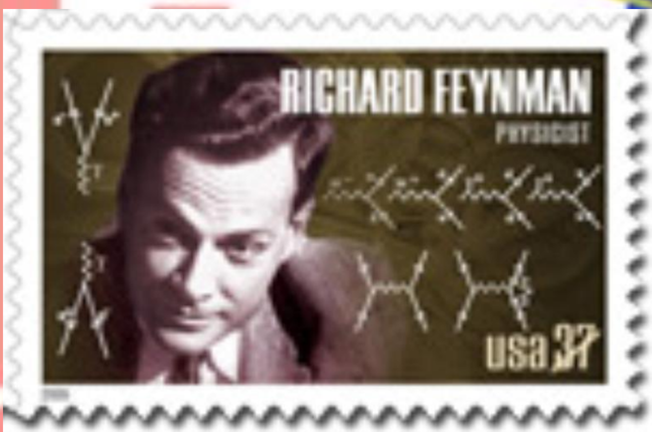
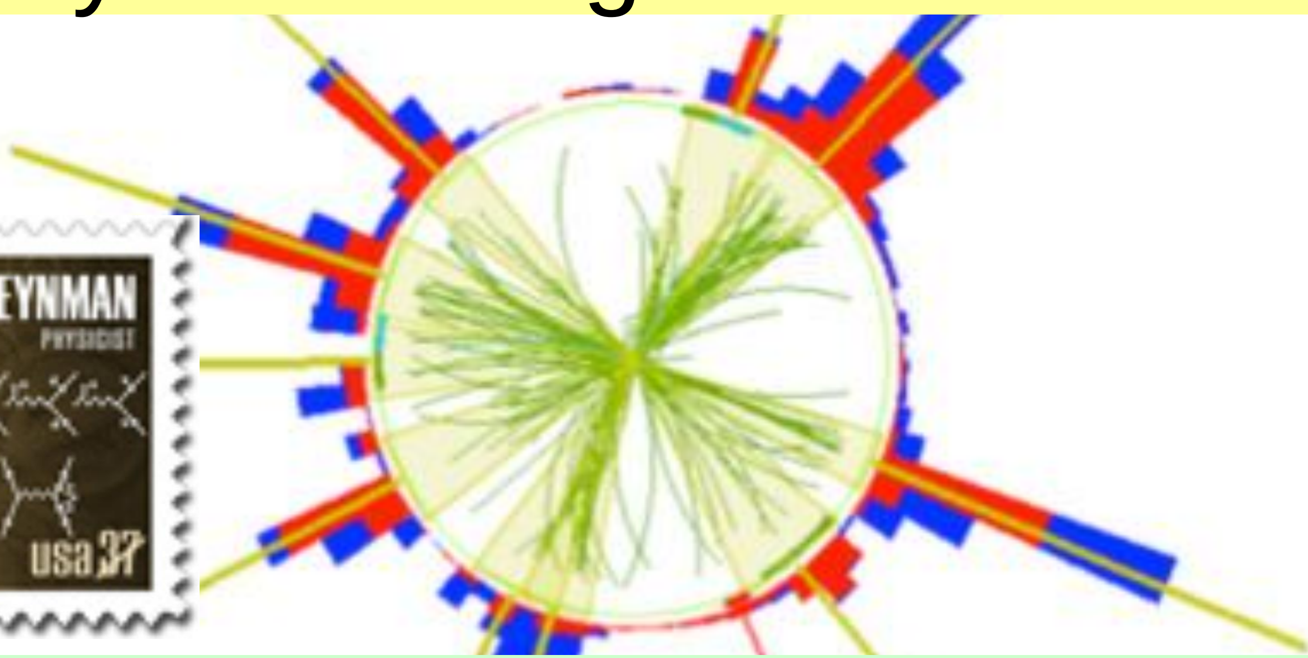
Seven Decades of Particle Scattering

from Feynman Diagrams to the LHC





Lance Dixon (SLAC)

Richard Feynman at 100

Nanyang Technological University

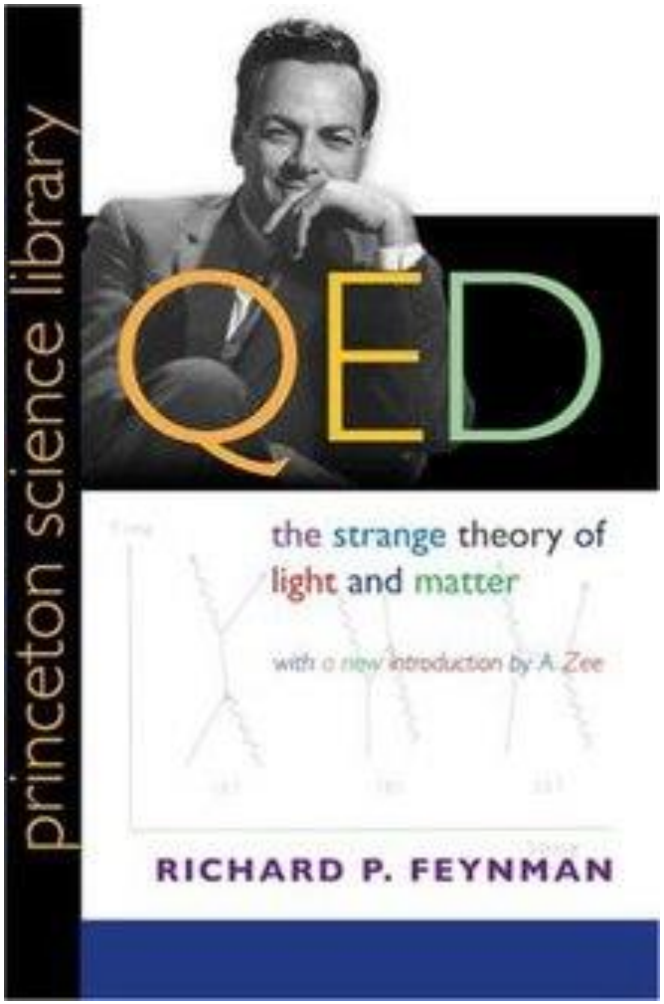
October 23, 2018

Outline

- Feynman, Feynman diagrams and QED
- Feynman and early QCD
- QCD at the LHC – beyond Feynman diagrams
- Particle scattering in other theories
- Conclusions

- **Disclaimers:**
- I'm not a historian
- Half of the 7 decades happened before I entered graduate school

Feynman's revolutionary insights into scattering of quantum particles



were initially for Quantum ElectroDynamics

Theory of how electrons interact with the particles associated with light or electromagnetism = photons

The most precise theory of all – good to a part per trillion!

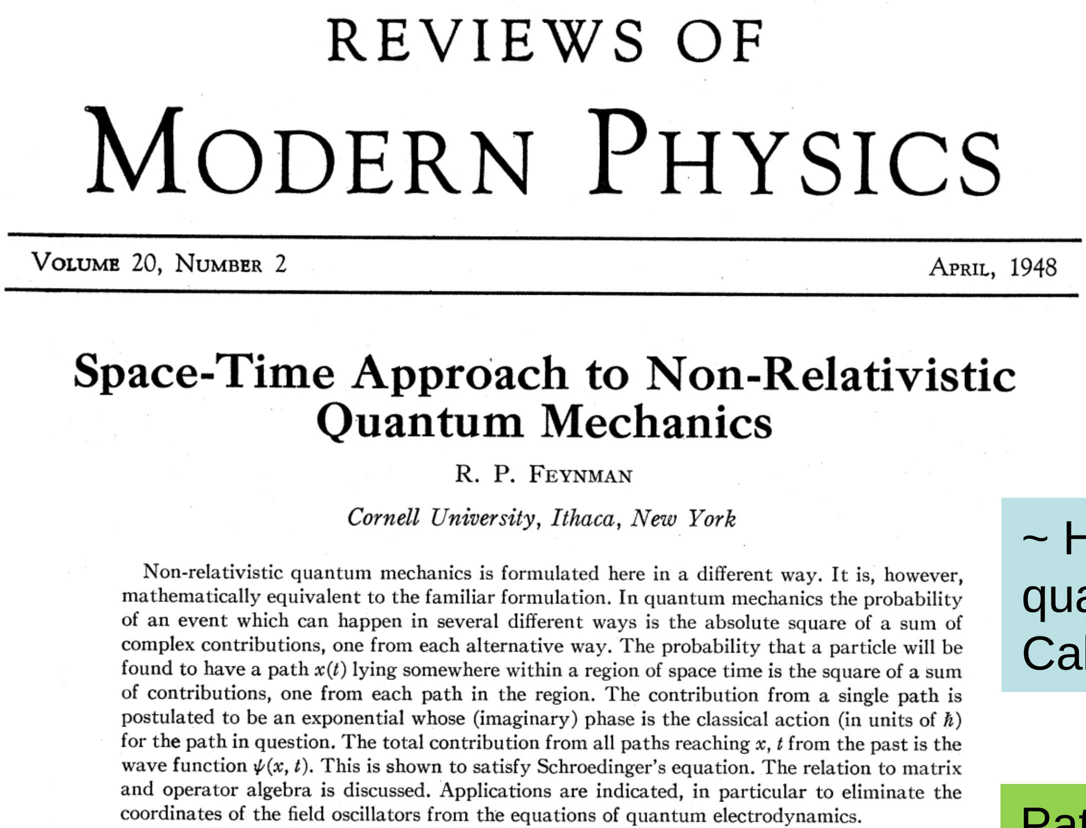
Shelter Island, June 1947

NAS Archives

Dirac theory of electron incomplete:

- Willis Lamb reports on Lamb shift between 2S and 2P hydrogen
- Isadore Rabi reports on electron anomaly [Nafe, Nelson, Rabi]

Feynman's thesis work
birthplace of the path integral

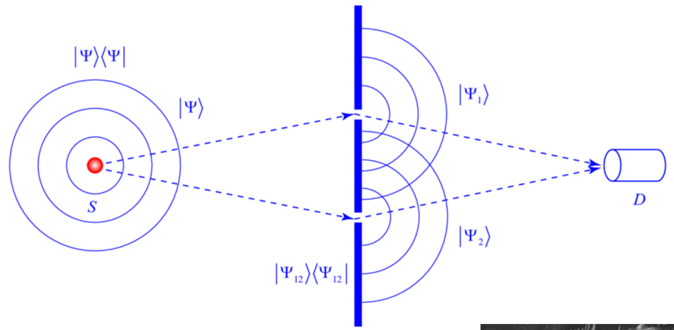

REVIEWS OF
MODERN PHYSICS
VOLUME 20, NUMBER 2 APRIL, 1948

Space-Time Approach to Non-Relativistic Quantum Mechanics

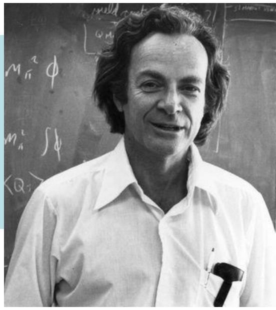
R. P. FEYNMAN
Cornell University, Ithaca, New York

Non-relativistic quantum mechanics is formulated here in a different way. It is, however, mathematically equivalent to the familiar formulation. In quantum mechanics the probability of an event which can happen in several different ways is the absolute square of a sum of complex contributions, one from each alternative way. The probability that a particle will be found to have a path $x(t)$ lying somewhere within a region of space time is the square of a sum of contributions, one from each path in the region. The contribution from a single path is postulated to be an exponential whose (imaginary) phase is the classical action (in units of $\hbar$) for the path in question. The total contribution from all paths reaching x, t from the past is the wave function $\psi(x, t)$. This is shown to satisfy Schroedinger's equation. The relation to matrix and operator algebra is discussed. Applications are indicated, in particular to eliminate the coordinates of the field oscillators from the equations of quantum electrodynamics.

$$\int [D\phi(t,x)] \exp\Big\{\frac{i}{\hbar} S[\phi(t,x)]\Big\}$$



~ How Feynman introduced quantum mechanics to us Caltech undergrads in 1979



Path integrals really come to life in lattice gauge theory
– talk by M. Creutz

The beginning of Feynman diagrams

PHYSICAL REVIEW VOLUME 76, NUMBER 6 SEPTEMBER 15, 1949

The Theory of Positrons

R. P. FEYNMAN
Department of Physics, Cornell University, Ithaca, New York
(Received April 8, 1949)

PHYSICAL REVIEW VOLUME 76, NUMBER 6 SEPTEMBER 15, 1949

Space-Time Approach to Quantum Electrodynamics

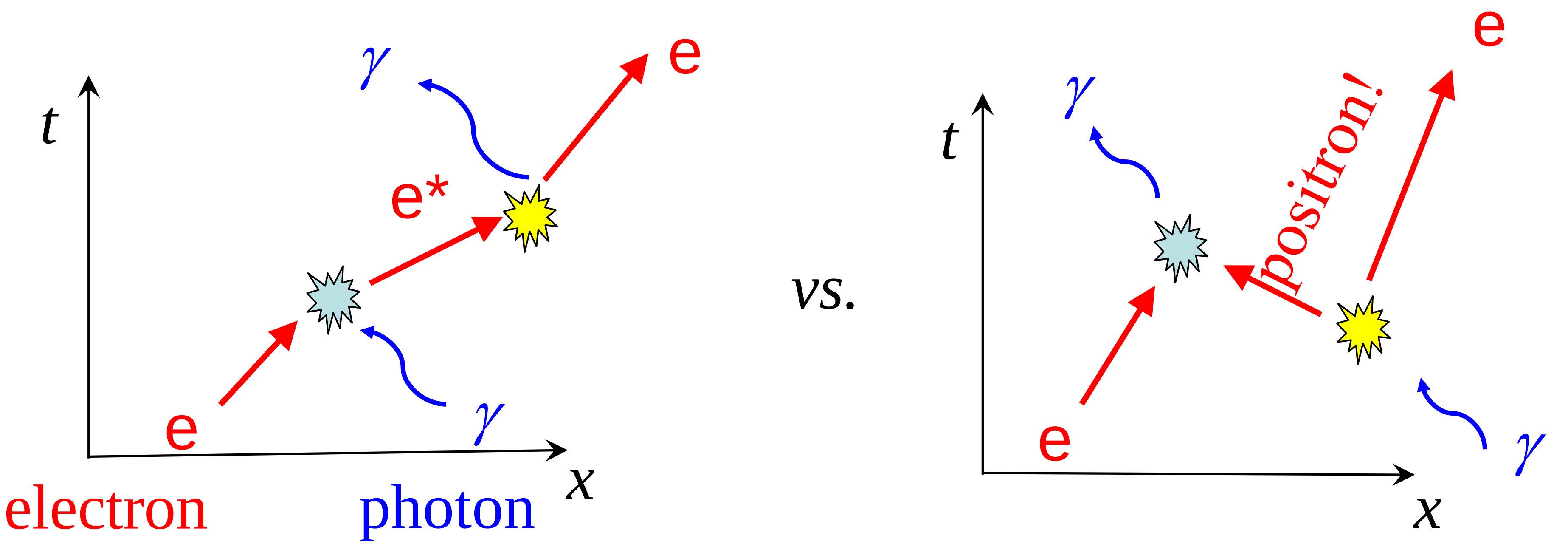
R. P. FEYNMAN
Department of Physics, Cornell University, Ithaca, New York
(Received May 9, 1949)

- Before Feynman, quantum-mechanical calculations were strictly ***time-ordered***, based on the Hamiltonian *H* which evolves states forward in time:

$$|\psi(t)\rangle = \exp(-iHt/\hbar)|\psi(0)\rangle$$

Covariance and positrons

Feynman realized that time ordering is ***ambiguous*** in special relativity: Two observers moving with respect to each other can see the same two events happen in different order.

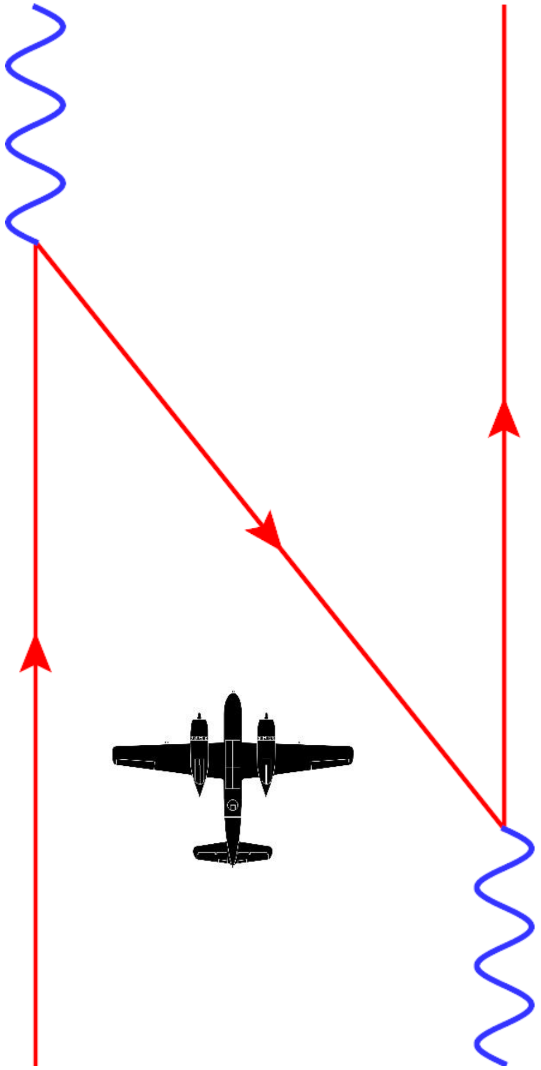


A positron is an electron moving backward in time

These two **time-ordered** contributions naturally belong together!

A holistic view

Following the charge rather than the particles corresponds to considering this continuous world line as a whole rather than breaking it up into its pieces. It is as though a bombardier flying low over a road suddenly sees three roads and it is only when two of them come together and disappear again that he realizes that he has simply passed over a long switchback in a single road.



On and off the “mass shell”

- Einstein: energy of a particle at rest is $E = mc^2$
- Energy of a particle in motion with momentum $\mathbf{p}$:
  $E^2 = (\mathbf{p}c)^2 + (mc^2)^2 = \mathbf{p}^2 + m^2$ for $c = 1.$
- Energy & momentum form a relativistic four vector, $p^\mu = (p^0, p^1, p^2, p^3) = (E,\mathbf{p})$
- Its relativistically invariant “length” is its mass:

$$p^2 = p^\mu p_\mu = E^2 - \mathbf{p}^2 = m^2$$

- Real particles are on-shell, $p^2 = m^2$
- Virtual particles are off-shell, $p^2 \neq m^2$

Neither advanced nor retarded

In order to combine the two contributions, Feynman needed to construct a new "propagator" – the rule for how the electron gets from point A to point B. It also had to move positrons (sometimes called negative energy solutions) backward in time from point B to point A.

Retarded propagator only propagates effects to later time, it is causal.

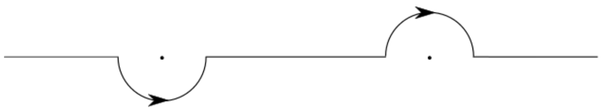
$$G_{\mathrm{ret}}(p) = \frac{1}{(p^0 + i\varepsilon)^2 - \mathbf{p}^2 - m^2}$$

Advanced propagator only propagates effects to earlier time, it's anti-causal

$$G_{\mathrm{adv}}(p) = \frac{1}{(p^0 - i\varepsilon)^2 - \mathbf{p}^2 - m^2}$$

Feynman propagator does either, depending on energy, it's covariant

$$G_F(p) = \frac{1}{p^2 - m^2 + i\varepsilon}$$



Freeman Dyson, interlocutor

PHYSICAL REVIEW VOLUME 75, NUMBER 3 FEBRUARY 1, 1949

The Radiation Theories of Tomonaga, Schwinger, and Feynman

F. J. Dyson
Institute for Advanced Study, Princeton, New Jersey

(Received October 6, 1948)

A unified development of the subject of quantum electrodynamics is outlined, embodying the main features both of the Tomonaga-Schwinger and of the Feynman radiation theory. The theory is carried to a point further than that reached by these authors, in the discussion of higher order radiative reactions and vacuum polarization phenomena. However, the theory of these higher order processes is a program rather than a definitive theory, since no general proof of the convergence of these effects is attempted.

The chief results obtained are (a) a demonstration of the equivalence of the Feynman and Schwinger theories, and (b) a considerable simplification of the procedure involved in applying the Schwinger theory to particular problems, the simplification being the greater the more complicated the problem.

Dyson as Ben Jonson to Feynman's Shakespeare

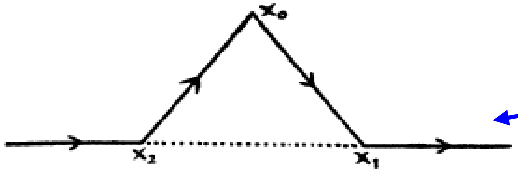


FIG. 1.

First Feynman diagram in print!

"Nature herself was proud of his designs, and joyed to wear the dressing of his lines."

