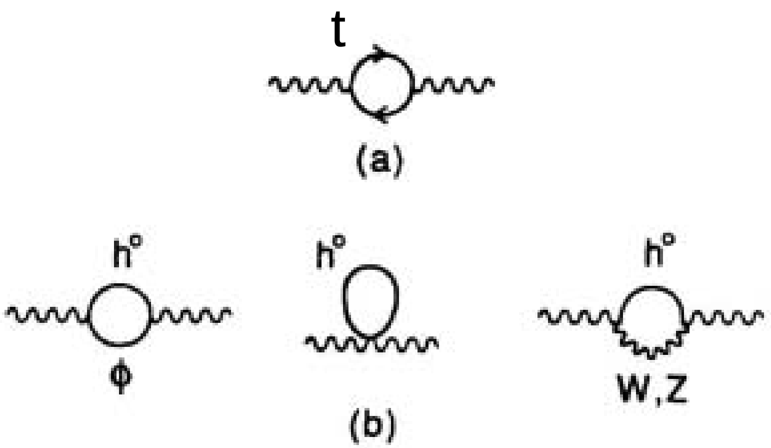
Tests are of sufficient accuracy that loop calculations are required
-> more parameters enter.

1) Strong coupling constant: $\alpha_S(m_Z) = 0.118 \pm 0.005$
   can be obtained from 3 jet rates.
2) Top quark mass: $m_t = 175 \pm 5$ GeV from the Tevatron
3) Higgs boson mass M_H = ???

One of the main goals of a fit to all the electroweak data

Problem is that while top quark contributions enters quadratically, $\sim \alpha_W (m_t/m_Z)^2$,

Higgs contribution only enters logarithmically,
$\sim \alpha_W \ln(m_H/m_Z)$



Left-right asymmetry for e^+e^- -> Z^0 -> anything

$$e_L^- \longrightarrow Z^0 \longleftarrow e^+$$

$$\sigma_L = \sigma(e_R^+ e_L^- \to Z^0) \propto (g_L^e)^2$$

vs.

$$e_R^- \longrightarrow Z^0 \longleftarrow e^+$$

$$\sigma_R = \sigma(e_L^+ e_R^- \to Z^0) \propto (g_R^e)^2$$

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = \frac{(g_L^e)^2 - (g_R^e)^2}{(g_L^e)^2 + (g_R^e)^2} = A_e = \frac{(-\frac{1}{2} + s_W)^2 - (s_W)^2}{(-\frac{1}{2} + s_W)^2 + (s_W)^2}$$

Experimentally, electron polarization is $P_e < 100\%$, measure:

$$A_{LR}^{exp} = \frac{N_L - N_R}{N_L + N_R} = \frac{[\frac{1}{2}(1+P_e)\sigma_L + \frac{1}{2}(1-P_e)\sigma_R] - [\frac{1}{2}(1+P_e)\sigma_R + \frac{1}{2}(1-P_e)\sigma_L]}{[\frac{1}{2}(1+P_e)\sigma_L + \frac{1}{2}(1-P_e)\sigma_R] + [\frac{1}{2}(1+P_e)\sigma_R + \frac{1}{2}(1-P_e)\sigma_L]}$$

$$A_{LR}^{exp} = P_e \cdot A_{LR}$$

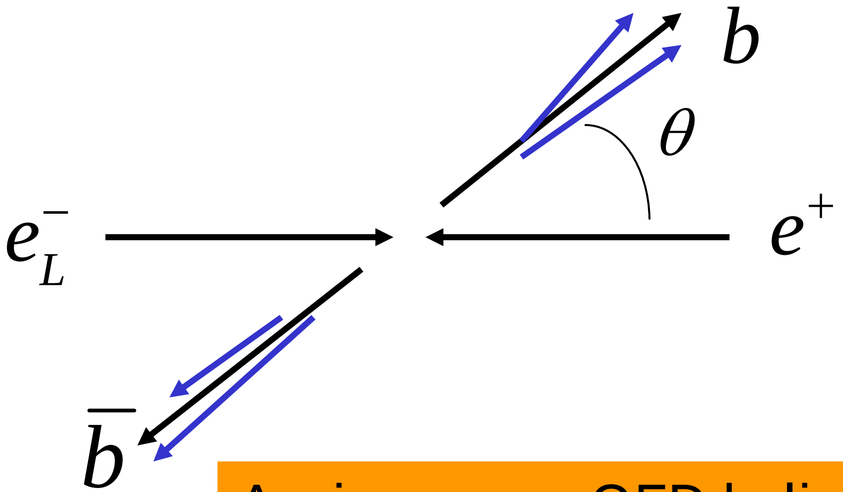
SLD, with 500,000 Z^0s, measured:

$$A_{LR} = 0.151 \pm 0.002$$

$$s_W = \sin^2\theta_W = 0.2310 \pm 0.0002$$

Through loop diagrams, constrains $m_{Higgs} < 200$ GeV

Polarized forward-backward asymmetry for $e^+e^- \to Z^0 \to b\bar{b}$



Measured at SLD

LEP measured the same asymmetry, but with unpolarized electrons & positrons.

