1.) $\gamma e \to \gamma e$

$$p_e = (E_e, 0, 0, -\sqrt{E_e^2 - m^2})$$

$$p_\gamma = (E_\gamma, 0, 0, E_\gamma)$$

$$p_{tot}^2 = (E_e + E_\gamma)^2 - \left(\sqrt{E^2 - m^2} - E_\gamma\right)^2$$

$$= 2E_\gamma E_e + 2E_\gamma E_e\left(1 - \frac{1}{2}\frac{m^2}{E_e^2}\right) + m^2$$

$$E_{cm}^2 = 4E_\gamma E_e + m^2 - \frac{E_\gamma}{E_e}m^2$$

$$\beta = \frac{|\vec{p}|}{E} = \frac{\sqrt{E_e^2 - m^2} - E_\gamma}{E_e + E_\gamma}$$

$$\gamma = (1 - \beta^2)^{-1/2} = \frac{E_e + E_\gamma}{E_{cm}}$$

in CM after collision,

$$p_{e_f} = (E_{e_f}, -\sqrt{E_{e_f}^2 - m^2}\,\hat{n})$$

$$p_{\gamma_f} = (E_{\gamma_f}, E_{\gamma_f}\hat{n})$$

$$E_{e_f} + E_{\gamma_f} = E_{cm}$$

$$E_{\gamma_f} = \sqrt{E_{e_f}^2 - m^2}$$

$$E_{\gamma_{f_{cm}}} = \frac{E_{cm}^2 - m^2}{2E_{cm}}$$

$$p_{\gamma_{f_{cm}}} = E_{\gamma_{f_{cm}}}(1, 0, \sin\theta, \cos\theta)$$

$$p_{\gamma_{f_{lab}}} = E_{\gamma_{f_{cm}}}\begin{pmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ \sin\theta \\ \cos\theta \end{pmatrix} \Rightarrow E_{\gamma_{f_{lab}}} = E_{\gamma_{f_{cm}}}(\gamma - \beta\gamma\cos\theta)$$

$E_{\gamma_{f_{lab}}}$ maxed at $\theta = \pi$ (& not $\theta = 0$ b/c $E_e > E_\gamma$ & γ comes out in opposite ~~direction~~ w/ which it went in)

$$E_{\gamma_{f_{lab_{max}}}} = E_{\gamma_{f_{cm}}}\gamma(1+\beta) = \frac{E_{cm}^2 - m^2}{2E_{cm}}\,\frac{E_e + E_\gamma}{E_{cm}}\,\frac{E_e + \sqrt{E_e^2 - m^2}}{E_e + E_\gamma}$$

$$E_{\gamma_{f_{lab_{max}}}} \simeq E_e\frac{E_{cm}^2 - m^2}{E_{cm}^2}$$

(for $E_\gamma \ll E_e$)

$$\simeq E_e\frac{4E_\gamma E_e}{4E_\gamma E_e + m^2}$$

$$E_{\gamma_{f_{lab_{max}}}} \simeq E_e\frac{1}{1 + \frac{m^2}{4E_e E_\gamma}}$$

$$\lambda = 1.06\,\mu m$$

$$E_\gamma = \frac{hc}{\lambda} = \frac{2\pi}{\lambda} \simeq 1.16 \cdot 10^{-9}\,GeV$$

$$m \simeq 5.11 \cdot 10^{-4}\,GeV$$

if $E_e = 50\,GeV$, $E_{\gamma_f} \simeq 23.5\,GeV$

if $E_e = 250\,GeV$, $E_{\gamma_f} \simeq 204\,GeV$

2.) $\sigma(\nu N) = G_F^2 s$

assume ν has cross sectional area σ

mean free path λ is st volume $\lambda\sigma$ = volume per nucleon of earth

$$\lambda\sigma = \frac{\frac{4}{3}\pi\left(\frac{d_E}{2}\right)^3}{(6\cdot 10^{24}\,\text{kg})\frac{6\cdot 10^{26}}{\text{kg}}}$$ ← vol of earth ← # of nucleons

$$\lambda \simeq \frac{\frac{4}{3}\pi\frac{1}{8}(1.3\cdot 10^4\,\text{km})^2(d_E)}{36\cdot 10^{50}} \frac{1}{(1.2\cdot 10^{-5}\,\text{GeV}^{-2})^2} \frac{1}{(.197\,\text{GeV}\cdot 10^{-18}\,\text{km})^2}\cdot\frac{1}{s}$$

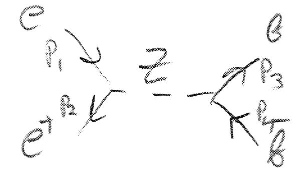
$$\simeq \frac{4400}{(s/\text{GeV}^2)} d_E$$

$s = 2m_p E_\nu + m_p^2 \Rightarrow$ if $E_\nu \simeq 1\,\text{GeV}$ (& $m_p \simeq 1\,\text{GeV}$) $\Rightarrow s \simeq 3\,\text{GeV}^2$

