

SEARCHING FOR ELECTRON-INTERACTING DARK MATTER WITH
THE SUPER CRYOGENIC DARK MATTER SEARCH

by

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Abstract

Abundant and diverse evidence suggests that around 85% of the matter in the Universe is non-luminous and non-baryonic. While this ‘dark matter’ interacts gravitationally with Standard Model matter, its nature and non-gravitational interactions are not yet understood. The SuperCDMS experiment uses cryogenic semiconductor detectors to search for interactions of dark matter with Standard Model matter. The next generation is currently under construction at SNOLAB, with plans to begin commissioning in 2024. Before their operation in the main experiment, the SuperCDMS SNOLAB detectors will be tested at the Cryogenic Underground TEst (CUTE) facility, a low background environment at SNOLAB constructed for this purpose. This thesis describes a dark matter search using data from SuperCDMS Soudan, in which constraints on dark photons and axion-like particles with masses as low as $40 \text{ eV}/c^2$ are presented. Also described are the projected sensitivities of SuperCDMS SNOLAB and upgrades to the facility for several electron-interacting dark matter candidates. Finally, a characterization study of an R&D detector at the CUTE facility is presented.

Statement of Personal Contribution

The work of this thesis is mostly my own, but being conducted in the context of the SuperCDMS collaboration, it often relies on contribution from others. Here I provide a summary of my personal contributions, but I also make an effort throughout this thesis to acknowledge the efforts of others and clearly set this apart from my own contributions. Chapter 4 describes a search for dark photons and axion like particles using iZIP data from SuperCDMS Soudan, as well as showing combined limits with data from CDMSlite. I performed the analysis work of redefining new quality cuts, producing an energy resolution model and energy scale calibration, setting the limits on the iZIP data, and combining the data with CDMSlite limits. The work of the CDMSlite limits was performed by my colleague R. Germond, and all of the limits are dependent on the photoelectric cross section, for which an extensive literature review was performed by M. Wilson. I additionally produced gamma simulations and an extension of the search using the dark Compton scattering process. My main contributions to the sensitivity projections for SuperCDMS SNOLAB and future upgrade scenarios in Chapter 5 are the adaptation of the sensitivity code to be able to use different dark matter models, implementing models for dark absorption and light dark matter electron scattering, and various minor bug fixes. In addition, I ran all of the projections for the electron-interacting dark matter models. The sensitivity code itself was originally developed by M. Pyle and T. Saab, but many people have contributed to it. Chapter 6 describes an analysis of data taken at the CUTE facility. I participated in the construction and commissioning of the facility, run planning for most of its runs so far and the data acquisition. These tasks were entirely collaborative efforts. The analysis work of this chapter

is my own, but relates to other analyses performed by R. Underwood, S. Watkins, and C. Fink that I refer to in the text.

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This dissertation is the result of collaborative work with many individuals and as such I will surely miss many people who ought to be acknowledged, thus I begin by issuing a general thank you (and an apology for not directly naming) to all those who I have missed.

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It has taken a lot of effort over the past few years to keep the CUTE facility running, and in particular I would like to thank the onsite support from SNOLAB personnel at CUTE over the COVID-19 pandemic. With restricted access and increased safety protocols, it has been a huge challenge to keep going. Thank you to Andy Kubik, Silvia Scorza, Marco Olivares, Jasmine Gauthier, Melissa Baiocchi, Tyler Reynolds, and Vijay Iyer, as well as many others. We also could not have run without the support of the data taking team, including Bruno Serfass, Ziqing Hong, Aditi Pradeep, Ryan Underwood, Sukee Dharani, Adam Mayer, Richard Germond, Emanuele Michielin, Yan Liu, and many others. The data analysis efforts at CUTE have also mostly been undertaken by those named already. I would also like to extend thanks to Sam Watkins, Caleb Fink, and Matt Pyle for use of their analysis tools, as well as their advice on detector behaviour. Thank you also to Gilles Gerbier, Philippe Camus, and Serge Nagorny.

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List of Acronyms

ADMX	Axion Dark Matter eXperiment
ALP	axion-like particle
AMS-02	Alpha Magnetic Spectrometer
AQUAL	AQUAdratic Lagrangian theory
BBN	Big Bang nucleosynthesis
BDT	boosted decision tree
BLIP	Berkeley Large Ionization- and Phonon-mediated
CAST	CERN Axion Solar Telescope
CCD	charge-coupled device
CDEX	China Dark matter EXperiment
CDM	cold dark matter
CDMS	Cryogenic Dark Matter Search
CDMSlite	CDMS low ionization threshold experiment
CJPL	China Jinping Underground Laboratory
CL	confidence level
CMB	Cosmic Microwave Background
CMS	Compact Muon Solenoid
CoGeNT	Coherent Germanium Neutrino Technology
COUPP	Chicagoland Observatory for Underground Particle Physics
CP	charge-parity
CPD	Cryogenic PhotoDetector
CRESST	Cryogenic Rare Event Search using Superconducting Thermometers
CT	charge trapping
CUTE	Cryogenic Underground TEst
DAMIC	Dark Matter in CCDs
DAMIC-M	Dark Matter in CCDs at Modane
DAQ	data acquisition

DMC	Detector Monte Carlo
EC	electron capture
EFT	Effective Field Theory
ELDER	elastically decoupling relic
ER	electron recoil
ERDM	electron-recoil dark matter
ETF	ElectroThermal Feedback
FGST	Fermi Gamma-ray Space Telescope
FIMP	feebly interacting massive particle
FNAL	Fermi National Accelerator Laboratory
FV	fiducial volume
HEMT	High Electron Mobility Transistor
HV	high voltage
II	impact ionization
IMPACT	Ionization Measurement with Phonons At Cryogenic Temperatures
IR	infrared
ISS	International Space Station
iZIP	interleaved Z-sensitive Ionization and Phonon
JFET	Junction gate Field Effect Transistor
LDM	light dark matter
LF	low frequency
LHC	Large Hadron Collider
LIGO	Laser Interferometer Gravitational-Wave Observatory
LIP	Lightly Ionizing Particle
LNGS	Laboratori Nazionali del Gran Sasso
LSBB	Laboratoire Souterrain Bas Bruit
LSM	Laboratoire Souterrain de Modane
LUX	Large Underground Xenon experiment
LZ	LUX-ZEPLIN
MACHO	MAssive Compact Halo Object
MOND	MOdified Newtonian dynamics
NEWS-G	New Experiments With Spheres-Gas
NIST	National Institute of Standards and Technology

NR	nuclear recoil
NRDM	nuclear-recoil dark matter
NSOF	Non-Stationary Optimal Filter
NTD	neutron transmutation doped
NTL	Neganov-Trofimov-Luke
OF	Optimal Filter
OI	optimum interval
OVC	Outer Vacuum Can
PBH	primordial black hole
PDF	probability distribution function
PFV	phonon fiducial volume
PICASSO	Project In CANada to Search for Supersymmetric Objects, or Projet d'Identification de CAndidats Supersymtriques SOmbres
piZIP	phonon iZIP
PLR	profile likelihood ratio
PMT	photo-electron multiplier tube
PQ	Pecci-Quinn
PSD	Power Spectral Density
QCD	quantum chromodynamics
QET	Quasiparticle-trap-assisted Electrothermal-feedback Transition-edge-sensor
QFV	charge fiducial volume
RAQUAL	relativistic AQUAL
RF	radio frequency
SD	spin dependent
SENSEI	Sub-Electron-Noise Skipper-CCD Experimental Instrument
SI	spin independent
SIMP	strongly interacting massive particle
SIMPLE	Superheated Instrument for Massive Particle Experiments
SM	Standard Model
SQUID	Superconducting QUantum Interference Device
SUF	Stanford Underground Facility
SUL	Soudan Underground Laboratory
SuperCDMS	Super Cryogenic Dark Matter Search
SUSY	supersymmetry

TES	Transition Edge Sensor
TeVS	Tensor-Vector-Scalar
TEXONO	Taiwan EXperiment On Neutrino
TFN	Thermal Fluctuation Noise
TPC	Time Projection Chamber
WIMP	Weakly Interacting Massive Particle
ZIP	Z-sensitive Ionization and Phonon

Chapter 1

Introduction

The Standard Model (SM) of particle physics paints a beautifully succinct picture of the elementary particles and three of the four fundamental forces of our Universe [1]. Despite its predictive power, there are a variety of as of yet unexplained phenomena; it turns out that this painting is not yet complete. An abundance of astronomical and cosmological evidence suggests that the majority of matter in the Universe is non-luminous, non-baryonic, cold dark matter [2]. Experimental efforts to find dark matter typically come in one of three forms: direct detection looking for interactions of dark matter with SM matter, indirect detection searching for the products of dark matter self-interactions or annihilations, and production of dark matter at colliders. Much of the search focus in the direct detection community has been on the canonical Weakly Interacting Massive Particle (WIMP), but lately there is growing interest in electron-recoil dark matter (ERDM), including bosonic dark matter and low-mass dark matter coupled to SM matter via a mediator.

The Super Cryogenic Dark Matter Search (SuperCDMS) experiment is a direct detection dark matter experiment operating cryogenic semiconductor detectors, typically germanium or silicon. Several world-leading results have been obtained over the years of operation in the Soudan Underground Laboratory (SUL) in Minnesota, USA, with the majority of research efforts focused on WIMPs. The next generation of the experiment is currently under construction at the SNOLAB facility, located in Ontario, Canada. SuperCDMS

SNOLAB will begin commissioning in 2024, and will be sensitive to low-mass dark matter, including nuclear-recoil dark matter (NRDM) and ERDM. The Cryogenic Underground TEst (CUTE) facility is a low background test facility located in the same experimental hall as SuperCDMS SNOLAB. It was designed for the testing of SuperCDMS SNOLAB detectors before their operation in the main experiment while minimizing exposure to cosmogenic radiation. During this time, the readout electronics, detector performance, and calibration can be tested. CUTE will eventually become a general test facility where other experiments can request facility time for testing cryogenic detectors in a low background environment.

The first two Chapters will provide the reader with the necessary background information and context for the work of this thesis. In Chapter 2 I motivate the existence of dark matter and describe several promising candidates, several of which will feature prominently in later chapters. I also describe the various experimental approaches to detecting dark matter, with an emphasis on direct detection, and provide an overview of the SuperCDMS experiment and CUTE. Chapter 3 focuses on the SuperCDMS detectors, providing an extensive overview of the physics principles as well as specific detector designs. A search for low-mass bosonic dark matter using data from SuperCDMS Soudan is presented in Chapter 4. I describe a reanalysis of the data, which had previously only been used for NRDM searches, in the context of ERDM. Constraints are set on the coupling strength of dark photons and axion-like particles (ALPs) with masses from $40 \text{ eV}/c^2$ to $500 \text{ keV}/c^2$. While the main analysis considers only the process of dark absorption, I also demonstrate how incorporating dark Compton interactions can improve the sensitivity at higher masses. In Chapter 5 I describe forecasting efforts for the expected sensitivity of SuperCDMS SNOLAB and a variety of potential background and detector upgrades using the same facility for several ERDM models. The overall forecasting effort can guide R&D efforts to optimize the sensitivity of any upgrade scenarios for both NRDM and ERDM. Finally, in Chapter 6 I describe an analysis of the performance of an R&D detector at the CUTE facility, demonstrating the importance of detector characterization for dark matter searches.

Chapter 2

Missing Matter

In the early 1900's astronomers were met with baffling evidence suggesting that galaxies behave as if there is more mass than implied by observations of luminous matter. At this point in time, the mass of stars and galaxies was inferred from measurements of the luminosity, and these observations implied the existence of some kind of nonluminous matter, termed dark matter. Lord Kelvin attempted to apply the theory of gases to the Milky Way in an attempt to estimate the amount of dark matter, establishing a relationship between the size of the system and the velocity dispersion of the stars [3]. Efforts were made by several astronomers in the 1920's and 1930's to estimate the local dark matter density from the kinematics of stars; a detailed review of the history of dark matter observations and theory can be found in Ref. [4].

Perhaps the most famous evidence for dark matter came in 1933 when Fritz Zwicky studied the redshifts of 8 galaxies in the Coma cluster, which had been published by Hubble and Humason in 1931 [5]. Using the apparent velocities of the galaxies, Zwicky applied the virial theorem to determine the mass of the cluster [6]. The high observed velocities of the galaxies implied a higher mass than determined from the total luminosity, indicating the presence of a significant amount of non-luminous mass. Zwicky measured a mass-to-light ratio of around 500, a result which was confirmed by a similar measurement on galaxies in the Virgo cluster by Sinclair Smith in 1936 [7]. While these measurements were later found

to be overestimates by an order of magnitude due to an incorrect measure of the Hubble constant [2, 5], the velocity dispersion relations still point to the existence of non-luminous matter.