Again can use QED helicity amplitudes, "dressed" with appropriate weak couplings:

$$\frac{d\sigma(e_R^+ e_L^- \to b\bar{b})}{d\cos\theta} \propto (g_L^e)^2\left[(g_L^b)^2(1+\cos\theta)^2 + (g_R^b)^2(1-\cos\theta)^2\right]$$

$$= \frac{1}{2}(g_L^e)^2\left[((g_L^b)^2 + (g_R^b)^2)(1+\cos^2\theta) + ((g_L^b)^2 - (g_R^b)^2)\cdot 2\cos\theta\right]$$

$$\propto (1+\cos^2\theta) + 2\cos\theta \cdot A_b$$

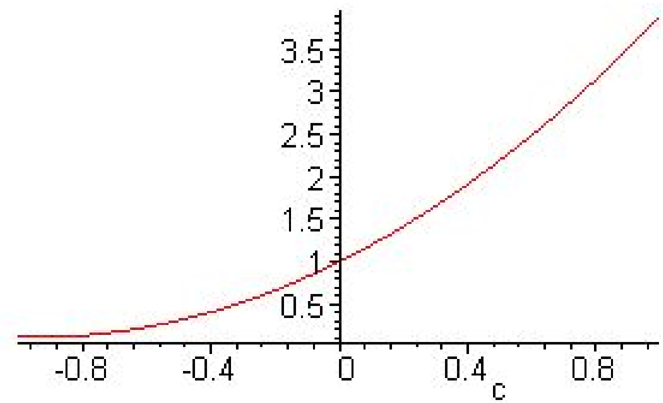
$$A_b = \frac{(g_L^b)^2 - (g_R^b)^2}{(g_L^b)^2 + (g_R^b)^2}$$

Polarized forward-backward asymmetry for $e^+e^- \rightarrow Z^0 \rightarrow b\bar{b}$

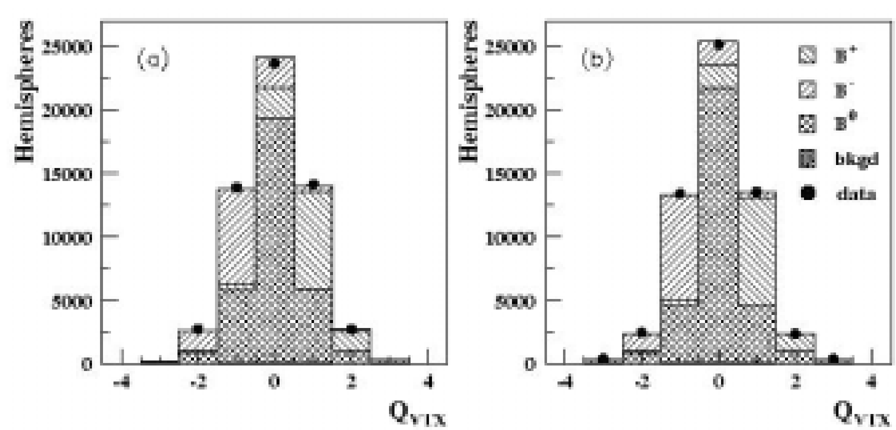
$$A_b = \frac{(g_L^b)^2 - (g_R^b)^2}{(g_L^b)^2 + (g_R^b)^2} = \frac{(-\frac{1}{2} + \frac{1}{3}s_W)^2 - (\frac{1}{3}s_W)^2}{(-\frac{1}{2} + \frac{1}{3}s_W)^2 + (\frac{1}{3}s_W)^2}$$

$A_b = 0.94$ for $s_W = \sin^2\theta_W = 0.230$

$$\frac{d\sigma}{d\cos\theta} \propto \left[(1+\cos^2\theta) + 2\cos\theta \cdot 0.94\right] \Rightarrow$$



But b vs. $\bar{b}$ tagging is not perfect. Use (for example) vertex charge, ~ 80% probability of getting b sign right.

