Problem Set 5 (revised) — due May 28 (or thereabouts)

Part I: as before

Creutz, problems 16.1, 16.2 and 16.4:

1. Show that the two-dimensional Ising model is self-dual.
2. Show that the three-dimensional Z_2 gauge theory is dual to the three-dimensional Ising model.
4. Show the self-duality of the Wilson Z_4 model with only the couplings β_0 and $\beta_1 = \beta_3$.

Part II:

Either write your own Monte Carlo program for the two-dimensional Ising model, or run my FORTRAN90 program. Mine is in the SLAC central UNIX directory (if you can't access it, let me know)

~ `lance/fortran/mc_ising`

and the required files are

`ising.f90` (main program)

`isinglib.f90` (contains random number generator)

`makefile` (use to compile and link files, via UNIX command "make ising")

`ising_input_sample` (sample input to program `ising`, in the correct format)

`ising` can be run interactively (see `ising_input_sample` to see what inputs it expects). But for longer jobs it may be better to run it as a batch job; e.g. in UNIX at SLAC, type

`bsub -i ising_input_sample -o ising_output ising`

Currently, `ising` just returns the spin-spin correlation function, $\langle s_{1,1}s_{1,j}\rangle$, where i,j are the coordinates of the spin $s_{i,j}$, and also the connected spin-spin correlation function,

$$\langle\langle s_{1,1}s_{1,j}\rangle\rangle \equiv \langle s_{1,1}s_{1,j}\rangle - \langle s_{1,1}\rangle\langle s_{1,j}\rangle \,,$$

but it is easy to add lines to measure other quantities.

For $\beta H = -J\sum_{\langle ab\rangle} s_a s_b$, the critical coupling/temperature is $J_c = \frac{1}{2}\ln(1+\sqrt{2}) = .4406867935$.

1a. For a J somewhat below J_c (T somewhat above T_c), we expect, for j sufficiently large,

$$\langle s_{1,1}s_{1,j}\rangle \sim C\exp(-j\times M_s)\,, \tag{1}$$

where M_s is the mass of the continuum limit of the spin field, in lattice units. Extract such a mass from your Monte Carlo data; comment on the deviations from eq. (1) that you see for very small j and very large j.

1b. The mass $M_s = M_s(T)$ should scale to zero with some exponent η as $T \to T_c$ ($J \to J_c$),

$$M_s(T) \propto (T - T_c)^\eta \propto (J_c - J)^\eta$$

Repeat the mass measurement in part 1a for at least two or three values of J, a little below J_c, and thus measure η.

2a. Right at the critical point, $J = J_c$, the theory is scale invariant (actually, conformally invariant), and all correlation functions have power law fall-off. In particular, the scaling dimension for the spin field is well-known to be 1/8, which means that we expect

$$\langle s_{1,1} s_{1,j} \rangle \sim C \times j^{-1/4} \,. \tag{2}$$

Verify this scaling behavior.

2b. Investigate the restoration of rotational invariance in the continuum limit by calculating, also at $J = J_c$, the correlation function on the diagonal,

$$\langle s_{1,1} s_{j,j} \rangle \sim C \times (\sqrt{2} j)^{-1/4} \,. \tag{3}$$