Throughout the 20th century, the Standard Model (SM) of particle physics was developed (see Ref. [1] for an overview of the SM). This theory describes all known elementary particles including leptons, bosons, and quarks, as well as three of the four fundamental forces. It has demonstrated great success in its predictive power, but cannot incorporate gravity as described by General Relativity and has various other outstanding problems. Additionally, the SM does not account for the existence of dark matter. Questions about the nature of dark matter have been raised: is it some kind of novel particle (or particles)? Does it interact via any of the other three fundamental forces (weak, strong, electromagnetic), or via some new force or mediator?

This also leads to the question of whether or not we can detect dark matter directly, rather than just implying its existence through gravitational measurements. The first direct detection dark matter searches were constructed in 1986 [8] to search for evidence of dark matter interactions directly with SM matter. The idea that dark matter could be observed indirectly through decay or interaction products was also introduced, leading to indirect detection searches [9–12]. It was also hypothesized that we could produce dark matter directly, leading to searches for dark matter production at colliders [13–15].

In the following Chapter I provide a foundation for the study of dark matter, starting with evidence for dark matter in Section 2.1. In Section 2.2 we will discuss various promising particle candidates for dark matter. Section 2.3 explores the various ways in which we can attempt to search for dark matter, and in Section 2.4 we explore the direction detection dark matter search experiment SuperCDMS.

2.1 Evidence for Dark Matter

2.1.1 Galaxy Dynamics

Galactic rotation curves are plots of the rotation profile, or orbital speed, of galactic components like stars and interstellar gas as a function of their distance from the galactic centre. The orbital speed of an object a distance r from the galactic centre is given by

$$v = \sqrt{\frac{GM(r)}{r}}, \quad (2.1)$$

where $M(r)$ is the mass enclosed within the distance r from the centre and G is the gravitational constant. It has been observed that there are low amounts of luminous matter at high galactic radii, thus the expected velocity profile should fall as $r^{-1/2}$.

Up until the 1960's there was a great deal of skepticism about the existence of dark matter when considering the observed mass-to-luminosity ratios. Arguments were made that the measured clusters were unstable and expanding, and thus the virial theorem could not be applied to them. However, this hypothesis was found to be in tension with the ages of the galaxies and of the Universe [16]. It was also proposed that regular matter, in the form of free hydrogen, that has simply been missed could account for the missing mass. However, though free hydrogen makes up a significant portion of the mass of galaxy clusters, it was found to be unable to account for all of the missing mass by various measurements [17, 18].

In 1970, a breakthrough occurred when Vera Rubin and Kent Ford used observations of optical light from the Andromeda (M31) Galaxy, measuring the velocity profile of ionized hydrogen [19]. They obtained the velocities by measuring the Doppler shift of the $H\alpha$ emission lines in molecular hydrogen. The velocity profile deviated from the expected $r^{-1/2}$ behaviour, instead remaining constant at high radius. Throughout the 1970's they catalogued rotation curves for galaxies of varying sizes, observing the same flat behaviour at high radius [20, 21] as shown in Figure 2.1.

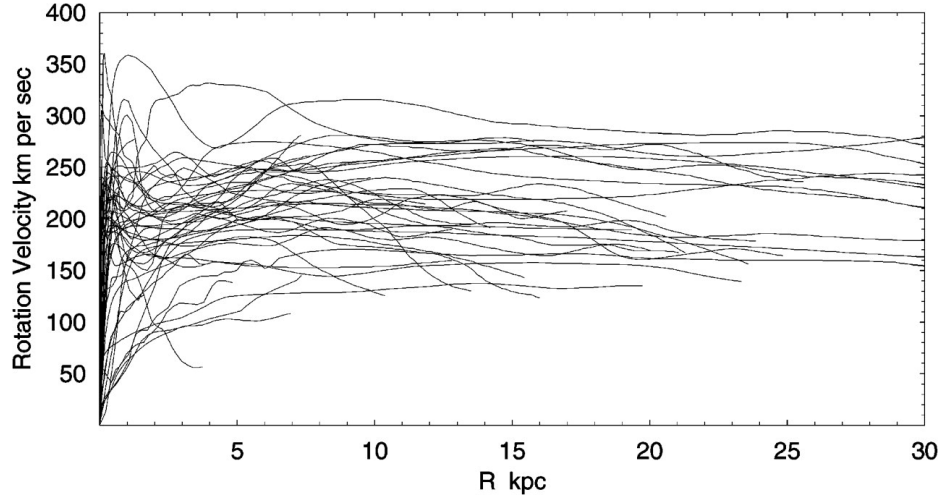


Figure 2.1: Rotation curves for various spiral galaxies showing the rotational velocity as a function of distance from the galactic centre, from Ref. [21].

The behaviour of the galactic rotation curves implies an oblate spheroid cloud of non-luminous matter surrounding each galaxy that accounts for the majority of the galactic mass. This phenomenon has since been described as a dark matter halo.

Our own galaxy is not exempt from the implied presence of dark matter. Estimates have been made of the so called *local dark matter density*. Estimates for a spherical halo put the local dark matter density between $0.2 - 0.4 \text{ GeV}/c^2/\text{cm}^3$ [22], and for the purpose of converting direct detection measurements to information about dark matter, the dark matter community has adopted a value of $0.3 \text{ GeV}/c^2/\text{cm}^3$. However, the shape of the halo may actually be flattened rather than spherical, in which case a local density between $0.3 - 0.7 \text{ GeV}/c^2/\text{cm}^3$ may be more likely [23].

2.1.2 Gravitational Lensing

General Relativity predicts that mass distributions in between a distant light source and an observer will deflect the incoming light, causing distortions in the image in an effect called *gravitational lensing*. The most basic example of a gravitational lens is the Schwarzschild lens [24]. A lens of point mass M deflects light by an angle α given by

$$\alpha = \frac{4GM}{rc^2}, \quad (2.2)$$

where r is the distance between the light source and the lens in the plane perpendicular to the observer, G is the gravitational constant, and c is the speed of light. Extreme examples of strong lensing include arcs and ring images of point sources; the latter occurs when the observer, lens, and light source are aligned [25]. With detailed modelling of these distortions, the lens' total mass and mass distribution can be estimated. This is a way to directly measure the total mass of a cluster or galaxy rather than using the dynamics of rotation curves.

However, these dramatic strong lensing effects are rare; instead we must look for more subtle lensing effects to infer the presence of dark matter. The majority of lensing effects are so called *weak lensing*, where the images of the background objects are only partially deformed or distorted [25, 26]. Measurements must be performed on multiple background objects to statistically determine the lens mass because the effect is too weak to observe on the level of individual objects.

One of the most striking examples of gravitational lensing which provides evidence for the existence of dark matter is the Bullet Cluster, which, despite its name, is actually two colliding galactic clusters. The clusters are composed of galaxies, X-ray emitting intergalactic plasma (which account for most of the SM mass of the Bullet Cluster), and (possibly!) dark matter. The stellar and plasma components of each cluster behave differently in the collision, with the galaxies acting like collisionless particles. In contrast, the colliding plasma is slowed dramatically because of electromagnetic interactions. Due to this difference in behaviour, the plasma and stellar components are spatially separated, a phenomenon which can be observed via optical and X-ray measurements, which are sensitive to the light emitted by each component. Stellar images were taken by the Hubble telescope and the plasma component was measured by the Chandra X-Ray Observatory [27].

The mass distribution of the Bullet Cluster has been independently measured via gravitational lensing by Clowe et al. [27]. One would expect, in the absence of non-SM matter, that the mass distribution would closely follow the distribution of the interstellar plasma. However, as seen in Figure 2.2, the true centres of mass do not align with the centres of mass of ordinary matter. There are two large distributions of non-luminous matter which have bypassed one another. The total mass distribution implies that the majority of the mass in the cluster is some unseen matter which is mostly collisionless – compelling evidence for dark matter. Limits can be set on the dark matter self interactions, which must be weak in strength. From observations of the Bullet Cluster, the self interaction cross section per unit mass must be less than $1.25 \text{ cm}^2/\text{g}$ [28].

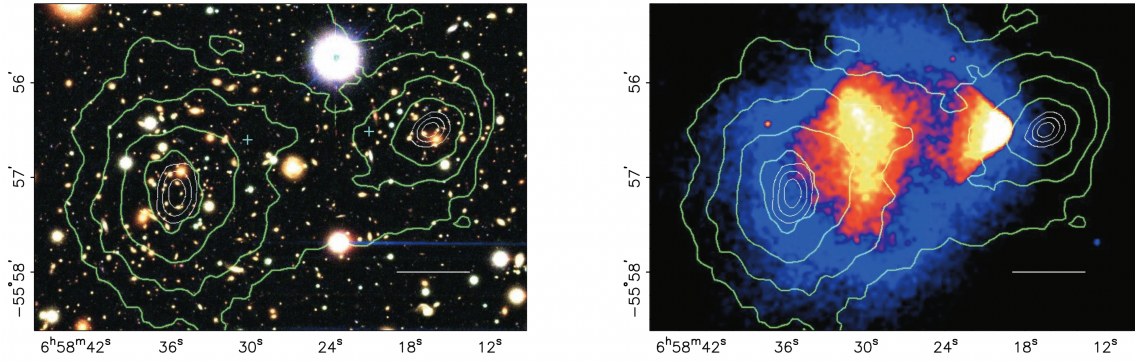


Figure 2.2: Images of the Bullet Cluster showing the stellar (left) and X-ray emitting plasma (right) components, from Ref. [27]. The green contours indicate the mass distribution as determined via gravitational lensing measurements. The centres of mass of the contours do not align with the distributions of the plasma component, which dominates the mass contribution of ordinary matter.

2.1.3 The Cosmic Microwave Background

Some of the most compelling evidence for the existence of dark matter comes from the Cosmic Microwave Background (CMB), primordial blackbody radiation from the epoch of recombination. Before the Universe cooled to the point where e^- and p^+ coalesced into neutral particles, these elementary particles were strongly coupled to photons [29]. At this

point, the Universe was roughly 378,000 years old and had a temperature of ~ 3000 K [30]. It was during recombination that matter and radiation became thermally decoupled. After decoupling, photons have travelled largely unimpeded through space, and today have a thermal black body distribution with temperature 2.72548 ± 0.00057 K [31]. It was initially thought that the CMB was a perfect blackbody spectrum. However, anisotropies were eventually found in the temperature and polarization at the 10^{-5} and 10^{-6} levels, respectively [32].

The composition of the Universe can be broken down into several components including matter, radiation, and dark energy. These components are parameterized by a dimensionless density parameter

$$\Omega_i = \frac{8\pi G}{3H^2} \rho_i, \quad (2.3)$$

where ρ_i is the contribution to the energy density parameter $\rho = \sum_i \rho_i$ from matter, radiation, and vacuum energy, and H is the present day Hubble parameter written in terms of the Hubble constant h as $H = 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$.

The total energy density of the Universe, Ω , is given by the Friedmann equation

$$\Omega = \Omega_M + \Omega_R + \Omega_\Lambda = 1 + \frac{k}{a^2 H^2}, \quad (2.4)$$

where Ω_M , Ω_R , and Ω_Λ are the matter, radiation, and vacuum energy contributions, respectively, and $k \in (-1, 0, 1)$ describes the curvature of the Universe as open, flat, or closed. Note that Ω_M is the combined density of baryonic matter Ω_b and dark matter Ω_c (here the subscript c represents cold dark matter (CDM), which will be explained further in this section). It turns out that these parameters have a significant impact on the temperature fluctuations of the CMB.

The temperature anisotropies measured over the full 4π sky map can be expressed in

terms of a spherical harmonic expansion

$$T(\theta, \phi) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\theta, \phi), \quad (2.5)$$

where $Y_{\ell m}(\theta, \phi)$ are spherical harmonics and $a_{\ell m}$ are coefficients of the expansion. The angular power spectrum of the anisotropies is defined

$$C_\ell = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} |a_{\ell m}|^2. \quad (2.6)$$

The band power, which is typically the plotted quantity, is defined

$$\mathcal{D}_\ell^{TT} = \frac{\ell(\ell + 1)C_\ell}{2\pi}, \quad (2.7)$$

where the superscript TT indicates that this is the temperature-temperature power map.

