Problem Set 4 — due May 12

1. Investigate the scheme-dependence of the beta function in a theory like (massless) ϕ^4 theory, in which g is the expansion parameter, and the beta function has the generic form

$$\gamma(g) = \gamma_0 g^2 + \gamma_1 g^3 + \gamma_2 g^4 + \ldots$$

That is, if $g = g' + c_1 g'^2 + c_2 g'^3 + \ldots$, find the c_i-dependence of the first scheme-dependent term in $\gamma'(g')$.

2. Consider $SU(N_c)$ gauge theory with n_f quarks in the limit that N_c and n_f are both very large, but with $n_f = 11N_c/2 - \epsilon$, where ϵ is of order 1. Show that this theory has an infrared-stable fixed-point at a perturbative (i.e. small) value of the coupling, g^*, and compute $g^*(N_c, \epsilon)$ to first order in ϵ/N_c.

3. (Problem 13.1 of Creutz.) Define $g_R^2(r)$ to be proportional to r^2 times the force between two quarks separated by a distance r. Argue that the corresponding renormalization-group function in the full theory of strong interactions including quark loops must exhibit a zero at non-vanishing g_R.