The Electroweak Interaction

Lecture 15 Physics 152/252

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(thanks again to Colin Jessop)

The Weak Force

Recall that there are particle decay processes $a \to b + c$

$$\Gamma = \frac{\langle | M |^2 \rangle}{2m} d\text{LIPS} \qquad \Gamma = \frac{1}{\tau}$$

Since the matrix element $M \propto g^2$

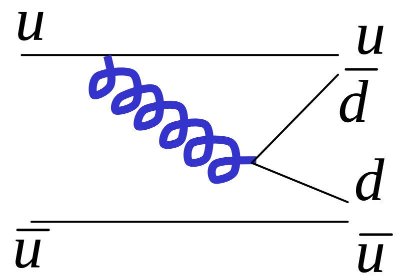
where g is the coupling strength of the interaction,
τ the lifetime of the particle, is

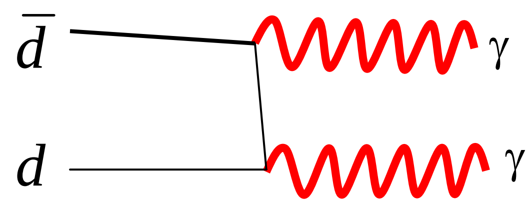
$$\tau \propto \frac{1}{g^4}$$

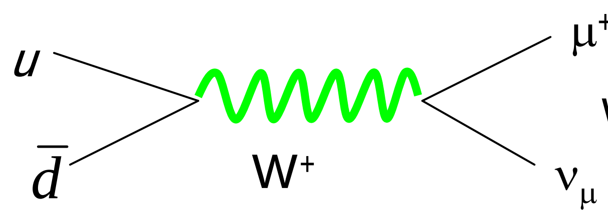
The lifetime of a particle indicates the strength of the decay mechanism

The Weak Force

$a \to b + c$	$\tau \propto \frac{1}{g^4}$	Force	
$\rho^0 \to \pi^+ \pi^-$	10^{-23}s	(diagram 1)	strong
$\pi^0 \to \gamma\gamma$	10^{-15}s	(diagram 2)	EM
$\pi^+ \to \mu^+ \nu_\mu$	10^{-8}s	(diagram 3)	weak







Characteristics of the Weak Force

The time scale of the decay is long. Radioactive decays must proceed by the weak force since the timescale ranges from 10^{-8} s to years

E.g. neutron β decay $$n \rightarrow p e^- \bar{\nu}_e$$ τ_n = 920 seconds

Weak decays often involve neutrinos
- do not interact by the EM force or the strong force
- cannot detect in conventional detectors
- can infer existence from conservation of E,p (Pauli, 1930)

Pauli's letter of the 4th of December 1930

Dear Radioactive Ladies and Gentlemen,

As the bearer of these lines, to whom I graciously ask you to listen, will explain to you in more detail, how because of … the continuous beta spectrum, I have hit upon a desperate remedy to save … the law of conservation of energy. Namely, the possibility that there could exist in the nuclei electrically neutral particles, that I wish to call neutrons, which have spin 1/2 and obey the exclusion principle and which further differ from light quanta in that they do not travel with the velocity of light. The mass of the neutrons should be of the same order of magnitude as the electron mass and in any event not larger than 0.01 proton masses. The continuous beta spectrum would then become understandable by the assumption that in beta decay a neutron is emitted in addition to the electron such that the sum of the energies of the neutron and the electron is constant ...

Unfortunately, I cannot appear in Tubingen personally since I am indispensable here in Zurich because of a ball on the night of 6/7 December.
With my best regards to you, and also to Mr. Back.

Your humble servant
W. Pauli

Neutrinos would not be directly detected for 25 years:
Reines & Cowan, using Savannah River nuclear reactor,

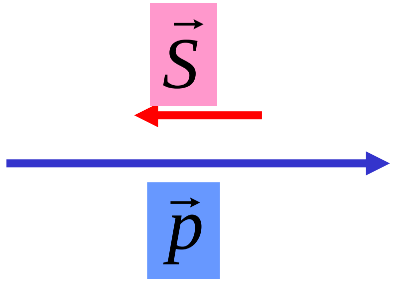
$$\overline{\nu}_e + p \rightarrow e^+ + n$$
$$(n + p \rightarrow d + \gamma)$$

Before discussing the weak interaction we need to review:

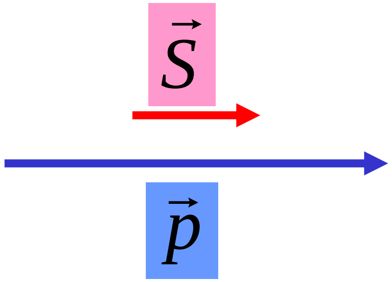
Helicity

Helicity is component of (spin) angular momentum along momentum vector. For fermions, the value is –1/2 or +1/2, depending on whether spin S is antiparallel or parallel to direction of motion p

Left handed -1/2



Right handed +1/2



Helicity (cont.)

