Physics 332, Spring 2008, Final Exam
— due at noon, Tuesday, June 10, Masoud Soroush's mailbox in Varian

This exam is governed by the Stanford Honor Code. You may use Peskin & Schroeder, my TASI lecture notes, and your class notes, but no other sources, and of course no aid from other individuals.

Problem 1 [10 pts]: Bob calculates the 3-loop β function for a theory with dimensionless coupling α to be

$$\beta(\alpha) = \mu\frac{\partial\alpha}{\partial\mu} = b_0\alpha^2 + b_1\alpha^3 + b_2\alpha^4 + \mathcal{O}(\alpha^5).$$

Sue, using a different renormalization scheme, and hence coupling definition,

$$\bar{\alpha} = \alpha + c_1\alpha^2 + c_2\alpha^3 + \mathcal{O}(\alpha^4),$$

finds

$$\bar{\beta}(\bar{\alpha}) = \mu\frac{\partial\bar{\alpha}}{\partial\mu} = \bar{b}_0\bar{\alpha}^2 + \bar{b}_1\bar{\alpha}^3 + \bar{b}_2\bar{\alpha}^4 + \mathcal{O}(\bar{\alpha}^5).$$

Show that Bob and Sue agree on the first two terms, $\bar{b}_0 = b_0$, and $\bar{b}_1 = b_1$, but disagree on the third term. Give a formula for Sue's $\bar{b}_2$ in terms of Bob's coefficients, c_1, and c_2.

Ralph uses a "strange" renormalization scheme, with coupling

$$\tilde{\alpha} = \alpha + d_1\alpha^2\ln\alpha + \mathcal{O}(\alpha^3).$$

What is the first order at which Ralph's computation begins to differ from Bob and Sue's, and what is the value of his coefficient $\tilde{b}_i$ at that order?

Problem 2 [20 pts]: The two-loop β function in $SU(N_c)$ gauge theory with N_f massless quark flavors is

$$\beta(\alpha) = -\Big(\frac{11}{3}N_c - \frac{2}{3}N_f\Big)\frac{\alpha^2}{2\pi} - \Big(\frac{17}{3}N_c^2 - \Big(\frac{13}{6}N_c - \frac{1}{2N_c}\Big)N_f\Big)\frac{\alpha^3}{4\pi^2} + \mathcal{O}(\alpha^4)$$

Consider the limit where N_c and N_f are both large, and $N_f = (11/2)N_c - \delta$, $\delta \ll 1$, such that the one-loop β function coefficient is almost zero. Working always to first order in δ, find the value of the fixed-point coupling α_* at which the β function vanishes. Sketch the β function from $\alpha = 0$ to the vicinity of α_*. Is α_* ultraviolet attractive or infrared attractive? What is the only nontrivial *relevant* gauge-invariant composite operator in this theory? Compute its one-loop anomalous dimension at the fixed point. If one perturbs away from the fixed point using this operator, how do physical masses scale with the coefficient of the operator? Also compute the one-loop anomalous dimension for the quark field. Use the result, and the Callan-Szymanzik equation, to write down the form of the quark propagator at the fixed point.

Problem 3 [30 pts]: When the LHC turns on later this year, one of its major tasks is to search for the Standard Model Higgs boson. The production cross section is dominated by gluon fusion, $gg \to h$, mediated by quark loops. Quarks couple to the Higgs boson through their Yukawa couplings,

$$\mathcal{L}_f = -m_f \bar{f} f \left(1 + \frac{h}{v} \right),$$

with $v = 246$ GeV. The time-reversed process, $h \to gg$, also contributes to the Higgs decay width, although it is not a channel that can be detected above the copious LHC backgrounds. There are four gluon-helicity configurations, $(++)$, $(+-)$, $(-+)$, $(--)$. Show that two of these vanish (why?), and the other two are related by a symmetry (which one?). Compute the one independent $h \to gg$ amplitude, and then the partial decay width $\Gamma(h \to gg)$, in terms of a sum over quark flavors, weighted by appropriate loop integrals. What is the numerical value of the ratio of the b quark contribution to the t quark contribution, if $m_h = 120$ GeV, $m_b = 5$ GeV and $m_t = 175$ GeV? (You may assume $m_b \ll m_h$ and $m_t \gg m_h$ for this part and below.) For $m_h = 120$ GeV, the dominant branching ratio of the Higgs boson is to b quark pairs, $h \to b\bar{b}$. What is the numerical value of the Higgs boson decay width for the above choice of masses? What is the branching ratio to a pair of gluons $\text{Br}(h \to gg)$, assuming that $\text{Br}(h \to b\bar{b}) \approx 1$? For $m_t \gg m_h$, the hgg interaction can be approximated by an effective operator. Write down this operator and its coefficient in the $m_t \gg m_h$ limit. For extra credit, compute the operator's anomalous dimension at order α_s (which enters the higher-order corrections to Higgs production and decay).