$$A_0^{\mathbf{Z}_{\mathrm{eff}}}[\mathbf{z}_a,2,3,4q] = 2\,T^{\mathbf{z}}\,\mathbf{y}^{\mathbf{z}}\Big\{\Big(-\mathbf{Q}^{\mathbf{z}}+\mathbf{v}^2_{\mathbf{z}_a}\mathbf{v}^2_{\mathbf{z}_b}\mathbf{y}_{\mathbf{z}_c}\Big)\,\mathbf{y}_{\mathbf{z}_a}$$
$$\times\Big[\mathcal{N}_{\mathbf{z}}\sum_{i=\mathbf{z},\mathbf{z}_a}\Big(\mathbf{v}^{(\mathbf{z}+\mathbf{z}_i)}\mathbf{v}^{(\mathbf{z}_a)}\big(\mathrm{i}_i{}^{\mathbf{z}}-A_{\mathbf{z}_a}\big)\mathbf{z}_a,\mathbf{z}\big)\,\mathbf{v}^{(\mathbf{z}_a)}\,\mathrm{i}_i{}^{\mathbf{z}}-A_{\mathbf{z}\mathbf{q}}\big)\mathbf{z}_a,\mathbf{q}_a,2,3\Big]$$
$$+\frac{\mathcal{N}_{\mathbf{z}}}{\sqrt{2}}\Big(-\mathbf{Q}^{\mathbf{z}}+\frac{1}{2}\mathbf{v}^2_{\mathbf{z}_a}\big(\mathbf{z}_a+\mathbf{z}_b\big)^2\mathbf{v}^2\mathbf{y}_{\mathbf{z}_c}\Big)\,\mathbf{y}_{\mathbf{z}_a}$$
$$\times\Big[\big(\mathbf{v}^{\mathbf{z}}\mathbf{v}^{(\mathbf{z}_a)}\mathrm{i}_i{}^{\mathbf{z}}+\big(\mathbf{v}^{\mathbf{z}}\mathbf{v}^{(\mathbf{z}_a)}+\mathbf{v}^{(\mathbf{z}_a)}\mathrm{i}_i{}^{\mathbf{z}}-\frac{1}{2}\mathbf{v}^{(\mathbf{z}+\mathbf{z}_a)}\,\mathrm{i}_i{}^{\mathbf{z}}\big)\,A^{\mathbf{z}}_{\mathbf{z}_a}\big(\mathbf{z}_a,\mathbf{q}_a\big)\mathbf{z},2,3\Big]$$
$$+\frac{\mathcal{N}_{\mathbf{z}}}{\sqrt{2}\,\mathrm{i}\,\mathbf{v}^{\mathbf{z}}}\,\mathbf{y}_{\mathbf{z}_a}\Big[\sum_{i=\mathbf{z},\mathbf{z}_a}\Big(\mathbf{v}^{(\mathbf{z}+\mathbf{z}_i)}\mathbf{v}^{(\mathbf{z}_a)}\big(\mathrm{i}_i{}^{\mathbf{z}}-\frac{1}{2}\mathbf{v}^{(\mathbf{z}+\mathbf{z}_a)}\,\mathrm{i}_i{}^{\mathbf{z}}\big)\,A^{\mathbf{z}}_{\mathbf{z}_a}\big(\mathbf{z}_a,\mathbf{q}_a\big)\mathbf{z},2,3\Big]$$
$$+\frac{1}{\sqrt{2}}\,\mathbf{v}^{(\mathbf{z}_a)}\,\mathrm{i}_i{}^{\mathbf{z}}\,A^{\mathbf{z}}_{\mathbf{z}_a}\big(\mathbf{z}_a,\mathbf{q}_a\big)\mathbf{z},2,3\Big]\Big\}\,, \tag{1}$$