You showed (ex. 7.7 of Griffiths) that the solutions of the Dirac equation

$$u^{(\pm)}(p,s) = \sqrt{\frac{E+m}{2p(p \pm p_z)}} \begin{pmatrix} u_A \\ \frac{\pm p}{E+m} u_A \end{pmatrix}$$

where

$$u_A = \begin{pmatrix} p_z \pm p \\ p_x + ip_y \end{pmatrix} \qquad p \equiv \; |\vec{\mathrm{p}}|$$

are **helicity eigenstates**:

$$\frac{1}{2}(\hat{p} \cdot \Sigma) u^{(\pm)}(p,s) = \pm \frac{1}{2} u^{(\pm)}(p,s)$$

For antiparticles the relation is reversed (because v(p) ~ u(-p)):

$$\frac{1}{2}(\hat{p} \cdot \Sigma) v^{(\pm)}(p,s) = \mp \frac{1}{2} v^{(\pm)}(p,s)$$

The γ^5 Matrix

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

satisfies

$$\gamma^{5+} = \gamma^5 \qquad \left(\gamma^5\right)^2 = \gamma^5 \qquad \gamma^5\gamma^\mu + \gamma^\mu\gamma^5 = 0$$

In the limit m=0 (E>>m), we saw earlier that

$$u^{(\pm)}(p,s) \cong \sqrt{\frac{1}{2(p \pm p_z)}}\begin{pmatrix} u_A \\ \pm u_A \end{pmatrix}$$

obeys

$$\gamma^5 u^{(\pm)}(p,s) = \pm u^{(\pm)}(p,s)$$

Also have

$$\gamma^5 v^{(\pm)}(p,s) = \pm v^{(\pm)}(p,s)$$

Helicity Projection Operator

Thus the following "chirality" projection operators are also helicity projection operators for m=0:

$$P_R = \frac{1}{2}(1+\gamma^5)$$

$$P_L = \frac{1}{2}(1-\gamma^5)$$

They satisfy

$$P_L^2 = P_R^2 = 1 \qquad P_L P_R = P_R P_L = 0 \qquad P_L + P_R = 1$$

$$\gamma^\mu P_L = P_R \gamma^\mu \qquad \gamma^\mu P_R = P_L \gamma^\mu$$

For m=0, $\boldsymbol{P_L}$ projects onto helicity –1/2 fermions but helicity +1/2 anti-fermions.

Helicity and the Electromagnetic Interaction

We can use the projection operators to split the electromagnetic current into 2 pieces:

$$j^\mu = e\overline{\psi}\gamma^\mu\psi = e(\overline{\psi}_L + \overline{\psi}_R)\gamma^\mu(\psi_L + \psi_R)$$

where $\psi_L = \tfrac{1}{2}(1-\gamma^5)\psi = P_L\psi$ $\qquad$ $\psi_R = P_R\psi$

$$\overline{\psi}_L = \psi^+ \tfrac{1}{2}(1-\gamma^5)^+\gamma^0 = \psi^+ \tfrac{1}{2}(1-\gamma^5)\gamma^0 = \overline{\psi}P_R$$

Since $\qquad P_R\gamma^\mu P_R = \gamma^\mu P_L P_R = 0$

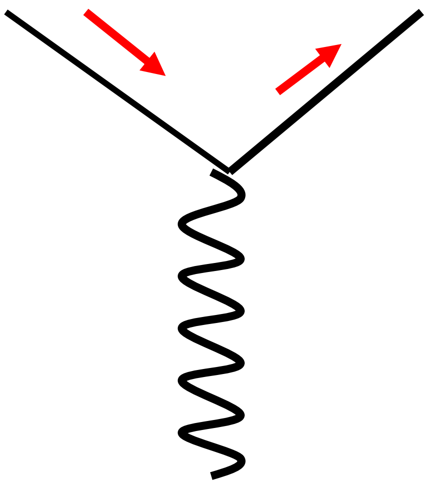
we have $\qquad \overline{\psi}_L\gamma^\mu\psi_R = 0 = \overline{\psi}_R\gamma^\mu\psi_L$

And so

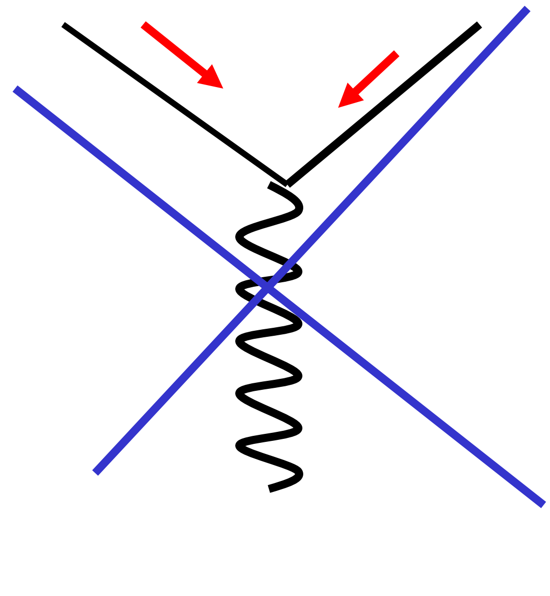
$$j^\mu = \overline{\psi}_L\gamma^\mu\psi_L + \overline{\psi}_R\gamma^\mu\psi_R$$

Helicity and the Electromagnetic Interaction

$$j^{\mu} = \overline{\psi}_L \gamma^{\mu} \psi_L + \overline{\psi}_R \gamma^{\mu} \psi_R$$