The monopole component ($\ell = 0$) corresponds to the blackbody component of the CMB, while the dipole ($\ell = 1$) corresponds to the Doppler shifting of the monopole component due to the relative movement of the Solar System and the CMB. Higher order moments are results of perturbations in the density of the early Universe, and give insight into the state and physics of the Universe at the time of decoupling. The power spectrum has been most precisely measured by the Planck satellite [2] and is one of the most powerful ways of studying the cosmology of the early Universe.

When the Universe became matter dominated (the matter density exceeded the photon density), initial over-densities in the matter-radiation fluid became gravitational wells, into which matter fell [33]. Baryons were still coupled to radiation; radiation pressure pushed them back out of the wells. The combination of the gravitational well and radiation pressure formed acoustic oscillations in the baryon-photon fluid; the resulting compressions of matter density led to variations in the temperature of the CMB which we can observe today. Matter that does not interact with photons, like dark matter, remained in the wells.

The CMB power spectrum as measured by the Planck satellite is shown in Figure 2.3. Important features are the peaks and troughs, the spacing between the peaks, and the location of the first peak [33]. These features are dependent on the initial conditions and energy content of the Universe at the time of recombination, and those after recombination. The location of the first peak is consistent with a flat Universe; that is, the current energy density of the Universe is equal to the critical density ρ_c [2]. Band power peaks correspond to oscillatory modes in the baryon-photon fluid which were at their maximum/minimum of oscillation when decoupling occurred. The position and distance between the peaks can be used to estimate Ω_Λ and Ω_M , as well as the Universe spatial curvature Ω_k . Note that the presence of baryons at decoupling decreases the heights of even number peaks compared to odd number peaks. Therefore, the difference in the height of peak 2 compared to peaks 1 and 3 gives a measure of Ω_b [33].

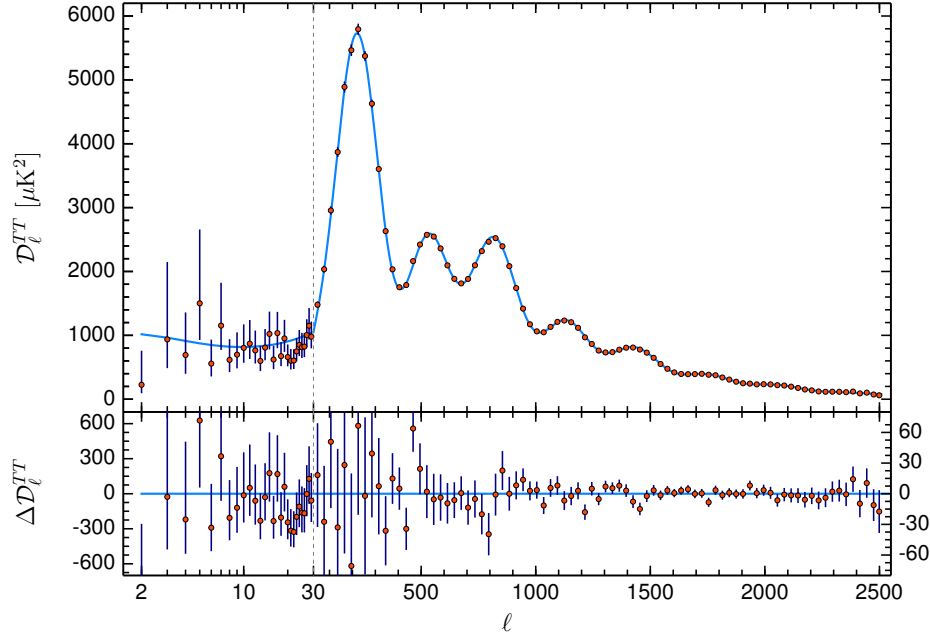


Figure 2.3: CMB power spectrum as measured by the Planck satellite with 1σ uncertainty and best fit curve (top). Also shown are the residuals for the best fit curve (bottom). From Ref. [2].

The Planck collaboration fit the measured spectrum with a model including dark energy (Λ) and *cold* (C) dark matter (DM) (Λ -CDM). The observed power spectrum shows that dark matter must be non-relativistic, i.e. cold, as hot (relativistic) or warm (having properties between hot and cold) dark matter has too much energy to be collected in the initial small density perturbations. The most recent results from Planck [2] for the various cosmological parameters are summarized in Table 2.1. The data are consistent with a flat Universe, i.e. $\Omega_k = 0.001 \pm 0.002$ [2].

Parameter	Symbol	Value
Matter Density	Ω_M	0.315 ± 0.007
Baryon Density	Ω_b	0.0493 ± 0.0008
CDM Density	Ω_c	0.264 ± 0.004
Dark Energy Density	Ω_Λ	0.6897 ± 0.0057
Hubble Parameter	h	0.674 ± 0.005

Table 2.1: Cosmological parameters from Planck [2].

The small observed baryon density, $\Omega_b \approx 0.05$, compared to the total matter density, $\Omega_M \approx 0.32$ [2], indicates that the baryon content does not account of all of the matter content, providing strong evidence for the existence of matter that was not coupled to photons prior to recombination.

2.1.4 Big Bang Nucleosynthesis

Big Bang nucleosynthesis (BBN) is a powerful probe of the early Universe, predicting the abundances of the light elements deuterium (D), ^3He , ^4He , and ^7Li , and providing constraints on the baryon energy density, Ω_b [32]. In the first few seconds after the Big Bang, the Universe was a hot plasma with energy density dominated by radiation, and protons and neutrons were kept in equilibrium by weak interactions. This equilibrium fixed the ratio of the neutron to proton number densities to $n_n/n_p \approx \exp(-Q/T)$ where $Q = 1.293 \text{ MeV}$ is the neutron-proton mass difference and T is the temperature of the Universe. As the Universe expanded, the radiation density and temperature both fell. As the temperature decreased,

the neutron-proton interconversion rate fell faster than the Hubble expansion rate. Weak interactions froze out at $T \sim 1$ MeV, with a corresponding neutron-proton number density ratio of $n_n/n_p \sim 1/6$. This ratio fell to $1/7$ due to free neutron decay prior to the synthesis of light elements commencing.

The first bound element to form was D, via the process $p(n, \gamma)\text{D}$. Despite a binding energy of 2.23 MeV, D production (and the production of other bound nuclei) was delayed until 0.1 MeV because the high photon to baryon ratio caused rapid photo-dissociation by high energy photons. The deuterium then underwent 2-body reactions to form He. Most neutrons ended up bound in ^4He , which is the most stable light element.

Nucleosynthesis primarily produced ^4He with $\sim 25\%$ primordial mass fraction (the ratio of the ^4He mass density to the total baryon mass density), compared to around 10^{-5} for ^3He and D and 10^{-10} for ^7Li . These primordial abundances depend almost entirely on one parameter, the baryon to photon ratio, η , and were fixed by the time the Universe was ~ 180 s old. However, we observe the abundances at significantly later epochs – after stellar nucleosynthesis, which both produces heavier elements and alters the primordial abundances of the light elements. It is necessary to measure the abundances at locations with low metal abundances to ensure that the values are as close to the primordial values as possible. In particular, D is essentially only produced by BBN, as it is destroyed when cycled into stars [34]. The D/H ratio currently provides the best constraints on the baryon density and baryon-to-photon ratio.

Predictions of the primordial abundances from BBN can be compared to the abundances implied from observed data. The D/H abundance is in excellent agreement with that observed for quasar absorption systems; a weighted mean of 11 of the most precise measurements gives $\text{D}/\text{H}|_p \times 10^6 = 25.47 \pm 0.25$ [32]. There is reasonable agreement for the He abundance determined from extragalactic HII systems (regions of ionized interstellar atomic hydrogen); the value of the ^4He abundance recommended by the Particle Data Group in 2022 based on recent measurements is $Y_p = 0.245 \pm 0.003$ [32]. However, there

is poor agreement for ${}^7\text{Li}$ abundance observed in atmospheres of halo dwarf stars; these metal-poor stars are the best suited objects for determining the primordial ${}^7\text{Li}$ abundance. This ‘Lithium Problem’ is still an unresolved mystery for BBN.

The concordant range of the baryon-to-photon ratio η is primarily determined by the D/H ratio [35] as

$$\eta = (6.143 \pm 0.190) \times 10^{-10}. \quad (2.8)$$

We also note that η is related to the baryon energy density, Ω_b , by

$$\Omega_b = 3.66 \times 10^7 \eta h^{-2}, \quad (2.9)$$

and the corresponding baryon density from the concordant range of η as determined from BBN is

$$\Omega_b h^2 = 0.02244 \pm 0.00069. \quad (2.10)$$

The Planck result of $\Omega_b h^2 = 0.02233 \pm 0.00015$ [2] translates to a baryon-to-photon ratio of $\eta = 6.09 \pm 0.06$. This demonstrates excellent agreement between the measurement of the baryon density from the CMB and that from BBN, indicating that $\sim 5\%$ of the energy density is baryonic. As previously discussed, this is significantly less than the total matter energy density of the Universe determined by the CMB measurements, $\Omega_m \approx 0.3$, indicating that the majority of matter is non-baryonic. The primordial abundances predicated by BBN are shown compared to the observed abundances in Figure 2.4, which demonstrates the good agreement between CMB and BBN measurements of the baryon density, as well as the discordance of the measured ${}^7\text{Li}$ abundance.

2.1.5 Alternatives to Dark Matter

While it might be natural to some to suggest that the phenomena of the mass-to-luminosity ratio discrepancy and the behaviour of galactic rotation curves can be accounted for by the presence of a new type of non-luminous matter, others have proposed that there may

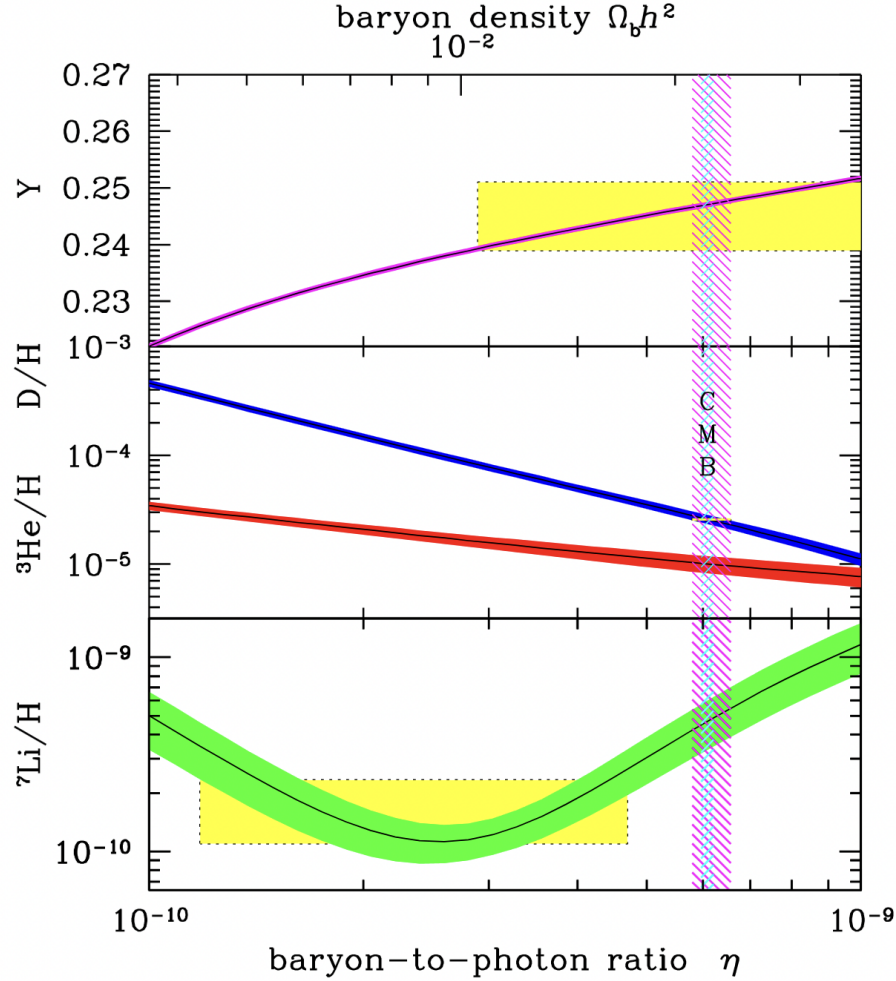


Figure 2.4: The primordial abundances of the light elements D, ${}^3\text{He}$, ${}^4\text{He}$, and ${}^7\text{Li}$ from Ref [30]. Lines show the predictions from the standard model of BBN with 95% CL bands, and the yellow boxes show observed abundances. The narrow blue band shows the baryon density as measured from the CMB at 95% CL, and the wide pink band shows the BBN $D+{}^4\text{He}$ concordance range at 95% CL.

be another explanation – what if our theory of gravity is incorrect on large scales? If the gravitational force falls as $1/r$ rather than $1/r^2$, then flat galactic rotation curves would be consistent with the observed mass distribution without the need for dark matter.

