Physics 332, Spring 2008, Problem Set 5
— due Wednesday, June 4, in class

Problem 1 [4 pts]: In class we showed that the color decomposition for the 2-quark-2-gluon tree amplitude is

$$\mathcal{A}_4^{\rm tree}(1_{\bar q}, 2_q, 3, 4) = g^2 \Big[(T^{a_3} T^{a_4})_{i_2}^{\;\bar\jmath_1} \, A_4^{\rm tree}(1_{\bar q}, 2_q, 3, 4) + (T^{a_4} T^{a_3})_{i_2}^{\;\bar\jmath_1} \, A_4^{\rm tree}(1_{\bar q}, 2_q, 4, 3) \Big] \,.$$

Work out the color-summed cross section $\sum_{\rm colors} \mathcal{A}_4^{{\rm tree}\,*} \mathcal{A}_4^{\rm tree}$ for this case, analogous to the 4-gluon case done in class [eq. (10) of hep-ph/9601359]. (Be careful not to drop the "$-1/N_c$" terms in this case.)

Problem 2 [3 pts]: Prove the Schouten identity,

$$\langle i\, j \rangle \langle k\, l \rangle \;=\; \langle i\, k \rangle \langle j\, l \rangle + \langle i\, l \rangle \langle k\, j \rangle \,,$$

or equivalently,

$$\lambda_i^\alpha \lambda_{j\,\alpha} \, \lambda_k^\beta \lambda_{l\,\beta} \;=\; \lambda_i^\alpha \lambda_{k\,\alpha} \, \lambda_j^\beta \lambda_{l\,\beta} + \lambda_i^\alpha \lambda_{l\,\alpha} \, \lambda_k^\beta \lambda_{j\,\beta} \,.$$

Hint: Use the fact that there are only 2 independent vectors in a 2-dimensional vector space; hence $\lambda_k^\alpha = c_1 \lambda_i^\alpha + c_2 \lambda_j^\alpha$ for some complex numbers c_1 and c_2. Next determine c_1 and c_2.

Problem 3 [3 pts]: Using color-ordered rules and the spinor-helicity formalism, compute the color-ordered helicity amplitude $A_4^{\rm tree}(1_{\bar q}^-, 2_q^+, 3^-, 4^+)$.