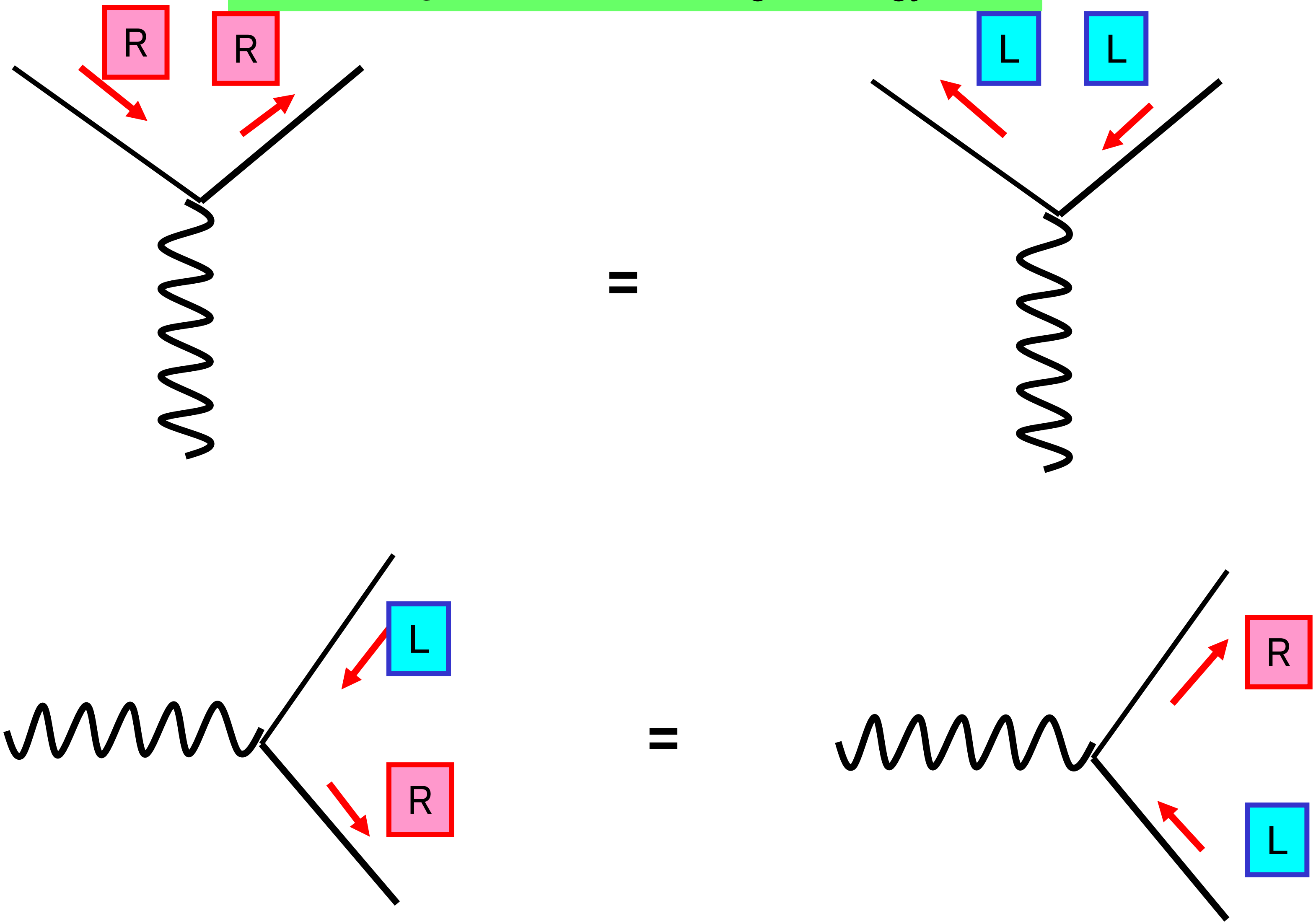


not



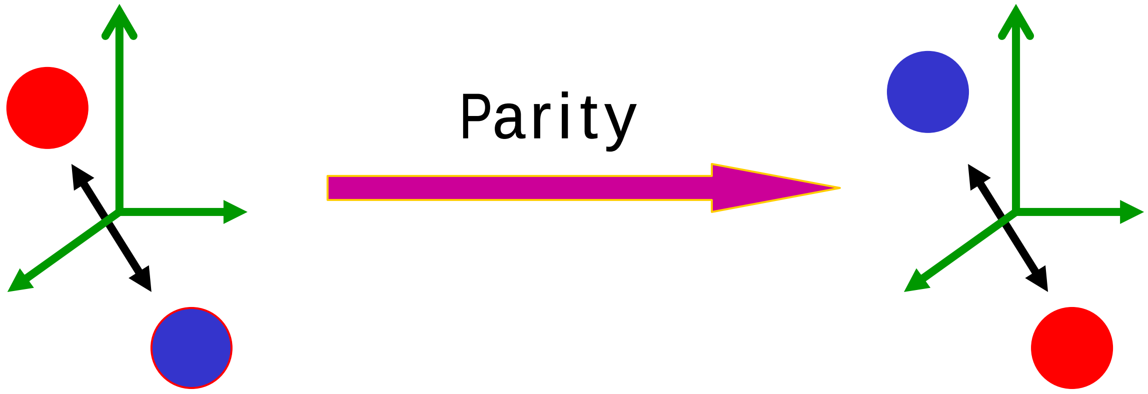
Helicity is conserved in the electromagnetic interaction in high energy (m=0) limit