The theory of MODified Newtonian dynamics (MOND) was first introduced in 1983 by Mordehai Milgrom [36], who proposed that instead of mass obeying Newton’s second law $F = ma$, in limit of low accelerations ($a \ll a_0 \sim 2 \times 10^{-10} \text{ m/s}^2$) the gravitational force

instead scales as

$$F = m\mu \left(\frac{a}{a_0} \right) a. \quad (2.11)$$

MOND was not initially intended as a realistic theory [4], but instead as a weak-field limit of some more complex framework. Over the years, new theories of modified gravity have been proposed. AQUAdratic Lagrangian theory (AQUAL) is a modification of the Newtonian gravity Lagrangian, but is not compatible with General Relativity. This theory was later modified to relativistic AQUAL (RAQUAL) [37].

However, these theories could not account for the amount of gravitational lensing observed around galaxy clusters if there is only luminous matter. That is, theories of modified gravity alone cannot address the dark matter problem on the scale of clusters. The first possible solution to this lensing problem, Tensor-Vector-Scalar (TeVeS) gravity, was proposed by Berkstein in 2004 [38]. TeVeS is flexible enough to have the potential to be consistent with gravitational lensing observations and other cosmological phenomena like the CMB. However, the flexibility of the theory limits its predictive power, and this theory cannot explain the Bullet Cluster and does not predict the ratio of the second and third peaks in the CMB angular power spectrum [4].

2.2 Dark Matter Candidates

With the conclusion from cosmological evidence that the majority of the mass in the Universe must be non-baryonic, stable (or long lived) CDM, a variety of possible particle dark matter candidates have been postulated. In this section, several prominent dark matter candidates are discussed, including the canonical WIMP, light dark matter (LDM), and bosonic dark matter.

2.2.1 Weakly Interacting Massive Particles

A strongly motivated dark matter candidate is the Weakly Interacting Massive Particle (WIMP), which has been the subject of much of the dark matter search effort over the last few decades. This CDM candidate would be produced thermally in the early Universe, and now moves with non-relativistic velocities. WIMPs arise naturally in various theories beyond the SM including e.g. as the lightest supersymmetric particle in supersymmetry (SUSY) theories [39, 40], or as the lightest Kaluza-Klein particle in extradimensional theories [41]. Neither of these theories were proposed to solve the problem of dark matter existence; the WIMP dark matter candidate is an exciting bonus.

WIMP Freeze-Out and Relic Abundance

The following summary of the WIMP relic abundance follows Refs. [29, 40]. Consider a massive non-baryonic dark matter particle χ in thermal equilibrium with the early Universe. At this time, the Universe was radiation dominated. The equilibrium is maintained by the annihilation of the dark matter with its antiparticle $\bar{\chi}$ into lighter particles ($\chi\bar{\chi} \rightarrow l\bar{l}$) and the reverse process, which is possible when the temperature T exceeds the dark matter mass, m_χ .

As the Universe cools to a temperature below the dark matter mass, the dark matter production process becomes impossible, and the equilibrium abundance decreases exponentially. Eventually, the annihilation reaction rate falls below that of the Hubble expansion rate as it becomes more and more unlikely that two particles will meet and annihilate. At this point the dark matter interactions ‘freeze-out’ and the relic abundance is fixed. The dark matter annihilation rate in early Universe is given by

$$\Gamma = n_\chi \langle \sigma v \rangle, \quad (2.12)$$

where n_χ is the dark matter number density and $\langle \sigma v \rangle$ is the thermally-averaged product

of the annihilation cross section and the relative velocity. As long as it occurs frequently enough, the annihilation process keeps the dark matter in thermal equilibrium with the SM bath. Decoupling occurs when $\Gamma \approx H$, the Hubble expansion rate.

In thermal equilibrium, the number density, n_χ^{eq} , is given by

$$n_\chi^{eq} = \frac{g}{(2\pi)^3} \int f(\mathbf{p}) d^3\mathbf{p}, \quad (2.13)$$

where g is the initial number of degrees of freedom for the particle and $f(\mathbf{p})$ is the Fermi-Dirac or Bose-Einstein distribution (depending on whether the particle is a fermion or boson, respectively). When $T \ll m_\chi$, n_χ^{eq} can be approximated as

$$n_\chi^{eq} \approx g \left(\frac{m_\chi T}{2\pi} \right)^{3/2} e^{-m_\chi/T}. \quad (2.14)$$

The time evolution of the number density, $n_\chi(t)$, is governed by the Boltzmann equation:

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle\sigma_A v\rangle [(n_\chi)^2 - (n_\chi^{eq})^2]. \quad (2.15)$$

Note that the second term on the left side of the equation accounts for the expansion of the Universe.

While there is not an analytic solution to the Boltzmann equation, a simple analytic approximation can be derived in a somewhat lengthy derivation that is not reproduced here. The relic abundance can be approximated in this manner as [40]

$$\Omega_\chi h^2 = \frac{m_\chi n_\chi}{\rho_c} \simeq \frac{3 \times 10^{-27} \text{ cm}^3 \text{ s}^{-1}}{\langle\sigma_A v\rangle}. \quad (2.16)$$

This result is largely independent of WIMP mass. Figure 2.5 shows numerical solutions to the Boltzmann equation for different values of the annihilation cross section. A larger annihilation cross section means that the WIMP stays in equilibrium for longer, leading to a decreased relic density.

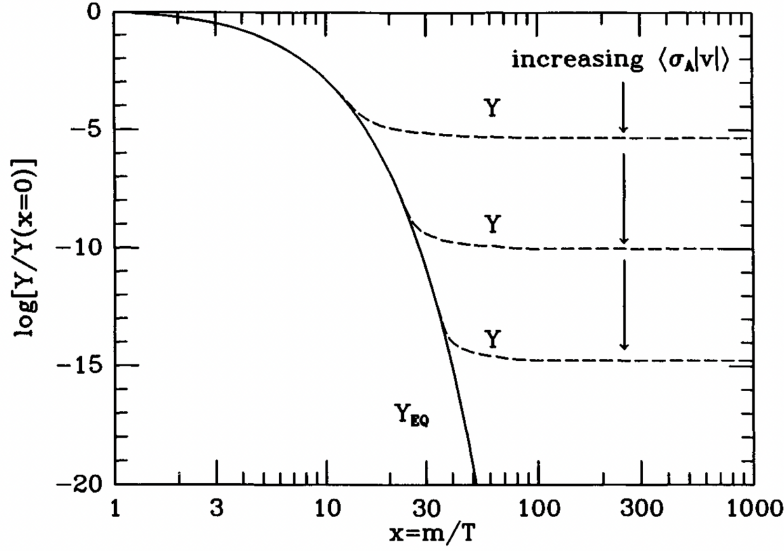


Figure 2.5: Evolution of the WIMP number density Y over time. The solid line indicates the equilibrium abundance, and the dashed lines indicate the true abundance for various $\langle\sigma v\rangle$. From Ref. [29].

Consider an estimate for the expected annihilation cross section for a new particle with weak-scale interactions [40]

$$\langle\sigma_A v\rangle \approx \alpha^2 (100 \text{ GeV})^{-2} \approx 10^{25} \text{ cm}^3 \text{ s}^{-1}. \quad (2.17)$$

This value is remarkably close to that required to achieve the relic dark matter density as shown in Equation 2.16, yet there is no a priori reason that weak-scale interactions should have anything to do with the cosmological density. This remarkable coincidence, referred to as the WIMP miracle, has motivated extensive experimental searches for weak-scale dark matter. Despite the weak scale of the WIMP interaction, the local dark matter flux is large enough that one could expect to see evidence of WIMP-SM matter interactions via elastic scattering with nuclei, with typical expected event rates of less than one event a year per kg of target material [42].

Interaction Rates in Direct Detection of WIMPs

The following description of the WIMP nucleus-interaction rate is based on the formalism in Refs. [22, 43]. We consider the interaction of a WIMP with mass m_χ with a nucleus of mass m_N . The elastic scattering is in the non-relativistic limit, and the energy of the nucleus after scattering is

$$E_R = \frac{\mu_N^2 v^2 (1 - \cos \theta^*)}{m_N}, \quad (2.18)$$

where $\mu_N = m_\chi m_N / (m_\chi + m_N)$ is the reduced dark matter-nucleus mass and θ^* is the scattering angle in the centre of mass reference frame. We can express the differential event rate in terms of the differential cross section, $\frac{d\sigma}{dE_R}$, as

$$\frac{dR}{dE_R} = \frac{\rho_0}{m_N m_\chi} \int_{v_{min}}^{\infty} v f(v) \frac{d\sigma}{dE_R}(v, E_R) dv. \quad (2.19)$$

The integration is taken over the range of WIMP speeds capable of causing a recoil of energy E_R : $v_{min} = \sqrt{m_N E_R / (2\mu_N^2)}$ is the minimum speed. The upper limit is technically infinite, but in practice is limited by the galactic escape velocity v_{esc} , above which dark matter is too fast moving to be gravitationally bound by the Milky Way. Recent measurements put $v_{esc} \approx 544$ km/s [44]. The function $f(v)$ describes the dark matter velocity distribution in the detector reference frame, normalized to unity. A Maxwellian distribution is typically assumed,

$$f(v) = A e^{-(v+v_E)^2/v_0^2}, \quad (2.20)$$

where v_E is the average velocity of the Earth relative to the dark matter, and v_0 is the orbital speed of stars in the galaxy. A collaborative effort between authors from various direct detection dark matter experiments was undertaken in 2021 in an attempt to unify various parameters and conventions used in direct dark matter searches [45]. This paper suggests values of $v_E = 238$ km/s, $v_0 = 253.7$ km/s, and $v_{esc} = 544$ km/s, though these values are not necessarily definitive.

The total observable event rate in a detector can be determined by integrating over all recoil energies

$$R = \int_{E_T}^{\infty} dE_R \frac{\rho_0}{m_N m_\chi} \int_{v_{min}}^{\infty} v f(v) \frac{d\sigma}{dE_R}(v, E_R) dv, \quad (2.21)$$

where E_T is the detector energy threshold, i.e. the smallest energy deposition that the detector is capable of measuring.

The WIMP-nucleus differential cross section can be separated into a spin independent (SI) and a spin dependent (SD) contribution as follows

$$\frac{d\sigma}{dE_R}(v, E_R) = \frac{m_N}{2\mu_N^2 v^2} [\sigma_0^{SI} F_{SI}^2(E_R) + \sigma_0^{SD} F_{SD}^2(E_R)], \quad (2.22)$$

where $\sigma_0^{SI/SD}$ are the spin-independent/dependent cross sections at zero momentum transfer and $F_{SI/SD}^2(E_R)$ are form factors encoding the dependence on the momentum transfer for spin-independent/dependent interactions. The SI cross section scales with the atomic mass A^2 , making heavy targets ideal for detectors. Direct detection experiments typically present results in the dark matter-nucleon cross section vs. m_χ plane to be able to compare experiments with different targets.

2.2.2 Light Dark Matter

With the lack of a dark matter discovery and exclusion of vast amounts of the WIMP parameter space in recent years, alternative candidates are becoming favourable. One well-motivated candidate is light dark matter (LDM), with masses in the keV to GeV range. There is a lower bound of $\sim 2 \text{ GeV}/c^2$ on the mass of the WIMP, the Lee-Weinberg bound [46], below which the WIMP dark matter relic density would overclose the Universe. A new force mediator is introduced to enable interactions between LDM and the SM matter, subverting the Lee-Weinberg constraint by which weak-force mediated dark matter is bound.

LDM-Electron Interactions

LDM interacts via both nuclear and electron scattering, but the most relevant signal is electron interactions. For this reason LDM is sometimes referred to as electron-scattering dark matter. To see why this is, consider that for a nucleus of mass m_N , the nuclear recoil energy of a dark matter scattering interaction with momentum transfer $\vec{q}$ is bounded by [47]

$$E_{NR} = \frac{q^2}{2m_N} \leq \frac{2\mu_{\chi N}^2 v^2}{m_N} \simeq 1 \text{ eV} \times \left(\frac{m_\chi}{100 \text{ MeV}} \right)^2 \left(\frac{20 \text{ GeV}}{m_N} \right) \quad (2.23)$$

where $\mu_{\chi N}$ is the reduced LDM–nucleus mass and a typical WIMP speed of 300 km/s is assumed. For a dark matter mass of 100 MeV, the nuclear recoil energy is already sub-eV for most materials used in direct detection experiments, below their typical detection thresholds.

