Physics 332, Spring 2008, Problem Set 1 — due Wednesday, April 16

Problem 1 [2 pts]: Use an integration by parts similar to that following eq. (7.32) of Peskin and Schroeder to show that $\delta_1 = \delta_2$ for QED at one loop in dimensional regularization; *i.e.* show that eq. (10.43) and (10.46) are the same.

Problem 2 [3 pts]: Problem 10.2 of Peskin and Schroeder, part (a) only.

Problem 3 [2pts]: Problem 10.3 of Peskin and Schroeder. (Note that $(4\pi)^2$ in the denominator should be $(4\pi)^4$.)

Problem 4 [3pts]: Consider a real scalar theory in $d = 4$ space-time dimensions with interaction $\mathcal{L}_{\text{int}} = g_4\phi^4 + g_6\phi^6$. Assuming that the only intrinsic scale is the cutoff momentum Λ, we can express the couplings as

$$g_4 = u_4 \, , \qquad g_6 = \frac{u_6}{\Lambda^2} \, , \tag{1}$$

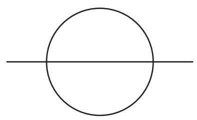
where u_4 and u_6 are dimensionless. As far as the S-matrix is concerned, this theory is equivalent to a ϕ^4 theory with an *effective* coupling constant. To illustrate this fact, consider the following one-particle irreducible correlation functions (shown on the next page):

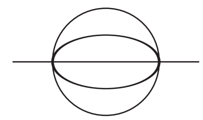
1. Consider the graphs for the irreducible self-energy G_2 and G_2'. Show that they have the same degree of divergence, as in pure ϕ^4 theory.

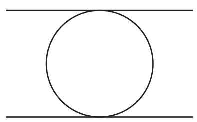
2. Show that the irreducible ϕ^4 vertex graphs G_4 and G_4' diverge like $\ln \Lambda$, as in pure ϕ^4 theory.

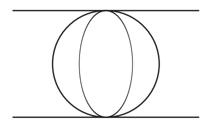
3. Show that the irreducible six-point function G_6 is convergent and the ones involving g_6, such as G_6', vanish when $\Lambda \to \infty$. This shows that the correlation functions depend on g_6 only through the four point functions, as vertex insertions.

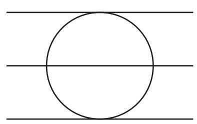
4. Can we renormalize the theory, with the assumed scaling (1)?

$G_2 =$ 

$G_2' =$ 

$G_4 =$ 

$G_4' =$ 

$G_6 =$ 

$G_6' =$