

Problem Set 1 — due April 16 (then April 21)

(Problems 3.1 to 3.4 of Creutz, *Quarks, Gluons and Lattices*)

1. Consider the harmonic oscillator with $V(x) = \frac{1}{2}kx^2$. Diagonalize the transfer matrix T . (Hint: find an operator of the form $\frac{1}{2}p^2 + \frac{1}{2}\omega^2 x^2$ which commutes with T and can thus be simultaneously diagonalized.)
2. In the above example, find the propagator $\langle x_i x_j \rangle$.
3. Show that in general $a^{-2}\langle (x_{i+1} - x_i)^2 \rangle$ diverges as a goes to zero. Show that the split-point product $a^{-2}\langle (x_{i+1} - x_i)(x_i - x_{i-1}) \rangle$ approaches $-\langle 0|p^2|0\rangle/m^2$ in the continuum limit. Where does the minus sign come from?
4. Calculate the fluctuations in the propagator for the free particle ($V(x) = 0$)

$$D^2(i, j) = \langle (x_i x_j)^2 \rangle - \langle x_i x_j \rangle^2.$$

Show that the fluctuations in the split point product of problem 3 diverge as a goes to zero. Derive the virial theorem for the continuum theory:

$$\frac{1}{m}\langle 0|p^2|0\rangle = \langle 0|xV'(x)|0\rangle.$$

This gives the average momentum squared without large fluctuations.