In contrast, consider a scattering interaction with a bound electron. The electron momentum may be arbitrarily high. The energy transferred to the electron, ΔE_e , is approximately equal to the energy lost by the LDM, ΔE_χ , as the energy transferred to the rest of the atom, ΔE_N , is very small. Energy conservation gives

$$\Delta E_e = -\Delta E_\chi - \Delta E_N = -\frac{|m_\chi \vec{v} - \vec{q}|^2}{2m_\chi} + \frac{1}{2}m_\chi v^2 - \frac{q^2}{2m_N} = \vec{q} \cdot \vec{v} - \frac{q^2}{2\mu_{\chi N}}. \quad (2.24)$$

In practice, as ΔE_N is so small we can replace $\mu_{\chi N}$ with m_χ . Maximizing ΔE_e with respect to $\vec{q}$ gives

$$\Delta E \leq \frac{1}{2}\mu_{\chi N} v^2 \simeq \frac{1}{2} \text{ eV} \times \left(\frac{m_\chi}{\text{MeV}} \right). \quad (2.25)$$

Thus LDM has a much larger possible energy transfer in an electron scattering interaction than for nuclear scattering. Electron scattering is of greater interest to direct detection experiments.

Here I focus on LDM coupled to the SM via a $U(1)_D$ gauge boson A' , i.e. a dark photon mediator. Following the formalism of [47], the LDM–electron scattering cross section can

be parameterized as follows:

$$\bar{\sigma}_e = \frac{16\pi\mu_{\chi e}^2\alpha\varepsilon^2\alpha_D}{(m_{A'}^2 + \alpha^2 m_e^2)^2} \simeq \begin{cases} \frac{16\pi\mu_{\chi e}^2\alpha\varepsilon^2\alpha_D}{m_{A'}^4} & m_{A'} \gg \alpha m_e \\ \frac{16\pi\mu_{\chi e}^2\alpha\varepsilon^2\alpha_D}{(\alpha m_e)^4} & m_{A'} \ll \alpha m_e, \end{cases}$$

where $\mu_{\chi e}$ is the LDM-electron reduced mass, $m_{A'}$ is the mass of the dark photon mediator, ε is the kinetic mixing parameter (the strength of the mediator coupling to the SM), and $\alpha_D \equiv g_D^2/4\pi$ (where g_D is the $U(1)_D$ gauge coupling).

The form factor for the interaction is dependent on the momentum transfer, q , and can be written as

$$F_{DM}(q) = \frac{m_{A'}^2 + \alpha^2 m_e^2}{m_{A'}^2 + q^2} \simeq \begin{cases} 1 & m_{A'} \gg \alpha m_e, q^2 \\ \frac{\alpha^2 m_e^2}{q^2} & m_{A'} \ll \alpha m_e. \end{cases}$$

In the following equations we simplify the energy transferred to the electron ΔE_e to E_e . The differential event rate for this process in a crystal target is

$$\frac{dR}{d\ln E_e} = \frac{\rho_\chi}{m_\chi} N_{cell} \bar{\sigma}_e \alpha \frac{m_e^2}{\mu_{\chi e}^2} \int d\ln q \left(\frac{E_e}{q} \eta(v_{min}(q, E_e)) \right) F_{DM}(q)^2 |f_{\text{crystal}}(q, E_e)|^2, \quad (2.26)$$

where $f_{\text{crystal}}(q, E_e)$ is the dimensionless crystal form factor containing the details of the electronic structure, $N_{cell} = M_{\text{target}}/M_{\text{cell}}$ is the number of unit cells in the crystal target and η is a function encoding the dark matter velocity profile

$$\eta(v_{min}) = \int \frac{d^3v}{v} g_\chi(v) \Theta(v - v_{min}). \quad (2.27)$$

Here, v_{min} is the minimum speed required for a dark matter particle for the electron to gain energy E_e with momentum transfer q , and may be written as

$$v_{min}(q, E_e) = \frac{E_e}{q} + \frac{q}{2m_\chi}. \quad (2.28)$$

The crystal form factor mentioned above is dependent solely on the properties of the

target material. There are publicly available electron recoil spectra which have been numerically calculated with a program called QEdark [47, 48], which also calculates the crystal form factor. These spectra are provided for a range of LDM masses from $0.1 \text{ MeV}/c^2$ to $10 \text{ GeV}/c^2$ and for different forms of $F_{DM}(q)$. The spectra are given in 0.1 eV bins across recoil energies from 0 to 50 eV . The analysis of Chapter 5 utilizes these spectra.

LDM Candidates

There are several well motivated LDM candidates which may be probed experimentally by direct detection dark matter searches identified in the 2017 US Cosmic Visions report [49]. These provide sharp theory targets in the LDM parameter space (typically the $\bar{\sigma}_e - m_\chi$ plane). Here I summarize five identified targets in terms of how they achieve the observed relic abundance and their expected parameter space target under the assumption of a fixed $m_{A'}$. These targets are probed in the analysis of Chapter 5.

The *elastic scalar* scenario considers a complex scalar particle, χ , of mass m_χ and dark photon mediator A' with mass $m_{A'} > 2m_\chi$ [47]. The correct relic abundance is achieved via the freeze-out of the direct annihilation process $\chi + \chi^* \leftrightarrow A'^* \rightarrow \text{SM} + \text{SM}$. The target cross section is a fixed line in the $\sigma_{DD} - m_\chi$ plane [49], where σ_{DD} is the direct detection cross section.

The elastic scalar scenario can be examined again but taking χ to be a Dirac fermion. This *asymmetric fermion* scenario obtains the correct relic abundance through an initial asymmetry, and is therefore an asymmetric dark matter candidate. A symmetric component is also produced by the direct annihilation process $\chi + \chi^* \rightarrow A'^* \leftrightarrow \text{SM} + \text{SM}$, the abundance of which is smaller for large cross sections. This symmetric component can annihilate; if the abundance is too large this distorts the CMB power spectrum and is ruled by CMB constraints. A lower bound is therefore set on the annihilation cross section and the direct detection cross section [47].

Elastically decoupling relics (ELDERs) have their relic abundance set by the decoupling

of the elastic scattering off of SM particles via A' exchange, $\chi + \text{SM} \leftrightarrow \chi + \text{SM}$, rather than annihilation into SM particles like the thermal freeze-out scenario [50]. There are two additional processes of importance: annihilation and the self annihilation process $\chi\chi \leftrightarrow \underbrace{\chi\cdots\chi}_n$, where $n \geq 3$. The $n = 3$ case is considered in Ref. [50]. At high temperatures in the early Universe, the χ is in thermal equilibrium with the SM. As the Universe cools below the dark matter mass the annihilation process decouples, leaving only self annihilation and the elastic scattering processes. In this scenario, the elastic scattering decouples *before* the self annihilation, and energy from self annihilation keeps the dark matter at a roughly constant temperature while the Universe expands. Eventually, this process also decouples. The constant dark matter temperature (and therefore density) during this ‘cannibalization period’ means that the relic abundance is fixed by the dark matter density when the elastic scattering decoupled; therefore, the abundance is fixed by the size of the elastic scattering cross section. This theory predicts a specific line in the $\sigma_{DD} - m_\chi$ plane.

Closely related to the ELDER is the strongly interacting massive particle (SIMP), whose relic abundance is set by the self annihilation process described above, reducing the number of particles in the dark sector [51, 52]. Specifically, $3 \rightarrow 2$ dark matter to dark matter annihilations are considered. The dark matter remains at the same temperature as the SM sector because of the elastic scattering off of SM particles and is in thermal equilibrium at freeze-out. The difference between SIMP and ELDER dark matter is that the self annihilation process is the first to decouple [50]. This process defines a region in the $\sigma_{DD} - m_\chi$ plane with the upper boundary set by the $2 \rightarrow 2$ dark matter to SM thermal relic line (i.e. the elastic scalar line) and the lower bound set by the ELDER line.

The next candidate is a Majorana fermion. This can achieve the correct relic abundance via thermal freeze-out, but the scattering cross section with SM particles is velocity suppressed. The expected line is just the elastic scalar line multiplied by $2(\mu_{\chi,e/n}^2/m_\chi^2)v_\chi^2$, where $\mu_{\chi,e/n}^2$ is the dark matter-electron or dark matter-nucleon reduced mass (depends on the interaction type, here we have only considered electron interactions), and v_χ is the

dark matter halo velocity [49]. This line lies lower in the $\sigma_{DD} - m_\chi$ plane than the other mentioned targets because of the low local dark matter velocity.

The final candidate is produced via *freeze-in*, and is the only candidate identified by Ref. [49] in the $F_{DM}(q) \propto 1/q^2$ regime ($m_{A'} \ll \text{keV}$ scale [53]). There is an initially empty hidden sector which remains thermally decoupled from the SM sector because the dark matter coupling is so small [50] (hence the name feebly interacting massive particle (FIMP) sometimes used to describe this candidate [53]). The hidden sector becomes populated when SM particles annihilate to hidden sector particles in the process $\text{SM} + \text{SM} \rightarrow A'^* \rightarrow \chi\bar{\chi}$. A fixed line in the $\sigma_{DD} - m_\chi$ plane is predicted [54]. The dark matter number density can be determined in the same manner as the freeze-out scenario, though in the case of freeze-in the production process dominates the dynamics, and annihilation can be ignored [55]. This means that the dark matter phase space distribution, which is not known a priori, can be neglected. As with freeze-out, the production process becomes Boltzmann suppressed when the temperature of the Universe drops below the dark matter mass, and the dark matter density ‘freezes in’. Figure 2.6 compares the evolution of the dark matter number density over time for the freeze-in and freeze-out models.

2.2.3 Bosonic Dark Matter

If we want to consider even lower mass dark matter, we find that sub-keV/ c^2 dark matter is constrained to be bosonic. The number density of sub-keV/ c^2 fermionic dark matter has Fermi degeneracy pressure high enough to prevent the formation of galactic substructure [49]. Example bosonic dark matter candidates include dark photons and ALPs.

Axion-like Particles

The axion was originally introduced as a solution to the strong charge-parity (CP) problem, but is also a well-motivated dark matter candidate. Quantum chromodynamics (QCD) is the standard theory of strong interactions, and predicts CP violation proportional to an

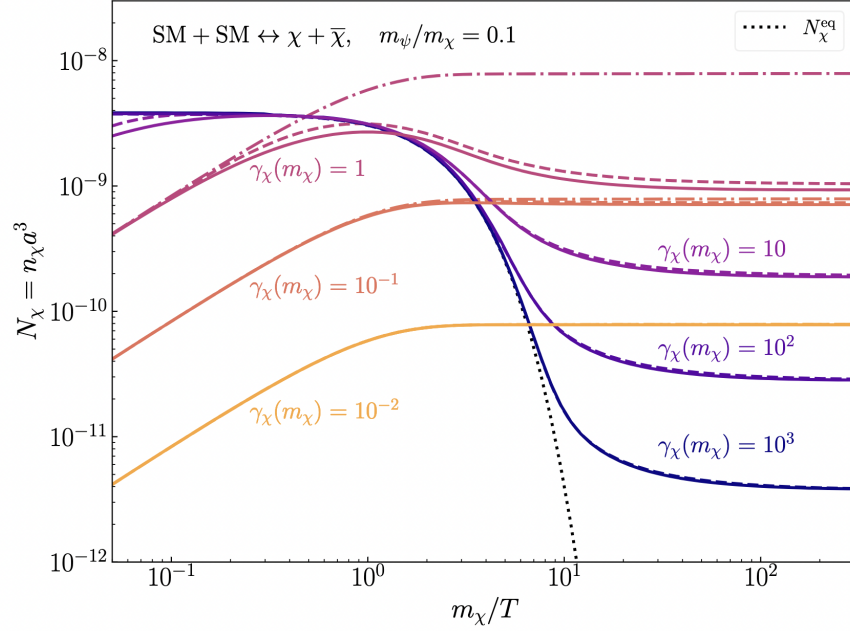


Figure 2.6: The dark matter comoving density for freeze-in and freeze-out models, from Ref. [55]. The interaction strength is characterized by $\gamma_\chi(m_\chi)$. The solid line is a numerical solution to the Boltzmann equation, the dashed line is an approximation assuming the dark matter follows a Boltzmann distribution and is in thermal equilibrium with the SM, and the dash-dotted line for the freeze-in scenario neglects dark matter annihilation. The dotted line indicates the equilibrium abundance.

angle θ . Most notably, this predicts a non-zero electric dipole moment of the neutron, $d_n = 2.4 \times 10^{-16} e \theta \text{ cm}$ [56]. However, this has been experimentally constrained $|d_n| < 1.8 \times 10^{-26} e \text{ cm}$ at 90% C.L. [57], corresponding to an upper bound on θ of $\sim 10^{-10}$. The unexpected smallness of this parameter is known as the *strong CP problem*. In 1977, Pecci and Quinn proposed an elegant mechanism to solve this problem, introducing a new Pecci-Quinn (PQ) symmetry, $U(1)_{PQ}$, which is spontaneously broken at a scale f_a [58, 59]. This symmetry is explicitly broken during the QCD phase transition (the point in the early Universe when quarks and gluons became bound into neutrons and protons, occurring when the Universe was around $1 - 10 \mu\text{s}$ old [60]), and the complex phase of the field develops a potential with minimum at $\theta = 0$. The pseudo Nambu Goldstone boson arising from the

symmetry breaking, the QCD axion [59, 61], is automatically produced in the early Universe by this misalignment mechanism. That is, the initial phase θ_i of $U(1)_{PQ}$ is dynamically driven to 0 during Hubble expansion. The field begins oscillating about its minimum, and the amplitude of these oscillations sets the relic axion density [62].

