

**Physics 332, Spring 2008, Problem Set 2**

— due Friday, April 25 (Masoud Soroush's mailbox in Varian)

**Problem 1:** (a) [6 pts] For the  $O(N)$  linear sigma model discussed in section 11.2 of Peskin and Schroeder, compute the finite one-loop (order  $\lambda^2$ ) correction to the mass of the  $\sigma$  particle,  $\delta^{(1)}m_\sigma^2 \equiv m_\sigma^2 - 2v^2\lambda^{\overline{MS}}(M)$ . Use the  $\overline{MS}$  scheme to renormalize  $\lambda$  (see p. 377), subtracting a multiple of  $2/\epsilon - \gamma + \ln(4\pi) - \ln(M^2)$  for the divergent counterterm. Here  $M$  is the  $\overline{MS}$  scale (introduced via  $\lambda \rightarrow \lambda M^\epsilon$  in  $d = 4 - \epsilon$ ). Impose the vanishing of the one-point function including all finite terms, however.

(b) [1 pt] Do you need to know the finite part of  $\delta_Z$ ?

(c) [1 pt] Explain how on-shell renormalization of  $\lambda$  would have made this computation much more difficult.

**Problem 2** [2 pts]: In the linear sigma model, no energy should be required to move the vacuum from  $\phi_i = (0, \dots, 0, v)$  to another nearby vacuum, parametrized by the  $N - 1$  pion fields  $\pi^k$ . Yet there is a term proportional to  $[(\pi^k)^2]^2$  in the shifted Lagrangian (11.9), which seems to cost energy for nonzero values of  $\pi^k$ . Explain what is going on.