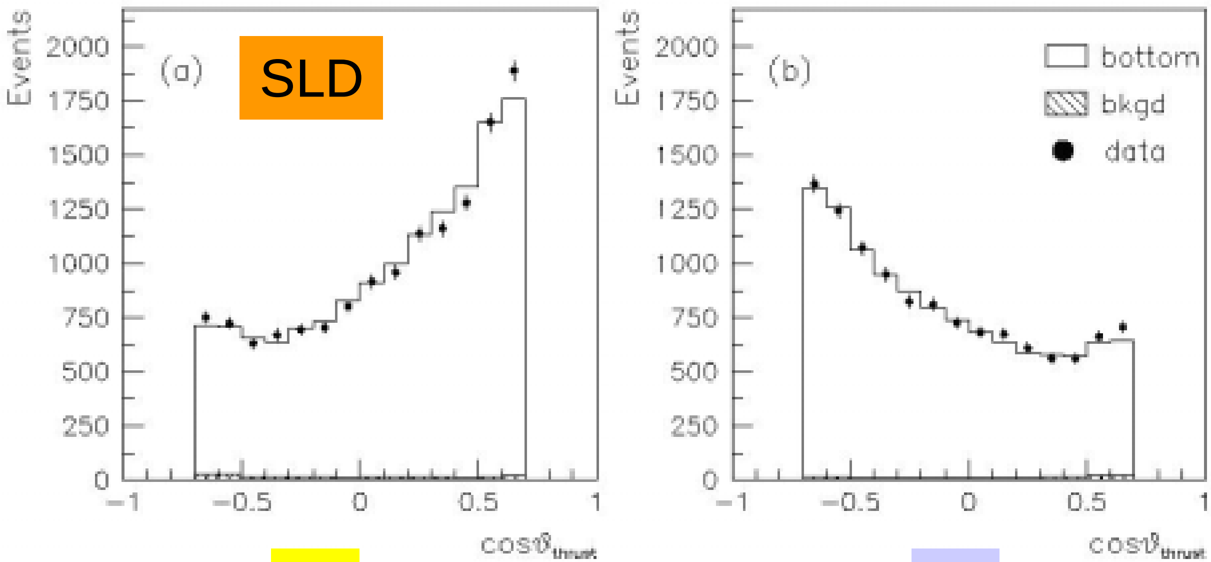


Polarized forward-backward asymmetry for $e^+e^- \to Z^0 \to b\bar{b}$

Including 80% tagging probability:

$$\frac{d\sigma^{\exp}}{d\cos\theta} \propto 0.8\left[(1+\cos^2\theta)+2\cos\theta\cdot 0.94\right]+0.2\left[(1+\cos^2\theta)-2\cos\theta\cdot 0.94\right]$$

$\Rightarrow$



e^-_L e^-_R

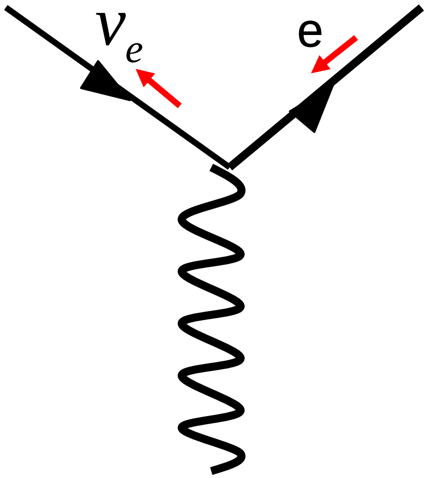
e_R same as e_L
but $\cos\theta$ <-> $-\cos\theta$,
and smaller
due to A_{LR}

Data provide one of best measurements of $Zb\bar{b}$ coupling

Quark mixing and the CKM matrix

Recall the charged leptonic weak current paired

$\nu_l \Leftrightarrow l$

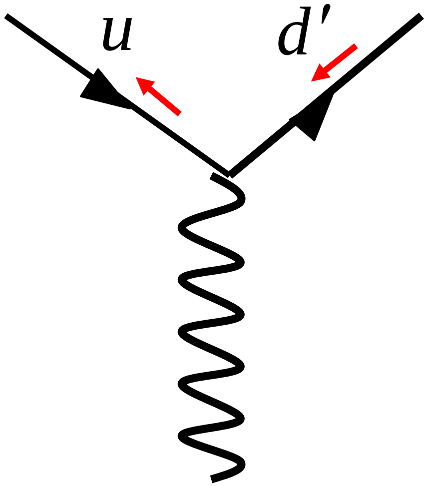


$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad (\nu_e)_R \quad (e)_R$$

$$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \quad (\nu_\mu)_R \quad (\mu)_R$$

$$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L \quad (\nu_\tau)_R \quad (\tau)_R$$

The charged quark weak current is a bit different



$$\begin{pmatrix} u \\ d' \end{pmatrix}_L \qquad (u)_R \qquad (d')_R$$

$$\begin{pmatrix} c \\ s' \end{pmatrix}_L \qquad (c)_R \qquad (s')_R$$

$$\begin{pmatrix} t \\ b' \end{pmatrix}_L \qquad (t)_R \qquad (b')_R$$

Same SU(2) structure for quarks, but lower component of doublet is a **linear combination** of d, s, b - denoted d',s', b'

Why is that?

Write mass terms for up- and down- type quarks as:

$\overline{u}_L^i (M_u)_{ij} u_R^j + h.c.$ with $u^i = (u, c, t)$

$\overline{d}_L^i (M_d)_{ij} d_R^j + h.c.$ with $d^i = (d, s, b)$

We diagonalize the mass terms by unitary transformations on the L and R quark fields:

$u_L^i \rightarrow (U_L^u)^i_j u_L^j$

$d_L^i \rightarrow (U_L^d)^i_j d_L^j$

$u_R^i \rightarrow (U_R^u)^i_j u_R^j$

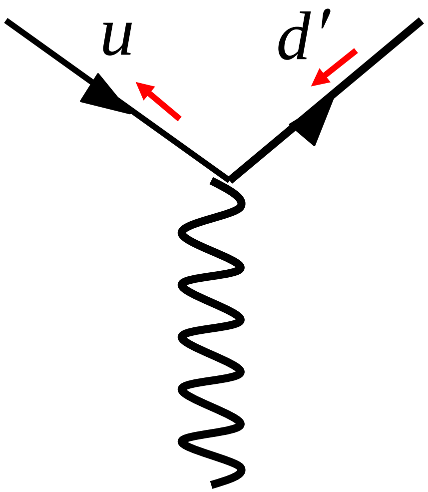
$d_R^i \rightarrow (U_R^d)^i_j d_R^j$

But then the charged weak current becomes off-diagonal:

$$j_w^{+\mu} = \frac{1}{\sqrt{2}} \overline{u}_L^i \gamma^\mu d_L^i \rightarrow \frac{1}{\sqrt{2}} \overline{u}_L^i \gamma^\mu [(U_L^u)^+ U_L^d]_{ij} d_L^j \equiv \frac{1}{\sqrt{2}} \overline{u}_L^i \gamma^\mu V_{ij} d_L^j$$

V is a unitary 3 x 3 matrix, called the Cabibbo-Kobayashi-Maskawa (CKM) matrix.

To summarize: The charged quark weak current is given by



$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$j^{\mu}_{weak} = \frac{1}{\sqrt{2}} (\bar{u} \quad \bar{c} \quad \bar{t}) \gamma^{\mu} \tfrac{1}{2}(1-\gamma^5) V \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Note that the same phenomenon does not happen for quark neutral currents because

$$\bar{u}^i_L \gamma^{\mu} u^i_L \rightarrow \bar{u}^i_L \gamma^{\mu} [(U^u_L)^+ U^u_L]_{ij} u^j_L \equiv \bar{u}^i_L \gamma^{\mu} u^i_L \qquad \text{etc.}$$

-> No tree-level flavor-changing neutral currents (GIM mechanism)

Experimental values of the CKM quark mixing matrix

$$V = \begin{pmatrix} |V_{ud}| = 0.9745 - 0.9760 & |V_{us}| = 0.217 - 0.224 & |V_{ub}| = 0.0018 - 0.0045 \\ |V_{cd}| = 0.217 - 0.224 & |V_{cs}| = 0.9737 - 0.9753 & |V_{cb}| = 0.036 - 0.042 \\ |V_{td}| = 0.004 - 0.013 & |V_{ts}| = 0.035 - 0.042 & |V_{tb}| = 0.9991 - 0.9994 \end{pmatrix}$$

Note that the diagonal elements are approximately 1
while the off-diagonal elements are 1,2 or 3 orders of magnitude smaller

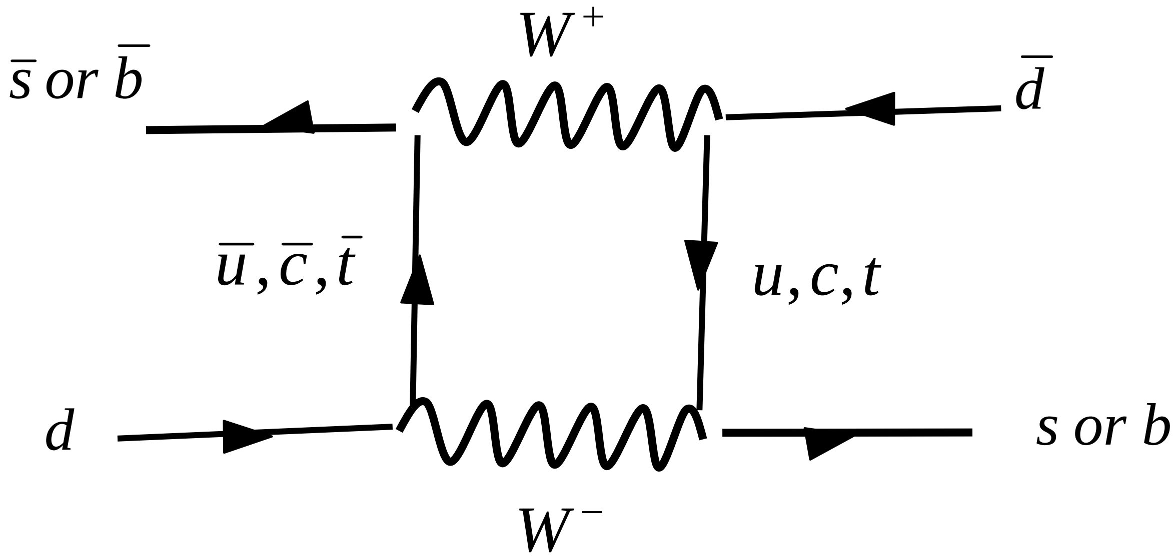
$u \rightarrow d, \quad c \rightarrow s, \quad t \rightarrow b$ transitions are favored

$s \rightarrow u, \quad b \rightarrow c, \quad b \rightarrow u$ transitions are suppressed

-> K and B mesons will be rather long-lived

The mixing happens because up and down quarks both have mass.
Since neutrinos have mass the same mixing is possible in the lepton sector
- called the Maki-Nakagawa-Sakata(-Pontecorvo) [MNS(P)] matrix.