$$A_{\mathbf{z}\mathbf{q}}\big[\mathbf{z}_a,2,3,4q\big] = A_0\big[\mathbf{z}_a,2,3,4q\big] + \frac{1}{\mathcal{N}_{\mathbf{z}}}\,A_0\big[\mathbf{z}_a,4q,2,3\big]$$
$$+\frac{\mathrm{i}\,\mathbf{v}^{(\mathbf{z}_a)}}{\mathcal{N}_{\mathbf{z}}}\,A_1\big[\mathbf{z}_a,2,3,4q\big]-\frac{\mathbf{v}^2_{\mathbf{z}_a}}{\mathcal{N}_{\mathbf{z}}}\,A_2^{\mathbf{z}}\big[\mathbf{z}_a,2,3,4q\big]+\frac{1}{\mathcal{N}_{\mathbf{z}}}\,A_3\big[\mathbf{z}_a,2,3,4q\big]\,,$$
$$A_{\mathbf{z}\mathbf{q}}\big[\mathbf{z}_a,4q,2,3\big] = A_0\big[\mathbf{z}_a,2,3,4q\big] + A_1\big[\mathbf{z}_a,2,3,4q\big] + A_2\big[\mathbf{z}_a,2,3,4q\big] + A_3\big[\mathbf{z}_a,2,3,4q\big]$$
$$+A_4\big[\mathbf{z}_a,4q,2,3\big] + A_5\big[\mathbf{z}_a,4q,2,3\big]\,, \tag{2}$$
$$A^{\mathbf{z}}_{\mathbf{z}_a}\big[\mathbf{z}_a,4q,2,3\big] = -\mathrm{i}\,\mathbf{v}^{(\mathbf{z}_a)}\big[\mathbf{z}_a,4q,2,3\big]-\mathbf{v}^2_{\mathbf{z}_a}\big[\mathbf{z}_a,2,3\big]\,,$$
$$A^{\mathbf{z}}_{\mathbf{z}_a}\big[\mathbf{z}_a,4q,2,3\big] = A^{\mathbf{z}}_{\mathbf{z}}\big[\mathbf{z}_a,2,3\big]\,,$$
$$A^{\mathbf{z}}_{\mathbf{z}_a}\big[\mathbf{z}_a,4q,2,3\big] = A^{\mathbf{z}}_{\mathbf{z}}\big[\mathbf{z}_a,2,3\big]\,,$$
$$A_0\big[\mathbf{z}_a,2,3,4q\big] = A^{\mathbf{z}}_{\mathbf{z}}\big[\mathbf{z}_a,2,3,4q\big]+A^{\mathbf{z}}_{\mathbf{z}}\big[\mathbf{z}_a,2,3,4q\big]\,,$$
$$A_1\big[\mathbf{z}_a,2,3,4q\big] = A^{\mathbf{z}}_{\mathbf{z}}\big[\mathbf{z}_a,2,3,4q\big]+A^{\mathbf{z}}_{\mathbf{z}}\big[\mathbf{z}_a,2,3,4q\big]\,,$$
$$A_2\big[\mathbf{z}_a,2,3,4q\big] = A^{\mathbf{z}}_{\mathbf{z}}\big[\mathbf{z}_a,2,3,4q\big]+A^{\mathbf{z}}_{\mathbf{z}}\big[\mathbf{z}_a,2,3,4q\big]\,, \tag{3}$$
$$A_3\big[\mathbf{z}_a,2,3,4q\big] = A^{\mathbf{z}}_{\mathbf{z}}\big[\mathbf{z}_a,2,3,4q\big]+A^{\mathbf{z}}_{\mathbf{z}}\big[\mathbf{z}_a,2,3,4q\big]\,,$$
$$A^{\mathbf{z}}_{\mathbf{z}}{}^{\mathbf{z}_a\mathbf{z}_b} = \varepsilon_{\mathbf{z}}\,\Big[\mathbf{z}^{\mathbf{z}_a}\mathbf{y}^{\mathbf{z}_b}+\mathbf{z}^{\mathbf{z}_b}\Big]\,, \tag{4}$$
$$\varepsilon_{\mathbf{z}} = \frac{1}{4\mathrm{i}\,\mathbf{v}^{(\mathbf{z}_a)}}\,\frac{\Gamma\big(1+\mathbf{z}\big)\,\Gamma\big(1-\mathbf{z}\big)}{\Gamma\big(1-\mathbf{z}_a\big)}\,, \tag{5}$$
$$\delta_{10} = \mathrm{i}\,\alpha-\mathrm{i}\,\alpha_1-\mathrm{i}\,\alpha_2\,,\qquad \delta_{11} = \mathrm{i}\,\alpha_1-\mathrm{i}\,\alpha_2-\mathrm{i}\,\alpha_3\,,\qquad \delta_{12} = \mathrm{i}\,\alpha_2-\mathrm{i}\,\alpha_3-\mathrm{i}\,\alpha_4\,,$$
$$\Delta_{\mathbf{z}} = \mathbf{z}_1^2+\mathbf{z}_2^2+\mathbf{z}_3^2-2\,\mathbf{z}_1\mathbf{z}_2-2\,\mathbf{z}_1\mathbf{z}_3-2\,\mathbf{z}_2\mathbf{z}_3\,,$$
$$\widetilde{\delta}_{10} = \mathrm{i}\,\alpha-\mathrm{i}\,\alpha_1-\mathrm{i}\,\alpha_2\,,\qquad \widetilde{\delta}_{11} = \mathrm{i}\,\alpha_1-\mathrm{i}\,\alpha_2-\mathrm{i}\,\alpha_3\,,\qquad \widetilde{\delta}_{12} = \mathrm{i}\,\alpha_2-\mathrm{i}\,\alpha_3-\mathrm{i}\,\alpha_4\,, \tag{6}$$
$$\widetilde{\Delta}_{\mathbf{z}} = \mathbf{z}_1^2+\mathbf{z}_2^2+\mathbf{z}_3^2-2\,\mathbf{z}_1\mathbf{z}_2-2\,\mathbf{z}_1\mathbf{z}_3-2\,\mathbf{z}_2\mathbf{z}_3\,.$$