The mass of the axion is set by and inversely proportional to the scale constant of the PQ symmetry breaking, f_a , and can be written as [61]

$$m_a^2 \approx \frac{f_\pi^2 m_\pi^2}{f_a^2} \frac{m_u m_d}{(m_u + m_d)^2}, \quad (2.29)$$

where f_π is the pion decay constant, m_π is the pion mass, and m_u and m_d are the masses of the up and down quarks, respectively. Because f_a can be very large, axions can be very light, and they are produced in low momentum modes, i.e. are non-relativistic [63]. They are therefore CDM candidates. However, the mass and the coupling strength to SM particles has a fixed relationship.

The simplest QCD axions have mass m_a constrained between 10^{-5} and 10^{-3} eV [64]. The upper limit is set by energy loss via Compton production ($\gamma e \rightarrow ae$) and Primakoff processes in stars, which is proportional to m_a^2 . The axion must therefore be light enough to not affect stellar evolution [65]. The lower mass bound is provided by cosmology: the energy density contained in axion oscillations is proportional to f_a , putting a lower bound on m_a to avoid overclosing the Universe. However, in pre-inflationary scenarios, the mass could be as low as 10^{-12} eV, where the constraint is set by ensuring f_a does not exceed the Planck scale [49].

Axions are weakly coupled to electromagnetism, and searches are typically performed by resonant cavity haloscopes such as the Axion Dark Matter eXperiment (ADMX) [66] and X-ray helioscopes such as the CERN Axion Solar Telescope (CAST) [67], taking advantage of axion-photon couplings. However, the low masses of the axion are out of the range of interest for direct detection experiments and are beyond the scope of this work.

A more general category of particles, axion-like particles (ALPs), do not constrain the

mass and coupling strength. These particles are similarly pseudo Nambu-Goldstone bosons of a new spontaneously broken $U(1)$ symmetry, but the symmetry breaking scale f_a is not related to the ALP mass m_a . The mass range of interest is considerably wider than the axion, and the eV – keV scale ALP parameter space is accessible to experiments searching for WIMPs and LDM.

The interaction mechanism of interest for this work is the axioelectric effect, where an ALP is absorbed by a bound electron in an atom and produces a liberated electron in a process similar to photoelectric absorption of SM photons. The strength of the coupling for this process is parameterized by the axioelectric coupling constant, g_{ae} . The cross-section for this interaction is [68]

$$\sigma_a(E_a) = \sigma_{p.e.}(E_a) \frac{g_{ae}^2}{\beta_a} \frac{3E_a^2}{16\pi\alpha m_e^2 c^4} \left(1 - \frac{\beta_a^{2/3}}{3}\right), \quad (2.30)$$

where E_A and v_a are the total energy and the velocity of the ALP, respectively, $\beta_a = v_a/c$ is the relativistic beta factor, α is the fine structure constant, and g_{ae} is the axioelectric coupling of ALPs to electrons. For non-relativistic ALPs, $E_a = m_a c^2$ and $\beta_a \ll 1$, and Equation 2.30 reduces to

$$\sigma_a(m_a) = \sigma_{p.e.}(m_a c^2) \frac{g_{ae}^2}{\beta_a} \frac{3m_a^2}{16\pi\alpha m_e^2}. \quad (2.31)$$

The interaction rate of ALP absorption, assuming all relic dark matter is composed of non-relativistic ALPs, is

$$R_a(m_a) = \rho_{DM} \sigma_{p.e.}(m_a c^2) \frac{3c}{16\pi\alpha m_e^2} g_{ae}^2 m_a. \quad (2.32)$$

Equation 2.32 can be used to set limits on g_{ae} as a function of ALP mass, m_a , as seen in Chapters 4 and 5. The expected signal for a given mass is a Dirac delta function at the mass-equivalent energy $m_a c^2$, smeared by the energy resolution of the detector.

Dark Photons

The dark photon is the hypothetical massive vector boson of a new dark $U(1)_D$ gauge group under which all SM particles are uncharged. This new dark, or hidden, photon can mix with the SM hypercharge $U(1)$ gauge group [69]. For masses of the dark photon significantly below the mass of the W boson and at energies well below the weak scale, the dominant mixing channel is between the dark photon and the SM photon, as seen in the addition to the SM Lagrangian [70]

$$\mathcal{L} \supset -\frac{1}{4}F'^{\mu\nu}F'_{\mu\nu} - \frac{\varepsilon}{2}F^{\mu\nu}F'_{\mu\nu} + \frac{1}{2}m_{A'}^2 A'^\mu A'_\mu, \quad (2.33)$$

where ε is the kinetic mixing parameter which we have already seen in our discussion of LDM, $m_{A'}$ is the dark photon mass, A' and $F'^{\mu\nu}$ are the new dark photon field and field strength tensor, respectively, and $F^{\mu\nu}$ is the field strength of the SM photon.

For $m_{A'}$ less than twice the electron mass and ε sufficiently small, the lifetime of the dark photon can be longer than the age of the Universe, making the dark photon a viable candidate to constitute all relic dark matter [70]. Heavier dark photons would decay into electron-positron pairs and be unable to produce the correct relic density. Like the ALP, the dark photon may interact with electrons in an absorption process comparable to the photoelectric absorption of the SM photon. Here I follow the models presented in Refs. [70, 71].

The interaction cross-section for dark absorption of the dark photon can be written in terms of the photon electric cross section

$$\sigma_{A'}(E_{A'}) = \frac{\varepsilon_{\text{eff}}^2}{v_{A'}} \sigma_{p.e.}(E_{A'}) n \hbar c, \quad (2.34)$$

where $E_{A'}$ and $v_{A'}$ are the total energy and the velocity of the dark photon, respectively, ε_{eff} is the effective kinetic mixing parameter, and n is the index of refraction.

The actual parameter of interest, ε , can be derived from ε_{eff} by accounting for in-medium

effects:

$$\varepsilon_{\text{eff}}^2 = \frac{\varepsilon^2 m_{A'}^2}{(m_{A'}^2 - 2m_{A'}\sigma_2 + \sigma_2^2 + \sigma_1^2)}, \quad (2.35)$$

where σ_1 and σ_2 are the real and complex parts of the complex conductivity, respectively. The real part of the complex conductivity is related to the photoelectric cross section:

$$\sigma_1(m_{A'}) = n \cdot \sigma_{p.e.}(m_{A'}) \cdot \rho \cdot \hbar c. \quad (2.36)$$

Here, ρ is the density of the material. In-medium effects impact ε only at low masses; above $\sim 20 \text{ eV}/c^2$, ε_{eff} approximates the kinetic mixing parameter.

For non-relativistic CDM, the mass-equivalent energy of the dark photon is approximately equal to the total energy, or $m_{A'}c^2 \sim E_{A'}$. The event rate of dark photon absorption, assuming that all relic dark matter is non-relativistic dark photons, is then given by

$$R_{A'}(m_{A'}) = \frac{\rho_{DM}}{m_{A'}} \varepsilon_{\text{eff}}^2 \sigma_{p.e.}(m_{A'}c^2) n \hbar c. \quad (2.37)$$

Equation 2.37 can be used to set limits on the kinetic mixing ε as a function of dark photon mass, $m_{A'}$, as seen in Chapters 4 and 5. The expected signal for a given mass is a Dirac delta function at the mass-equivalent energy $m_{A'}c^2$, smeared by the energy resolution of the detector.

Dark Compton Scattering of Bosonic Dark Matter

There is a process other than the absorption channel, typically neglected in absorption searches, where the dark matter undergoes a Compton-like process, interacting with electrons and emitting a SM photon in the final state. Figure 2.7 shows tree-level Feynman diagrams for this process.

Here I follow the formalism of Ref. [72], considering a dark photon of mass m_V and energy ω . In this formalism, we will use units of $c = 1$ to simplify the equations; here masses generally represent mass-equivalent-energies. The energy of the emitted SM photon

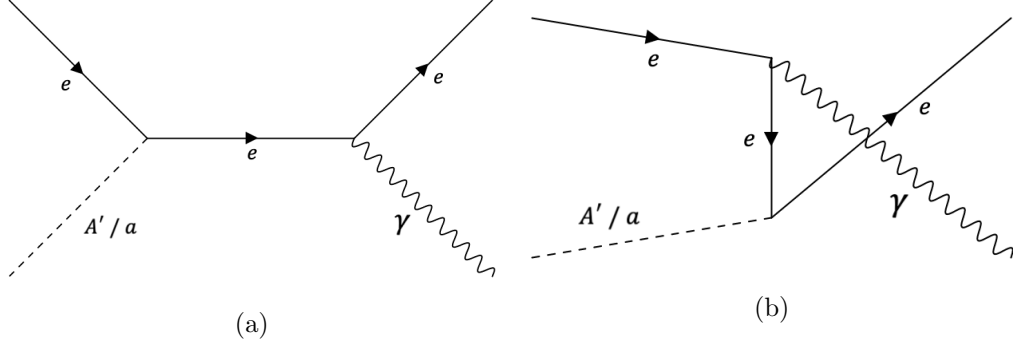


Figure 2.7: Tree-level Feynman diagrams of dark Compton scattering, where a dark boson is converted to a SM photon via electron scattering.

following a dark Compton interaction is

$$\omega' = \frac{m_{A'}^2 + 2m_e\omega}{2 \left(m_e + \omega - \sqrt{\omega^2 - m_{A'}^2} \cos \theta \right)}, \quad (2.38)$$

where θ is the scattering angle. For slow moving dark matter, $\omega \approx m_{A'}$, and the outgoing SM photon energy is fixed, as is the electron recoil energy:

$$E_{recoil} = \omega - \omega' = \frac{m_V^2}{2(m_V + m_e)}. \quad (2.39)$$

The scattering rate is related to the kinetic mixing parameter,

$$R_{C,A'}(m_{A'}) = \frac{n_e}{\rho_T} \frac{\rho_{DM}}{m_{A'}} \frac{e^4 \varepsilon^2}{24\pi m_e^2} \frac{(m_{A'} + 2m_e)(m_{A'}^2 + 2m_{A'}m_e + m_e^2)}{(m_{A'} + m_e)^3}, \quad (2.40)$$

where n_e and ρ_T are the electron number density and the mass density, respectively, of the target material. The same formalism holds true for ALPs. In this case, consider an ALP of mass m_a . The dark Compton scattering rate is related to the axioelectric coupling constant by

$$R_{C,a}(m_a) = \frac{n_e \rho_{DM}}{\rho_T} \frac{e^2 g_{ae}^2 m_a (m_a + 2m_e)^2}{16\pi m_e^2 (m_a + m_e)^4}. \quad (2.41)$$

For both dark photons and ALPs, the electron recoil energy spectrum is a delta function at energy

$$T_C = \frac{m_{A'/a}^2}{2(m_{A'/a} + m_e)}. \quad (2.42)$$

The actual signal measured by the detector is dependent on what happens to the produced photon. In a very large detector, the initial dark Compton interaction will be followed by secondary interactions of the photon with the detector material. The photon is fully absorbed by the detector, and the total energy deposited in the detector is the initial dark boson energy, $T_A = m_{DM}$. In contrast, for a very thin detector, the photon will most likely escape the detector without interacting, and only the initial electron recoil, T_C , contributes to the total energy deposited in the detector.

For intermediate sized detectors, the photon may undergo secondary interactions but still be able to escape without depositing all of its energy. The total energy deposited is somewhere in between T_C and T_A . Figure 2.8 illustrates the behaviour of the SM photon for the different detector sizes.