Neutral Meson Mixing via Box Diagrams
(c.f. presentations by Evan Kirby (K) & Adam Edwards (B))



$$K^0 \leftrightarrow \overline{K}^0$$

or

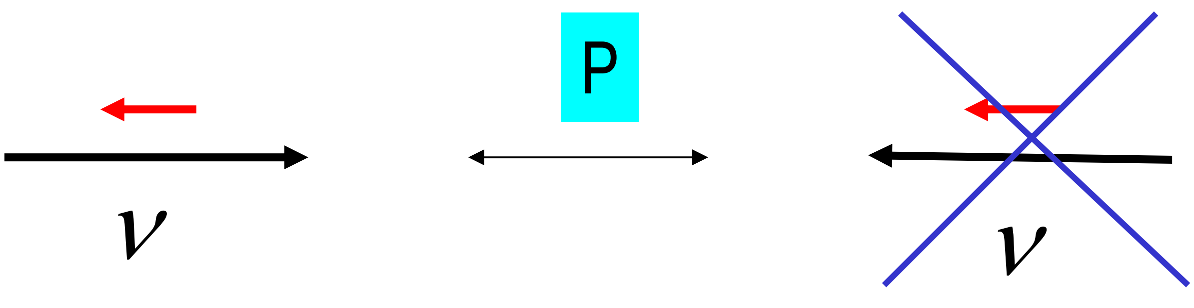
$$B^0 \leftrightarrow \overline{B}^0$$

The weak interactions (and the long lifetimes of K and B) allow neutral mesons to be converted to anti-mesons

CP Violation

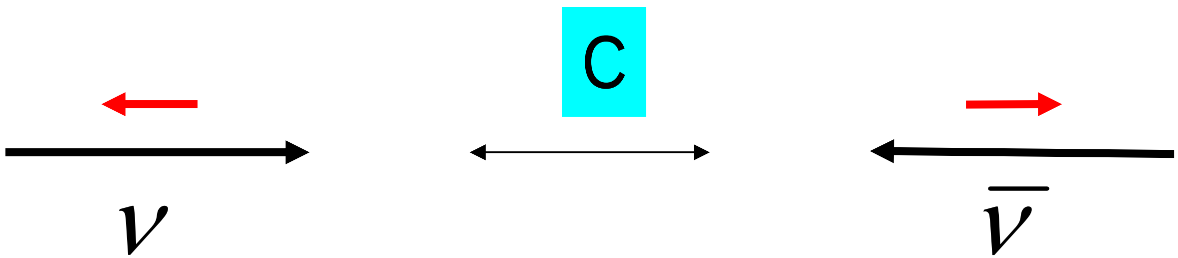
Recall that **parity violation** in the weak interactions was related to the fact that left-handed fermions interact, right-handed fermions don't. Opposite for anti-fermions.
In fact, there is no right-handed neutrino (left-handed anti-neutrino) in the Standard Model.

Parity turns interacting left-handed neutrino into non-interacting right-handed neutrino (which need not exist) -> not a symmetry:



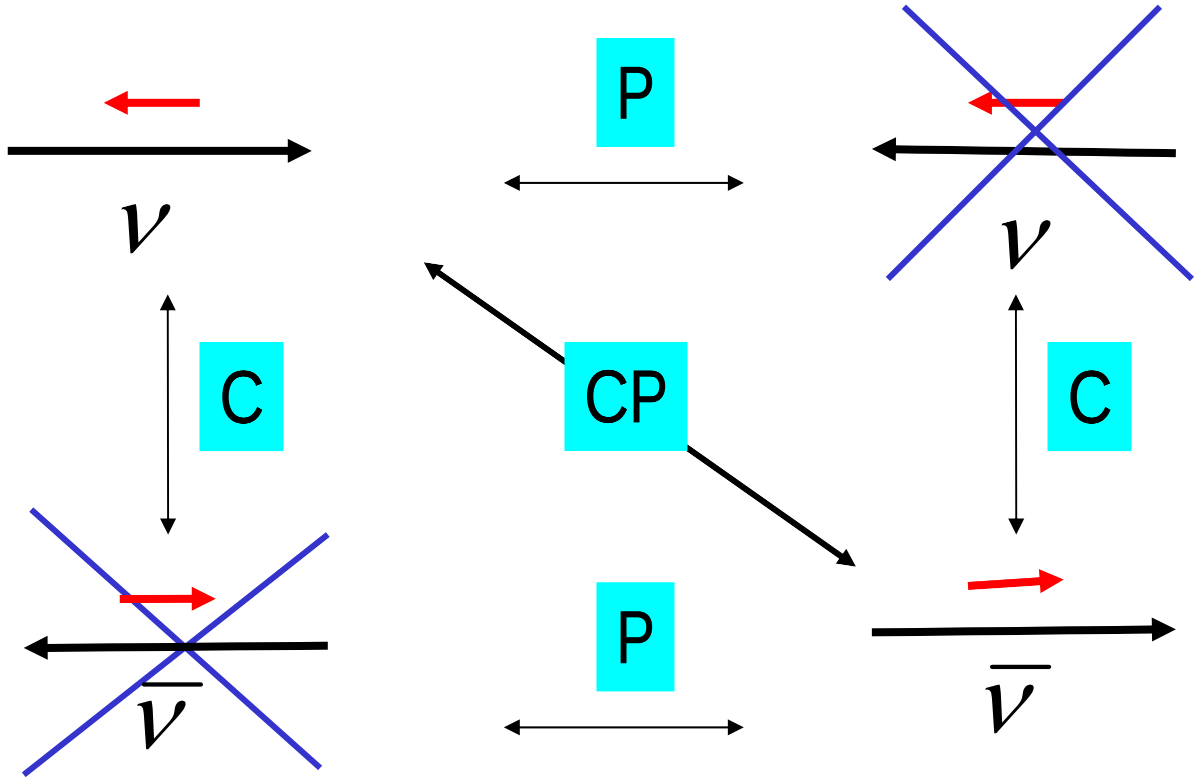
Charge Conjugation Symmetry

Another discrete symmetry is called charge conjugation symmetry (C). This is the operation of turning particle into antiparticle. The charge is reversed but the helicity is unchanged. For convenience we also rotate the neutrino direction around:



Combine C and P to get CP

CP turns left-handed neutrino into right-handed antineutrino which does exist and interacts weakly.
Weak interactions are **almost** invariant under CP.



How is CP violated in the Standard Model?

The only violation (apart from the "strong CP problem", about which we shall say no more) is in the charged current weak interaction, **if** the CKM matrix V contains a **nonzero phase**, so it **cannot be made real**.

$$CP: \quad j_w^{+\mu} = \frac{1}{\sqrt{2}} \bar{u}_L^i \gamma^\mu V_{ij} d_L^j \rightarrow \frac{1}{\sqrt{2}} \bar{d}_L^i \gamma^\mu (V^T)_{ij} u_L^j = j_w^{-\mu} (V \leftrightarrow V^*)$$

Useful approximate form for V (Wolfenstein):

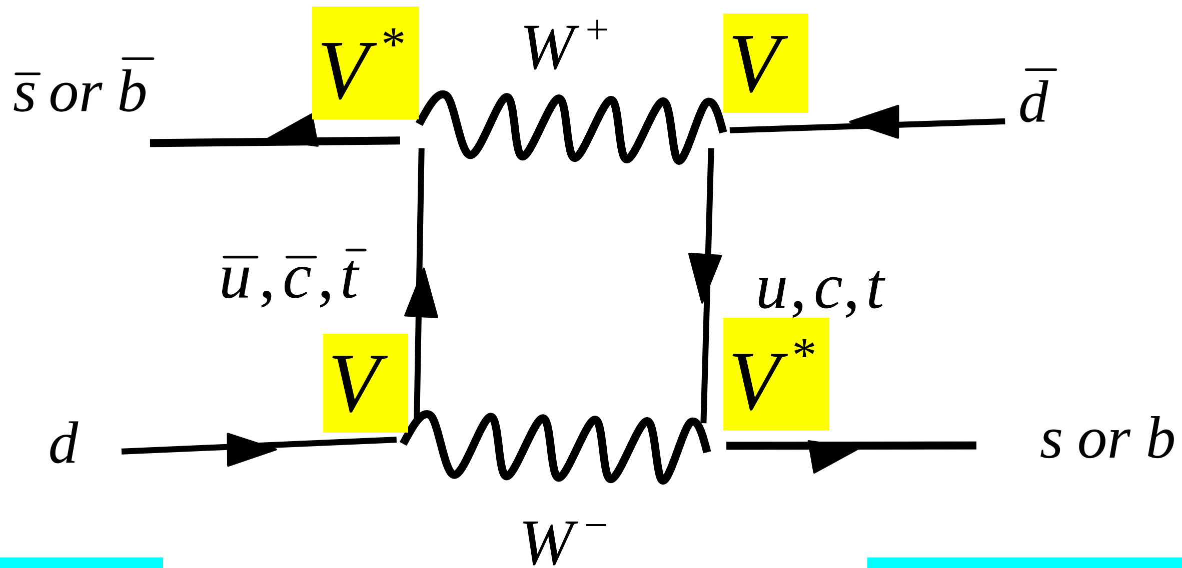
$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \cong \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

Puts only phase, η, into V_{ub} which is known to be small:

$$\lambda \equiv \sin\theta_c = 0.22$$

$$A = 0.8$$

Neutral Meson Mixing revisited



In $K^0 \leftrightarrow \overline{K}^0$ CP violating part goes like $V_{ts}^* V_{td} = -A^2 \lambda^5 (1 - \rho - i\eta)$

(much smaller than CP conserving part)

In $B^0 \leftrightarrow \overline{B}^0$ CP violating part goes like $V_{tb}^* V_{td} = A \lambda^3 (1 - \rho - i\eta)$

(comparable to CP conserving part)

CP symmetry of Interactions

strong	C	P	CP
electromagnetic	C	P	CP
Weak	$\not{C}$	$\not{P}$	$CP(almost)$

Weak interactions almost conserve CP so beginning and final states of weak decays must be eigenstates of CP but in mixing $K^0 \leftrightarrow \overline{K}^0$

$$CP : \left| K^0 \right\rangle \to -\left| \overline{K}^0 \right\rangle \qquad CP : \left| \overline{K}^0 \right\rangle \to -\left| K^0 \right\rangle$$