$$\mathrm{i}\mathbf{p}_2:\ 1\leftrightarrow 4,\quad 3\leftrightarrow 3,\quad 5\leftrightarrow 6,\quad \{\alpha\}\leftrightarrow\{\alpha\}\,, \tag{7}$$
$$\mathrm{i}\mathbf{p}_3:\ 1\leftrightarrow 2,\quad 3\leftrightarrow 4,\quad 5\leftrightarrow 6,\quad \{\alpha\}\leftrightarrow\{\alpha\}\,, \tag{8}$$
$$\mathrm{i}\mathbf{p}_4:\ 1\leftrightarrow 3,\quad 2\leftrightarrow 4,\quad 5\leftrightarrow 6,\quad \{\alpha\}\leftrightarrow\{\alpha\}\,, \tag{9}$$
$$\mathrm{i}\mathbf{p}_{\mathrm{ij}}:\ 1\leftrightarrow 1,\qquad 1\leftrightarrow\mathrm{i},\qquad \{\alpha\}\leftrightarrow\{\alpha\}\,,$$
$$\alpha\mathrm{i}\mathbf{p}_{\mathrm{ij}}:\ 1\leftrightarrow 4,\quad 2\leftrightarrow 3\,, \tag{10}$$

$$M_1[1,2,3,4] = \frac{[12]}{[1|(2+4)|3]\,[1|(2+3)|4]\,\Delta_{\mathbf{z}}}\Big\{\big[1|(2+3)|3\big]\,\delta_{12}\,\delta_{10}+\mathbf{s}_{23}\,\delta_{12}+\big[1|(2+3)|4\big]\,\big(\big[3|4\big]\,\delta_{10}-\big[2|3\big]\,\delta_{12}\big)$$
$$+\mathbf{s}_{24}\big[34\big]\,\big[1|2\big]\,\big[4|6\big]\,\delta_{12}-\big[1|(2+3)|3\big]\,\big(\big[3|6\big]\,\big[1|(2+3)|4\big]\,\big)\,\delta_{\mathbf{z}}^{\mathbf{z}_a}(\mathbf{s}_{23},\mathbf{s}_{23},\mathbf{s}_{23})$$
$$+\frac{1}{\big[1|(2+4)|3\big]\,\big[1|(2+3)|4\big]}\,\frac{\big[1|2\big]}{\Delta_{\mathbf{z}}}\,\Big(\big[1|(2+4)|3\big]-\big[1|(2+3)|4\big]\Big)\,\Big(\big[1|(2+3)|3\big]\,\delta_{10}+\big[1|(2+3)|4\big]\,\big(\big[1|4\big]\,\delta_{10}$$
$$-\frac{\big[4|1\big]\,\big[1|4\big]}{\big[1|4\big]}\,\Big(\big[1,\mathrm{i}-1\mathrm{i}\big]\,\big(\big[1|2\big]\,\delta_{12}-\big[1|3\big]\,\delta_{12}\big)+\big[1|1\big]\,\big(\big[1|3\big]\,\delta_{12}-\big[1|2\big]\,\delta_{12}\big)\Big)\Big)$$
$$+\big[2|3\big]\,\big[1|1\big]\,\big[1|3\big]\,\big[1,\mathrm{i}-1\mathrm{i}\big]\Big)\ln\Big(\frac{\mathbf{s}_{23}}{\mathbf{t}}\Big)\Big\}\,. \tag{11}$$

$$M_2[1,2,3,4] = \frac{1}{\big[1|(2+4)|3\big]\,\big[1|(2+3)|4\big]}\Big\{\frac{\big[12\big]\,\big[1|(2+3)|4\big]\,\big[1|(2+3)|3\big]\,\big(\big[3|1\big]\,\delta_{12}-2\,\big[3|4\big]\,\big[1|$$
$$+\frac{1}{\Delta_{\mathbf{z}}}\Big(\frac{\big[1|3\big]\,\big[4|1\big]\,\big[1|(2+3)|3\big]}{\big[3|4\big]}-\frac{\big[12\big]\,\big[4|1\big]}{\big[3|4\big]}\,\Big(\big[1|1\big]\,\big[4|1\big]\,\big[1|(2+3)|4\big]\,\delta_{12}$$
$$-\frac{\big[2|4\big]\,\big[1|1\big]}{\big[1|4\big]}\,\Big(\big[1|(2+3)|4\big]-\big[1|3|4\big]\Big)\,\Big(\frac{\big[4|1\big]}{\big[1|4\big]}\Big)\,\Big(\big[1|(2+3)|4\big]\,\delta_{10}-\big[1|3|4\big]\,\delta_{12}\Big)+\big[1|3|4\big]\,\big[1|1\big]\,\big[1|(2+3)|4\big]\Big)$$
$$-\big[1|3|4\big]\,\big[1|1\big]\,\big[1|(2+3)|4\big]\,\big(\delta_{12}-\delta_{10}\big)\Big)\ln\Big(\frac{\mathbf{s}_{23}}{\mathbf{t}}\Big)\Big\}$$
$$+\frac{1}{\big[1|(2+4)|3\big]\,\big[1|(2+3)|4\big]\,\big[1|(2+3)|3\big]}\Big\{\big[1|(2+3)|3\big]\,\big(\big[3|1\big]\,\delta_{12}-2\,\big[3|4\big]\,\big[1|$$
$$-\frac{\big[1|3\big]\,\big[4|1\big]}{\big[3|4\big]}\,\Big(\big[1|(2+4)|3\big]-\big[1|(2+3)|4\big]\Big)\,\Big(\frac{\big[4|1\big]}{\big[1|4\big]}\Big)\,\Big(\big[1|(2+3)|4\big]\,\delta_{10}-\big[1|3|4\big]\,\delta_{12}\Big)$$
$$+\big[1|(2+3)|4\big]\,\big[1|1\big]\,\big[1|(2+3)|4\big]\,\big(\delta_{12}-\delta_{10}\big)\Big)\ln\Big(\frac{\mathbf{s}_{23}}{\mathbf{t}}\Big)\Big\}\,. \tag{12}$$