The most iconic Feynman diagram

772 Phys. Rev. 76, 769 R. P. FEYNMAN

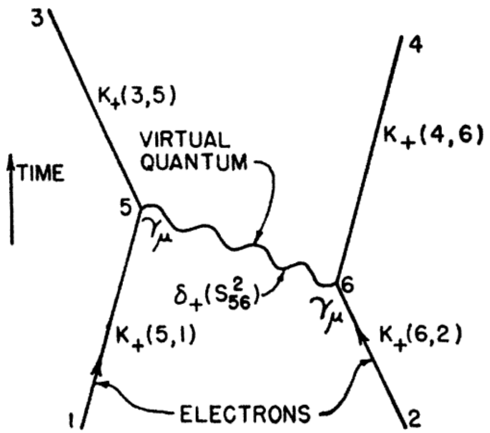


FIG. 1. The fundamental interaction Eq. (4). Exchange of one quantum between two electrons.

electron-electron scattering in QED

But it can be repurposed to also describe the most important processes in the Standard Model

Carved in stone in Tuva (courtesy of Glen Cowan, Ralph Leighton)

Feynman parameters

RPF, Phys. Rev. 76, 769

The integrals so far only contain one factor in the denominator. To obtain results for two factors we make use of the identity

$$a^{-1}b^{-1}=\int_0^1 dx(ax+b(1-x))^{-2}, \tag{14a}$$

(suggested by some work of Schwinger's involving Gaussian integrals). This represents the product of two reciprocals as a parametric integral over one and will therefore permit integrals with two factors to be expressed in terms of one. For other powers of a, b, we make use of all of the identities, such as

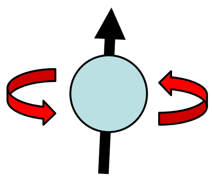
$$a^{-2}b^{-1}=\int_0^1 2xdx(ax+b(1-x))^{-3}, \tag{15a}$$

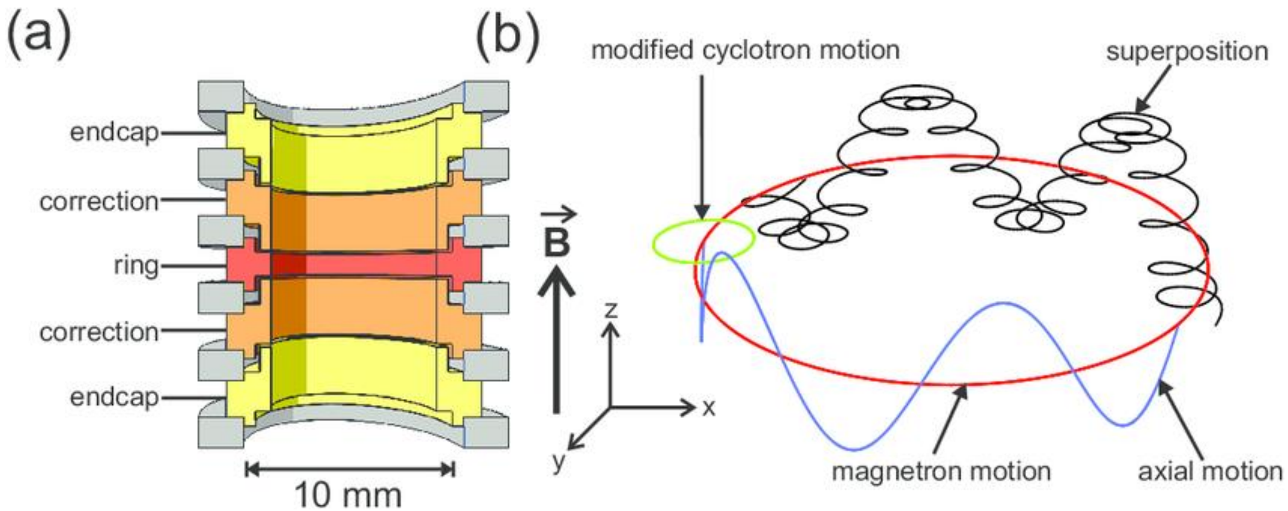
deducible from (14a) by successive differentiations with respect to a or b.

A mathematical trick, but an incredibly useful one.

The electron anomalous magnetic moment, a (precious) “baby” scattering amplitude

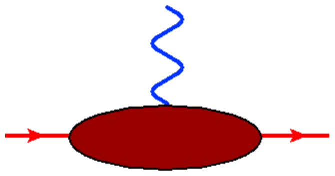
$$\vec{\mu}_e = g_e \frac{e\hbar}{2m_e c}\vec{S}_e$$





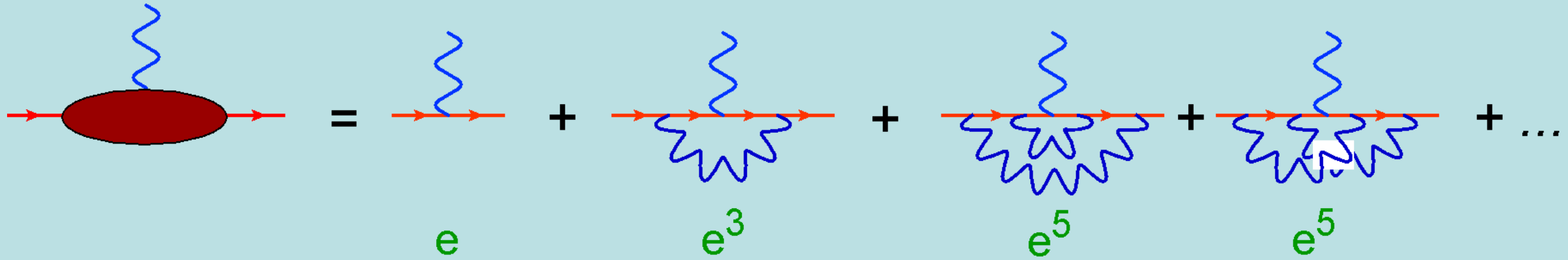
BASE, Eur. Phys. J. ST 224, 16, 3055 (2015)

Measurement doesn’t look much like particle scattering, but $a_e = (g_e - 2)/2$ can be computed from spin-flip part of $\gamma e \rightarrow e$ process as photon momentum → 0.



The loop expansion

- **Feynman:** Draw all diagrams with specified incoming and outgoing particles, weight them by coupling factors at each vertex. For a given process, extra powers of coupling for each closed loop.



In QED, each additional loop suppressed by the
fine structure constant

$$\frac{e^2}{4\pi\hbar c} \equiv \alpha = \frac{1}{137.035999\ldots}$$

By 3 loops, 72 diagrams!

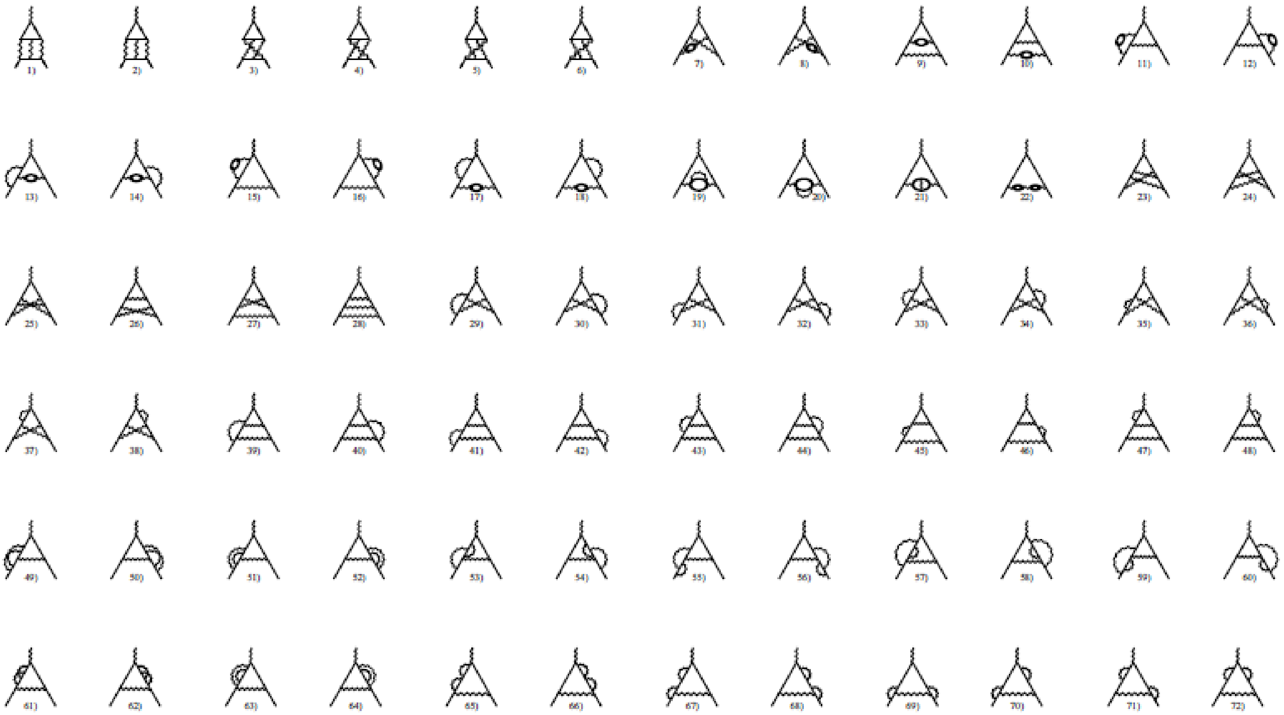


Fig. 7. The universal third order contribution to a_μ. All fermion loops here are muon–loops (first 22 diagrams). All non-universal contributions follow by replacing at least one muon in a closed loop by some other fermion

Without Feynman's methods, hopeless.

Even with Feynman diagrams, reaching this precision would take decades.

QED state of art today: 5 loops, 12,672 diagrams

30 gauge invariant sets

The most difficult set, 6354 diagrams, leading to 389 integrals. Evaluated numerically after Feynman Parameterization.

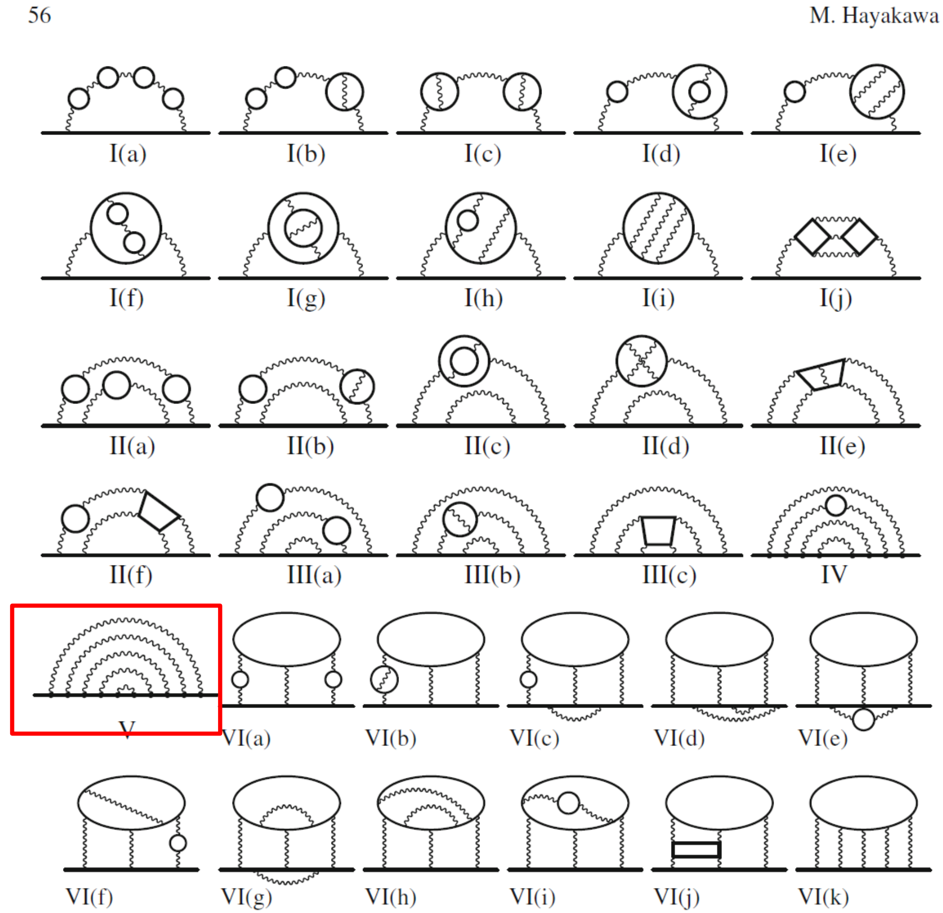


Fig. 2.7 Gauge-invariant subsets of self-energy-like diagrams at the tenth order

Aoyama, Hayakawa, Kinoshita, Nio, Watanabe, 2006-2017

7 decades of g_e-2 theory

fully analytic

$$\zeta_p = \sum_{k=1}^{\infty} \frac{1}{k^p}$$

$$\mathrm{Li}_4(\tfrac{1}{2}) = \sum_{k=1}^{\infty} \frac{1}{2^k\, k^4}$$

$$a_e = \frac{\alpha}{\pi}\cdot\frac{1}{2}$$

← Schwinger 1948

$$+ \left(\frac{\alpha}{\pi}\right)^2 \left[\frac{197}{144} + \frac{\pi^2}{12} - \frac{\pi^2}{2}\ln 2 + \frac{3}{4}\zeta_3\right]$$

← Karplus, Kroll 1950; Petermann 1957; Sommerfield 1957

$$+ \left(\frac{\alpha}{\pi}\right)^3 \left[\frac{28259}{5184} + \frac{17101}{810}\pi^2 - \frac{298}{9}\pi^2\ln 2 + \frac{139}{18}\zeta_3 - \frac{239}{2160}\pi^4 + \frac{100}{3}\left\{\mathrm{Li}_4\left(\frac{1}{2}\right) + \frac{1}{24}(\ln^4 2 - \pi^2\ln^2 2)\right\} + \frac{83}{72}\pi^2\zeta_3 - \frac{215}{24}\zeta_5\right] + \ldots$$

← Kinoshita, Cvitanovic 1972; Laporta, Remiddi 1996

numerical

$$= 0.5\frac{\alpha}{\pi}$$

$$- 0.328478965579 1\ldots\left(\frac{\alpha}{\pi}\right)^2$$

$$+ 1.181241456587 2\ldots\left(\frac{\alpha}{\pi}\right)^3$$

$$- 1.912245764926 4\ldots\left(\frac{\alpha}{\pi}\right)^4$$

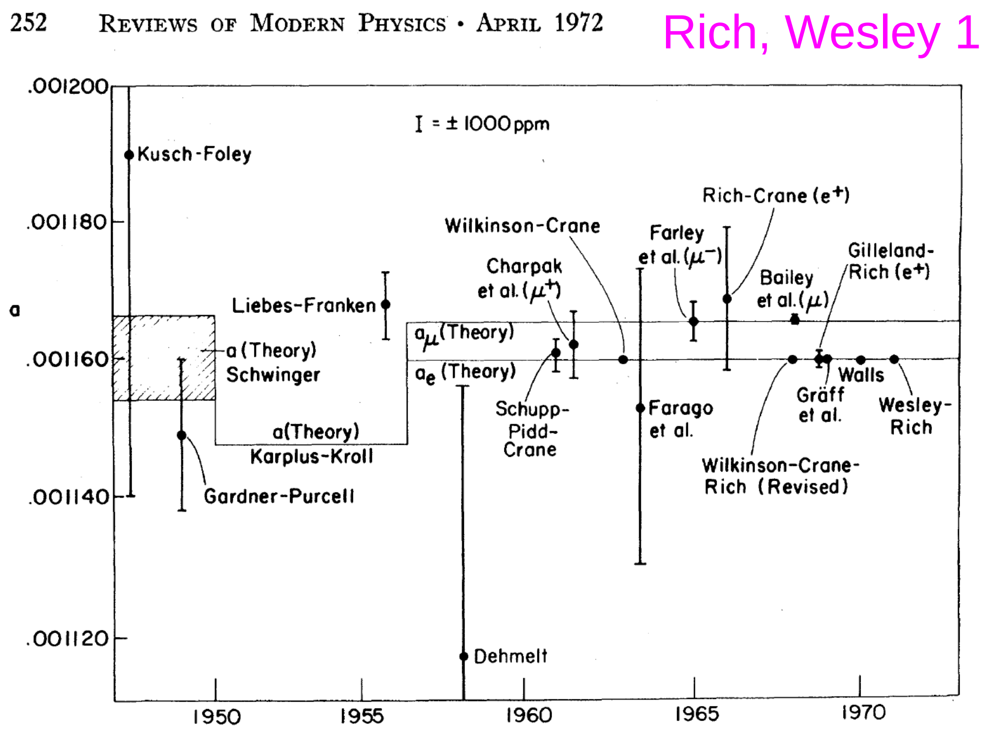
← Aoyama, Hayakawa, Kinoshita, Nio, 2005-2007; Laporta arXiv:1704.06996

(+ mass-dep.)

$$+ 6.7(\pm 0.2)\left(\frac{\alpha}{\pi}\right)^5$$

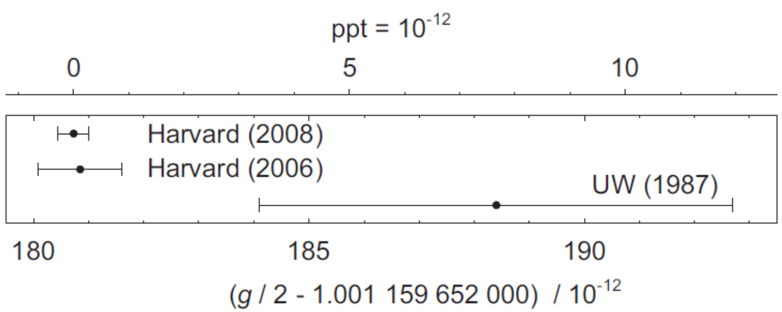
← Aoyama, Hayakawa, Kinoshita, Nio, Watanabe, 2006-2017

All needed to match incredible improvements in experimental precision



Rich, Wesley 1972

Van Dyck, Schwinberg, Dehmelt, 1977-1987



Hanneke, Hoogerheide, Gabrielse, 2006-2010

$$g/2 = 1.001\ 159\ 652\ 180\ 73\ (28) \quad [0.28\ \text{ppt}]$$

Magnetic anomaly anomalies?

- New measurement of fine structure constant in cesium:

$$\alpha^{-1}(\mathrm{Cs}) = 137.035999046(27)$$

- Leads to 2.4σ discrepancy for electron

Davoudiasl, Marciano
arXiv:1806.10252

$$\begin{aligned}\Delta a_e &\equiv a_e^{\exp} - a_e^{\mathrm{SM}} \\ &= [-87 \pm 28\,(\exp) \pm 23\,(\alpha) \pm 2\,(\mathrm{theory})] \\ &\times 10^{-14},\end{aligned}$$

Measuring Earth-Moon distance to width of human hair: 10^{-13}

- Opposite in sign to better known 3.7σ discrepancy for muon

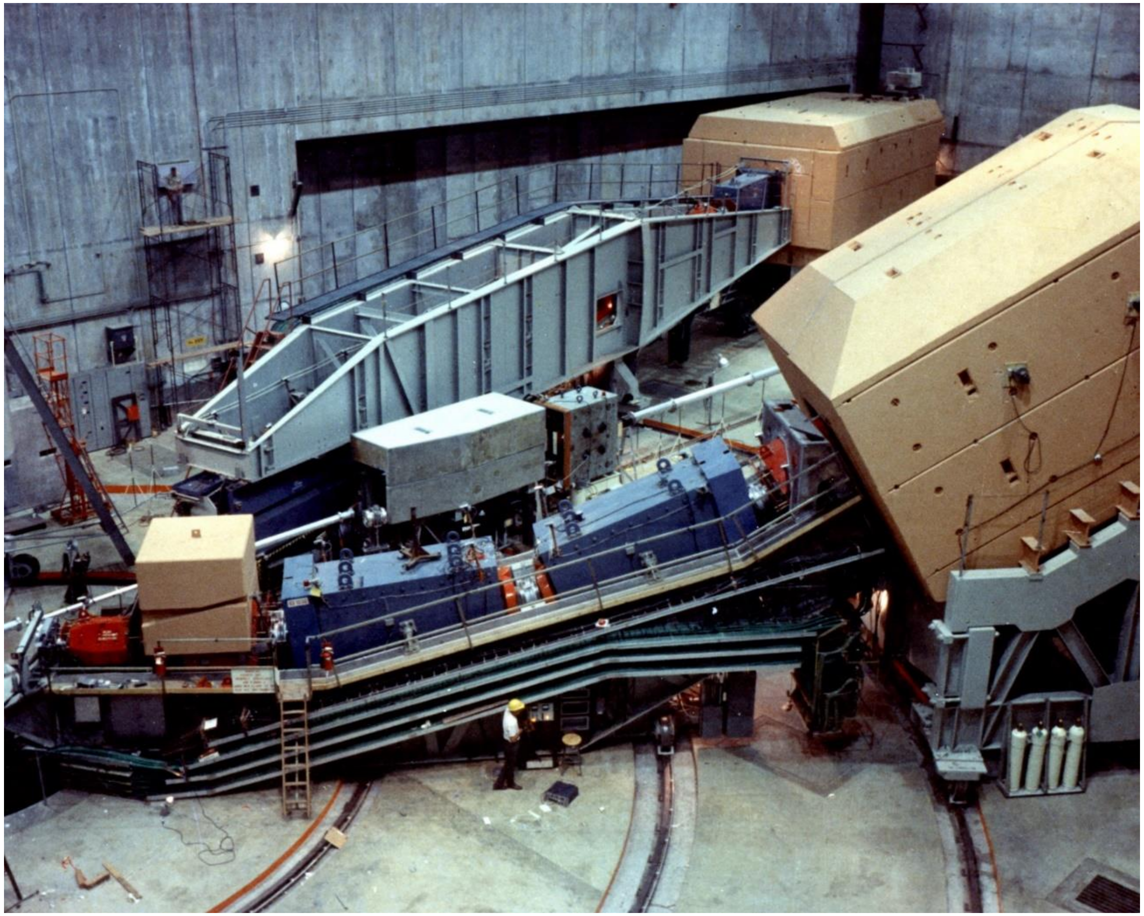
$$\Delta a_\mu \equiv a_\mu^{\exp} - a_\mu^{\mathrm{th}} = (2.74 \pm 0.73) \times 10^{-9}$$

- Could one or both of these be harbingers of new physics?
- Or statistical fluctuations or other issues?