Allowed QED vertices in high energy limit



Equalities are due to parity

The Weak Interaction and Parity



$$r = (x, y, z) \rightarrow r' = (-x, -y, -z)$$ i.e mirror reflection

Parity is the inversion of the system through the origin

Parity

Position/Momentum are odd under parity

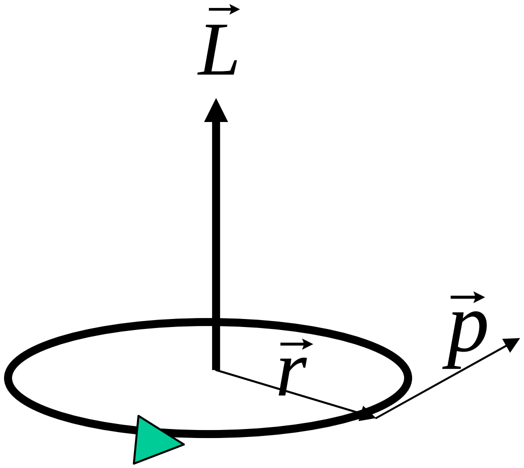
$$P : \vec{r} \rightarrow -\vec{r} \qquad P : \vec{p} \rightarrow -\vec{p}$$

Angular momentum/spin is even under parity
- said to be an “axial vector”

$$P : \vec{L} = \vec{r} \times \vec{p} \rightarrow \vec{L} = -\vec{r} \times -\vec{p}$$

Helicity:

$$P : \vec{S} \cdot \hat{p} \rightarrow -\vec{S} \cdot \hat{p}$$

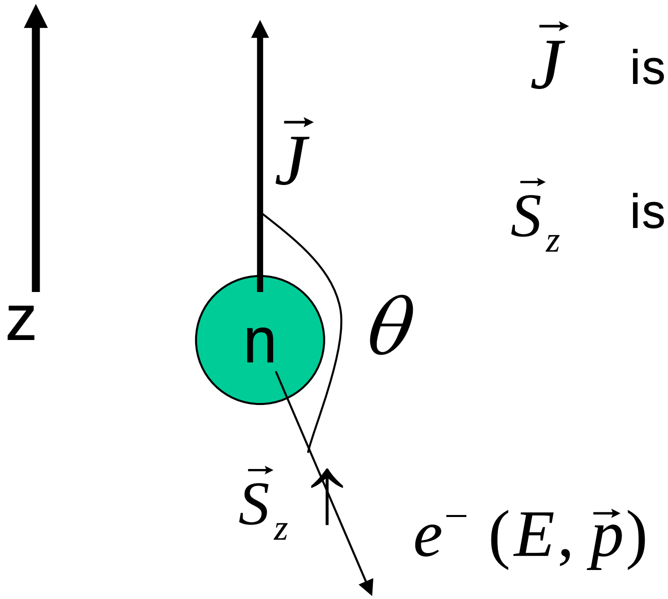


So Helicity is reversed under parity:

$$L \Leftrightarrow R$$

Parity and the Weak Interaction

Consider β decay $n \rightarrow p e^- \overline{\nu}_e$



$\vec{J}$ is *n* spin direction, oriented along z by *B* field

$\vec{S}_z$ is component of electron spin in z direction

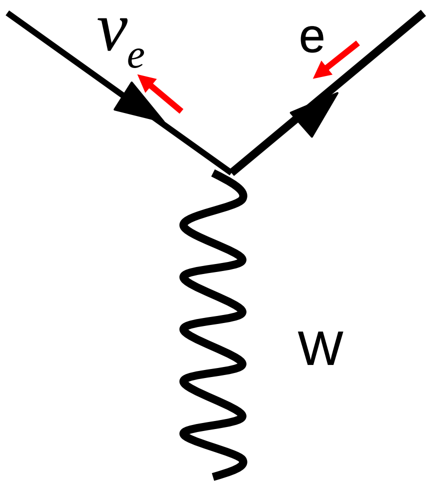
S_z / decay angle distribution is found to be $\propto 1 - \frac{\vec{S}_z \cdot \vec{p}}{E} = 1 - \frac{v}{c}\cos\vartheta$

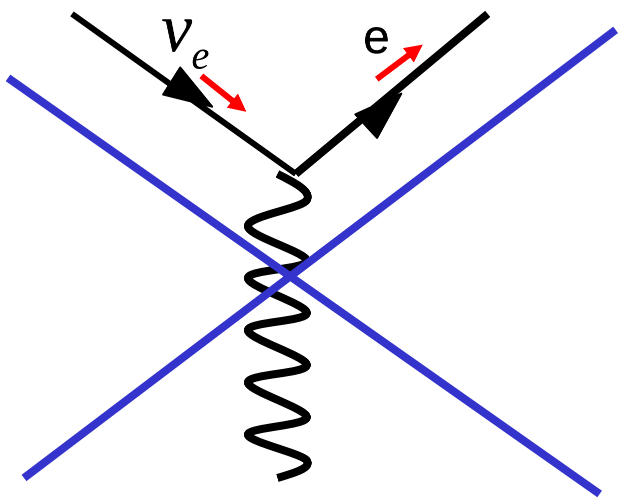
This is odd under Parity

$$P : (1 - \frac{\vec{S}_z \cdot \vec{p}}{E} = 1 - \frac{v}{c}\cos\vartheta) \rightarrow (1 + \frac{\vec{S}_z \cdot \vec{p}}{E} = 1 + \frac{v}{c}\cos\vartheta)$$

For v=c, electron is never emitted in the direction of its spin (cosθ=1)!