$$M_3[1,2,3,4] = \frac{1}{\big[1|(2+4)|3\big]\,\big[1|(2+3)|4\big]}\Big\{\frac{\big[12\big]\,\big[1|(2+3)|4\big]\,\big[1|(2+3)|3\big]\,\big(\big[3|1\big]\,\delta_{12}-2\,\big[3|4\big]\,\big[1|$$
$$+\frac{1}{\Delta_{\mathbf{z}}}\Big(\frac{\big[1|3\big]\,\big[4|1\big]}{\big[3|4\big]}-\frac{\big[12\big]\,\big[4|1\big]}{\big[3|4\big]}\,\Big(\big[1|1\big]\,\big[4|1\big]\,\big[1|(2+3)|4\big]\,\delta_{12}$$
$$-\frac{\big[2|4\big]\,\big[1|1\big]}{\big[1|4\big]}\,\Big(\big[1|(2+3)|4\big]-\big[1|3|4\big]\Big)\,\Big(\frac{\big[4|1\big]}{\big[1|4\big]}\Big)\,\Big(\big[1|(2+3)|4\big]\,\delta_{10}-\big[1|3|4\big]\,\delta_{12}\Big)$$
$$-\big[1|3|4\big]\,\big[1|1\big]\,\big[1|(2+3)|4\big]\,\big(\delta_{12}-\delta_{10}\big)\Big)\ln\Big(\frac{\mathbf{s}_{23}}{\mathbf{t}}\Big)\Big\}\,. \tag{13}$$

$$M_4[1,2,3,4] = \frac{1}{\big[1|(2+4)|3\big]\,\big[1|(2+3)|4\big]}\Big\{\frac{\big[12\big]\,\big[1|(2+3)|4\big]\,\big[1|(2+3)|3\big]\,\big(\big[3|1\big]\,\delta_{12}-2\,\big[3|4\big]\,\big[1|$$
$$+\frac{\big[12\big]\,\big[4|1\big]}{\Delta_{\mathbf{z}}}\Big(\frac{\big[1|3\big]\,\big[4|1\big]\,\big[1|(2+3)|4\big]}{\big[3|4\big]}-\frac{\big[12\big]\,\big[4|1\big]}{\big[3|4\big]}\,\Big(\big[1|1\big]\,\big[4|1\big]\,\big[1|(2+3)|4\big]\,\delta_{12}$$
$$-\frac{\big[2|4\big]\,\big[1|1\big]}{\big[1|4\big]}\,\Big(\big[1|(2+3)|4\big]-\big[1|3|4\big]\Big)\,\Big(\frac{\big[4|1\big]}{\big[1|4\big]}\Big)\,\Big(\big[1|(2+3)|4\big]\,\delta_{10}-\big[1|3|4\big]\,\delta_{12}\Big)$$
$$-\big[1|3|4\big]\,\big[1|1\big]\,\big[1|(2+3)|4\big]\,\big(\delta_{12}-\delta_{10}\big)\Big)\ln\Big(\frac{\mathbf{s}_{23}}{\mathbf{t}}\Big)\Big\}\,. \tag{14}$$

$$M_5[1,2,3,4] = \frac{1}{\big[1|(2+4)|3\big]\,\big[1|(2+3)|4\big]}\Big\{\frac{\big[12\big]\,\big[1|(2+3)|4\big]\,\big[1|(2+3)|3\big]\,\big(\big[3|1\big]\,\delta_{12}-2\,\big[3|4\big]\,\big[1|$$
$$+\frac{\big[12\big]\,\big[4|1\big]}{\Delta_{\mathbf{z}}}\Big(\frac{\big[1|3\big]\,\big[4|1\big]\,\big[1|(2+3)|4\big]}{\big[3|4\big]}-\frac{\big[12\big]\,\big[4|1\big]}{\big[3|4\big]}\,\Big(\big[1|1\big]\,\big[4|1\big]\,\big[1|(2+3)|4\big]\,\delta_{12}$$
$$-\frac{\big[2|4\big]\,\big[1|1\big]}{\big[1|4\big]}\,\Big(\big[1|(2+3)|4\big]-\big[1|3|4\big]\Big)\,\Big(\frac{\big[4|1\big]}{\big[1|4\big]}\Big)\,\Big(\big[1|(2+3)|4\big]\,\delta_{10}-\big[1|3|4\big]\,\delta_{12}\Big)$$
$$-\big[1|3|4\big]\,\big[1|1\big]\,\big[1|(2+3)|4\big]\,\big(\delta_{12}-\delta_{10}\big)\Big)\ln\Big(\frac{\mathbf{s}_{23}}{\mathbf{t}}\Big)\Big\}\,. \tag{15}$$