On to real (violent) particle scattering

- Feynman's role in understanding structure of matter: the proton as a bound state of more fundamental objects, quarks and gluons.
- Gell-Mann and Zweig proposed quarks in early 1960s, but were they real, or a mathematical tool to represent symmetries?
- SLAC, a lab built in the 1960s to scatter electrons off protons at record energies, could answer question directly

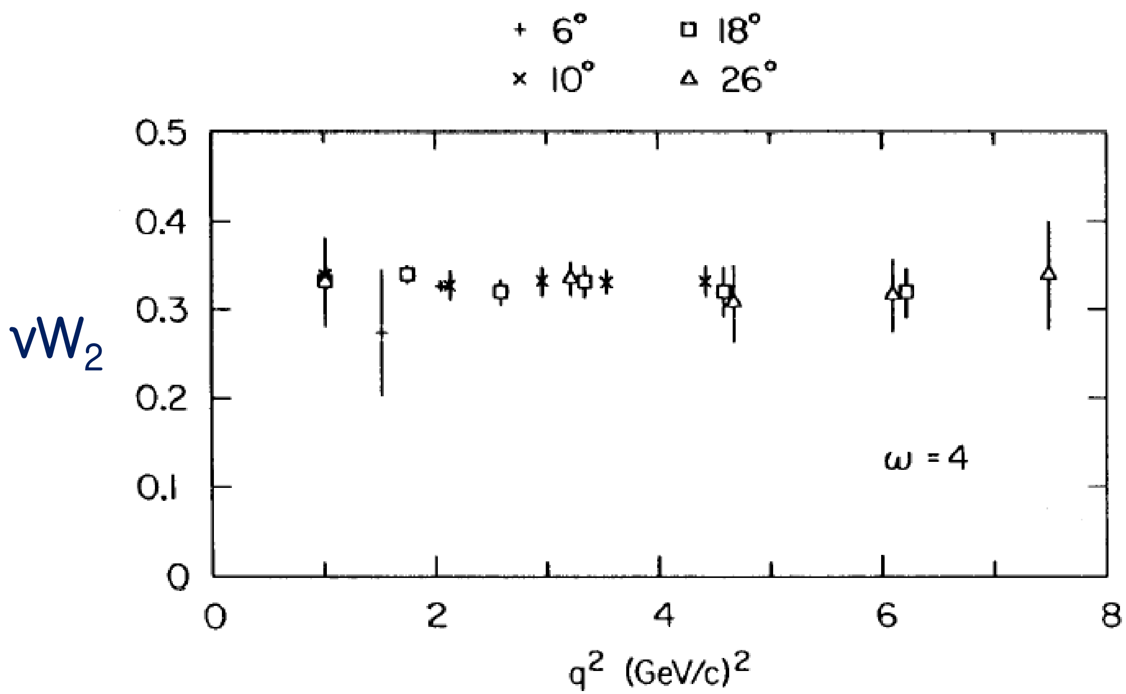
Where quarks were found

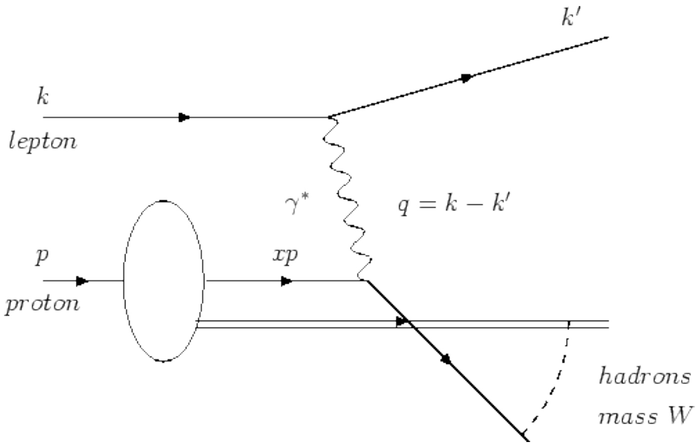


End Station A at SLAC, where "deep inelastic" scattering experiments were performed that revealed "Bjorken Scaling"

Talk by Marty Breidenbach
at SLAC Summer Institute 2018,
"50 years of the Standard Model"

Scaling: νW_2 for fixed ω vs q^2

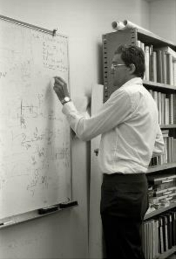




q^2 = photon virtuality

$$\omega = \frac{2p \cdot q}{q^2} = \frac{2M_p \nu}{q^2} = \frac{1}{x}$$

M. Breidenbach, SSI 2018

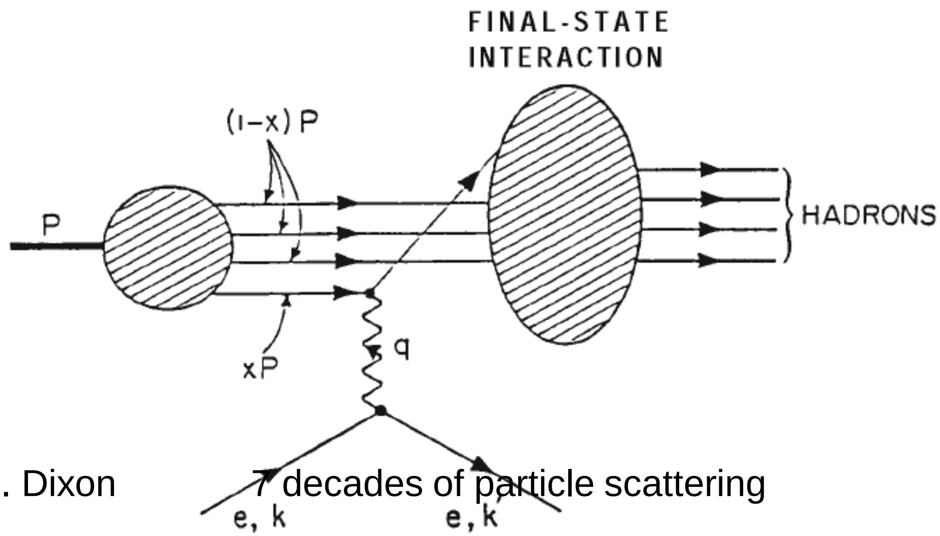


Bj Bjorken

M. Breidenbach, SSI 2018

Partons

- Many of us did not understand bj's current algebra motivation for scaling
- Feynman visited SLAC in August 1968. He had been working on hadron-hadron interactions with point like constituents called partons. We showed him the early data on the weak q^2 dependence and scaling – and he (instantly!) explained the data with his parton model.
- In an infinite momentum frame, the point like partons were slowed, and the virtual photon simply elastically scatters from one parton without interactions with the other partons – the impulse approximation.
- This was a wonderful, understandable model for us.



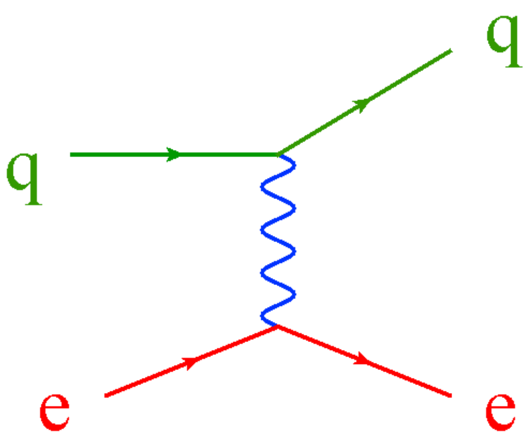
$$W_2^{(i)}(\nu,q^2)=Q^2_i\delta(\nu-q^2/2MX_i)$$
$$=Q^2_i x_i/\nu\ \delta(x_i-q^2/2M\nu)$$

$$\nu W_2(\nu,q^2)=\sum_N \mathcal{P}(N)\left\{\sum_{i=1}^{N} Q_i^{\,2}\right\} x f_N(x)=F_2(x)$$

$$x=\frac{q^2}{2M\nu}=\frac{1}{\omega}$$

"pdf"

Same iconic Feynman diagram, now with quarks



Two deep inelastic structure functions, F_1 and F_2
But this diagram depends on spin of partons, and for spin ½, get Callan-Gross relation, $F_2 = 2xF_1$

$$W_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) F_1(x,Q^2) + \frac{\hat{P}_\mu \hat{P}_\nu}{P \cdot q} F_2(x,Q^2)$$

Confirmed experimentally early on
(at large x where gluon can be neglected)

Scaling → asymptotic freedom
→ nonabelian gauge theories
→ QuantumChromoDynamics, QCD (SU(3) color)

Gross, Wilczek; Politzer 1973

Fritzsch, Gell-Mann 1972;
Weinberg 1973

Feynman and the weak interaction

V – A left-handed structure

also Sudarshan, Marshak

PHYSICAL REVIEW VOLUME 109, NUMBER 1 JANUARY 1, 1958

Theory of the Fermi Interaction

R. P. FEYNMAN AND M. GELL-MANN
California Institute of Technology, Pasadena, California

(Received September 16, 1957)

The representation of Fermi particles by two-component Pauli spinors satisfying a second order differential equation and the suggestion that in β decay these spinors act without gradient couplings leads to an essentially unique weak four-fermion coupling. It is equivalent to equal amounts of vector and axial vector coupling with two-component neutrinos and conservation of leptons. (The relative sign is not determined theoretically.) It is taken to be "universal"; the lifetime of the μ agrees to within the experimental errors of 2%. The vector part of the coupling is, by analogy with electric charge, assumed to be not renormalized by virtual mesons. This requires, for example, that pions are also "charged" in the sense that there is a direct interaction in which, say, a π^0 goes to π^- and an electron goes to a neutrino. The weak decays of strange particles will result qualitatively if the universality is extended to include a coupling involving a Λ or Σ fermion. Parity is then not conserved even for those decays like $K \to 2\pi$ or 3π which involve no neutrinos. The theory is at variance with the measured angular correlation of electron and neutrino in He^6, and with the fact that fewer than 10^{-4} pion decay into electron and neutrino.

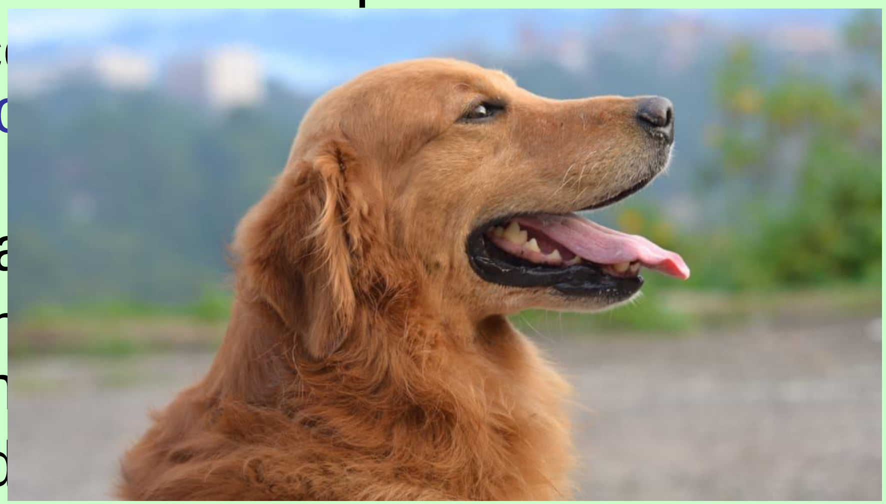
Later completed to $SU(2)_L$ x U(1)
with neutral currents
and a Higgs mechanism

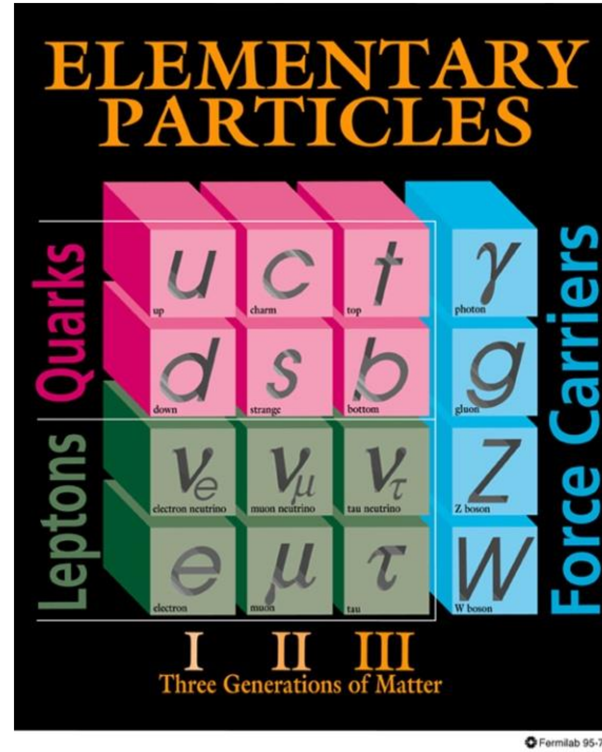
Glashow, Weinberg, Salam 1961-1968

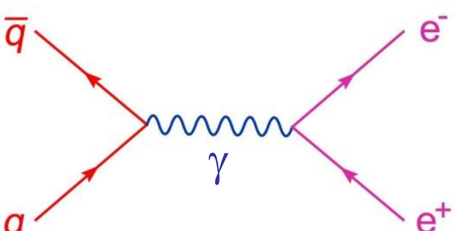
Brout, Englert; Higgs;
Guralnik, Hagen, Kibble 1964

Standard Model

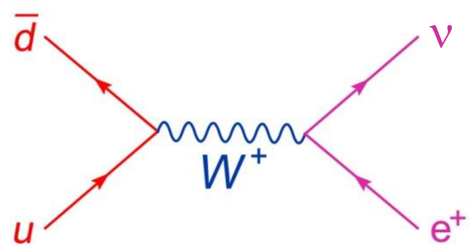
- All elementary forces except gravity in same basic framework
- Matter made of spin ½ fermions
- Forc
  vecto
- Add
  expla
  → fir
  quar
- Solid



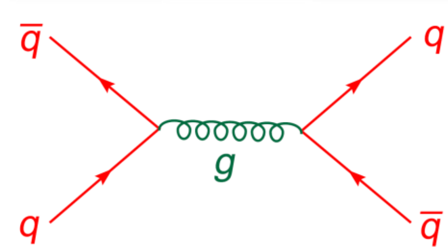




electromagnetism (QED)

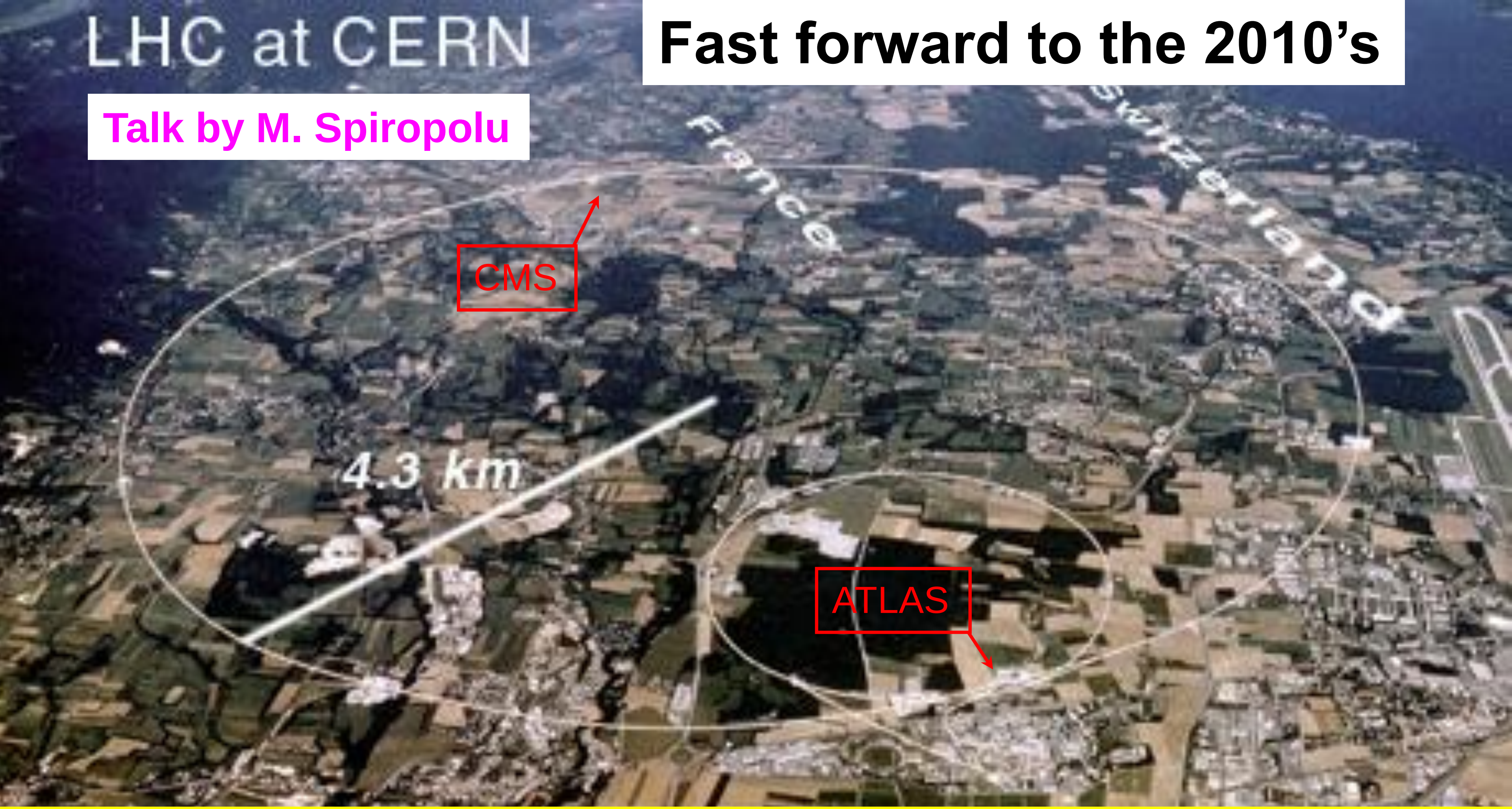


weak



strong (QCD)

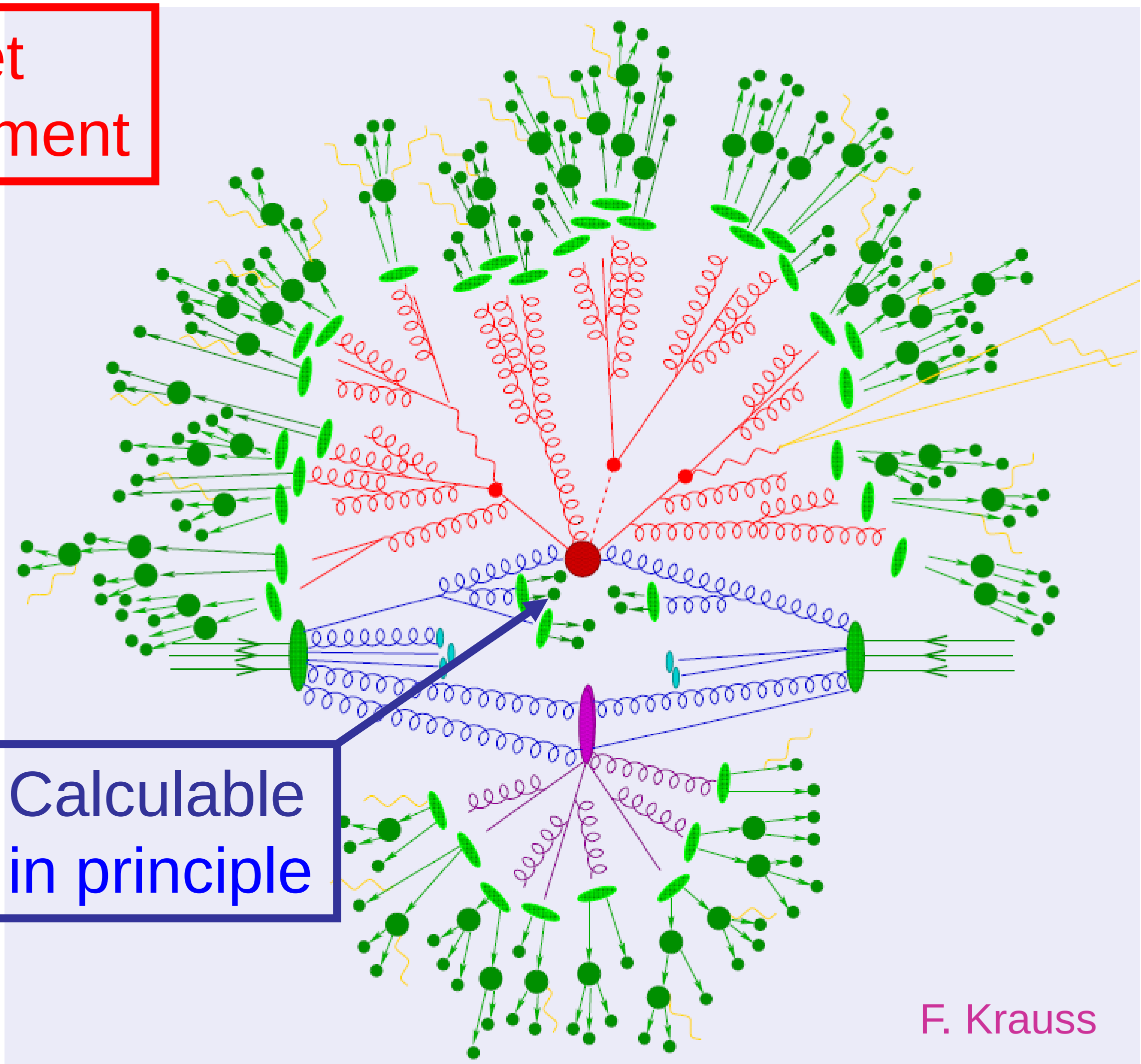
Fast forward to the 2010's



- **The current "Energy Frontier" of particle physics**
- **Make particles that cannot be made anywhere else**
- **Higgs boson discovered in 2012, but properties still need exploring**
- **Will accumulate ~ 20 times as many collisions by ~ 2030**

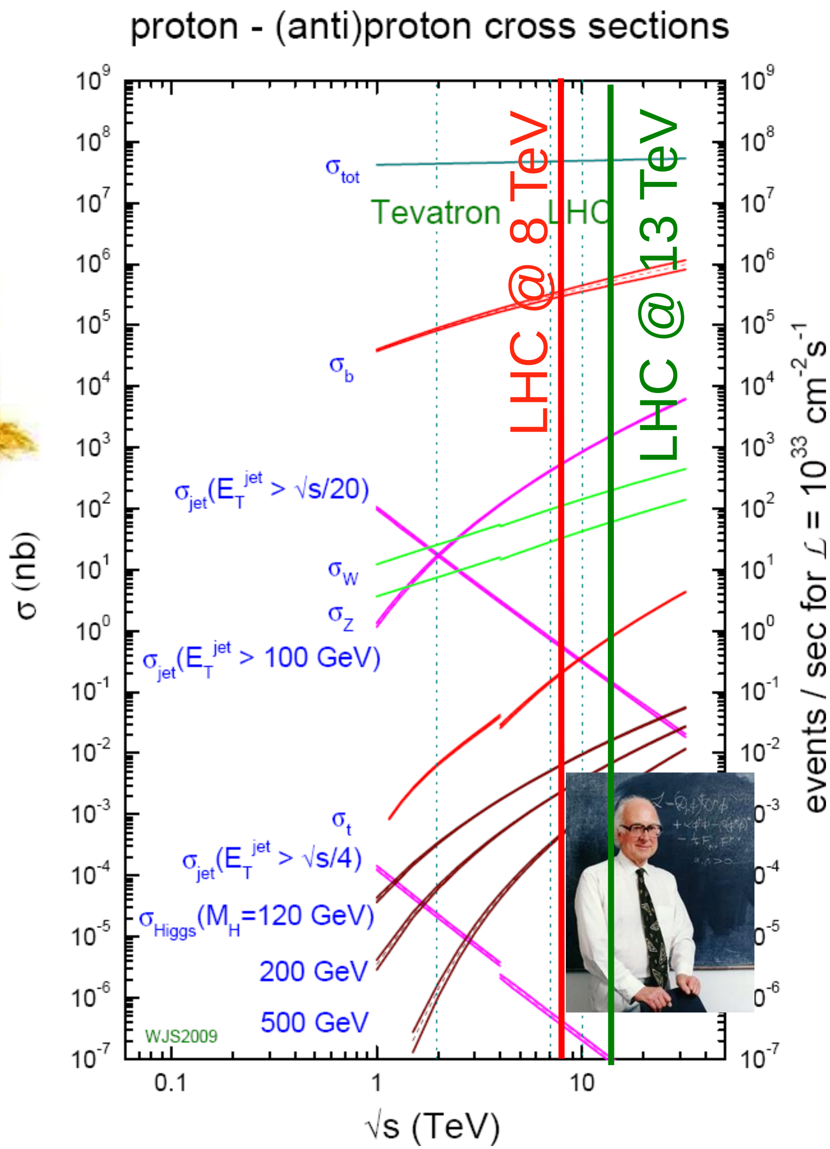
Typical LHC Collision

Model or get from experiment



Calculable in principle

LHC Data Dominated by Jets



Jets from quarks and gluons.