The charged weak interaction violates parity **maximally**





By analogy to EM we associate the charged weak interactions with a current, which is purely left-handed:

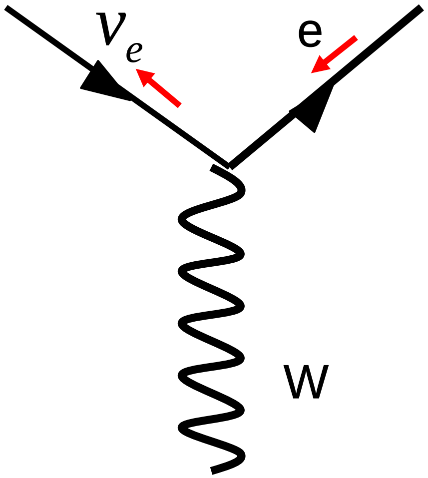
$$j_W^{-\mu} = \frac{1}{\sqrt{2}} \overline{\psi}_L(e) \gamma^\mu \psi_L(\nu_e) \equiv \frac{1}{\sqrt{2}} \overline{e}_L \gamma^\mu \nu_L$$

(also $j_W^{+\mu} = \frac{1}{\sqrt{2}} \overline{\nu}_L \gamma^\mu e_L$)

The charged weak interaction only couples to left-handed leptons (e,μ,τ,ν_i).
(Also, only couples to left-handed quarks.)
It couples only to right-handed anti-fermions.

The right handed neutrino (if it exists!) does not interact weakly (or strongly, or electromagnetically) -> "sterile" neutrino.

The charged weak current



$$E_L \equiv \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \qquad (\nu_e)_R \qquad (e)_R$$

?

SU(2) symmetry relates the left handed electron and neutrino. The left handed components form a doublet representation and the right handed components are singlets (non-interacting). Called weak isospin symmetry. We can derive the interaction from the gauge principle:

$$L = i\bar{E}_L \gamma^\mu (\partial_\mu - igA_\mu^a T^a) E_L$$

$$= i\bar{E}_L \gamma^\mu E_L + g(W_\mu^+ j_w^{+\mu} + W_\mu^- j_w^{-\mu} + W_\mu^3 j_w^{3\mu})$$

SU(2) generators:

$T^a = \sigma_a / 2$

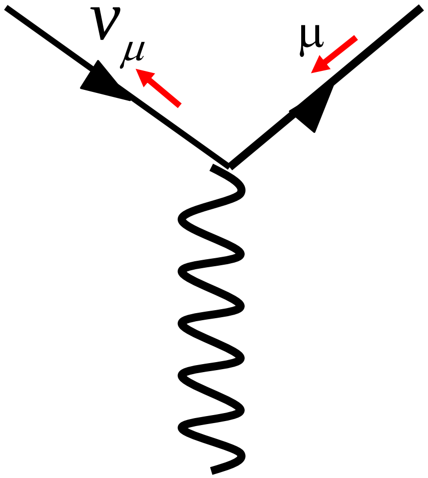
σ_a = Pauli matrices

$g \equiv g_w$

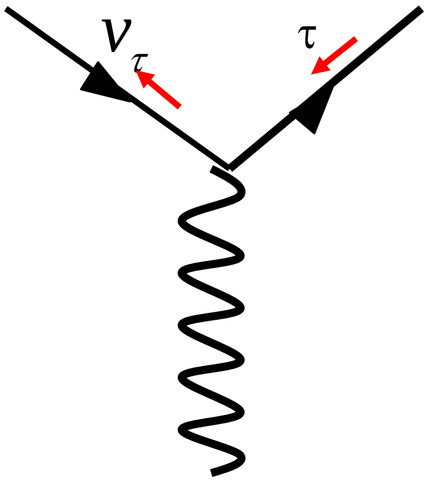
return to later

$$W_\mu^\pm = \tfrac{1}{\sqrt{2}}(A_\mu^1 \mp iA_\mu^2)$$

The charged weak current has exactly the same form for the heavier leptons

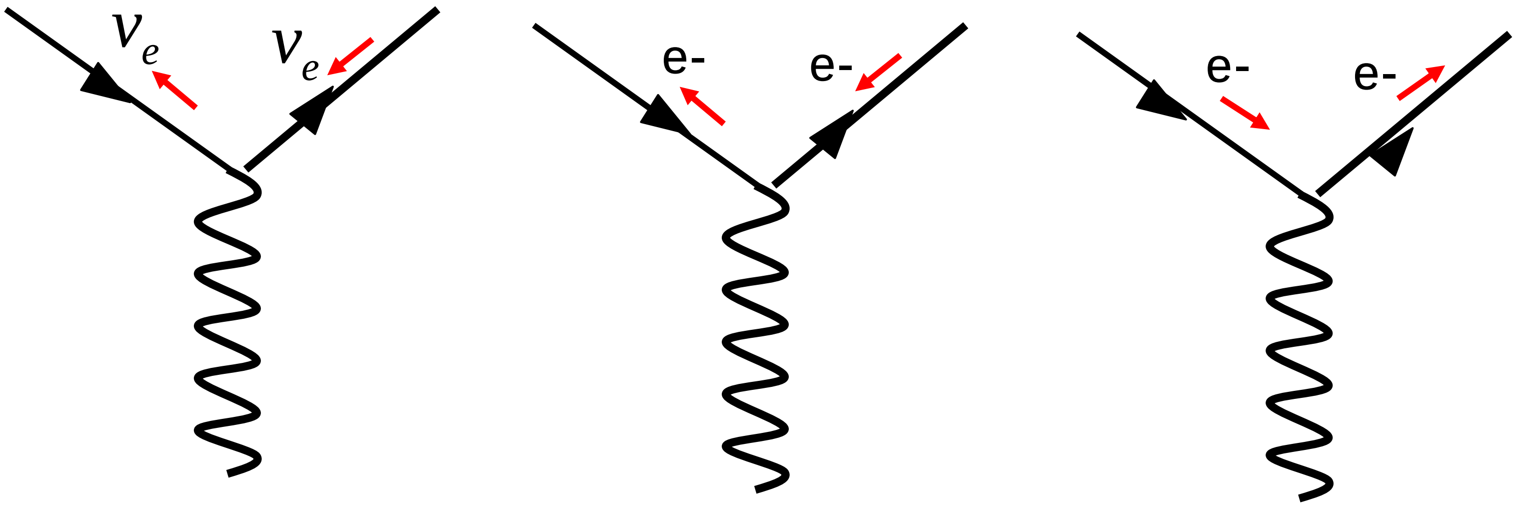


$$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \qquad (\nu_\mu)_R \qquad (\mu)_R$$



$$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L \qquad (\nu_\tau)_R \qquad (\tau)_R$$