$\Rightarrow \lambda \simeq 1500\, d_E$

for $\lambda = d_E$, $s = 4400\,\text{GeV}^2 \Rightarrow E_\nu \simeq \frac{s}{2m_p} \simeq 2.2\,\text{TeV}$

3)



(assuming all c's are $\mathbb{R}$)

$$\langle |M|^2 \rangle \propto tr\left(\not{p}_2 \gamma^\mu (c_v^e - c_A^e \gamma^5) \not{p}_1 \gamma^\nu (c_v^e - c_A^e \gamma^5)\right)$$

$$\times tr\left(\not{p}_3 \gamma_\mu (c_v^b - c_A^b \gamma^5) \not{p}_4 \gamma_\nu (c_v^b - c_A^b \gamma^5)\right)$$

$$\propto \left(({c_v^e}^2 + {c_A^e}^2) tr(\not{p}_2 \gamma^\mu \not{p}_1 \gamma^\nu) - 2 c_v^e c_A^e \, tr(\not{p}_2 \gamma^\mu \not{p}_1 \gamma^\nu \gamma^5)\right)$$

$$\times \left(({c_v^b}^2 + {c_A^b}^2) tr(\not{p}_3 \gamma_\mu \not{p}_4 \gamma_\nu) - 2 c_v^b c_A^b \, tr(\not{p}_3 \gamma_\mu \not{p}_4 \gamma_\nu \gamma^5)\right)$$

$$tr(\not{p} \gamma^\alpha \not{q} \gamma^\beta) = 4(p^\alpha q^\beta - g^{\alpha\beta}(p \cdot q) + q^\alpha p^\beta)$$

$$tr(\not{p} \gamma^\alpha \not{q} \gamma^\beta \gamma^5) = 4i \epsilon^{\gamma\alpha\delta\beta} p_\gamma q_\delta$$

let $w = c_v^e$, $x = c_A^e$, $y = c_v^b$, $z = c_A^b$

$$\langle |M|^2 \rangle \propto (w^2 + x^2)(y^2 + z^2)\left(2(p_2 \cdot p_3)(p_1 \cdot p_4) + 2(p_2 \cdot p_4)(p_1 \cdot p_3)\right)$$

$$- 4wxyz \, \epsilon^{\alpha\mu\beta\nu} \epsilon_{\gamma\mu\delta\nu} \, p_{2\alpha} p_{1\beta} p_3^\gamma p_4^\delta$$

$$\propto (w^2 + x^2)(y^2 + z^2)\left(\frac{u^2}{2} + \frac{t^2}{2}\right)$$

$$+ 8wxyz\left((p_2 \cdot p_3)(p_1 \cdot p_4) - (p_2 \cdot p_4)(p_1 \cdot p_3)\right)$$

$$\propto (w^2 + x^2)(y^2 + z^2)(u^2 + t^2) + 4wxyz(u^2 - t^2)$$

$$u \propto 1 + \cos\theta$$

$$t \propto 1 - \cos\theta$$

3.)

$$u^2+t^2=(1+\cos\theta)^2+(1-\cos\theta)^2=2+2\cos^2\theta$$
$$u^2-t^2=(1+\cos\theta)^2-(1-\cos\theta)^2=4\cos\theta$$

$$\frac{d\sigma}{d\cos\theta}\propto\langle|M|^2\rangle\propto 2(c_v^{e\,2}+c_A^{e\,2})(c_v^{b\,2}+c_A^{b\,2})(1+\cos^2\theta)$$
$$+16\,c_v^e c_A^e c_v^b c_A^b\cos\theta$$

$$\frac{d\sigma}{d\cos\theta}\propto 1+\cos^2\theta+2\left(\frac{2c_v^e c_A^e}{c_v^{e\,2}+c_A^{e\,2}}\right)\left(\frac{2c_v^b c_A^b}{c_v^{b\,2}+c_A^{b\,2}}\right)\cos\theta$$

$$=1+\cos^2\theta+2A_eA_b\cos\theta$$

$$\text{for } A_i=\frac{2c_v^i c_A^i}{c_v^{i\,2}+c_A^{i\,2}}=\frac{g_L^{i\,2}-g_R^{i\,2}}{g_L^{i\,2}+g_R^{i\,2}}$$

$$A_{FB}=\frac{\int_0^1 d\cos\theta\,\frac{d\sigma}{d\cos\theta}-\int_{-1}^0 d\cos\theta\,\frac{d\sigma}{d\cos\theta}}{\int_{-1}^1 d\cos\theta\,\frac{d\sigma}{d\cos\theta}}$$

$$\frac{\int_0^1 dx(1+x^2+ax)-\int_{-1}^0 dx(1+x^2+ax)}{\int_{-1}^1 dx(1+x^2+ax)}$$

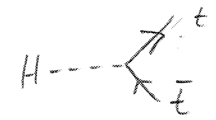
$$=\frac{2(1+\frac{1}{3}+\frac{a}{2})}{2+\frac{2}{3}}=1+\frac{3}{8}a$$

$$A_{FB}=\frac{3}{8}\cdot 2A_eA_b$$

$$=\frac{3}{4}A_eA_b$$

4.) let's first get all the necessary $\langle |M|^2 \rangle$'s.

