

Problem Set 2 — due April 28

1. (\approx problems 4.3 of Creutz, *Quarks, Gluons and Lattices*) One might consider as a non-perturbative cutoff [for scalar field theory, say] disregarding a field's Fourier components which carry momentum larger than some cutoff parameter. How does this compare to the lattice cutoff in real space?
2. The group structure constants are defined by $[T^a, T^b] = if^{abc}T^c$. Normalizing the generators T^a by $\text{Tr}(T^a T^b) = k\delta^{ab}$, show that the f^{abc} are totally antisymmetric. From associativity of the matrix product, prove the Jacobi identity,

$$f^{abd}f^{cde} + f^{cad}f^{bde} + f^{bcd}f^{ade} = 0.$$

3. The transformations of the generators T^a themselves define the adjoint representation, $g^{-1}T^a g \equiv R_{\text{adj}}^{ab}(g)T^b$, and the generators $(v^a)_{bc}$ are defined by $R_{\text{adj}}^{ab}(e^{i\omega^a T^a}) = e^{i\omega^a v^a}$. Show that the v^a obey the same commutation relations as the T^a , $[v^a, v^b] = if^{abc}v^c$, and find an explicit formula for the v^a . (*Hint*: Consider the above Jacobi identity.)
4. Give an explicit description of the *center* of the group $SU(N_c)$. The center is defined to be the maximal subgroup of a group, such that all elements of the subgroup commute with all elements of the group. How does the \mathbf{N}_c representation transform under the group element that generates the center of $SU(N_c)$? How does the adjoint representation transform?
5. (problem 7.3 of Creutz) Show that the fermionic terms in the QCD lattice action,

$$\frac{a^3}{2} \sum_{\langle ij \rangle} \bar{\psi}_i (1 + \gamma_\mu e_\mu) U_{ij} \psi_j + (a^4 m_0 + 4a^3) \sum_i \bar{\psi}_i \psi_i,$$

have the correct classical continuum limit.