- *q*,*g* from decay of new particles?
- Or old physics?

new physics →

- Every process shown also with one more jet at ~ 1/5 the rate
- Need **accurate production rates** for

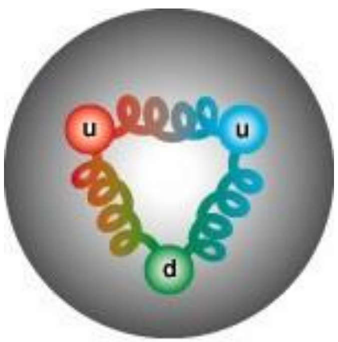
  X + *1,2,3,…* jets

  in Standard Model

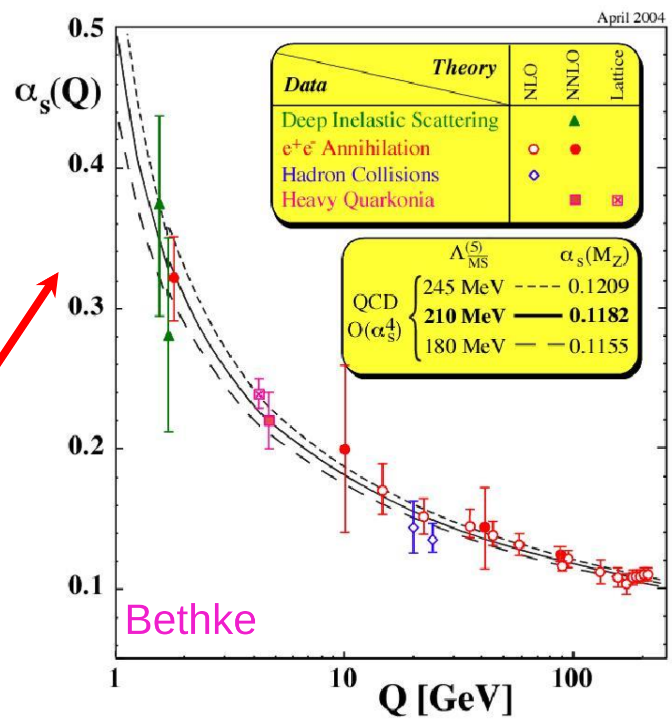
Asymptotic Freedom

Gross, Wilczek, Politzer (1973)

Gluons anti-screen charge: Interaction strength α_s for QCD is a small number at short distances, so we can expand in this small parameter → perturbation theory → Feynman diagrams



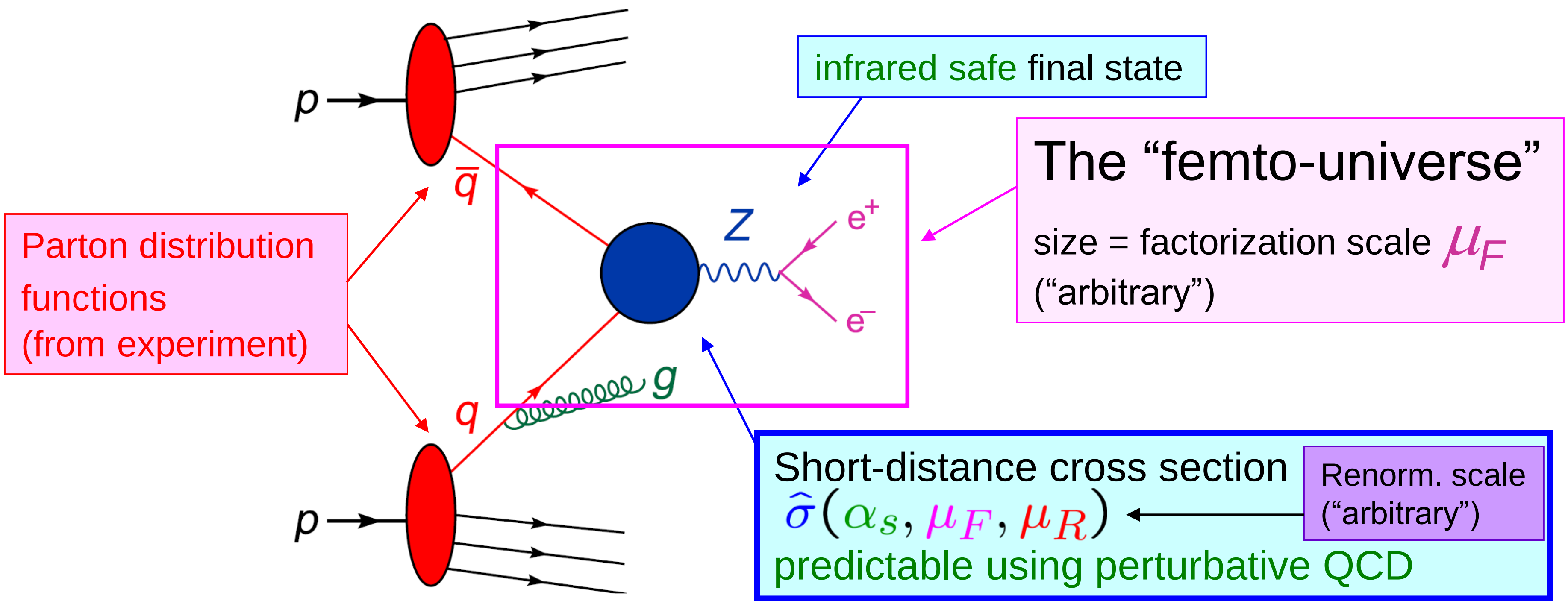
confining



calculable

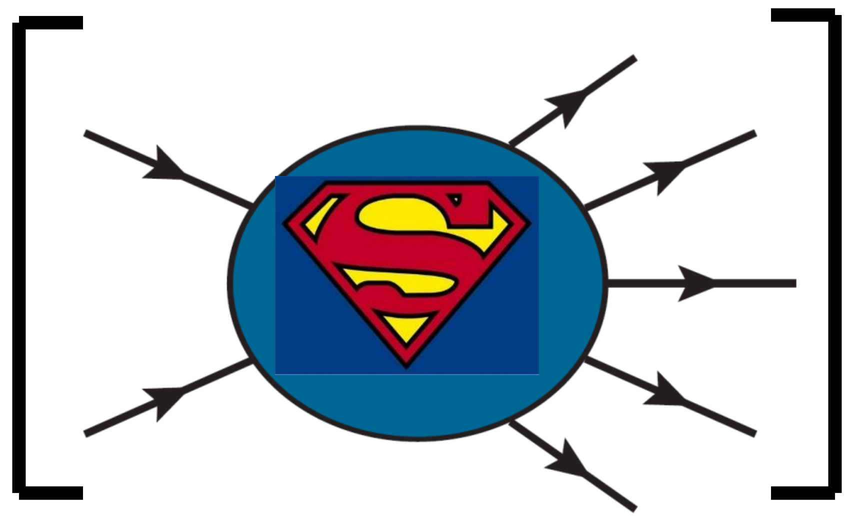
QCD Factorization & Parton Model

At short distances, quarks and gluons (partons) in proton are almost free.
Sampled "one at a time" due to impulse approximation



Short-distance cross sections built out of scattering amplitudes, *S*-matrix elements

$$\sigma = \int$$



Short-Distance Cross Section in Perturbation Theory

$$\hat{\sigma}(\alpha_s, \mu_F, \mu_R) = [\alpha_s(\mu_R)]^{n_\alpha} \left[\hat{\sigma}^{(0)} + \frac{\alpha_s}{2\pi} \hat{\sigma}^{(1)}(\mu_F, \mu_R) + \left(\frac{\alpha_s}{2\pi}\right)^2 \hat{\sigma}^{(2)}(\mu_F, \mu_R) + \cdots \right]$$

LO NLO NNLO

- **Problem:** Leading-order (LO) predictions only qualitative due to **poor convergence** of expansion in $\alpha_s(\mu) \sim 0.1$
- Can easily get ~ 50-100% uncertainties in LO predictions

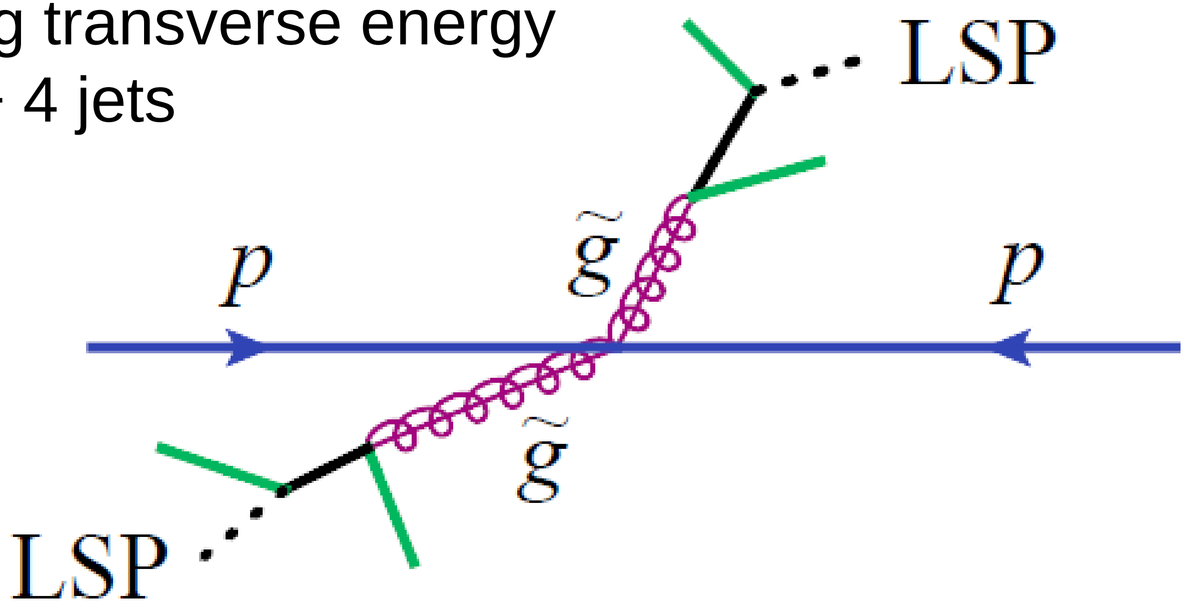
- Uncertainties brought under much better control with NLO corrections: ~ 50-100% → ~ 15-20%

- NNLO becoming increasingly available
→ ~ 3-8% uncertainties

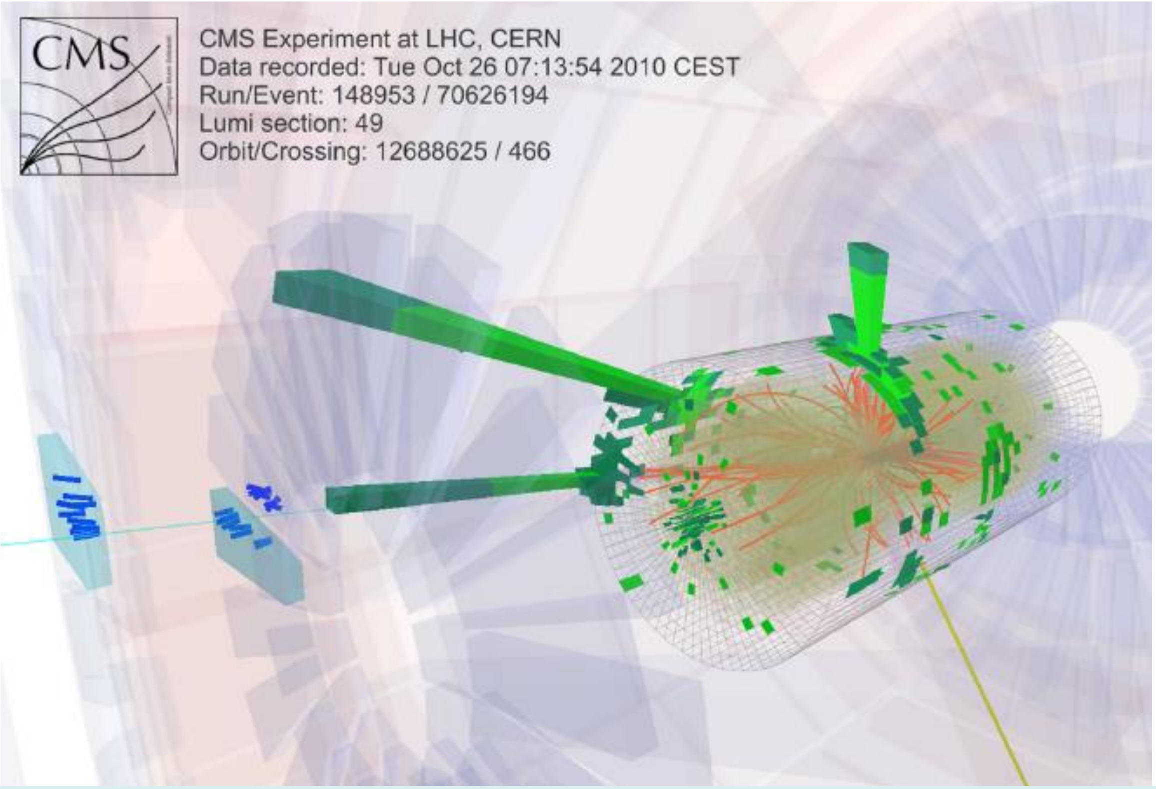
LHC might be making dark matter

Heavy colored particles could decay rapidly to stable Weakly Interacting Massive Particle (WIMP = LSP) plus jets

→ Missing transverse energy
MET + 4 jets



Is LHC already making dark matter?

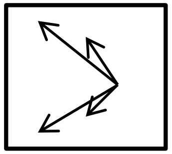


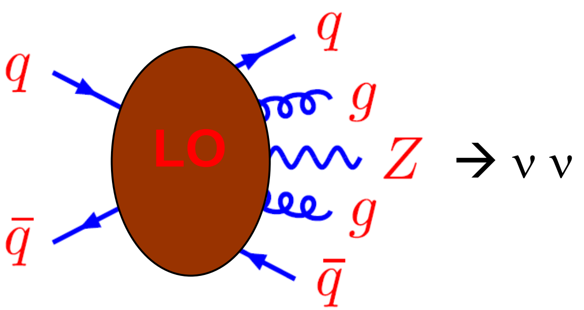
- 5 jets
- sum of jet transverse momenta H_T= 1132 GeV
- missing transverse energy H_{TMiss} = 693 GeV

No! Happens in Standard Model too

- **MET + 4 jets from**
  pp → Z + 4 jets,
  Z → neutrinos

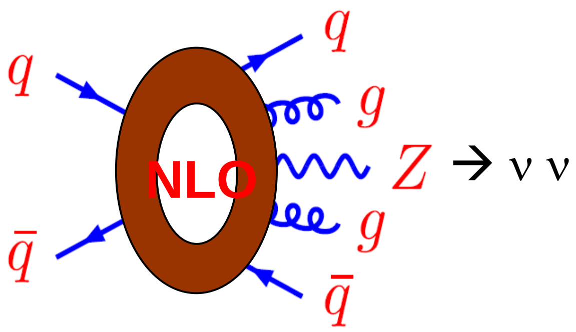
Neutrinos escape detector.
Irreducible background.





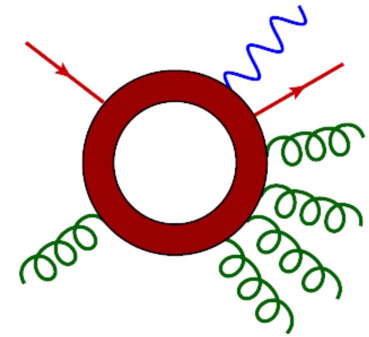
State of art for Z + 4 jets was based on
Leading Order (LO) approximation in QCD
→ **normalization uncertain**

Now available at
Next to Leading Order,
greatly reducing
theoretical uncertainties

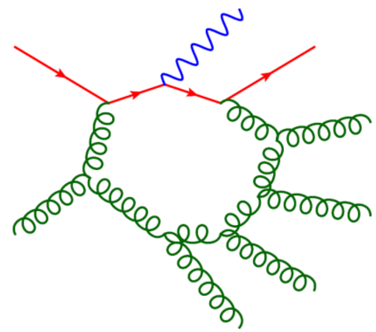


A Better Way to Compute?

- Backgrounds (and many signals) at NLO require one-loop scattering amplitudes for **many** ultra-relativistic (“massless”) particles – especially quarks and gluons of QCD



- **Feynman** told us how to do this – in principle



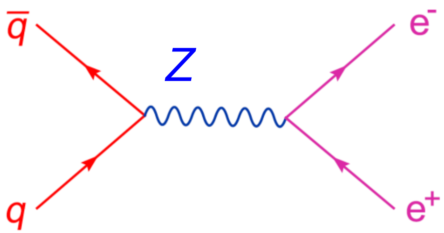
+ 256,264 more

- **Feynman diagrams**, while very general and **powerful**, are not optimized for these processes
- **There are much more efficient methods**

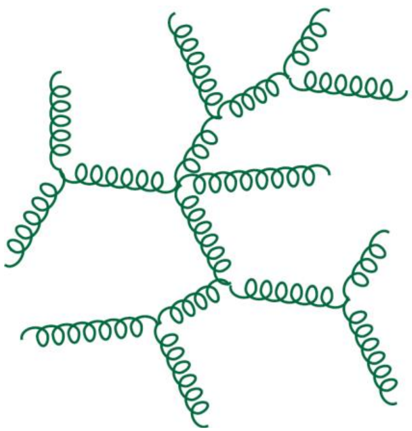
LO = Trees

LO cross section uses only Feynman diagrams with no closed loops – tree diagrams.
Here's a very simple one:

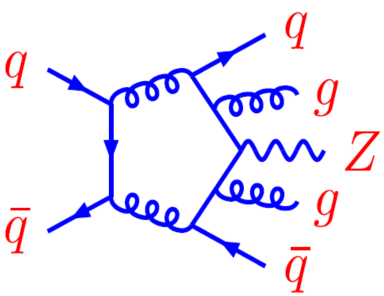
$\hat{\sigma}^{(0)}$



Although there are many kinds of trees, some harder than others, "textbook" methods often suffice



+ …



NLO = Loops

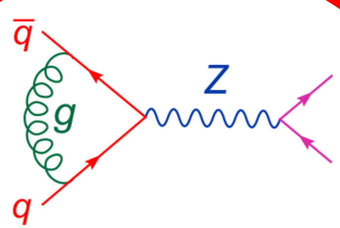
NLO cross section needs Feynman diagrams with exactly one closed loop

Textbook methods quickly fail, even with very powerful computers

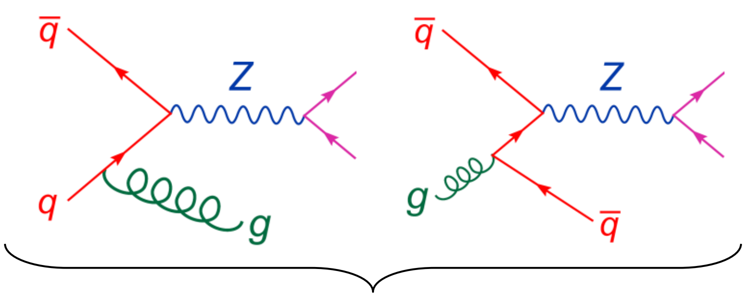
- NLO also needs tree-level amplitudes with one more parton
- Both terms infinite(!) – combine them to get a finite result
- Now also a solved problem

$\hat{\sigma}^{(1)}$

NLO



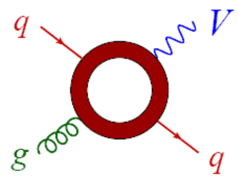
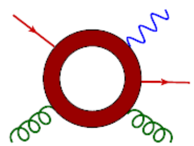
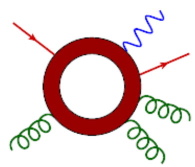
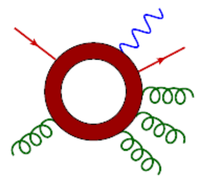
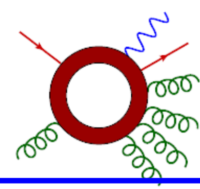
1 loop



tree + 1 parton

Quantifying the one-loop QCD challenge

***pp* → *W* + *n* jets** (amplitudes with most gluons)

# of jets		# 1-loop Feynman diagrams
1		11
2		110
3		1,253
4		16,648
5		256,265

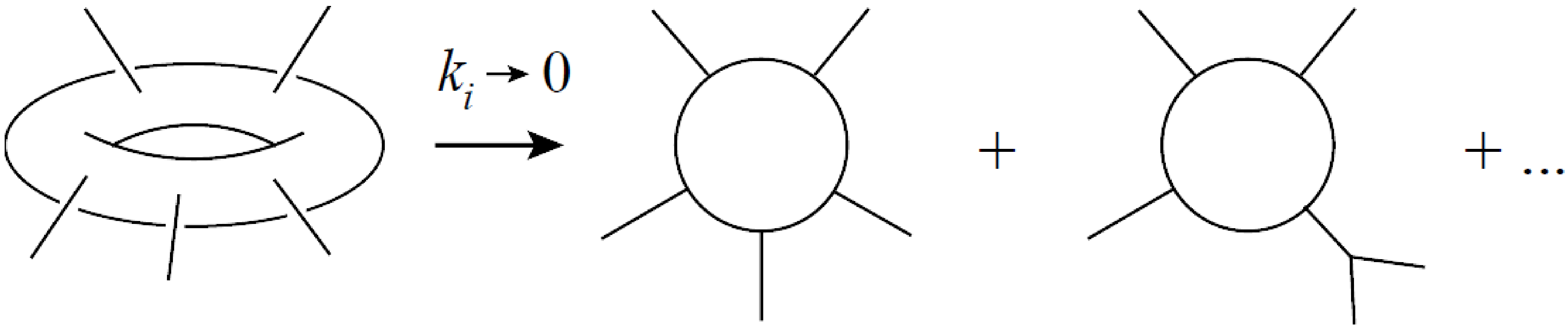
Current limit with Feynman diagrams (up to 2 jets)