So the CP eigenstates are linear combinations

$$CP : \left| K_1 \right\rangle = \frac{1}{\sqrt{2}}\left(\left| K^0 \right\rangle - \left| \overline{K}^0 \right\rangle \right) \to +\left| K_1 \right\rangle$$

$$CP : \left| K_2 \right\rangle = \frac{1}{\sqrt{2}}\left(\left| K^0 \right\rangle + \left| \overline{K}^0 \right\rangle \right) \to -\left| K_2 \right\rangle$$

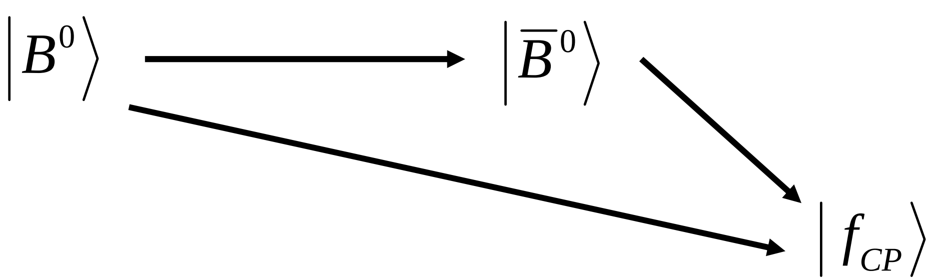
CP Violation in K system (c.f. Evan Kirby's presentation)

Cronin et al.'s observation of $K_L \rightarrow \pi\pi$
can be interpreted as CP violation due to mixing:

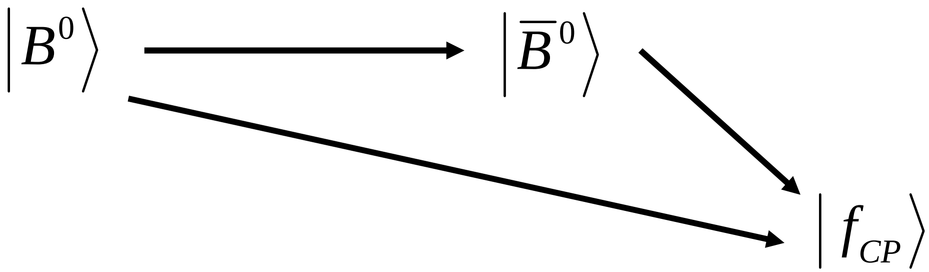
$$|K_L\rangle = \frac{1}{\sqrt{(1+|\varepsilon|^2}}\left(|K_2\rangle + \varepsilon|K_1\rangle\right) \qquad |\varepsilon| = 2.3\times10^{-3}$$

CP Violation in B system

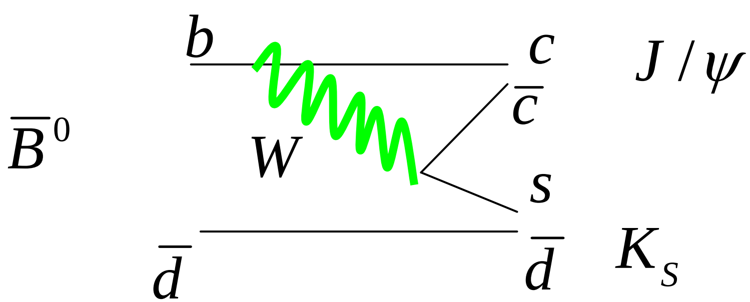
Described in analogous way. However, $|B_S\rangle$ and $|B_L\rangle$ have almost identical lifetimes, so you can't wait for $|B_S\rangle$ to decay away. Instead, look for CP violation in
interference between mixing and decay to CP eigenstate *f*

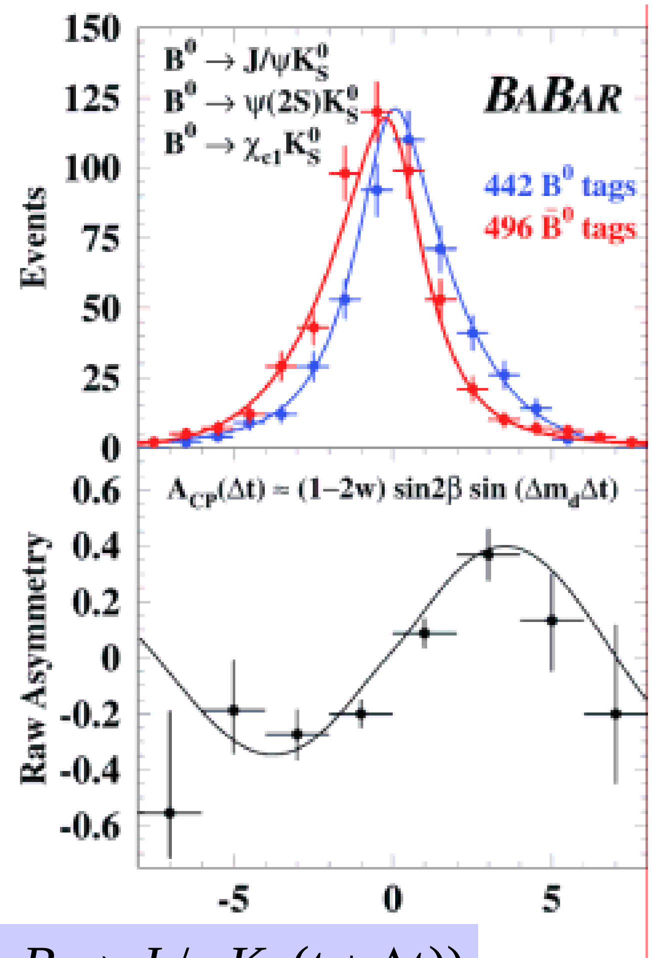


CP Violation in B system (cont.)