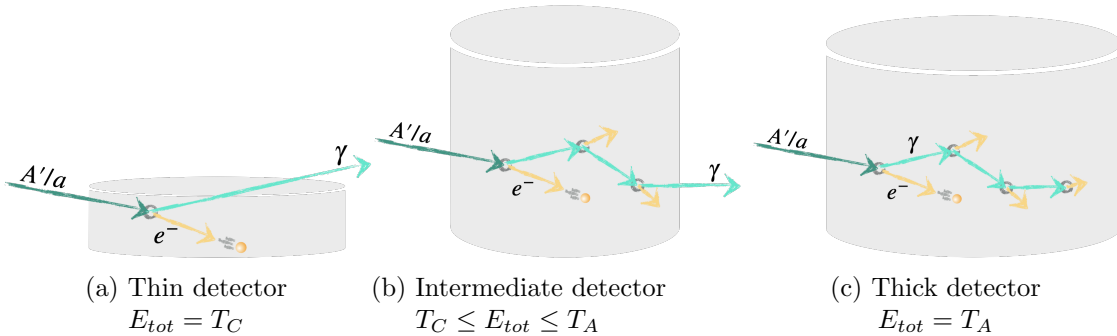


Figure 2.8: Illustrations of the behaviour of the produced SM photon in a dark Compton interaction for detectors of varying sizes. E_{tot} is the total energy deposited in the detector. Figure provided by B. von Krosigk.

The dark Compton scattering rate is most relevant compared to the absorption channel at higher dark matter masses when the photoelectric cross section decreases. The rates

become comparable around $100 \text{ keV}/c^2$ and at higher masses the dark Compton process begins to dominate. Thus for low mass searches the inclusion of dark Compton scattering provides no benefit.

2.2.4 Other Candidates

This work by no means provides a comprehensive list of all possible dark matter candidates, but we will touch briefly on a few additional models of interest. Sterile neutrinos are theorized massive fermions with right handed chirality, called sterile because they do not participate in the electroweak force [73]. They do, however, mix with the SM neutrinos and can radiatively decay to a neutrino plus a photon [74]. Sterile neutrinos in the keV to MeV mass range are natural dark matter candidates. They are non-thermal relics; they can never have been in equilibrium otherwise they would overclose the Universe, thus their interaction is suppressed. Sterile neutrino dark matter would be indirectly detected by their X-ray emission products, but are heavily constrained [75].

MAssive Compact Halo Objects (MACHOs) represent a class of non-luminous baryonic dark matter candidates including black holes, red, brown, and white dwarf stars, and planets. Microlensing measurements of the Large Magellanic Cloud from the EROS-2 survey constrain the fraction of the halo mass in MACHOs to be less than 8% based on a single observed candidate event [76]. Additionally, constraints from the CMB power spectrum require that the majority of dark matter is non-baryonic, thus MACHOs can only account for a small fraction of the dark matter, unless they are primordial black holes (PBHs).

PBHs are an example of non-particle dark matter. These theoretical objects are produced in the early Universe before baryogenesis, and are a viable non-baryonic CDM candidate, though their production requires physics beyond the SM [77]. For example, they could be the result of density perturbations in the early Universe. Recent gravitational wave measurements by Laser Interferometer Gravitational-Wave Observatory (LIGO) and Virgo of binary black hole mergers has sparked renewed interest in PBHs as dark matter

candidates [78, 79].

2.3 Dark Matter Detection

Despite the plethora of evidence for the presence of dark matter and decades of dedicated dark matter searches, we have yet to make an actual discovery. Dark matter experiments are typically grouped into one of three different search approaches: direct detection, indirect detection, and production. These different approaches provide competition and complementarity; a discovery by one channel should be corroborated by the others.

2.3.1 Direct Detection

Direct detection experiments attempt to directly measure interactions of dark matter with a SM target material as the Earth moves through a flux of dark matter from the local halo. These experiments look for small (usually keV scale and below) energy depositions occurring at a low rate. The rarity of these dark matter/SM interactions necessitates minimizing backgrounds and maximizing exposures. For several decades, the majority of the experimental efforts were focused on WIMP-nucleon interactions. However, other interaction channels have gained attention in the last few years, including couplings to electrons. Other dark matter models are also receiving more attention in the absence of a WIMP discovery.

Experiments searching for these dark matter-SM matter interactions are diverse, though most fall into specific categories in terms of the types of evidence of particle interactions that they are sensitive to. These channels include heat (in the form of phonons, the quanta of vibration), charge, or light. Many experiments utilize a combination of these signal types to improve their sensitivity or discriminate recoil type. Refs. [62, 80] provide recent reviews of the status of direct detection searches.

Cryogenic Solid State Detectors

Cryogenic solid state detectors measure heat in the form of phonons from particle interactions in the crystalline target material. The SuperCDMS experiment uses Ge and Si detectors instrumented by W-Transition Edge Sensors (TESs) read out by Superconducting QUantum Interference Device (SQUID) amplifiers for phonon energy measurement. A high voltage mode was also developed to achieve even lower energy thresholds, gaining sensitivity to $< 10 \text{ GeV}/c^2$ WIMP masses [81]. The previous generation of the experiment operated in Soudan, MN, and the next phase is under construction at SNOLAB. I describe the experiment in significantly more detail in later sections as the subject of this thesis.

EDELWEISS is an experiment operating at Laboratoire Souterrain de Modane (LSM) in France using high purity Ge semiconductor detectors. The surfaces are instrumented with electrodes for ionization measurement and neutron transmutation doped (NTD) Ge sensors for phonon measurement. Nuclear recoil (NR)/electron recoil (ER) discrimination is achieved through the ratio of the phonon to charge signals. The EDELWEISS-III array consists of 24 cylindrical detectors ranging from 820 to 890 g in mass. They set competitive limits on SI nucleon-coupled WIMP dark matter in the $4\text{--}30 \text{ GeV}/c^2$ mass range [82]. The same array was used to set constraints for ALP and dark photon dark matter with masses from 0.8 to $500 \text{ keV}/c^2$ [83]. Another ERDM search was performed with a small 33.4 g detector with 0.53 electron-hole pair resolution, setting the most stringent limits for dark photons in the mass range from 6 to $9 \text{ eV}/c^2$ and competitive constraints for LDM [84].

The Cryogenic Rare Event Search using Superconducting Thermometers (CRESST) at Laboratori Nazionali del Gran Sasso (LNGS) uses scintillating CaWO_4 crystals with silicon-sapphire wafers to measure the scintillation light. The crystal and light detectors are instrumented with W-TESs. Discrimination between interaction types is possible from the ratio of the energy measured from each signal type. These detectors are sensitive to lower mass WIMPs because the light oxygen atoms have relatively favourable scattering kinematics compared to Si or Ge. The current iteration, CRESST-III, has achieved thresholds as

low as 30.1 eV for nuclear recoils with a 23.6 g detector, and is currently leading the low mass ($< 2 \text{ GeV}/c^2$) SI WIMP-nucleon coupling parameter space, achieving sensitivity to WIMP masses as low as $160 \text{ MeV}/c^2$ [85].

The China Dark matter EXperiment (CDEX) experiment, located at China Jinping Underground Laboratory (CJPL), uses p-type point contact Ge detectors. Sub-keV thresholds are achievable due to the low capacitance combined with large detector mass. CDEX-10 operated a 10 kg detector array immersed in liquid nitrogen, setting constraints on both SI and SD couplings [86]. They achieved an analysis threshold of 160 eV_{ee} (electron-equivalent, an energy scale based on assuming the interaction is an ER) and excluded new parameter space for $4\text{--}5 \text{ GeV}/c^2$ mass WIMPs for SI nucleon couplings. Data from two of the detectors were used to set constraints on dark photon dark matter for masses from 0.1 to $4.0 \text{ keV}/c^2$ in Ref. [87]. Similar detectors are used by the Coherent Germanium Neutrino Technology (CoGeNT) and the Taiwan EXperiment On Neutrino (TEXONO) [88] collaborations. The CoGeNT Collaboration observed a low energy excess [89] during operation at the SUL, which was investigated as a potential dark matter signal [90]. However, the potential dark matter discovery window has since been excluded by several experiments.

Noble Liquid Detectors

Noble liquid detectors measure scintillation from particle interactions in the liquid target, which produce heat, excited atoms, and ionized atoms. The excited states combine with neutral atoms, forming excimer states which decay, emitting UV light. Xenon and argon are both excellent scintillators, emitting light with wavelengths of 178 nm and 128 nm, respectively [91]. Typically, the emitted light is shifted to optical wavelengths by a wavelength shifter, though in LXe the photons can be directly measured using sensors with quartz windows, which are transparent to the LXe scintillation light [92]. Xenon experiments currently provide the strongest constraints on SI WIMP-nucleon couplings in the canonical WIMP mass range. Xenon has an average energy required to produce an electron-ion pair

of 15.6 eV, and the high atomic number $A = 131.3$ means that Xe is highly sensitive to SI interactions. Additionally, the prevalent isotopes ^{129}Xe and ^{131}Xe have a single unpaired neutron, providing sensitivity to SD couplings. The other noble liquids are candidates for future dark matter searches. While krypton has too high intrinsic backgrounds from long lived radioactive isotopes [80], the low atomic numbers of neon and helium make them extremely promising targets for low mass dark matter searches.

There are two main detector designs for noble liquid detectors. Single phase liquid noble experiments measure only the scintillation signal usually using a photo-electron multiplier tube (PMT) array surrounding the target liquid. Particle-type discrimination is possible from the pulse shape due to differences in the scintillation photon time distribution [93]. The excimer states from interactions have characteristic lifetimes, and are produced in different amplitudes depending on the nature of the interaction. This leads to a ‘prompt’ signal and a ‘late’ signal, the ratio of which can be used to discriminate recoil type. Dual phase Time Projection Chambers (TPCs) have a cylindrical target volume and utilize a PMT array below the liquid portion of the detector and another in the gaseous phase above [80]. The primary scintillation signal (termed S1) is measured by both arrays. An electric field drifts the liberated electrons, pulling them into the gas phase where they produce a secondary scintillation signal (S2) from collisions with the gas atoms. This signal is proportional to the number of electrons and is also measured by the PMT arrays. The simultaneous measurement of both signals allows for position reconstruction, and the ratio of the S1/S2 signals is used to discriminate ERs and NRs.

Notable LXe TPC experiments include the XENON collaboration, with XENON10 (15 kg LXe) [94], XENON100 (62 kg LXe) [95], and XENON1T (1995 kg LXe) at LNGS. Constraints on the WIMP-nucleon SI coupling from XENON1T are currently some of the most stringent constraints for masses larger than $6 \text{ GeV}/c^2$ [96]. XENONnT is an upgrade of XENON1T with 5.9 tonnes of LXe currently undergoing commissioning at LNGS; with a goal exposure of 20 t.y, the expected SI interaction sensitivity is $1.4 \times 10^{-48} \text{ cm}^2$ for a

50 GeV/ c^2 WIMP at 90% C.L [97]. Another LXe experiment of note is the LUX-ZEPLIN (LZ) experiment (7 t, 5.5 t fiducial volume) [98], which is the merger of two previous collaborations, Large Underground Xenon experiment (LUX) and ZEPLIN. Constraints on the SI WIMP-nucleon coupling set with 60 live days of data are the most stringent limits to date for masses above 9 GeV/ c^2 , at $6.5 \times 10^{-48} \text{ cm}^2$ for a 30 GeV/ c^2 WIMP mass. PandaX-II, located at CJPL, has a 580 kg sensitive LXe volume and has set competitive constraints on SI WIMP-nucleon interactions [99], LDM [100], and ALPs [101]. The next generation experiment DARWIN is currently in the R&D stage with a planned 50 t LXe volume (40 t active volume) [102].

LAr experiments include the single phase experiment DEAP-3600 (3200 kg LAr) at SNO-LAB [103] and the dual phase TPC DarkSide-50 at LNGS [104], which performed an analysis using the ionization signal only to achieve sensitivity to lower mass WIMPs, setting constraints surpassing even solid state detectors down to 1.8 GeV/ c^2 [105]. The next generation project DarkSide-20k (49.7 t active volume) will be a two phase LAr detector at LNGS [62].

Superheated Liquid Detectors

Superheated bubble chambers have been demonstrated to be sensitive to both spin independent and spin dependent dark matter interactions. The target material is a fluid kept in a superheated state. Energy depositions above a certain threshold will form bubbles, the position and number of which are recorded with stereoscopic camera readout. These detectors can be operated with a variety of different target fluids, though most contain the isotope ^{19}F . The unpaired proton of this isotope means that this is an excellent target for spin-dependent dark matter interactions.