Current limit with the new methods (up to 5 jets)

Back to the early 1990's

Bern, Kosower (1991)

- Zvi Bern, David Kosower used string theory, which has all Feynman diagrams in one package:



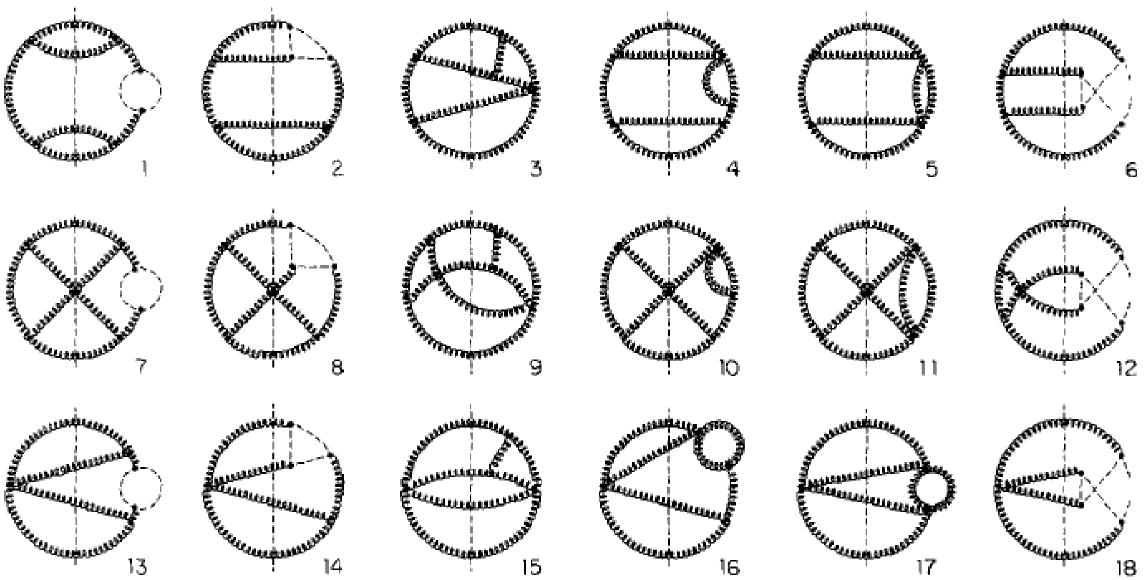
- Low-energy limit (large string tension limit) → Bern-Kosower rules.
- Computed *gg* → *gg* at 1-loop in terms of helicity amplitudes

$gg \rightarrow gg$ computed previously

Ellis, Sexton (1986)

- Using Feynman diagrams, and as interference with tree amplitude, summed over all colors and helicities

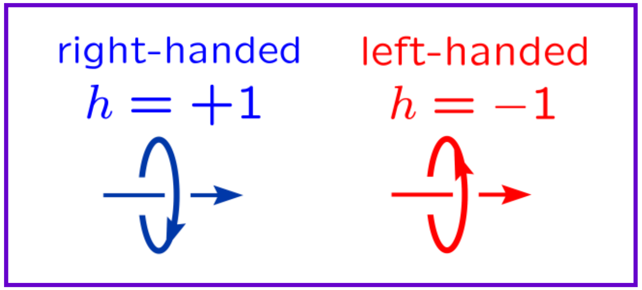
R.K. Ellis, J.C. Sexton / QCD radiative corrections

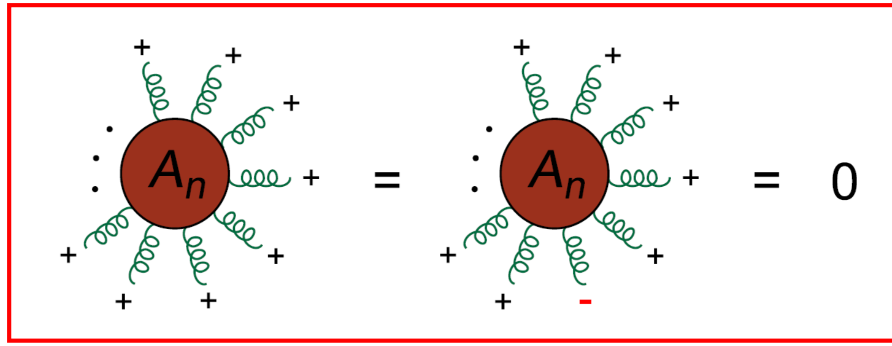


- Powerful and important result; however, sum obscured underlying simplicity of helicity amplitudes

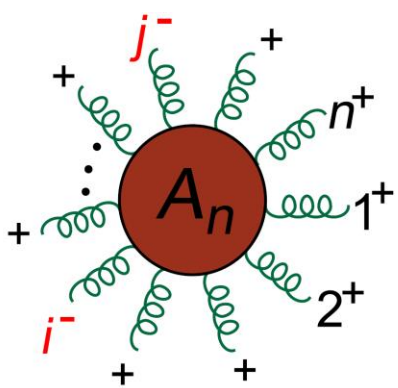
Helicity Formalism Exposes Tree-Level Simplicity in QCD

Many tree-level helicity amplitudes either vanish or are very short





Very opaque from Feynman diagrams



$$= \frac{\langle i\, j \rangle^4}{\langle 1\,2\rangle\langle 2\,3\rangle \cdots \langle n\,1\rangle}$$

Parke-Taylor formula (1986)

Spinor helicity formalism (1980s)

Instead of Lorentz products: $s_{ij} = 2k_i \cdot k_j = (k_i + k_j)^2$

Use spinor products:

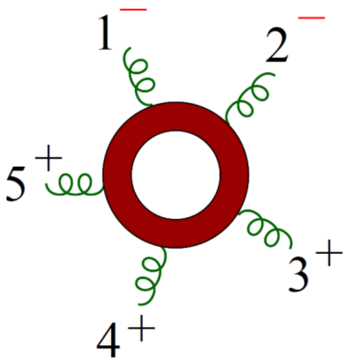
$$\langle i\, j \rangle = \sqrt{s_{ij}}\, e^{i\phi_{ij}} \qquad [j\, i] = \sqrt{s_{ij}}\, e^{-i\phi_{ij}}$$

If momenta are real, they are complex square roots of Lorentz products.
Complex variables, natural for complex amplitudes.

A new result: *gg → ggg*

Bern, LD, Kosower (1993)

- String formalism allowed us to push to one more leg.
- First 2 → 3 loop amplitude. “Just complicated enough.”
- Could start to see clearly how it was organized.



$$V^f = -\frac{5}{2\epsilon} - \frac{1}{2}\left[\ln\left(\frac{\mu^2}{-s_{23}}\right) + \ln\left(\frac{\mu^2}{-s_{51}}\right)\right] - 2, \qquad V^s = -\frac{1}{3}V^f + \frac{2}{9}$$

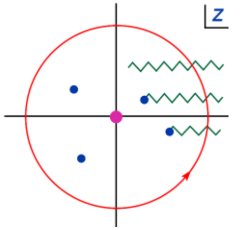
$$F^f = -\frac{1}{2}\frac{\langle 1\,2\rangle^2\left(\langle 2\,3\rangle[3\,4]\langle 4\,1\rangle + \langle 2\,4\rangle[4\,5]\langle 5\,1\rangle\right)}{\langle 2\,3\rangle\langle 3\,4\rangle\langle 4\,5\rangle\langle 5\,1\rangle}\frac{\mathrm{L}_0\left(\frac{-s_{23}}{-s_{51}}\right)}{s_{51}}$$

$$F^s = -\frac{1}{3}\frac{[3\,4]\langle 4\,1\rangle\langle 2\,4\rangle[4\,5]\left(\langle 2\,3\rangle[3\,4]\langle 4\,1\rangle + \langle 2\,4\rangle[4\,5]\langle 5\,1\rangle\right)}{\langle 3\,4\rangle\langle 4\,5\rangle}\frac{\mathrm{L}_2\left(\frac{-s_{23}}{-s_{51}}\right)}{s_{51}^3} - \frac{1}{3}F^f$$
$$- \frac{1}{3}\frac{\langle 3\,5\rangle[3\,5]^3}{[1\,2][2\,3]\langle 3\,4\rangle\langle 4\,5\rangle[5\,1]} + \frac{1}{3}\frac{\langle 1\,2\rangle[3\,5]^2}{[2\,3]\langle 3\,4\rangle\langle 4\,5\rangle[5\,1]} + \frac{1}{6}\frac{\langle 1\,2\rangle[3\,4]\langle 4\,1\rangle\langle 2\,4\rangle[4\,5]}{s_{23}\langle 3\,4\rangle\langle 4\,5\rangle s_{51}}$$

Only log arguments: s_{23} and s_{51} – not s_{12}, s_{34} or s_{45} Why??

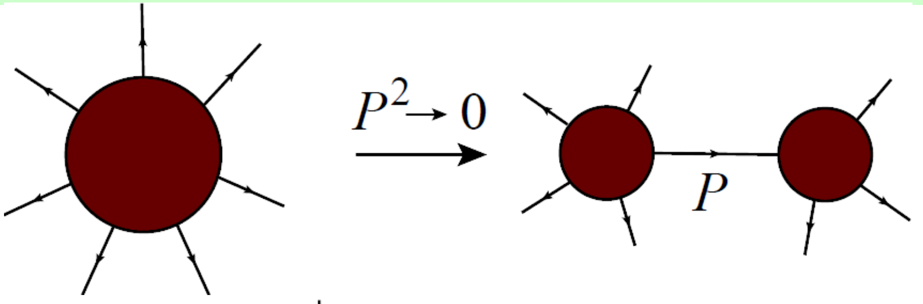
denominators related to collinear limits

1960's Analytic *S*-Matrix



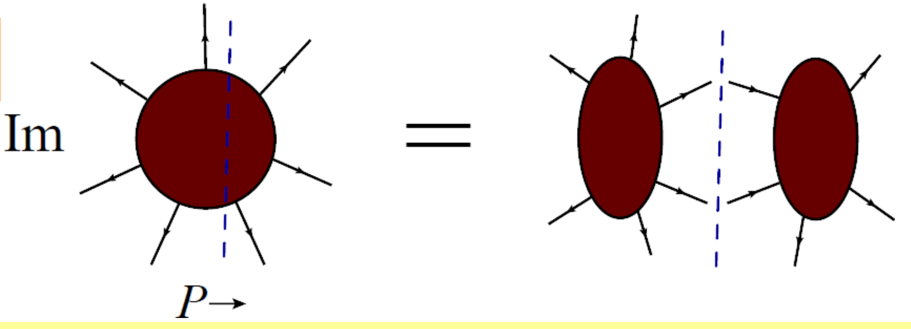
No QCD, no Lagrangian or Feynman rules for strong interactions.
Bootstrap program: Reconstruct scattering amplitudes **directly** from **analytic properties: "on-shell" information**

- Poles



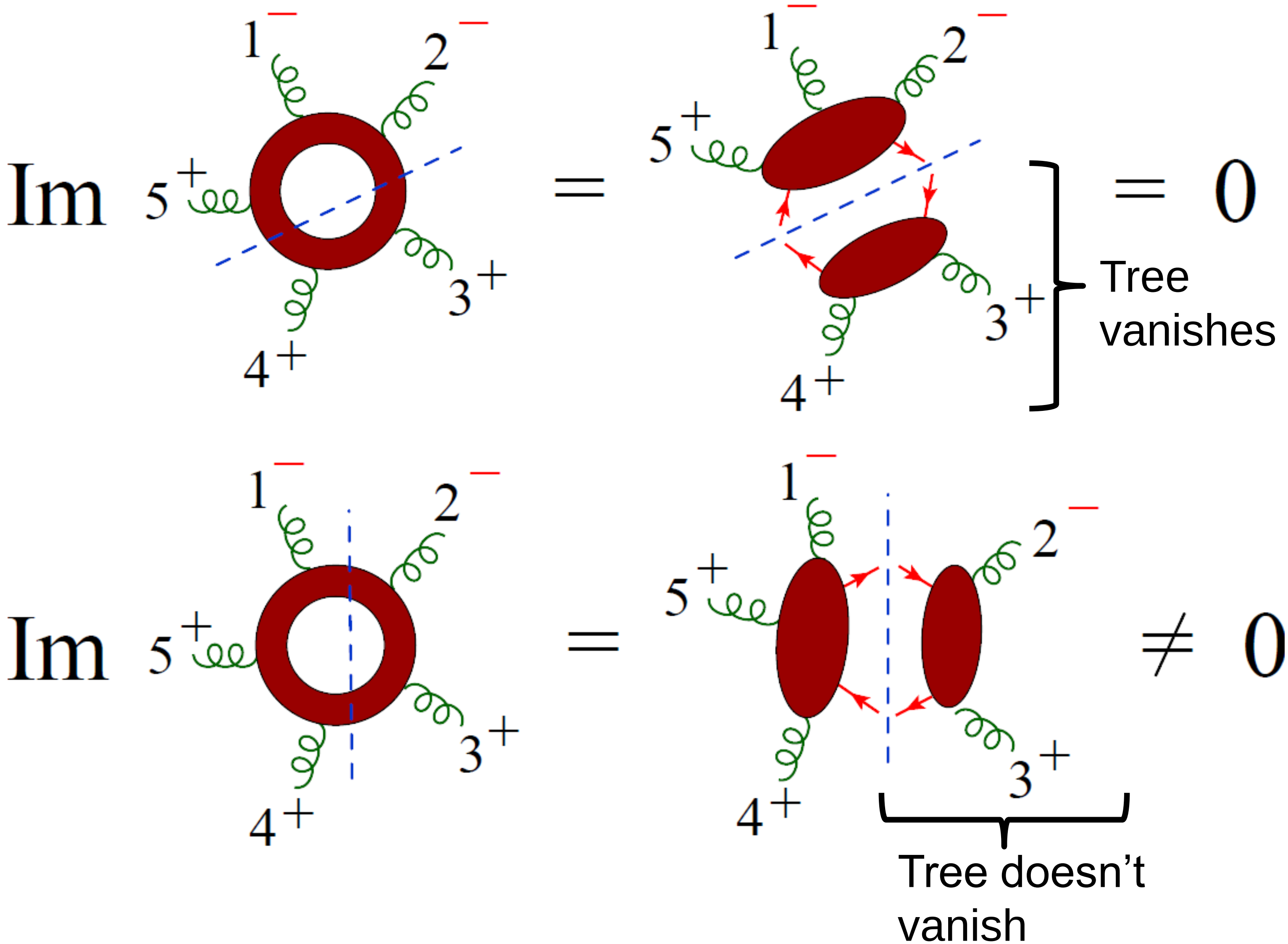
Landau; Cutkosky; Chew, Frautschi, Mandelstam; Eden, Landshoff, Olive, Polkinghorne; Veneziano; Virasoro, Shapiro; … (1960s)

- Branch cuts



Analyticity fell out of favor in 1970s with the rise of QCD & Feynman rules

Resurrect it for computing amplitudes in **perturbative** QCD
– as **alternative to Feynman diagrams!** **Unitarity method.**
Perturbative information would assist analyticity.



With a bit more work – far less than the original computation – we could compute the correct nonzero value by multiplying together simple trees

Recurrent theme

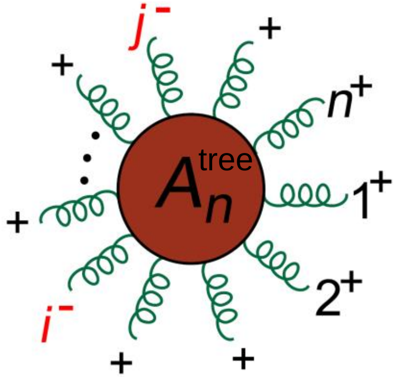
1. A little more computational power leads to new results
2. Explaining the patterns in the new results leads to new relations
3. The new relations can be exploited for even more computational power
4. See step 1.

Another recurrent theme

Bern, LD, Dunbar, Kosower (1994)

- When you are lacking computational power in QCD, first try its cousin, N=4 super-Yang-Mills theory   Brink, Schwarz, Scherk; Gliozzi, Scherk, Olive (1977)
- Using just one unitarity cut, could compute an infinite sequence of 1-loop amplitudes in this theory – the 1-loop Parke-Taylor amplitudes
- Remarkably, all made out of "easy 2 mass" scalar box integrals

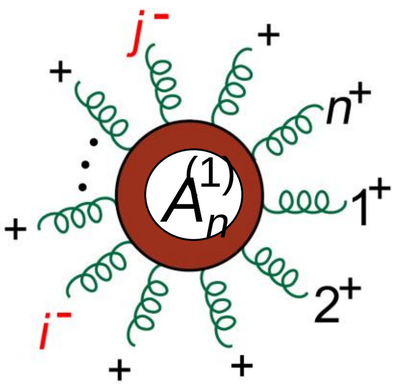
From trees to loops



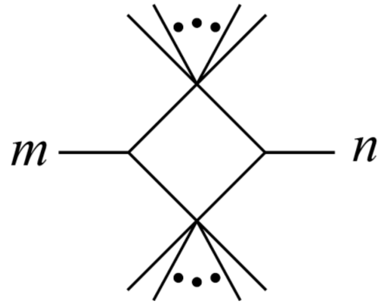
$$= \frac{\langle i\, j \rangle^4}{\langle 1\, 2\rangle \langle 2\, 3\rangle \cdots \langle n\, 1\rangle}$$

Parke-Taylor formula (1986)

N=4 SYM



$$= \frac{\langle i\, j \rangle^4}{\langle 1\, 2\rangle \langle 2\, 3\rangle \cdots \langle n\, 1\rangle} \sum_{m,n}$$



BDDK (1994)

For Efficient Computation

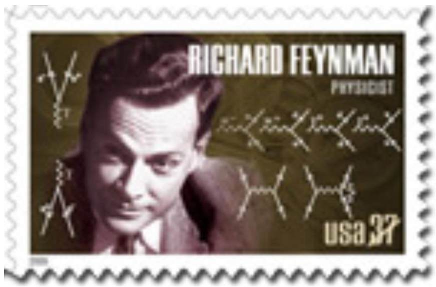
Reduce
the number of "diagrams"