$$\mathbf{v}^{\mathbf{z}\mathbf{z}} = -\frac{1}{\mathbf{t}^2}\Big(\Big(\frac{\partial^2}{\partial\,\mathbf{z}_{12}}\Big)^2+\Big(\frac{\partial^2}{\partial\,\mathbf{z}_{23}}\Big)^2+\Big(\frac{\partial^2}{\partial\,\mathbf{z}_{31}}\Big)^2\Big)-\frac{1}{\mathbf{t}}\Big(\frac{\partial^2}{\partial\,\mathbf{z}_{12}\,\partial\,\mathbf{z}_{23}}\Big)-6\,, \tag{16}$$

$$\mathbf{v}^{\mathbf{z}\mathbf{z}} = \frac{1}{\mathbf{t}^2}\Big(\Big(\frac{\partial^2}{\partial\,\mathbf{z}_{12}}\Big)^2+\Big(\frac{\partial^2}{\partial\,\mathbf{z}_{23}}\Big)^2+\Big(\frac{\partial^2}{\partial\,\mathbf{z}_{31}}\Big)^2\Big)-\frac{1}{\mathbf{t}}\Big(\frac{\partial^2}{\partial\,\mathbf{z}_{12}\,\partial\,\mathbf{z}_{23}}\Big)-6\,, \tag{17}$$

$$\mathbf{v}^{\mathbf{z}\mathbf{z}} = \frac{1}{\mathbf{t}^2}\Big(\Big(\frac{\partial^2}{\partial\,\mathbf{z}_{12}}\Big)^2+\Big(\frac{\partial^2}{\partial\,\mathbf{z}_{23}}\Big)^2+\Big(\frac{\partial^2}{\partial\,\mathbf{z}_{31}}\Big)^2\Big)-\frac{1}{\mathbf{t}}\Big(\frac{\partial^2}{\partial\,\mathbf{z}_{12}\,\partial\,\mathbf{z}_{23}}\Big)-6\,, \tag{18}$$

$$\mathbf{v}^{\mathbf{z}\mathbf{z}} = \frac{1}{\mathbf{t}^2}\Big(\Big(\frac{\partial^2}{\partial\,\mathbf{z}_{12}}\Big)^2+\Big(\frac{\partial^2}{\partial\,\mathbf{z}_{23}}\Big)^2+\Big(\frac{\partial^2}{\partial\,\mathbf{z}_{31}}\Big)^2\Big)-\frac{1}{\mathbf{t}}\Big(\frac{\partial^2}{\partial\,\mathbf{z}_{12}\,\partial\,\mathbf{z}_{23}}\Big)-6\,, \tag{19}$$

$$\mathbf{v}^{\mathbf{z}\mathbf{z}} = \frac{1}{\mathbf{t}^2}\Big(\Big(\frac{\partial^2}{\partial\,\mathbf{z}_{12}}\Big)^2+\Big(\frac{\partial^2}{\partial\,\mathbf{z}_{23}}\Big)^2+\Big(\frac{\partial^2}{\partial\,\mathbf{z}_{31}}\Big)^2\Big)-\frac{1}{\mathbf{t}}\Big(\frac{\partial^2}{\partial\,\mathbf{z}_{12}\,\partial\,\mathbf{z}_{23}}\Big)-6\,, \tag{20}$$

$$\mathbf{v}^{\mathbf{z}\mathbf{z}} = \frac{1}{\mathbf{t}^2}\Big(\Big(\frac{\partial^2}{\partial\,\mathbf{z}_{12}}\Big)^2+\Big(\frac{\partial^2}{\partial\,\mathbf{z}_{23}}\Big)^2+\Big(\frac{\partial^2}{\partial\,\mathbf{z}_{31}}\Big)^2\Big)-\frac{1}{\mathbf{t}}\Big(\frac{\partial^2}{\partial\,\mathbf{z}_{12}\,\partial\,\mathbf{z}_{23}}\Big)-6\,, \tag{21}$$