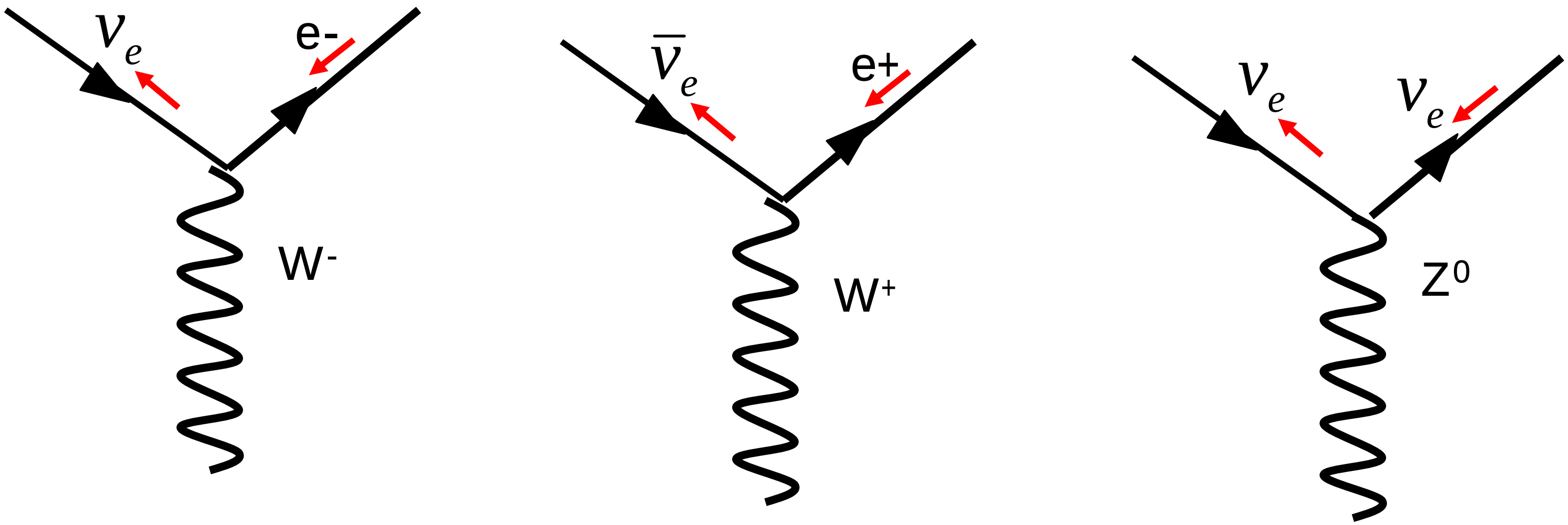
The weak neutral current



Experimental evidence showed that there is also a weak neutral current that couples to right and left-handed charged leptons, but only left-handed neutrinos

The $SU(2)_L \times U(1)_Y$ group

There are three vector bosons W^+, W^-, and Z^0.
They are different from the photon and gluon in that they have mass.



They correspond to the generators T^a of SU(2) = angular momentum algebra.
W^+ corresponds to $J^+ = J_x + iJ_y$ W^- to $J^- = J_x - iJ_y$ and W^3 to J_z.
Z^0 is a mixture of W^3 and the $U(1)_Y$ gauge boson, B^0, which couples to both left and right.
This is why the Z^0 couplings are not purely left-handed.
The other linear combination is the photon, γ.

The Higgs mechanism in brief (c.f. Kathryn Todd's presentation)

1) Postulate a scalar field ϕ whose potential energy forces it to lie away from the origin, $V(\phi) = \lambda \, (|\phi|^2 - v^2)^2$, $|\phi| = v \;\; (+\, \delta\phi\,)$

2) Give ϕ gauge interactions under $SU(2)_L \times U(1)_Y$:

$$L_{\text{Higgs}} = |\, \partial_\mu \phi - igA_\mu^a T^a \phi - ig' Y_H B_\mu \phi \,|^2$$

SU(2) doublet with $Y_H = +\frac{1}{2}$

Inserting $\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \text{v} \end{pmatrix}$ turns L_{Higgs} into gauge boson mass terms:

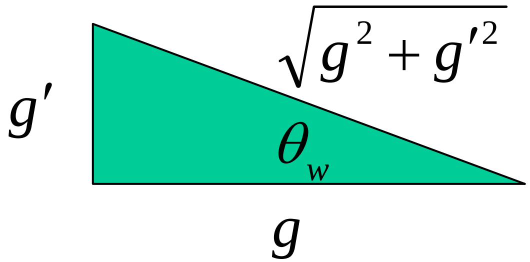
$$L_{\text{Higgs}} \Rightarrow \tfrac{1}{8}\,(0 \quad \text{v})(gA_\mu^a \sigma^a + g'B_\mu)(gA_\mu^b \sigma^b + g'B_\mu) \begin{pmatrix} 0 \\ \text{v} \end{pmatrix}$$

$$= \frac{1}{2}\frac{\text{v}^2}{4}\left[g^2 (A_\mu^1)^2 + g^2 (A_\mu^2)^2 + (-gA_\mu^3 + g'B_\mu)^2\right] = m_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} m_Z^2 Z_\mu Z^\mu$$

where $W_\mu^\pm = \tfrac{1}{\sqrt{2}}(A_\mu^1 \mp iA_\mu^2)$ $\quad Z_\mu = \frac{1}{\sqrt{g^2 + g'^2}}(gA_\mu^3 - g'B_\mu)$

with masses $m_W = g\frac{\text{v}}{2}$ $\quad m_Z = \sqrt{g^2 + g'^2}\,\frac{\text{v}}{2}$

Rewrite formulas using the Weak Mixing Angle



$$\sin\theta_w = \frac{g'}{\sqrt{g^2 + g'^2}}$$

$$\cos\theta_w = \frac{g}{\sqrt{g^2 + g'^2}}$$

$$Z_\mu = \cos\theta_w A_\mu^3 - \sin\theta_w B_\mu$$

Photon (m=0) is orthogonal combination, $$A_\mu = \sin\theta_w A_\mu^3 + \cos\theta_w B_\mu$$

To determine how fermions couple to gauge bosons, rewrite covariant derivative in terms of mass eigenstates:

$$D_\mu = \partial_\mu - igA_\mu^a T^a - ig'YB_\mu$$

$$= \partial_\mu - i\frac{g}{\sqrt{2}}(W_\mu^+ T^+ + W_\mu^- T^-) - i\frac{g}{\cos\theta_w} Z_\mu (T^3 - Q\sin^2\theta_w) - ieQA_\mu$$

with $Q = T^3 + Y$ and $e = g\sin\theta_w$

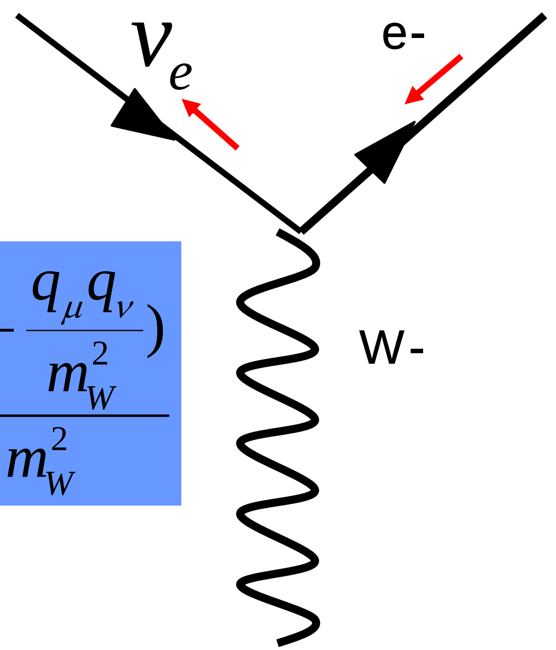
Gauge boson masses become

$$m_W = \frac{\mathrm{ev}}{2\sin\theta_w}$$

$$m_Z = \frac{\mathrm{ev}}{2\sin\theta_w \cos\theta_w}$$

The Feynman rules for the leptonic charged weak interaction

vertex $$\frac{-ie}{\sqrt{2}\sin\theta_w}\gamma^{\mu}\cdot\tfrac{1}{2}(1-\gamma^5)$$

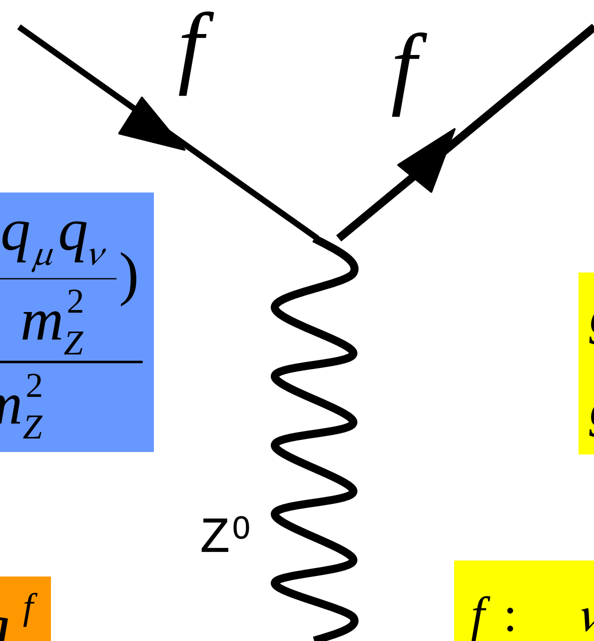


propagator $$\frac{i\left(-g_{\mu\nu}+\frac{q_\mu q_\nu}{m_W^2}\right)}{q^2-m_W^2}$$

Can calculate as for QED but with vertex and propagator modification

The weak neutral current (for any fermion)

vertex $\dfrac{-ie}{\sin\theta_w \cos\theta_w}\gamma^\mu\left[g_L^f \tfrac{1}{2}(1-\gamma^5) + g_R^f \tfrac{1}{2}(1+\gamma^5)\right]$