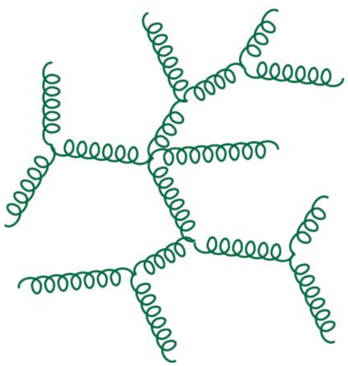
Reuse
building blocks over & over

Recycle
lower-point (1-loop) & lower-loop (tree)
on-shell amplitudes

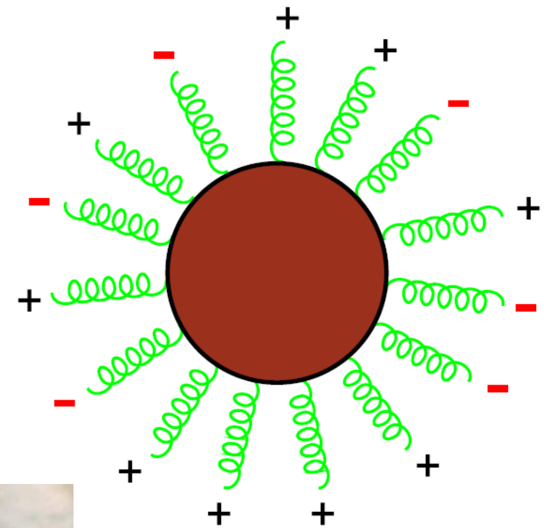
Recurse

Granularity vs. Fluidity





+ …

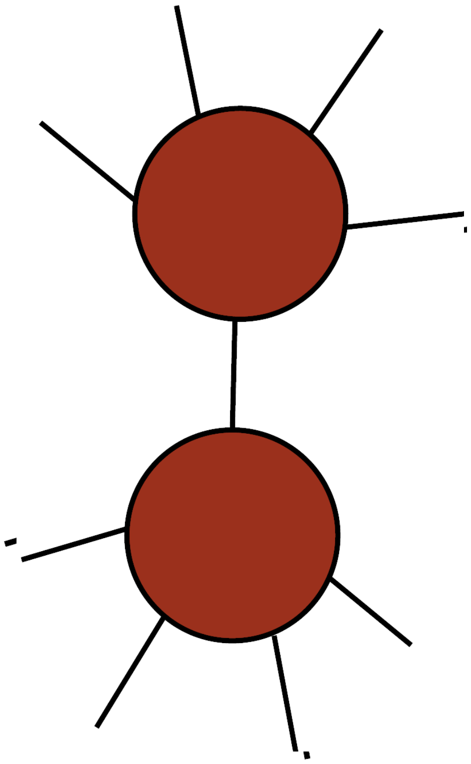


Recycling Fluid Amplitudes

Amplitudes fall apart into simpler ones in special limits – pole information

Picture leads directly to BCFW (on-shell) recursion relations

Britto, Cachazo, Feng, Witten, hep-th/0501052

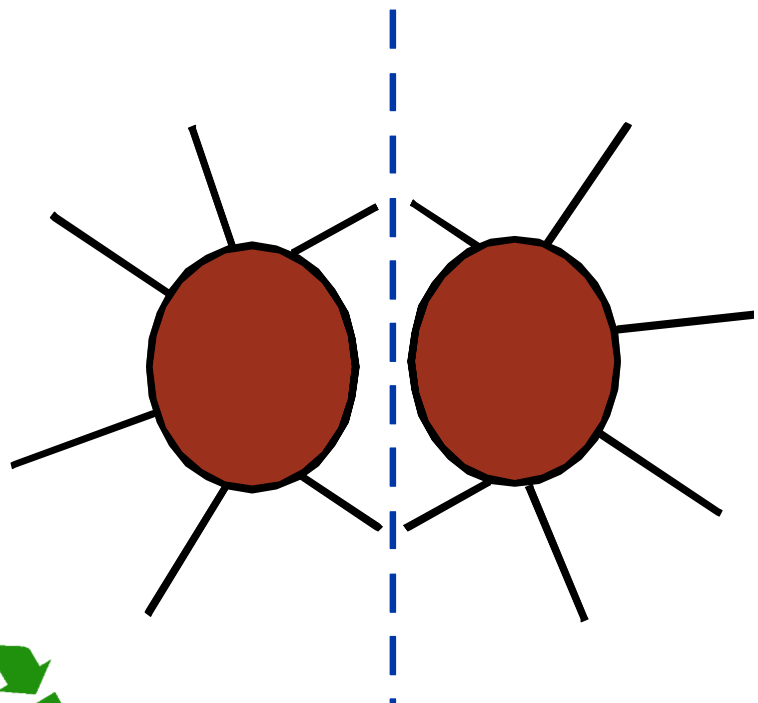


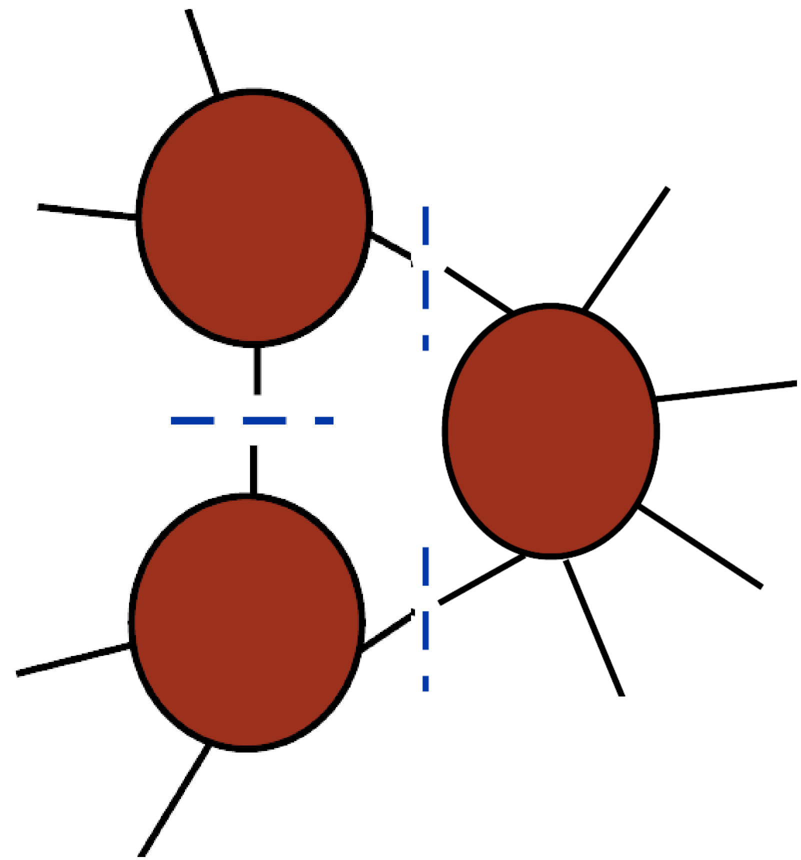
Trees recycled into trees

Branch cut information → Generalized Unitarity: Fluidity of the Loop Integrand

Ordinary unitarity:
put 2 particles on shell

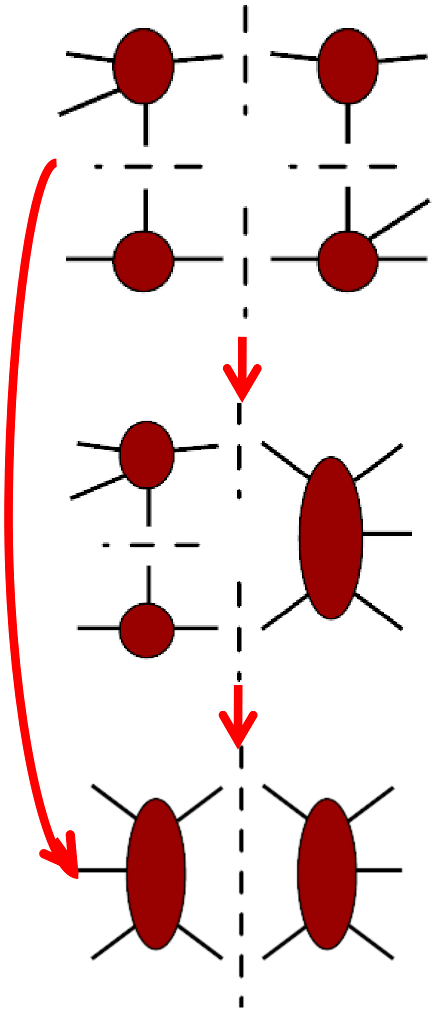
Generalized unitarity:
put 3 or 4 particles on shell





Trees recycled into loops!

One-loop amplitudes determined hierarchically



As a result…

Dramatic increase recently
in rate of NLO QCD predictions
for new processes!

Automated On-Shell One Loop Programs for LHC

circa 2011

Blackhat: Berger, Bern, LD, Diana, Febres Cordero, Forde, Gleisberg, Höche, Ita, Kosower, Maître, Ozeren, 0803.4180, 0808.0941, 0907.1984, 1004.1659, 1009.2338…
+ **Sherpa** → NLO *W,Z* + 3,4,5 jets   pure QCD 4 jets

CutTools: Ossola, Papado… Pittau, 0711.3596
NLO *WWW, WWZ*, ... …OPP, 0804.0350
NLO $t\bar{t}b\bar{b}$, $t\bar{t}$ + 2 jets,…
Bevilacqua, Czakon, Papa… …723; 1002.4009

MadLoop: Hirsch… …altoni, Pittau 1103.0621
HELAC-NLO: Bevilacqua et al, 1110.1499

Rocket: Giele, Zanderighi, 0805.2152
…, Giele, Kunszt, Melnikov, Zanderighi, 0810.2762
NLO *W* + 3 je… Ellis, Melnikov, Zanderighi, 0901.4101, 0906.1445
$W^+W^\pm$ + 2 jets Melia, Melnikov, Rontsch, Zanderighi, 1007.5313, 1104.2327

Current QCD Frontiers:
One loops, n > 6-8 external legs
Two loops, n > 4 external legs

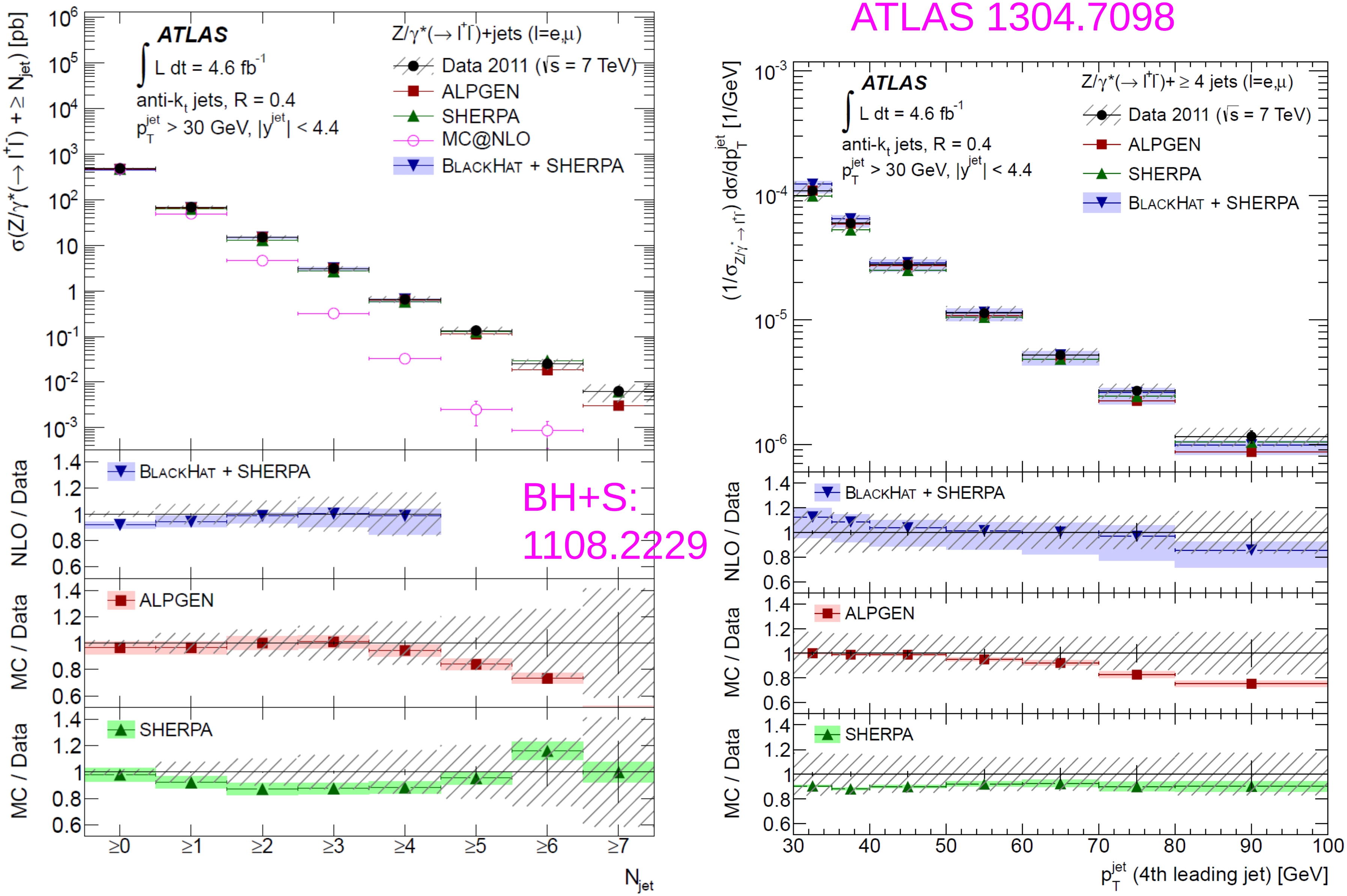
SAMURAI: Mastrolia, Ossola, Reiter, Tramontano, 1006.0710

NGluon: Badger, Biedermann, Uwer, 1011.2900,1209.0098

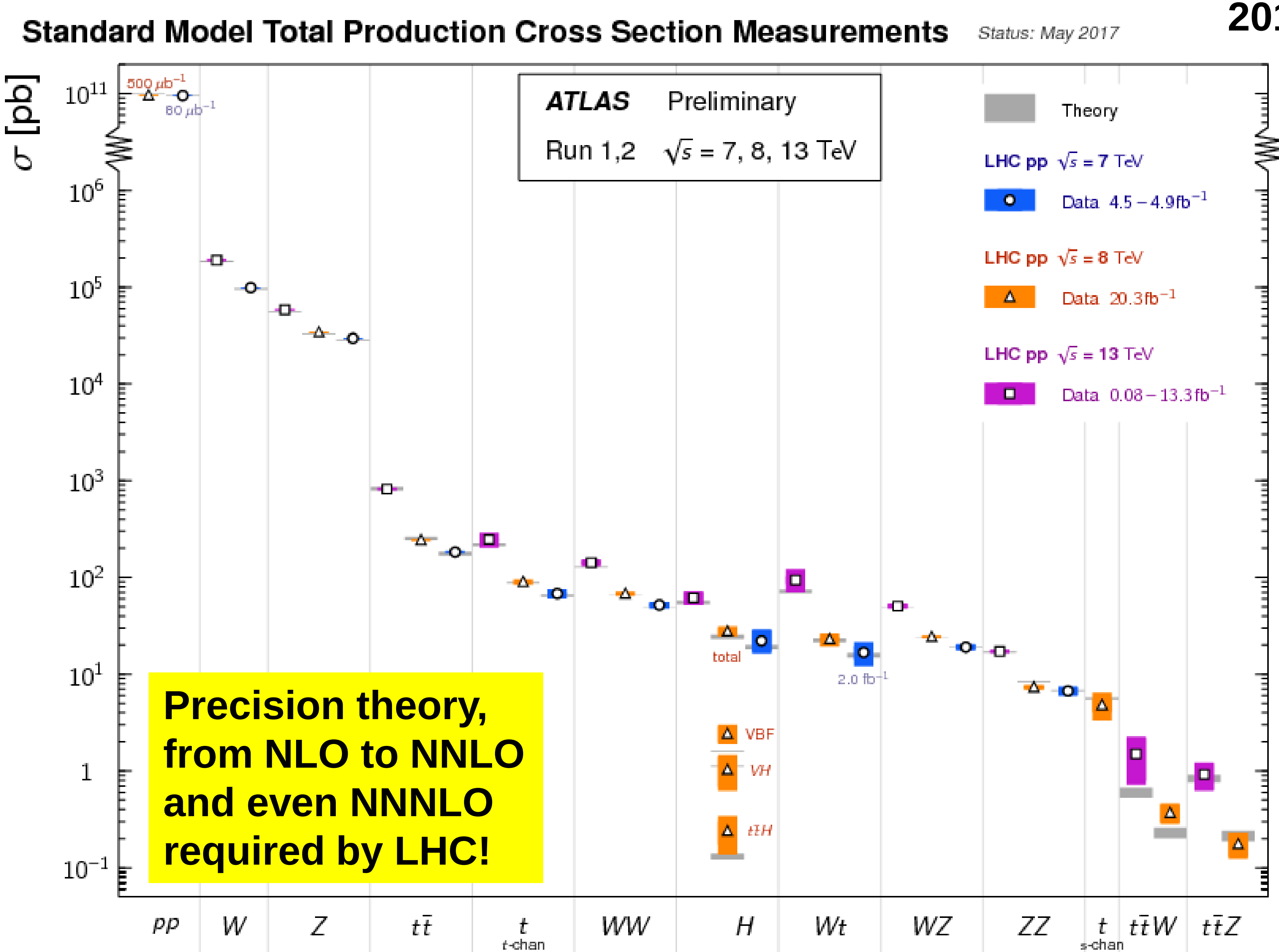
Open Loops: Cascioli, Maierhofer, Pozzorini, 1111.5206

NLO $pp \rightarrow Z + 1,2,3,4$ jets vs. ATLAS 2011 data

ATLAS 1304.7098

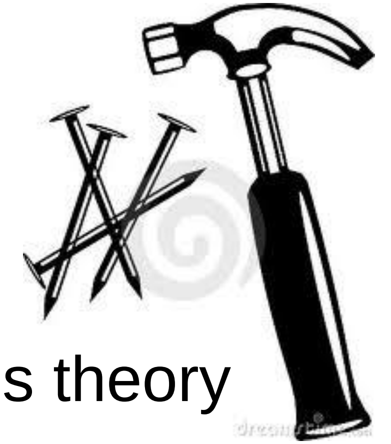


2017

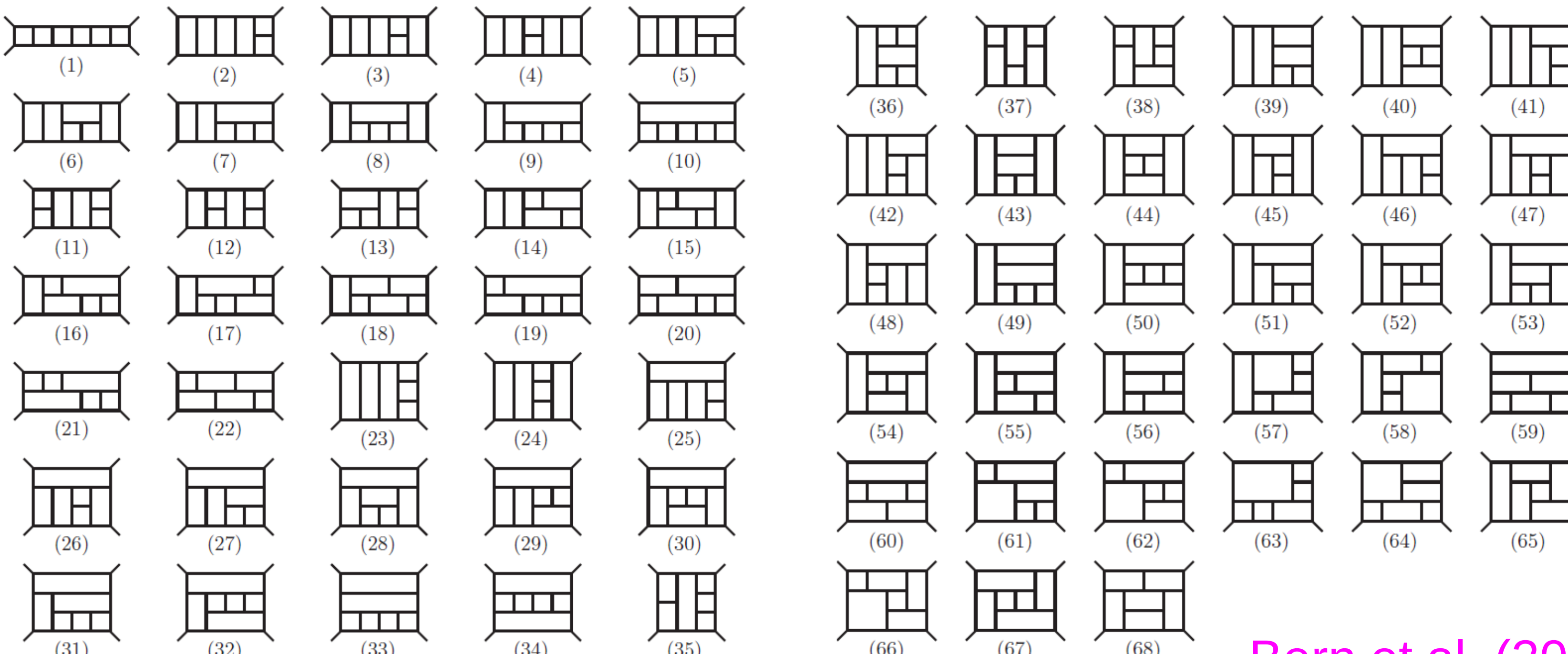


Precision theory, from NLO to NNLO and even NNNLO required by LHC!

In simpler theories can go to many loops



- 6-loop 4-gluon amplitude in N=4 super-Yang-Mills theory in planar limit of a large number of colors:

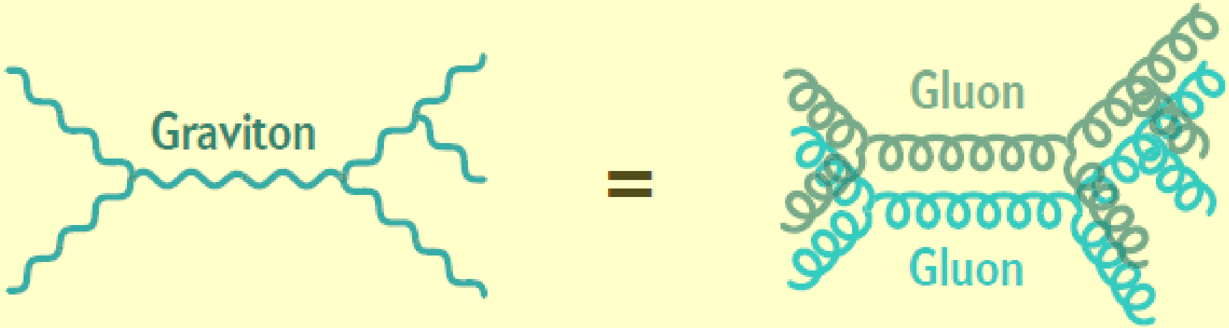


Bern et al. (2012)

Striking patterns emerge

- Including remarkable relations between gauge theory and gravity

Bern, Carrasco, Johansson (2008,2010)



- N=8 supergravity exceptionally well-behaved for a point-like theory of quantum gravity.
- Through 4 loops, no worse behaved than a finite gauge theory, N=4 SYM

Bern, Carrasco, LD, Johansson, Kosower, Roiban (2007-2012)

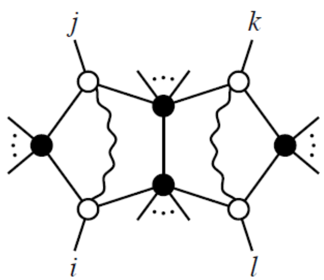
- However, this property breaks at 5 loops!

Bern, Carrasco, Chen, Edison, Johansson, Parra-Martinez, Roiban, Zeng, 1804.09311

Connections to Mathematics

- Planar N=4 super-Yang-Mills loop integrand: a volume form on the positive Grassmannian – the amplituhedron
  Arkani-Hamed, Bourjaily, Cachazo, Goncharov, Postnikov, Trnka, 1212.5605; Arkani-Hamed, Trnka, 1312.2007, 1312.7878

$$\mathcal{A}_{\rm MHV}^{2-{\rm loop}} = \frac{1}{2}\sum_{i<j<k<l<i}$$



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- The amplituhedron represents a geometric formulation of scattering, in which causality is emergent
- Whereas causality is hard-wired into Feynman diagrams.
- But this also means that amplituhedron integrand is not easy to integrate – need to rewrite so one can apply the $+i\varepsilon$ from Feynman

Connections to Math (cont.)

- After loop integration, often get iterated integrals,

$$G(a_1, \ldots, a_n; z) = \int_0^z \frac{dt}{t - a_1} G(a_2, \ldots, a_n; t) \,,$$

which possess a Hopf algebra co-product Brown, Goncharov

- Use structure to simplify amplitudes ranging from planar N=4 SYM Goncharov, Spradlin, Vergu, Volovich, 1006.5703

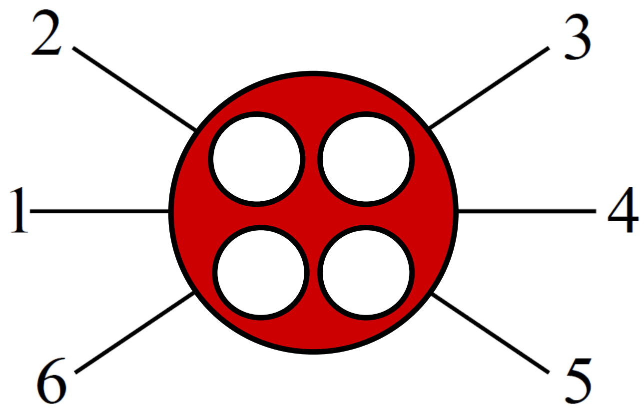
  to Higgs boson production Duhr,1203.0454

  to electron g-2 Laporta,1704.06996; Schnetz, 1711.05118

- Especially powerful in planar N=4 SYM where one can avoid even looking at the loop integrand!