$$\mathbf{v}^{\mathbf{z}\mathbf{z}} = \Big(\frac{[12]^2\,[45]^2}{[12|(2+3)|4]^2\,\delta_{12}\,[45|(4+5+3)|1]}+\frac{[12]^2\,[3|1+2+3|]^2}{[12|(2+3)|4]^2\,\delta_{12}\,[1|(2+3)|4]}\Big)$$
$$-\frac{[12]\,[12]\,[3|1+2+3|]^2}{[12|(2+3)|4]^2\,\delta_{12}\,[1|(2+3)|4]}\,\mathrm{Li}_{-1}\Big(\frac{\mathbf{s}_{23}}{\mathbf{t}},\frac{\mathbf{s}_{23}}{\mathbf{u}}\Big)$$
$$+\Big(\frac{[12]^2\,[45]^2\,\delta_{23}}{[12|(2+3)|4]^2\,\delta_{12}\,[45|(4+5+3)|1]}+\frac{[12]^2\,[3|1+2+3|]^2}{[12|(2+3)|4]^2\,\delta_{12}\,[1|(2+3)|4]}\Big)\,\mathrm{Li}_{-1}\Big(\frac{\mathbf{s}_{23}}{\mathbf{t}},\frac{\mathbf{s}_{23}}{\mathbf{u}}\Big)$$
$$+\frac{1}{\delta_{12}\,[12|(2+3)|4]}\,\mathrm{Li}_2^{\mathbf{Z}_{\mathrm{eff}}}(\mathbf{s}_{23},\mathbf{s}_{23},\mathbf{s}_{23})$$
$$+\Big[-2\,\frac{[12|(2+3)|4]\,[3|1+2+3|]^2}{\delta_{12}\,[12|(2+3)|4]}+\frac{[12|(2+3)|4]\,[3|1+2+3|]^2}{[12|(2+3)|4]^2\,\delta_{12}}\Big]\,\mathrm{Li}_2^{\mathbf{Z}_{\mathrm{eff}}}(\mathbf{s}_{23},\mathbf{s}_{23},\mathbf{s}_{23})$$
$$+\Big[-2\,\frac{[12|(2+3)|4]\,[3|1+2+3|]^2}{\delta_{12}\,[12|(2+3)|4]}+\frac{[12|(2+3)|4]\,[3|1+2+3|]^2}{[12|(2+3)|4]^2\,\delta_{12}}\Big]\,\mathrm{Li}_2^{\mathbf{Z}_{\mathrm{eff}}}(\mathbf{s}_{23},\mathbf{s}_{23},\mathbf{s}_{23})$$
$$-\Big(\frac{[12|(2+3)|4]\,[3|1+2+3|]^2}{[12|(2+3)|4]^2\,\delta_{12}}+\frac{[12|(2+3)|4]\,[3|1+2+3|]^2}{[12|(2+3)|4]\,\delta_{12}}\Big)\,\mathrm{Li}_2^{\mathbf{Z}_{\mathrm{eff}}}(\mathbf{s}_{23},\mathbf{s}_{23},\mathbf{s}_{23})$$
$$-\Big(\frac{[12|(2+3)|4]\,[3|1+2+3|]^2}{[12|(2+3)|4]^2\,\delta_{12}}+\frac{[12|(2+3)|4]\,[3|1+2+3|]^2}{[12|(2+3)|4]\,\delta_{12}}\Big)\,\mathrm{Li}_2^{\mathbf{Z}_{\mathrm{eff}}}(\mathbf{s}_{23},\mathbf{s}_{23},\mathbf{s}_{23})$$
$$\times\,\mathrm{Li}_2^{\mathbf{Z}_{\mathrm{eff}}}(\mathbf{s}_{23},\mathbf{s}_{23},\mathbf{s}_{23})+\Big(\frac{[12|(2+3)|4]\,[3|1+2+3|]^2}{[12|(2+3)|4]^2\,\delta_{12}}+\frac{[12|(2+3)|4]\,[3|1+2+3|]^2}{[12|(2+3)|4]\,\delta_{12}}\Big)\,\mathrm{Li}_{-1}\Big(\frac{\mathbf{s}_{23}}{\mathbf{t}},\frac{\mathbf{s}_{23}}{\mathbf{u}}\Big)$$
$$+\frac{[1|(2+4)|3]^2}{[12|(2+4)|3]\,[12|(2+3)|4]}\,\mathrm{Li}_{-1}\Big(\frac{\mathbf{s}_{23}}{\mathbf{t}},\frac{\mathbf{s}_{23}}{\mathbf{u}}\Big)+\frac{[12]^2\,[3|1]^2}{[12|(2+4)|3]\,[12|(2+3)|4]}$$
$$+\mathbf{v}^{\mathbf{z}}\,\delta_{\mathbf{z}}^{\mathbf{z}_a}(\mathbf{s}_{23},\mathbf{s}_{23},\mathbf{s}_{23})+\widetilde{\mathbf{v}}^{\mathbf{z}}\,\delta_{\mathbf{z}}^{\mathbf{z}_a}(\mathbf{s}_{23},\mathbf{s}_{23},\mathbf{s}_{23})-M_1[1,2,3,4]\Big|_{\mathrm{i}\mathbf{p}_{\mathrm{ij}}}\Big]$$
$$-2\,\frac{[12]\,[3|4]\,[1|(2+4)|3]\,[1|(2+3)|4]}{[3|4]\,[1|(2+4)|3]\,[1|(2+3)|4]}\,\frac{\mathrm{Li}_2\Big(\frac{\mathbf{s}_{23}}{\mathbf{t}}\Big)}{\delta_{12}}+\frac{1}{2}\,\frac{[3|4]\,[1|(2+4)|3]\,[1|(2+3)|4]}{[3|4]\,[1|(2+4)|3]\,[1|(2+3)|4]}\,\frac{\mathrm{Li}_2\Big(\frac{\mathbf{s}_{23}}{\mathbf{t}}\Big)}{\delta_{12}}$$
$$+\frac{1}{2}\,\frac{[3|4|1]\,[1|(2+4)|3]\,[1|(2+3)|4]}{[3|4]\,[1|(2+4)|3]\,[1|(2+3)|4]}\,\frac{\mathrm{Li}_2\Big(\frac{\mathbf{s}_{23}}{\mathbf{t}}\Big)}{\delta_{12}}+\frac{[3|4|1]\,[1|(2+3)|4]}{[3|4]\,[1|(2+3)|4]}\,\frac{\mathrm{Li}_2\Big(\frac{\mathbf{s}_{23}}{\mathbf{t}}\Big)}{\delta_{12}}$$
$$-2\,\frac{[1|(2+3)|4]\,[1|(2+3)|4]}{[1|(2+4)|3]}\,\ln^2\Big(\frac{\mathbf{s}_{23}}{\mathbf{t}}\Big)\,. \tag{40}$$