let's start w/ $H \to t\bar{t}$



$$iM = -i\frac{m_t}{v}\,\bar{u}_t v_{\bar{t}}$$

$$\langle |M|^2 \rangle = \frac{m_t^2}{v^2}\,tr\left((\not{p}_t + m_t)(\not{p}_{\bar{t}} - m_t)\right) = \frac{m_t^2}{v^2}\left(4 p_t\cdot p_{\bar{t}} - 4m_t^2\right)$$

$$p_t = \left(\frac{m_H}{2}, \sqrt{\left(\frac{m_H}{2}\right)^2 - m_t^2}\right)$$

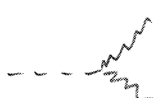
$$p_{\bar{t}} = \left(\frac{m_H}{2}, -\sqrt{\left(\frac{m_H}{2}\right)^2 - m_t^2}\right)$$

$$p_t \cdot p_{\bar{t}} = \left(\frac{m_H}{2}\right)^2 + \left(\frac{m_H}{2}\right)^2 - m_t^2 = \frac{1}{2}m_H^2 - m_t^2$$

$$= \frac{m_t^2}{v^2}\left(2m_H^2 - 8m_t^2\right)$$

$$= \frac{2m_t^2}{v^2}\left(m_H^2 - 4m_t^2\right)$$

now $H \to V^\mu V^\mu$ (vectors)



$$iM = 2i\frac{m_V^2}{v} g^{\mu\nu}\epsilon_\mu^{\lambda_1}\epsilon_\nu^{\lambda_2}$$

(you get a $g^{\mu\nu} g^{\alpha\beta} \sum_{\lambda'}\sum_{\lambda_2} \epsilon_\mu^{\lambda_1}\epsilon_\alpha^{\lambda_1 *}\epsilon_\nu^{\lambda_2}\epsilon_\beta^{\lambda_2 *}$)

$$\langle |M|^2 \rangle = \frac{4m_V^4}{v^2} g^{\mu\nu} g^{\alpha\beta}\left(g_{\mu\alpha} - \frac{q_V^\mu q_V^\alpha}{m_V^2}\right)\left(g_{\nu\beta} - \frac{q_{\bar{V}}^\nu q_{\bar{V}}^\beta}{m_V^2}\right)$$

$$= \frac{4m_V^4}{v^2}\left(g_{\mu\alpha} - \frac{q_V^\mu q_V^\alpha}{m_V^2}\right)\left(g^{\mu\alpha} - \frac{q_{\bar{V}}^\mu q_{\bar{V}}^\alpha}{m_V^2}\right)$$

F O I L

$$= \frac{4m_V^4}{v^2}\left(4 - 1 - 1 + \frac{(\frac{1}{2}m_H^2 - m_V^2)^2}{m_V^4}\right) = \frac{4m_V^4}{v^2}\left(2 + 1 - \frac{m_H^2}{m_V^2} + \frac{1}{4}\frac{m_H^4}{m_V^4}\right)$$

4.)

$$\Gamma = \frac{S|\vec{p}|}{8\pi m_H^2} \langle |m|^2 \rangle$$

for colors

$$\Gamma_{tt} = \frac{\sqrt{\left(\frac{m_H}{2}\right)^2 - m_t^2}}{8\pi m_H^2} \frac{2m_t^2}{v^2}\left(m_H^2 - 4m_t^2\right) \times 3$$

$$\Gamma_{W^+W^-} = \frac{\sqrt{\left(\frac{m_H}{2}\right)^2 - m_W^2}}{8\pi m_H^2} \frac{4m_W^4}{v^2}\left(3 - \frac{m_H^2}{m_W^2} + \frac{1}{4}\frac{m_H^4}{m_W^4}\right)$$

$$\Gamma_{ZZ} = \frac{\sqrt{\left(\frac{m_H}{2}\right)^2 - m_Z^2}}{8\pi m_H^2} \frac{4m_Z^4}{v^2}\left(3 - \frac{m_H^2}{m_Z^2} + \frac{1}{4}\frac{m_H^4}{m_Z^4}\right) \times \frac{1}{2}$$

b/c ZZ are indistinguishable bosons

$$\Gamma = \Gamma_{tt} + \Gamma_{W^+W^-} + \Gamma_{ZZ}$$

$v = 246$ GeV

$m_H = 400$ GeV

$m_W = 80$ GeV

$m_Z = 91$ GeV

$m_t = 175$ GeV

$\Gamma_{tt} \simeq 2.74$ GeV

$\Gamma_{WW} \simeq 16.6$ GeV

$\Gamma_{ZZ} \simeq 7.72$ GeV

$\Gamma_{tot} \simeq 27.1$ GeV

$\frac{\Gamma_{tt}}{\Gamma_{tot}} \simeq 10.1\%$

$\frac{\Gamma_{WW}}{\Gamma_{tot}} \simeq 61.3\%$

$\frac{\Gamma_{ZZ}}{\Gamma_{tot}} \simeq 28.5\%$