Hexagon function bootstrap

Use analytical properties of perturbative amplitudes in planar N=4 SYM to determine them directly, without ever peeking inside the loops



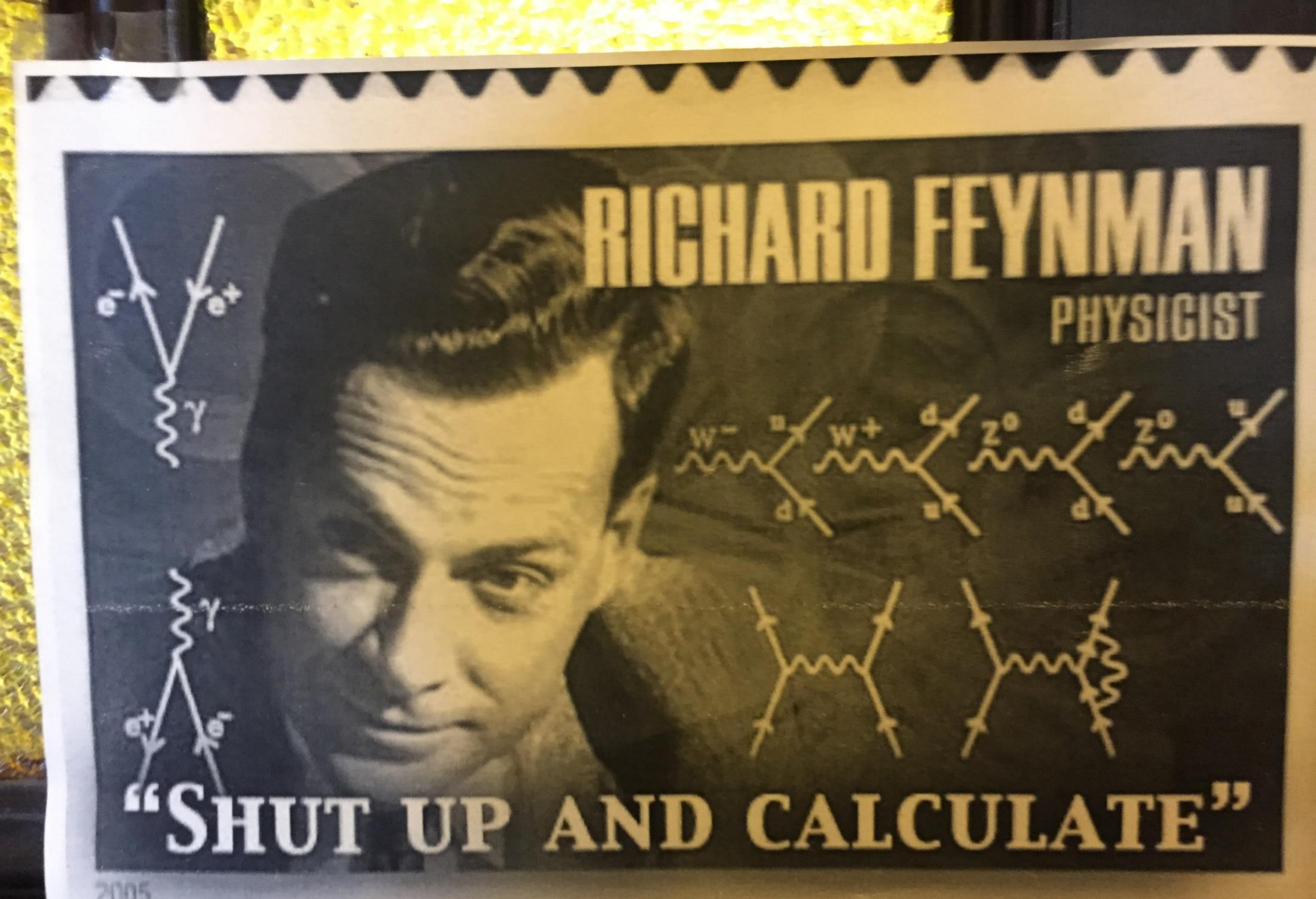
First step toward doing this nonperturbatively (no loops to peek inside) for general kinematics

Works to an astounding 6 loops for the first nontrivial amplitude (6 gluon scattering)

Caron-Huot, LD, Dulat, von Hippel, McLeod, Papathanasiou, to appear

70 years of particle scattering…

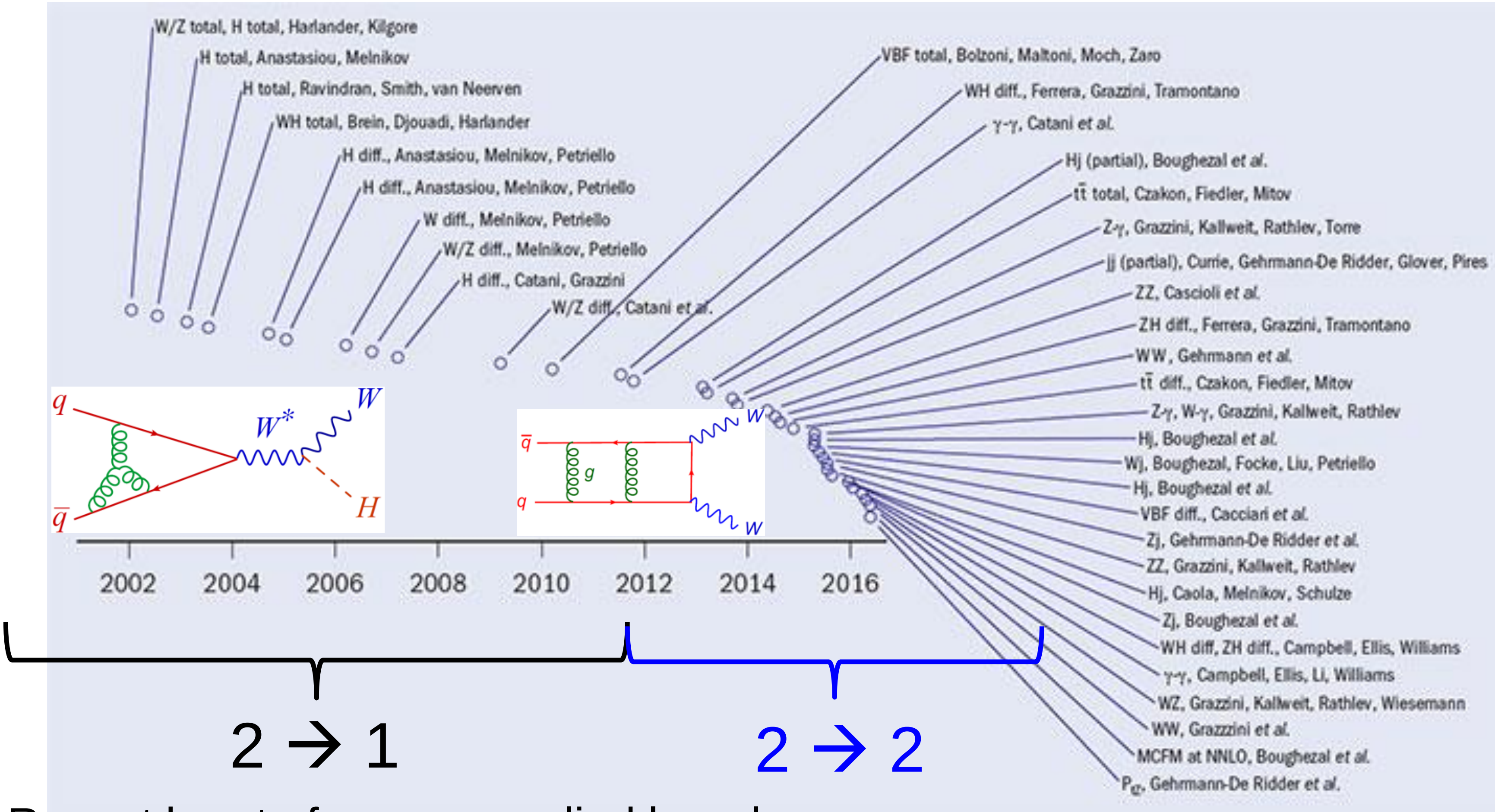
- Feynman's insights in making perturbation theory in QED covariant meant "anyone could compute" perturbative scattering – later on, including computers
- Feynman diagrams at the heart of almost all quantitative comparisons between theory and experiment since 1947
- LHC demands for theory → reorganize Feynman diagrams to incorporate unitarity (as well as many other advances in loop and phase space integration).
- Spawned many other novel "amplitudes" developments
- Standard Model, built by Feynman and others, has survived every quantitative test to date, but it is still not the last word!


RICHARD FEYNMAN
PHYSICIST
"SHUT UP AND CALCULATE"
2005

Extra Slides

NNLO timeline

G. Zanderighi
CERN Courier (2017)



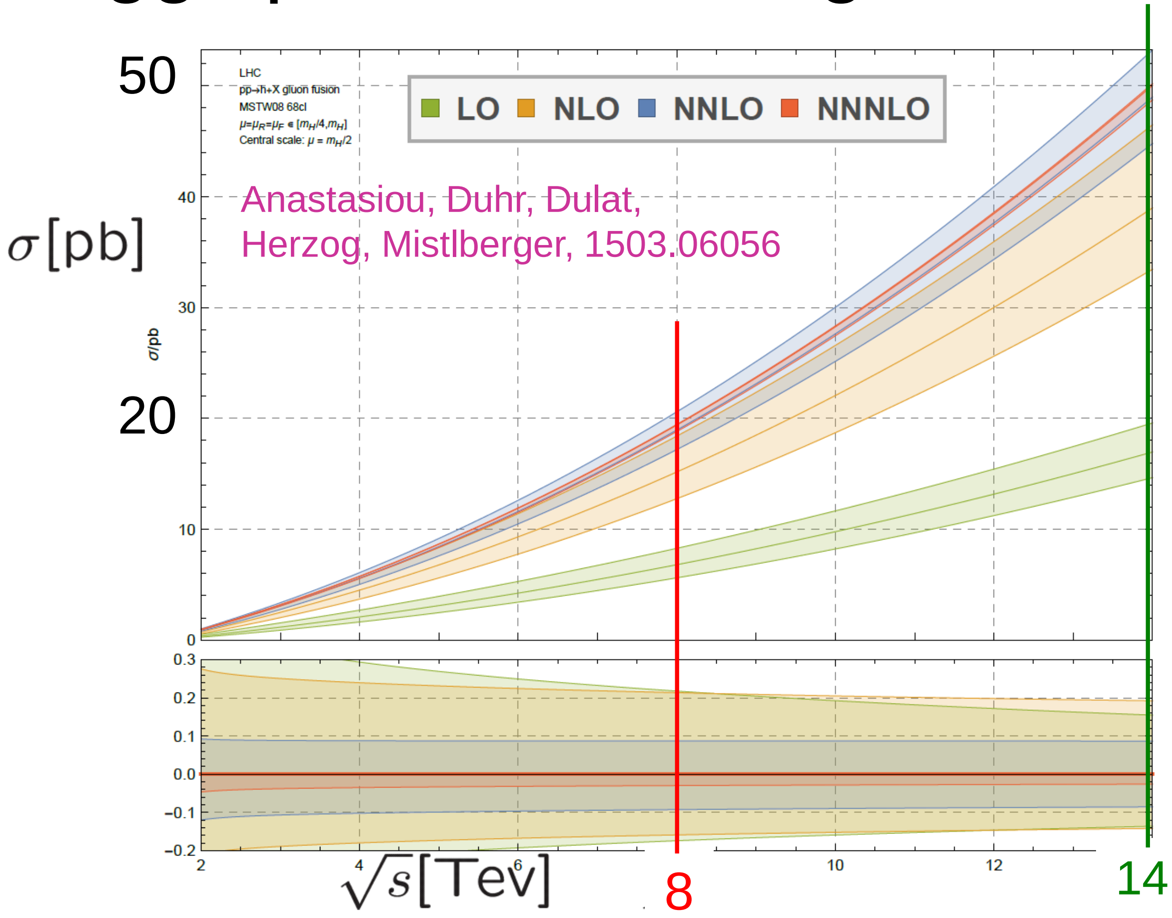
Recent burst of progress relied largely on:

a) 2 loop 2 → 2 amplitudes, new loop integration methods

b) NNLO subtraction (or slicing) methods

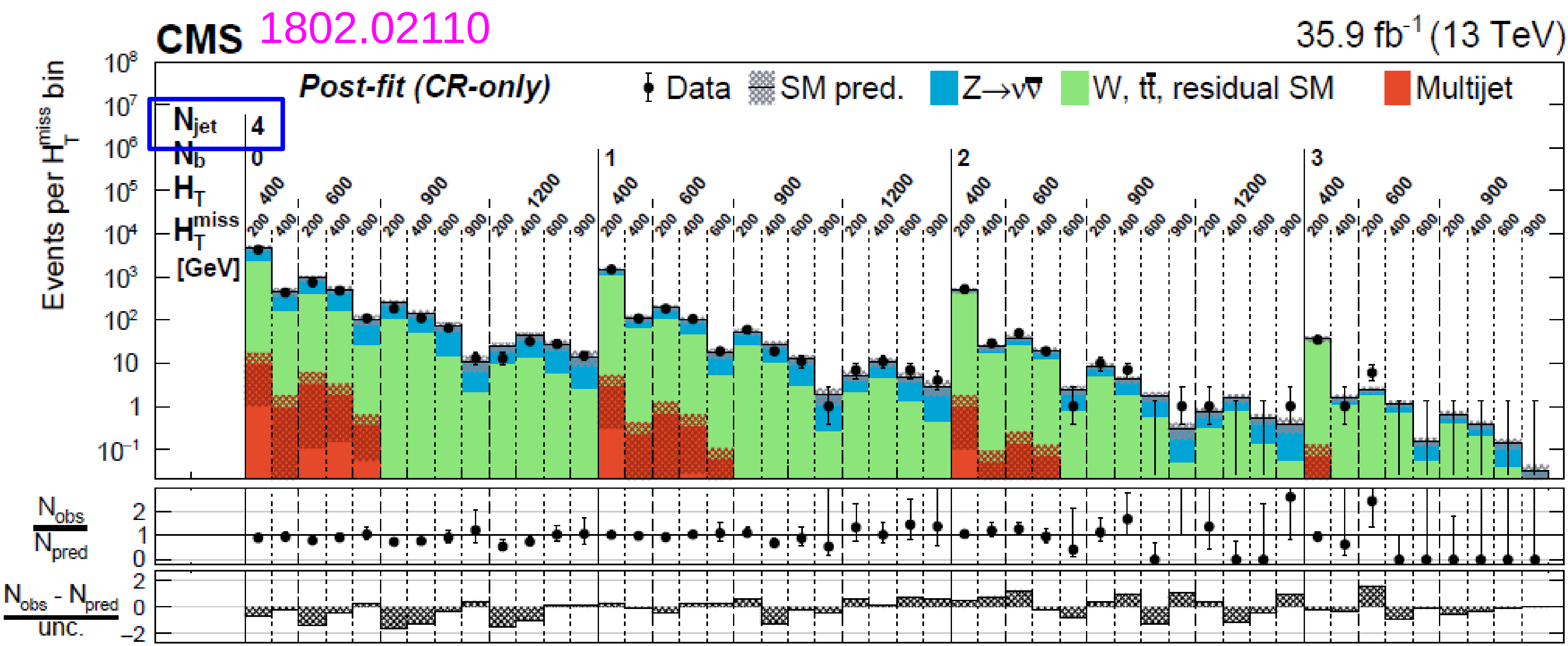
The one high p_T LHC cross section known at NNNLO

Higgs production via gluon fusion



LO → NNNLO gives factor of 2.7 increase!

SUSY (and other) searches now very advanced



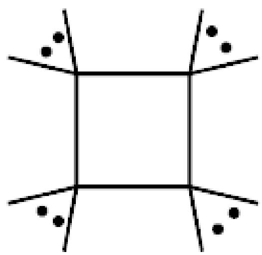
- **Also N_{jet} = 1,2,3,5,6**
- **No significant excesses seen, so set lower limits on masses of superparticles.**
- **Can better simulations, (N)NLO + parton showers, help further?**

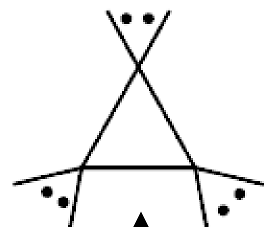
One-Loop Amplitude Decomposition

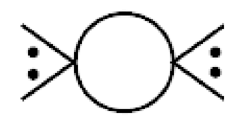
Bern, LD, Dunbar, Kosower (1994)

Merges analytic *S*-matrix with perturbative structure

coefficients can be determined from products of trees using (generalized) unitarity

$$A^{1\text{-loop}} = \sum_i d_i$$


$$+ \sum_i c_i$$


$$+ \sum_i b_i$$


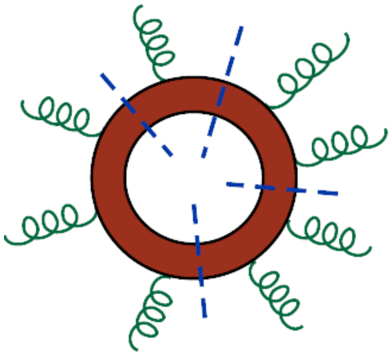
$$+ R + \mathcal{O}(\epsilon)$$

Known functions (integrals), same for all amplitudes

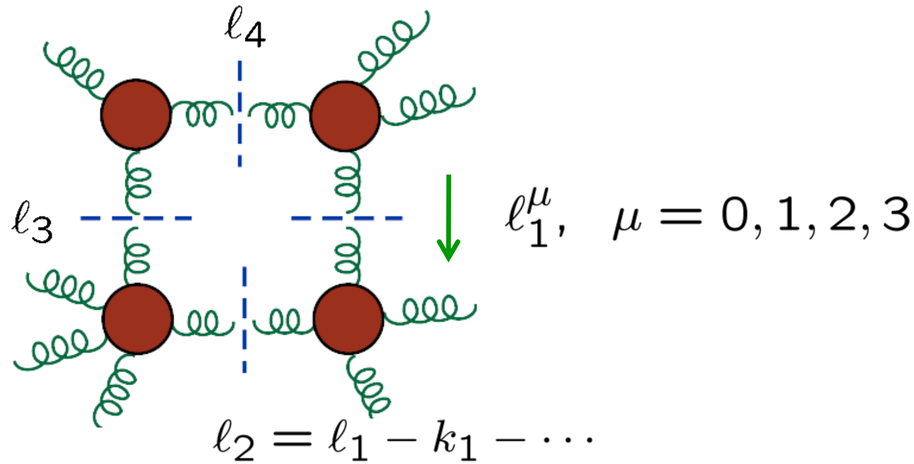
rational part; from D-dimensional trees, or recursively

Generalized Unitarity for Box Coefficients d_i

Britto, Cachazo, Feng, hep-th/0412103



$\Rightarrow$

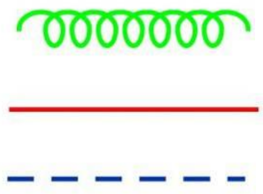


$$d_i = \sum_{\pm} A_1^{\text{tree}\pm} \times A_2^{\text{tree}\pm} \times A_3^{\text{tree}\pm} \times A_4^{\text{tree}\pm}$$

Just multiply together 4 different tree amplitudes, evaluated at appropriate “quadruple cut” kinematics

$$\ell_1^2 = \ell_2^2 = \ell_3^2 = \ell_4^2 = 0$$

N=4 SYM particle content



massless spin 1 gluon
4 massless spin 1/2 gluinos
6 massless spin 0 scalars

SUSY
Q_a, a=1,2,3,4
shifts helicity
by 1/2 $\longleftrightarrow$

$\mathcal{N}=4$	1	$\longleftrightarrow$	4	$\longleftrightarrow$	6	$\longleftrightarrow$	4	$\longleftrightarrow$	1
	g^-		$\lambda_{\bar{i}}^-$		$\bar{\phi}_{\bar{i}\bar{j}}, \phi_{ij}$		λ_i^+		g^+
helicity	-1		$-\frac{1}{2}$		0		$\frac{1}{2}$		1

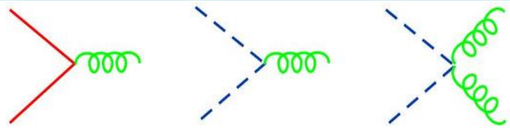
all in adjoint representation

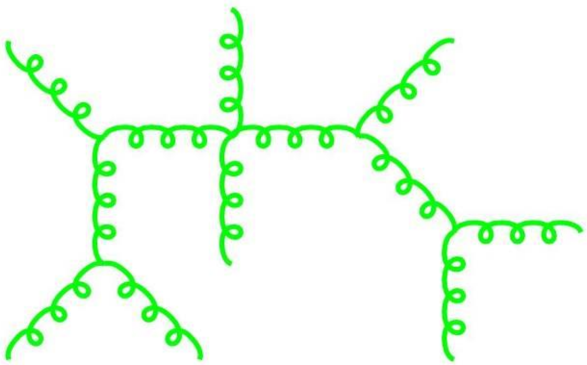
How are QCD and N=4 SYM related?

At tree level they are essentially identical

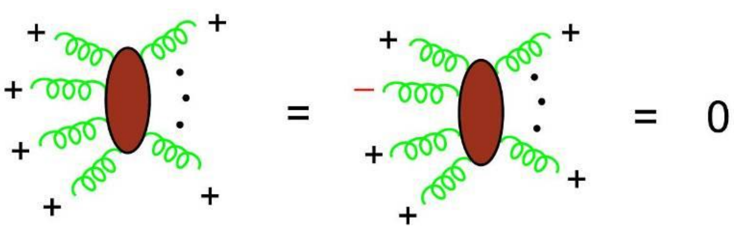
Consider a tree amplitude for *n* gluons.