Let $s_w \equiv \sin^2\theta_w$

propagator $\dfrac{i\left(-g_{\mu\nu} + \dfrac{q_\mu q_\nu}{m_Z^2}\right)}{q^2 - m_Z^2}$

$$g_L^f = I_3^f - Q^f \sin^2\theta_w$$
$$g_R^f = -Q^f \sin^2\theta_w$$

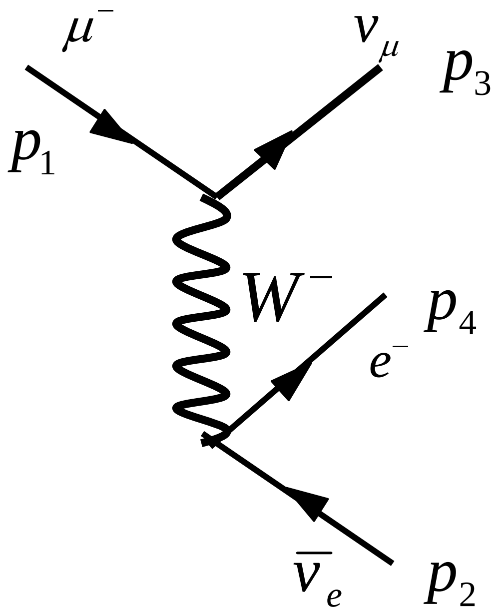
Note: $c_V = g_L^f + g_R^f$

$c_A = g_L^f - g_R^f$

f:	ν	e	u	d
g_L^f:	$\frac{1}{2}$	$(-\frac{1}{2}+s_w)$	$(\frac{1}{2}-\frac{2}{3}s_w)$	$(-\frac{1}{2}+\frac{1}{3}s_w)$
g_R^f:	0	s_w	$-\frac{2}{3}s_w$	$\frac{1}{3}s_w$

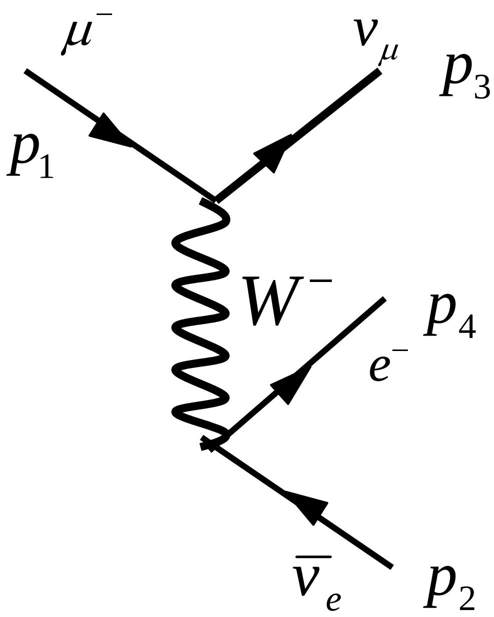
The decay of the muon

$$\mu^- \to e^- \overline{\nu}_e \nu_\mu$$



$$M = \frac{e^2}{2\sin^2\theta_w}\overline{u}_L(p_4)\gamma^\mu u_L(p_2)\frac{(g_{\mu\nu} - \frac{q_\mu q_\nu}{m_w^2})}{q^2 - m_w^2}\overline{u}_L(p_3)\gamma^\nu u_L(p_1)$$

The decay of the muon



Simplifies because M_W = 80 GeV much larger than q< (100 MeV).

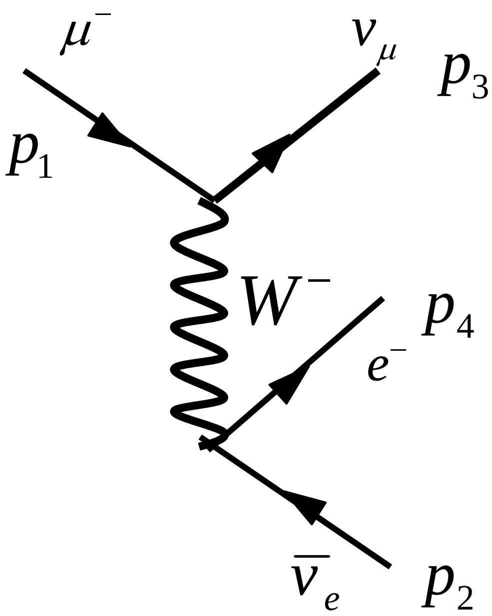
Weak force is weak because boson propagator is massive, not because coupling strength is weak.

$$M = \frac{e^2}{2m_w^2 \sin^2\theta_w} \overline{u}_L(p_4)\gamma^\mu u_L(p_2)\ \overline{u}_L(p_3)\gamma_\mu u_L(p_1)$$

Except for overall factors, M has same form as QED helicity amplitude worked out earlier in class (in $p_i \cdot p_j$ form):

$$M = \frac{2e^2}{m_w^2 \sin^2\theta_w}\sqrt{(p_1 \cdot p_2)(p_3 \cdot p_4)}$$

The decay of the muon



After squaring M, integrating over phase space (see Griffiths):

$$\Gamma_\mu = \frac{e^4 m_\mu^5}{12(8\pi)^3 m_w^4 \sin^4\theta_w}$$

Since $m_W = \frac{ev}{2\sin\theta_w}$ Γ_μ only depends on v.

Often rewritten in terms of Fermi constant, $G_F = \frac{1}{\sqrt{2}v^2}$

as $\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3}$

Experimental value of muon lifetime,

$\tau_\mu = 1/\Gamma_\mu = 2.2 \times 10^{-6}$ sec

used to determine $G_F = 1.166 \times 10^{-5}\ \text{GeV}^{-2}$ or $v = 246\ \text{GeV}$