$$\mathcal{V} = \frac{[34]^2\,[1|(2+4)|3]}{[12]\,\delta_{12}}+\frac{[12]}{\delta_{12}\,\delta_{23}}\Big(\frac{[6|(2+3)|1]\,[1|(2+4)|3]\,[1|(2+3)|6]}{[1|(2+3)|6]}-[12]\,[34]\,[43]\,[3|6]\Big)\,, \tag{41}$$

$$\widetilde{\mathcal{V}} = \frac{5}{2}\,\frac{\delta_{12}\,\delta_{10}+2\,\delta_{11}\,\delta_{23}}{\delta_{10}}\Big(\frac{[4|(1+2)|4]^2}{[12]\,[34]\,[1|(2+3)|4]}+\frac{[12]^2\,[3|1]^2}{[12]\,[34]\,[1|(2+3)|4]}\Big)$$
$$-2\,\frac{[4|(2+3)|1]\,[3|1]}{[12]\,\delta_{10}\,[1|(2+3)|4]}\,\frac{[4|(2+3)|1]\,[3|1]}{[12]\,\delta_{10}\,[1|(2+3)|4]}+\frac{[1|(2+3)|3|4]}{[12]\,[34]}$$
$$+\frac{[34]^2\,[6|(2+3)|1]\,[1|(2+3)|6]}{[12]\,[34]}+\frac{[34]\,[4|(1+2)+(1+3)|6]}{[12]\,[34]}+\frac{[12]\,[34]\,[1|(1+3)|1]}{[12]\,[34]}$$
$$-\frac{[1|(2+4)|3]}{[12]}\,\Big(\big[34\big]\,\big[1|\mathbf{s}_{23}+\mathbf{s}_{24}\big]+\big[3|1\big]\,\big[1|(2+3)|4\big]\Big)\Big)\,. \tag{42}$$

$$\mathbf{v}^{\mathbf{z}\mathbf{z}} = \frac{1}{\mathbf{t}^2}\Big(\Big(\frac{\partial^2}{\partial\,\mathbf{z}_{12}}\Big)^2+\Big(\frac{\partial^2}{\partial\,\mathbf{z}_{23}}\Big)^2+\Big(\frac{\partial^2}{\partial\,\mathbf{z}_{31}}\Big)^2\Big)-\frac{1}{\mathbf{t}}\Big(\frac{\partial^2}{\partial\,\mathbf{z}_{12}\,\partial\,\mathbf{z}_{23}}\Big)-6\,, \tag{43}$$

$$\mathbf{v}^{\mathbf{z}\mathbf{z}} = \frac{1}{\mathbf{t}^2}\Big(\Big(\frac{\partial^2}{\partial\,\mathbf{z}_{12}}\Big)^2+\Big(\frac{\partial^2}{\partial\,\mathbf{z}_{23}}\Big)^2+\Big(\frac{\partial^2}{\partial\,\mathbf{z}_{31}}\Big)^2\Big)-\frac{1}{\mathbf{t}}\Big(\frac{\partial^2}{\partial\,\mathbf{z}_{12}\,\partial\,\mathbf{z}_{23}}\Big)-6\,, \tag{44}$$

$$\mathbf{v}^{\mathbf{z}\mathbf{z}} = \frac{1}{\mathbf{t}^2}\Big(\Big(\frac{\partial^2}{\partial\,\mathbf{z}_{12}}\Big)^2+\Big(\frac{\partial^2}{\partial\,\mathbf{z}_{23}}\Big)^2+\Big(\frac{\partial^2}{\partial\,\mathbf{z}_{31}}\Big)^2\Big)-\frac{1}{\mathbf{t}}\Big(\frac{\partial^2}{\partial\,\mathbf{z}_{12}\,\partial\,\mathbf{z}_{23}}\Big)-6\,, \tag{45}$$