“Golden Mode”: $f_{CP} = (J/\psi)\ K_s$



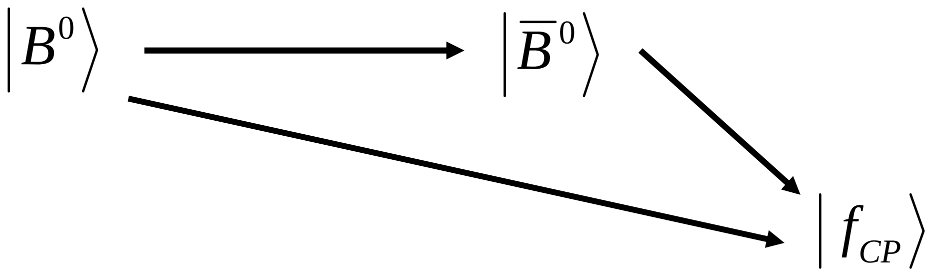


$$A_{CP}(\Delta t) = \frac{N(B^0(t); B \to J/\psi K_S(t+\Delta t)) - N(\overline{B}^0(t); B \to J/\psi K_S(t+\Delta t))}{N(B^0(t); B \to J/\psi K_S(t+\Delta t)) + N(\overline{B}^0(t); B \to J/\psi K_S(t+\Delta t))}$$

$$= \sin 2\beta \sin \Delta m \Delta t$$

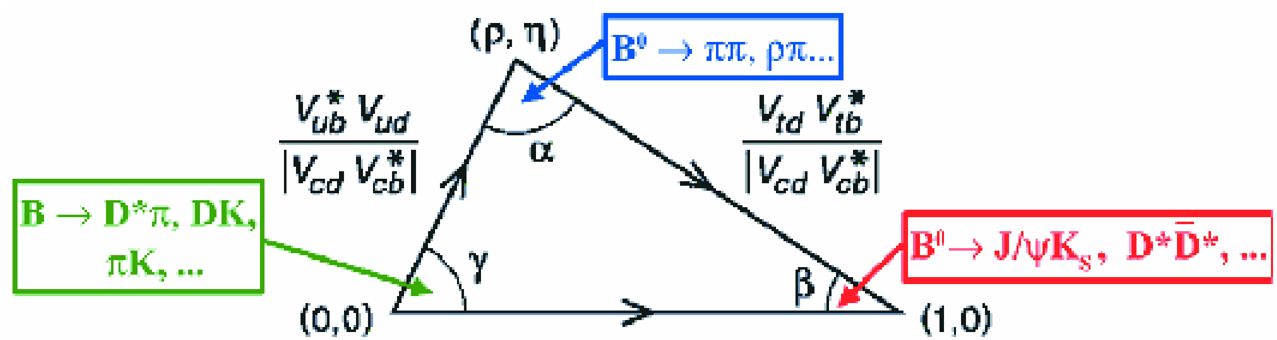
CP odd. Measures one of the angles, β, of the “unitarity triangle”

CP Violation in B system (cont.)



Different f_{CP} asymmetries measure different angles of the "unitarity triangle" - apex is at (ρ,η).

$$V^+V = 1 \quad \Rightarrow V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$$



There are several other ways to measure the sides, etc.
- goal is to overconstrain it, look for a contradiction.
- Some hints already, but nothing definitive yet.

Why is there CP violation?

The laws of physics are symmetric in matter <-> antimatter except for CP violation. So if Big Bang starts without a matter-antimatter asymmetry, we would not exist: there would remain equal amounts of matter and antimatter, which would annihilate efficiently, if not segregated on large scales.

However, we observe universe to made entirely of matter (as far as we can measure or infer it astronomically)

CP violation seems to be required to allow our universe to exist. On the other hand, the CP violation in the Standard Model does **not** appear to be sufficient. This is one of the prime motivations for the CP violation studies being carried out at the SLAC and KEK B factories (and in other experiments now in planning or construction).

Conclusion

We have outlined the basic ingredients of modern particle physics

- Relativistic Quantum Mechanics
- QED, QCD and the Electroweak interaction
- The underlying symmetries to make sense of the structure of the interactions
- The experimental devices used to establish these theories

There are many open questions:

- Where is the Higgs boson?
- Why are there 3 generations?
- Can we understand the origin of matter-antimatter asymmetry?

But we do have a firm foundation for understanding the basic interactions of the constituents of matter:

The Standard Model of Particle Physics: $SU(3)_c \times SU(2)_L \times U(1)_Y$

Final Thoughts