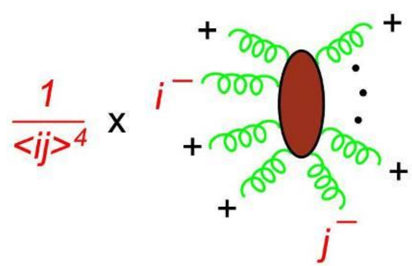
Fermions and scalars cannot appear because they are produced in pairs





Hence the amplitude is the **same** in QCD and N=4 SYM.
So the QCD tree amplitude "secretly" obeys
all identities of N=4 supersymmetry:





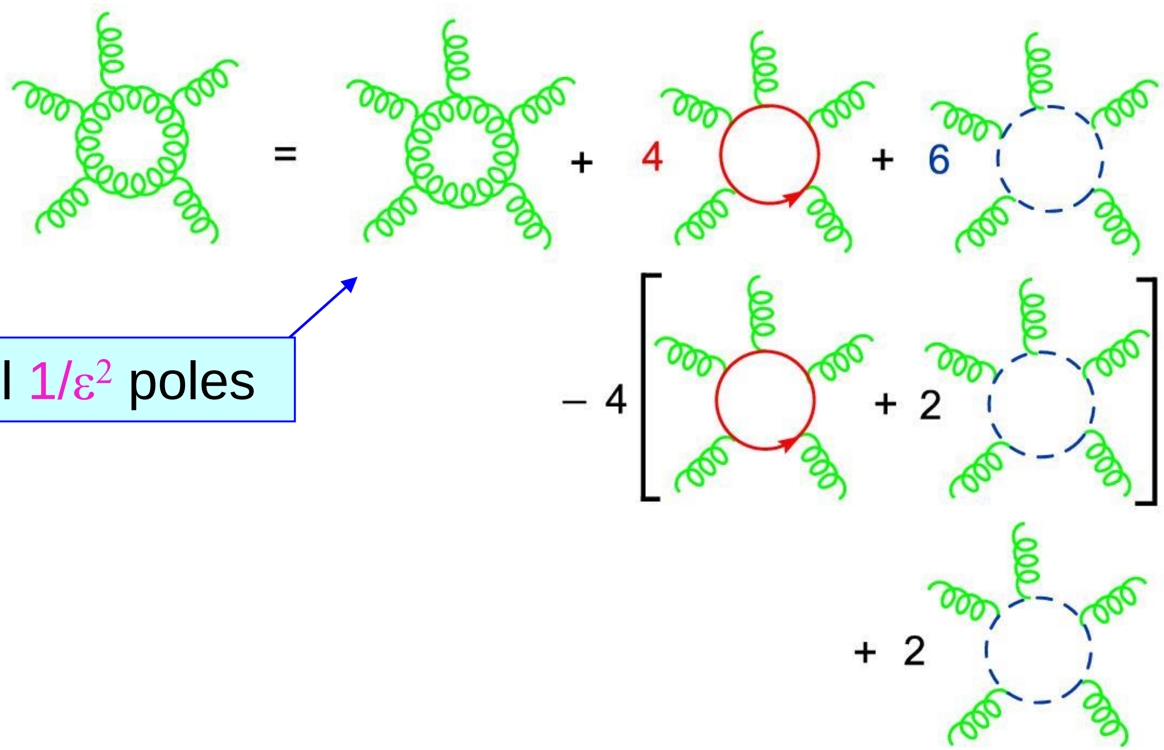
$$\frac{\langle i\, j\rangle^4}{\langle 1\, 2\rangle\langle 2\, 3\rangle\cdots\langle n\, 1\rangle}$$

independent of *i,j*

At loop level, QCD and N=4 SYM differ

However, it is profitable to rearrange the QCD computation to exploit supersymmetry

gluon loop



N=4 SYM

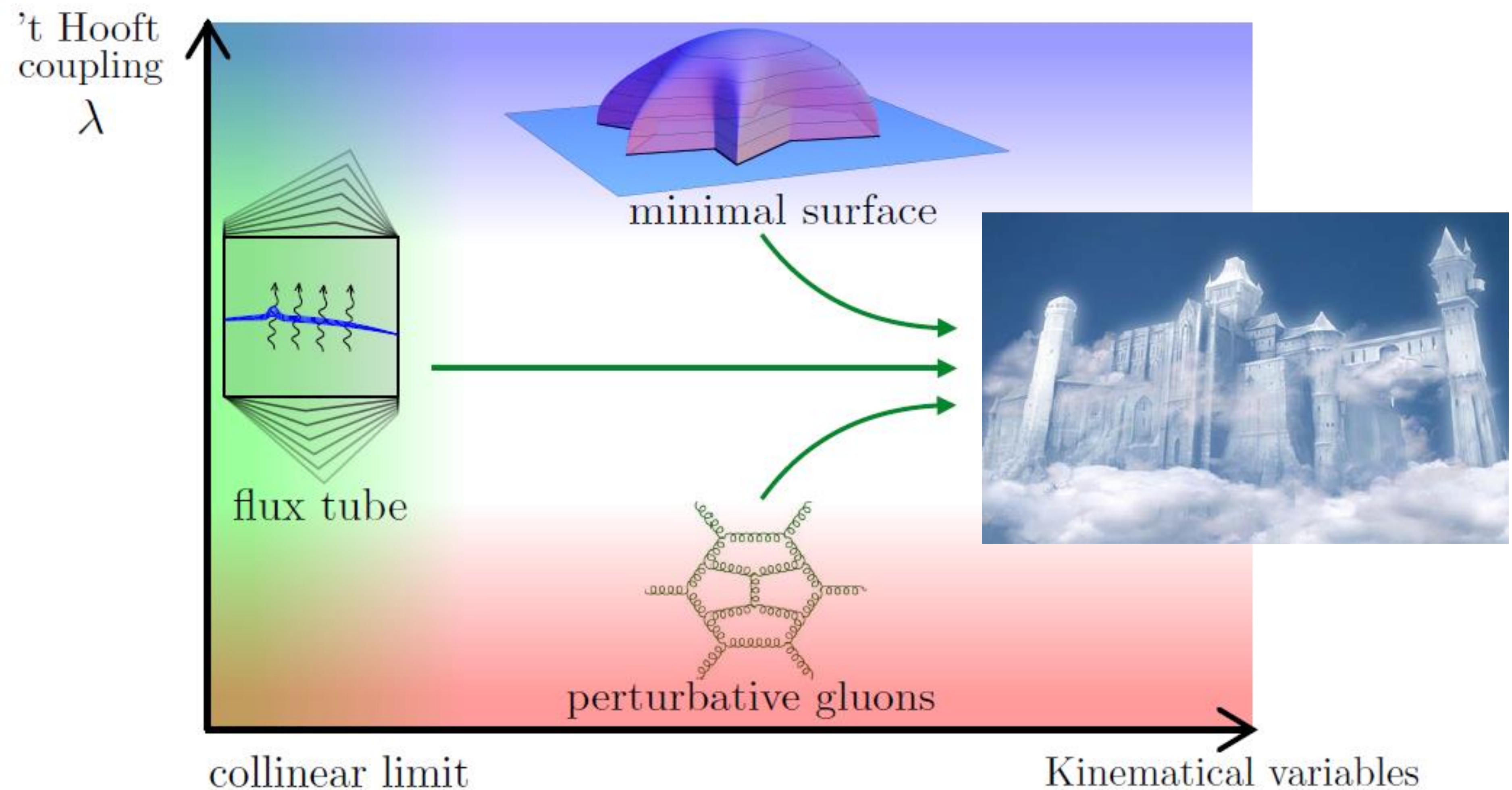
N=1 multiplet

scalar loop --- no SUSY, but also no spin tangles

contains all $1/\varepsilon^2$ poles

Solving Planar N=4 SYM

Images: A. Sever, N. Arkani-Hamed



Strong coupling and soap bubbles

Alday, Maldacena, 0705.0303

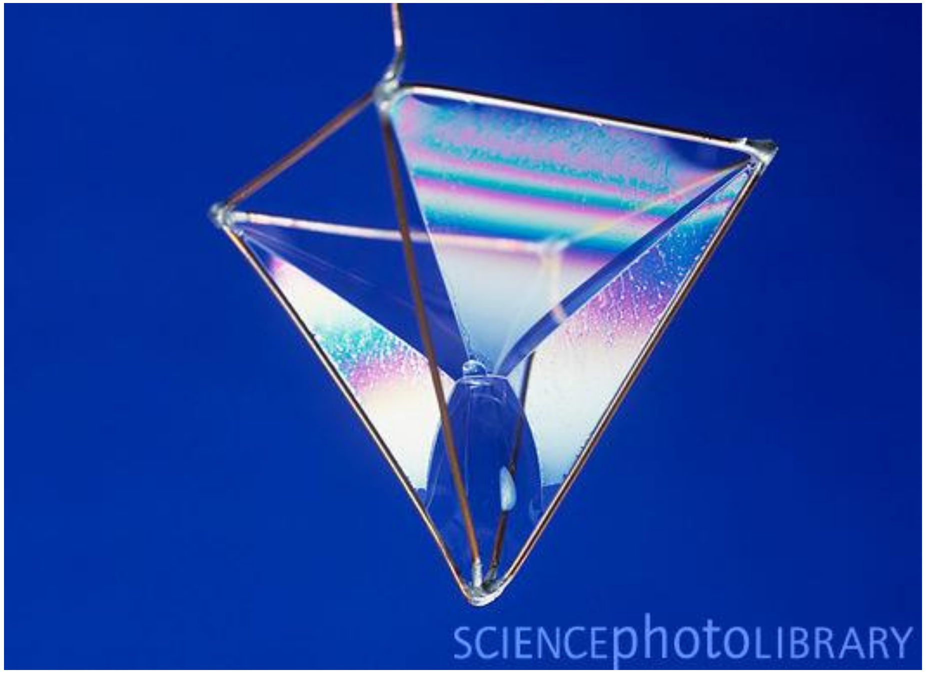
- Use AdS/CFT to compute scattering amplitude
- High energy scattering in string theory semi-classical:

two-dimensional string world-sheet stretches a long distance, classical solution minimizes area

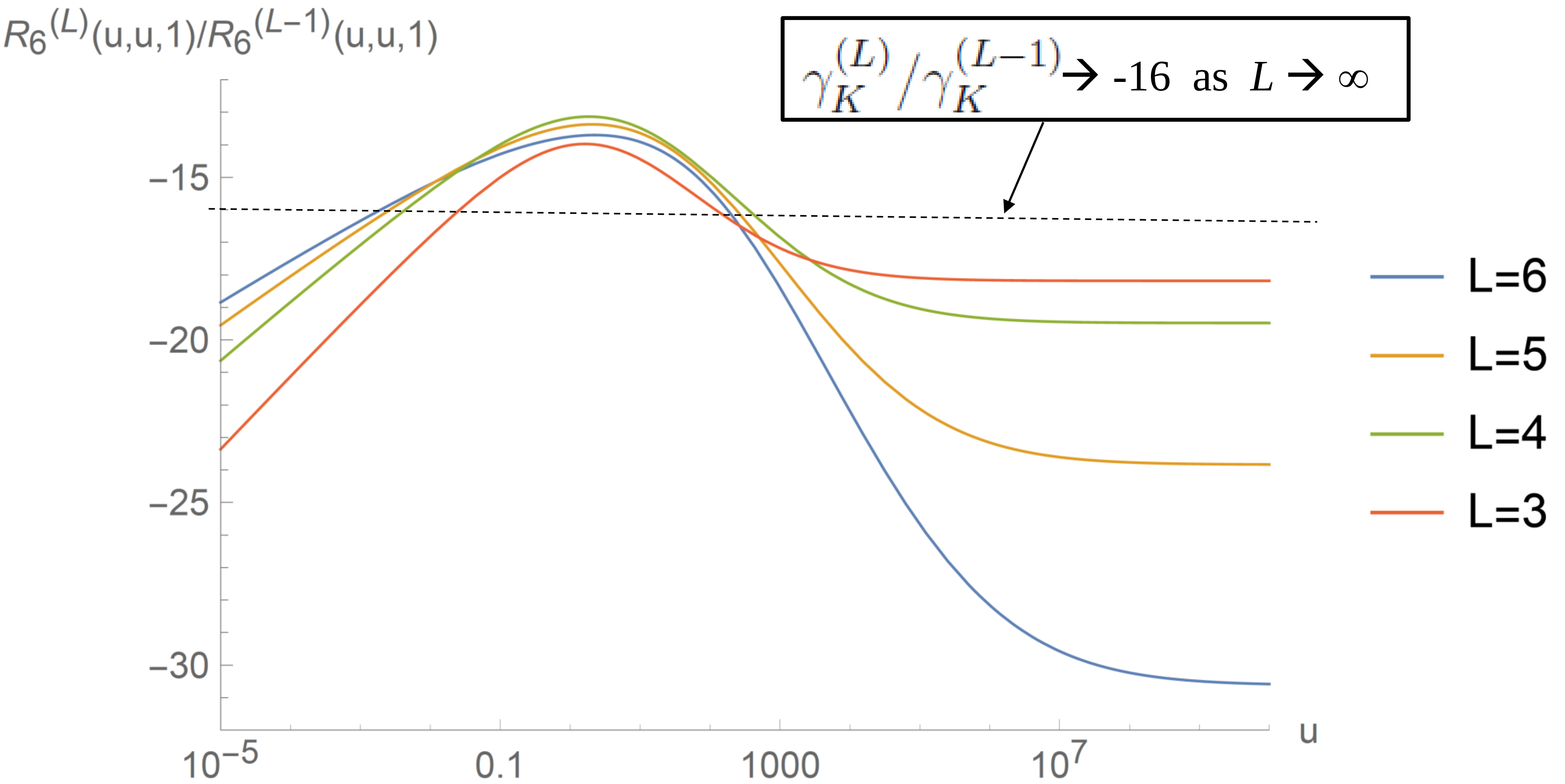
Gross, Mende (1987,1988)

Classical action imaginary
→ exponentially suppressed tunnelling configuration

$$A_n \sim \exp[-\sqrt{\lambda} S_{\rm cl}^{\rm E}]$$

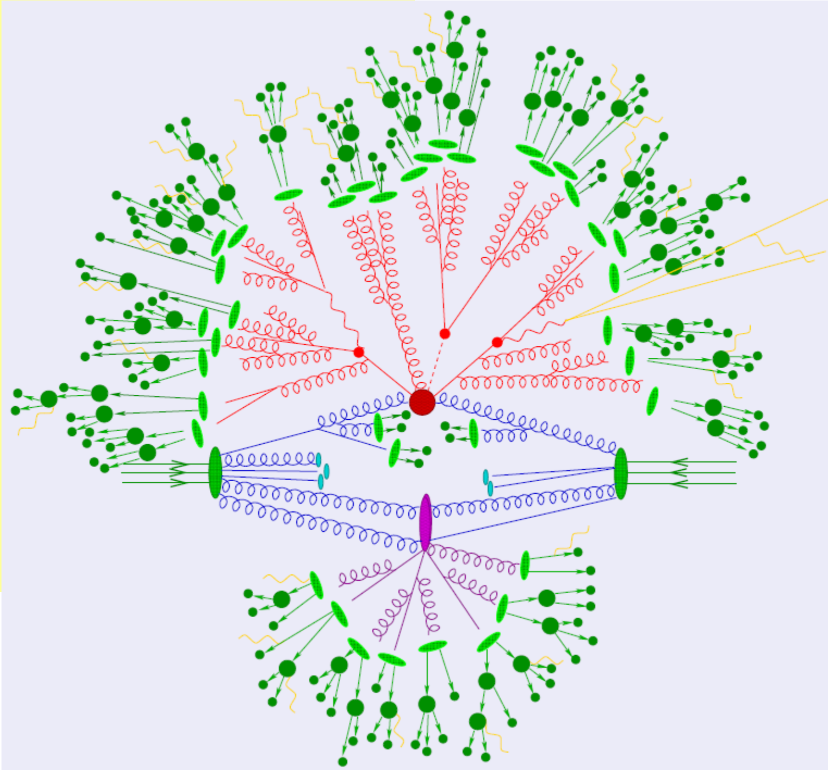


Ratio of successive loop orders for planar N=4 SYM 6-gluon amplitude for kinematics $(u,u,1)$ through 6 loops



Fixed order vs. Monte Carlo

- Previous plots were NLO but fixed-order, few partons: no model of long-distance effects included; cannot pass through a detector simulation
- Methods available for matching NLO parton-level results to parton showers, accuracy:
  - **MC@NLO** Frixione, Webber (2002) ; …; SHERPA implementation
  - **POWHEG** Nason (2004); Frixione, Nason, Oleari (2007)
  - **SHERPA** Krauss et al. (2012,…)
- Recently implemented for increasingly complex final states

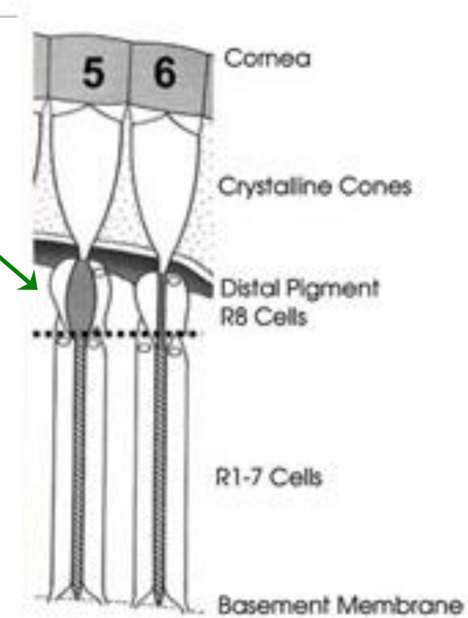


The tail of the mantis shrimp

- Reflects left and right circularly polarized light differently

- Led biologists to discover that its eyes have differential sensitivity
- It communicates via the helicity formalism

$\lambda/4$ plate



"It's the most private communication system imaginable. No other animal can see it."

- Roy Caldwell (U.C. Berkeley)

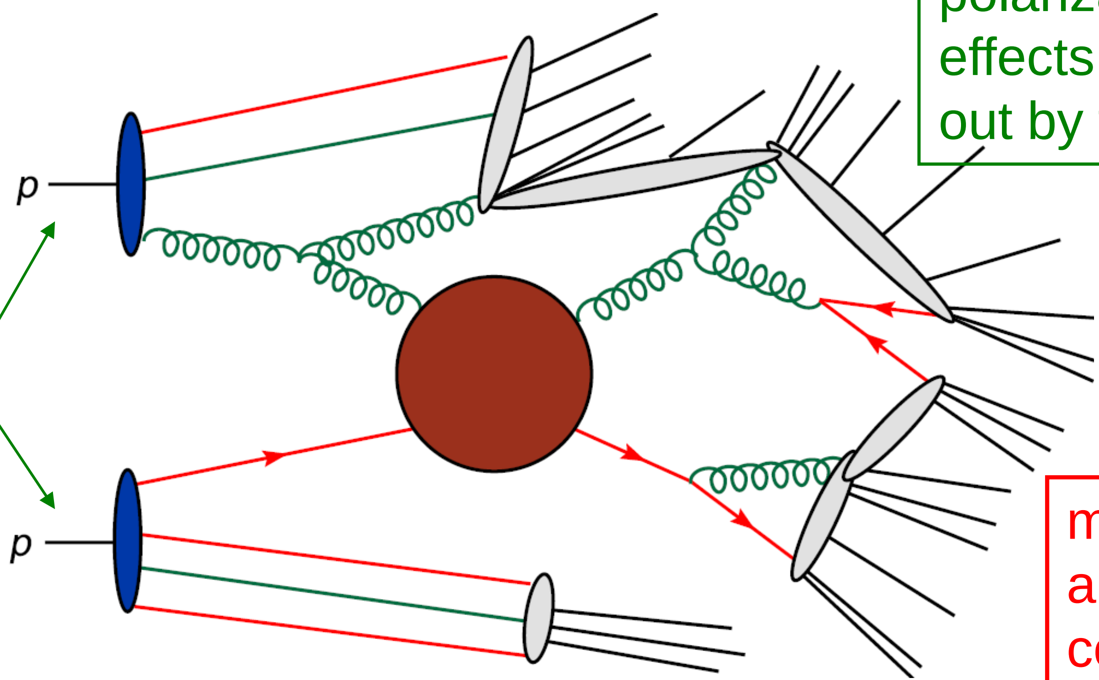
What the biologists didn't know

Particle theorists have also evolved capability to communicate results via helicity formalism

LHC experimentalists are blind to it

any final-state polarization effects washed out by fragmentation

unpolarized



must sum over all helicity configurations

Possible future of numerical approaches

- Computation is getting much cheaper
- Still, integrals needed at state-of-art, for either fixed-order, or high-precision resummation, or matched Monte Carlos, are getting more difficult.
- Find better ways to integrate approximate integrands?

Possible future of numerical approaches (cont.)

- With all the analytic and numerical knowledge we have accumulated, can machine learning be used in pQCD?
- For example, to predict approximate amplitudes?
- Already being used in jet substructure analyses to optimize separation of new physics from QCD.

Possible future of analytical approaches

- More sophisticated mathematics for complicated loop integrals
- Polylogarithms → elliptic integrals → hyperelliptic integrals → …
- Lots of interesting math including algebraic geometry and number theory.

Multiple Zeta Values (MZVs)

- Classical polylogs

evaluate to Riemann zeta values

$$\text{Li}_n(u) = \int_0^u \frac{dt}{t}\text{Li}_{n-1}(t) = \sum_{k=1}^{\infty} \frac{u^k}{k^n}$$

$$\text{Li}_n(1) = \sum_{k=1}^{\infty} \frac{1}{k^n} = \zeta(n) \equiv \zeta_n$$

- HPL's evaluate to nested sums called multiple zeta values (MZVs):

$$\zeta_{n_1,n_2,\ldots,n_m} = \sum_{k_1>k_2>\cdots>k_m>0}^{\infty} \frac{1}{k_1^{n_1} k_2^{n_2} \cdots k_m^{n_m}}$$

Weight $n = n_1 + n_1 + \ldots + n_m$

- MZV's obey many identities, e.g. stuffle

$$\zeta_{n_1}\zeta_{n_2} = \zeta_{n_1,n_2} + \zeta_{n_2,n_1} + \zeta_{n_1+n_2}$$

- All reducible to Riemann zeta values until weight 8.
  Irreducible MZVs: $\zeta_{5,3},\ \zeta_{7,3},\ \zeta_{5,3,3},\ \zeta_{9,3},\ \zeta_{6,4,1,1},\ \ldots$

One-loop QCD, cast of dozens

BlackHat, past and present:
Berger, Bern, Diana, LD, Febres Cordero, Forde, Gleisberg, Höche, Ita, Kosower, Lo Presti, Maître, Ozeren

Other key contributors:
Anastasiou, Badger, Bevilacqua, Biedermann, Britto, Cachazo, Cascioli, Czakon, Dunbar, Ellis, Feng, Frederix, Frixione, Garzelli, Giele, Hirschi, Kardos, Kunszt, Maltoni, Mastrolia, Melnikov, Ossola, Papadopoulos, Pittau, Pozzorini, Reiter, Schulze, Tramontano, Uwer, van Hameren, Weinzierl, Winter, Witten, Worek, Zanderighi, …