$$\mathbf{v}^{\mathbf{z}\mathbf{z}} = \Big(\frac{[12]^2\,[45]^2\,\delta_{23}}{[12|(2+3)|4]^2\,\delta_{12}\,[45|(4+5+3)|1]}+\frac{[12]^2\,[3|1+2+3|]^2}{[12|(2+3)|4]^2\,\delta_{12}\,[1|(2+3)|4]}\Big)\,\mathrm{Li}_{-1}\Big(\frac{\mathbf{s}_{23}}{\mathbf{t}},\frac{\mathbf{s}_{23}}{\mathbf{u}}\Big)$$
$$+\Big(\frac{[12]^2\,[45]^2\,\delta_{23}}{[12|(2+3)|4]^2\,\delta_{12}\,[45|(4+5+3)|1]}+\frac{[12]^2\,[3|1+2+3|]^2}{[12|(2+3)|4]^2\,\delta_{12}\,[1|(2+3)|4]}\Big)\,\mathrm{Li}_{-1}\Big(\frac{\mathbf{s}_{23}}{\mathbf{t}},\frac{\mathbf{s}_{23}}{\mathbf{u}}\Big)$$
$$+\frac{1}{\delta_{12}\,[12|(2+3)|4]}\,\mathrm{Li}_2^{\mathbf{Z}_{\mathrm{eff}}}(\mathbf{s}_{23},\mathbf{s}_{23},\mathbf{s}_{23})$$
$$+\Big[-2\,\frac{[12|(2+3)|4]\,[3|1+2+3|]^2}{\delta_{12}\,[12|(2+3)|4]}+\frac{[12|(2+3)|4]\,[3|1+2+3|]^2}{[12|(2+3)|4]^2\,\delta_{12}}\Big]\,\mathrm{Li}_2^{\mathbf{Z}_{\mathrm{eff}}}(\mathbf{s}_{23},\mathbf{s}_{23},\mathbf{s}_{23})$$
$$+\Big[-2\,\frac{[12|(2+3)|4]\,[3|1+2+3|]^2}{\delta_{12}\,[12|(2+3)|4]}+\frac{[12|(2+3)|4]\,[3|1+2+3|]^2}{[12|(2+3)|4]^2\,\delta_{12}}\Big]\,\mathrm{Li}_2^{\mathbf{Z}_{\mathrm{eff}}}(\mathbf{s}_{23},\mathbf{s}_{23},\mathbf{s}_{23})$$
$$-\Big(\frac{[12|(2+3)|4]\,[3|1+2+3|]^2}{[12|(2+3)|4]^2\,\delta_{12}}+\frac{[12|(2+3)|4]\,[3|1+2+3|]^2}{[12|(2+3)|4]\,\delta_{12}}\Big)\,\mathrm{Li}_2^{\mathbf{Z}_{\mathrm{eff}}}(\mathbf{s}_{23},\mathbf{s}_{23},\mathbf{s}_{23})$$
$$-\Big(\frac{[12|(2+3)|4]\,[3|1+2+3|]^2}{[12|(2+3)|4]^2\,\delta_{12}}+\frac{[12|(2+3)|4]\,[3|1+2+3|]^2}{[12|(2+3)|4]\,\delta_{12}}\Big)\,\mathrm{Li}_{-1}\Big(\frac{\mathbf{s}_{23}}{\mathbf{t}},\frac{\mathbf{s}_{23}}{\mathbf{u}}\Big)$$
$$+\mathbf{v}^{\mathbf{z}}\,\delta_{\mathbf{z}}^{\mathbf{z}_a}(\mathbf{s}_{23},\mathbf{s}_{23},\mathbf{s}_{23})+\widetilde{\mathbf{v}}^{\mathbf{z}}\,\delta_{\mathbf{z}}^{\mathbf{z}_a}(\mathbf{s}_{23},\mathbf{s}_{23},\mathbf{s}_{23})+\frac{1}{\mathbf{t}}\,\frac{[3|1+2+3|]^2}{[12|(2+3)|4]\,\delta_{12}}\Big(-\frac{[12|(2+3)|4]\,[3|1+2+3|]^2}{[12|(2+3)|4]}+\frac{[12|(2+3)|4]\,[3|1+2+3|]^2}{[12|(2+3)|4]^2}\Big)$$
$$+\frac{1}{\mathbf{t}}\,\frac{[3|1+2+3|]^2}{[12|(2+3)|4]\,\delta_{12}}\Big(-\big[1\mathrm{i}\big]\,\delta_{12}\,\big[3|1\big]\big(1+\big[3|1+2+3|\big]\big)$$
$$+\big[3|1+4|1\big]\,\big[3|1+2+3|\big]\big(4-\big[3|1\big]\