The PICO experiment was formed as a merger of two previous experiments: the Project In Canada to Search for Supersymmetric Objects, or Projet d'Identification de CAndidats Supersymtriques SOmbres (PICASSO) and the Chicagoland Observatory for Underground Particle Physics (COUPP). COUPP operated at FermiLab from 2011-2012, using the same

bubble chamber they operated at SNOLAB from 2010-2011 [106]. PICASSO achieved an exposure of 231.4 kg d using C_4F_{10} and set constraints on SI and SD WIMP interactions [107]. PICO-60 achieved an energy threshold of 2.45 keV with a 1404 kg day exposure of 52 kg of C_3F_8 and set the strongest constraints on the WIMP-proton spin-dependent cross section at $3.2 \times 10^{-41} \text{ cm}^2$ for a 25 GeV/ c^2 WIMP mass [108]. The next generation, PICO-500, is currently under construction at SNOLAB. The Superheated Instrument for Massive ParticLe Experiments (SIMPLE) at Laboratoire Souterrain Bas Bruit (LSBB) in France uses superheated droplet detectors, where each droplet acts as as a mini bubble chamber [109]. This is the same technique used by PICASSO.

Ionization-based Detectors

New Experiments With Spheres-Gas (NEWS-G) operates room temperature spherical proportional counters with light noble gases to search for low mass WIMPs. These detectors have low noise and high gain, achieving low energy thresholds of 10–40 eV_{ee} (electron equivalent recoil energy) and single electron sensitivity. A 60 cm detector was operated at LSM with a mixture of Ne + CH_4 (0.7%), excluding cross sections above $4.4 \times 10^{-37} \text{ cm}^2$ at 90% CL for a 0.5 GeV/ c^2 mass WIMP [110]. A larger 140 cm detector has been constructed and is now running at SNOLAB [111].

Charge-coupled devices (CCDs) are detectors which detect ionization over a pixel array. They are well optimized for low mass dark matter searches, and are able to reconstruct the position of energy depositions in 3D space, as well as the particle type. The Dark Matter in CCDs (DAMIC) experiment at SNOLAB uses Si CCDs, setting constraints on low mass WIMPs [112] and hidden photon dark matter [113]. Dark Matter in CCDs at Modane (DAMIC-M) will use a novel skipper readout to achieve single electron resolution; a prototype is already operating at Modane [114]. The Sub-Electron-Noise Skipper-CCD Experimental Instrument (SENSEI) at Fermilab also uses Si skipper CCDs, and has achieved world leading sensitivity to electron-coupled dark matter including LDM, dark photons, and

ALPs [115] with a 2 g CCD. A larger 13 g version of the experiment is running at SNOLAB, with planned upgrades to 100 g.

NaI Scintillators and Annual Modulation

As discussed in Section 5.4.4, the WIMP interaction event rate is dependent on the Earth's velocity with respect to the local dark matter halo; a consequence is that the Earth's motion around the Sun introduces a seasonal variation in the total dark matter event rate. This annual modulation has a one year period, with a maximum rate at June 2. The expected effect is an amplitude modulation on the order of $\sim 5\%$ [32]. A rate modulation on a one year timescale would help to confirm a dark matter detection.

The DAMA/LIBRA experiment at LNGS operates highly radiopure NaI(Tl) crystal detectors, measuring the scintillation light from particle interactions with PMTs. In an annual modulation search of DAMA/LIBRA-phase2 they claimed to have observed a signal consisted with dark matter at 9.5σ C.L. in the 2–6 keV range [116]. Figure 2.9 shows the measured modulation with best fit curve, including data from DAMA/LIBRA-phase1 [117].

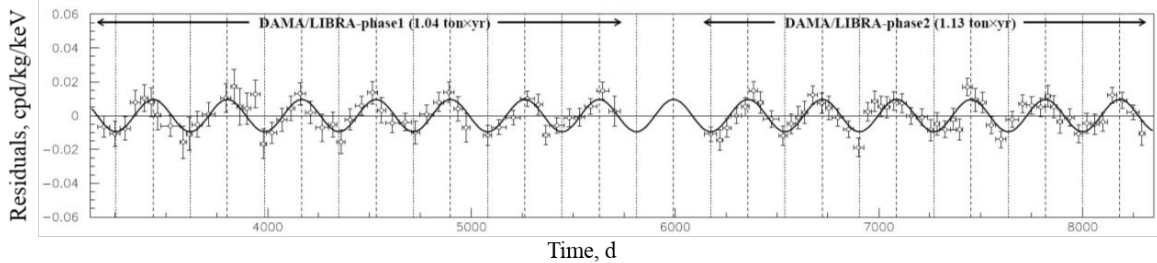


Figure 2.9: The residual rate measured by DAMA/LIBRA phase 1 and phase 2 as a function of time. The superimposed curve is of the form $A \cos(\omega(t - t_0))$ with period $T = 2\pi/\omega = 1$ y and phase $t_0 = 152.5$ d (corresponds to June 2). The modulation amplitude, A , is from the best fit to data from both phase 1 and phase 2. From Ref. [116].

Similar experiments such as COSINE-100, which also uses NaI(Tl) detectors, have been unable to reproduce the annual modulation result [118]. Explanations have been proposed such as contamination of the nitrogen purge gas with radioactive argon, giving a seasonal

variation of 2.8 keV X-rays [119]. More recently, it has been suggested that the effect may be statistical: an analysis of COSINE-100 data using the DAMA/LIBRA analysis techniques found an induced modulation signal [120].

The Current WIMP Dark Matter Landscape

The current landscape of direct detection experiments for SI nucleon-coupled WIMP parameter space is shown in Figure 2.10. The liquid noble detectors can be seen to dominate at high mass, while solid state detectors provide sensitivity at low masses. The next generation experiments are focusing on two directions: the push down to the ‘neutrino floor’, the point at which dark matter becomes hidden by the irreducible neutrino background, at high mass, and pushing to lower masses.

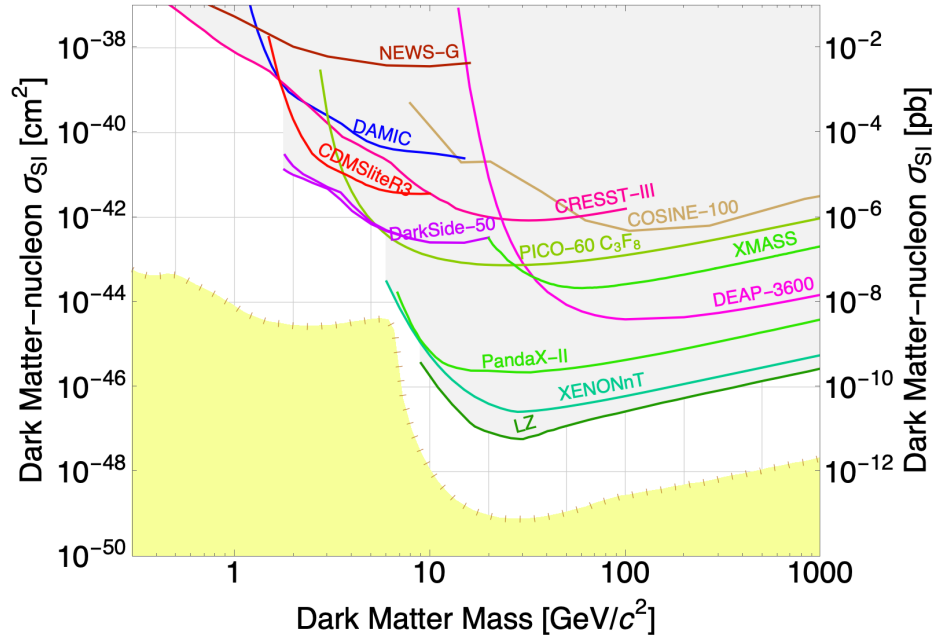


Figure 2.10: Current state of the SI WIMP-nucleon interaction parameter space for direct detection experiments [85, 99, 103, 108, 110, 112, 121–126]. The yellow shaded region represents the neutrino floor for a Xe target [127]. Figure produced using v5.18 of the publicly available SuperCDMS limit plotter [128].

2.3.2 Indirect Detection

Indirect detection searches look for products of dark matter self annihilation or decay, which are generally expected to be high energy gammas, neutrinos, or SM particle/antiparticle pairs. Experiments often look for an excess of these decay products around massive objects such as stars or black holes, where the high dark matter density increases the probability of self annihilation. Discrepancies between the measured flux and expectation based on astrophysical backgrounds are a potential dark matter signature. A signature of note would be decay into photons, producing a spectral line. The Fermi Gamma-ray Space Telescope (FGST) has searched for gamma ray [9] and X-ray [10] excesses, while the Alpha Magnetic Spectrometer (AMS-02) telescope on the International Space Station (ISS) currently leads hadronic [11] and leptonic [12] channels. The IceCube experiment has set constraints using searches for neutrino excesses from dark matter annihilation and decay in the Galactic Centre [129].

2.3.3 Production

The final method of dark matter detection is by directly producing it. The high energy interactions of colliders produce a variety of particle products, among which could be dark matter. Due to the negligible interactions of dark matter with SM particles, the dark matter products would not be directly detectable; instead, experiments look for missing mass and momentum among the collision products. Constraints are set on the dark matter mass and coupling cross sections to allow for comparison in a model independent manner with direct detection experiments. At the Large Hadron Collider (LHC), the Compact Muon Solenoid (CMS) and ATLAS experiments have set constraints examining the products of proton-proton collisions [13]. The Belle II experiment has similarly looked for evidence of dark matter using the SuperKEKB electron-positron collider [14].

Fixed target production with beam dump experiments is also possible, where particle beams are directed onto dense target materials. dark matter produced in the resulting

interactions travels downstream to a detector where it has a chance to interact and be directly detected. The MiniBooNE experiment at Fermi National Accelerator Laboratory (FNAL) is a neutrino detector downstream of a proton beam dump and has published constraints on dark matter-nucleon couplings [15].

2.4 SuperCDMS

The SuperCDMS experiment and its predecessor, the Cryogenic Dark Matter Search (CDMS), are direct detection dark matter experiments using cryogenic semiconductor detectors to search for evidence of dark matter interactions with SM matter. This section will provide a brief history of CDMS and the previous iteration of SuperCDMS at Soudan, as well as an overview of the future generation of SuperCDMS under construction at SNOLAB. The SuperCDMS detector technology will be covered in detail in Chapter 3.

2.4.1 The Origins of CDMS

The CDMS experiment laid the groundwork for what would eventually become the modern SuperCDMS experiment. This experiment operated in a shallow underground site at the Stanford Underground Facility (SUF) at Stanford University in 1998 [130], where neutrons produced by cosmic ray muons were a limiting background.

There were two main detector types: the Berkeley Large Ionization- and Phonon-mediated (BLIP) detector and the Z-sensitive Ionization and Phonon (ZIP) detector. BLIPs measured phonon signals via calorimetric temperature change, while ZIPs measured athermal phonons for position information. Data from three Ge BLIP detectors are in tension with the DAMA 3σ allowed region at 75% confidence level (CL) [130], though the ZIP would eventually become the mainstay of CDMS (and eventually SuperCDMS) detector technology. The ratio of the simultaneous measurements of phonon and charge signals from particle interactions provides discrimination between interaction types. With detector improvements, superior neutron shielding, and a longer exposure time, CDMS used 6 new ZIP

detectors (4 Ge, 2 Si) to exclude more parameter space for WIMPs with masses between 8 and $20 \text{ GeV}/c^2$ [131].

With the proof of principle in hand and additional funding secured, the next generation of the experiment, CDMS-II, moved to a new site much further underground [132]. CDMS-II operated at the SUL in Minnesota, USA from 2003 to 2008. Located in an old iron mine, SUL provided a 780 m (2090 m water equivalent) overburden, reducing the muon flux by a factor of 5×10^4 . Important aspects of the experiment included shielding by the rock overburden, a dilution refrigerator, an active muon veto detector, passive shielding, a μ -metal (a soft magnetic alloy with very high magnetic permeability) shield, copper cryostat cans, Ge and Si detectors, and a data acquisition system.

Initially, the payload consisted of the same six ZIP detectors previously operated at the SUF shallow site, though this was eventually upgraded to 19 Ge ($\sim 239 \text{ g}$ each) and 11 Si ($\sim 106 \text{ g}$ each) ZIPs [133]. The detectors were operated at 50 mK with a cryogenic system similar to the original CDMS experiment.

World-leading constraints on nucleon-coupled SI WIMP dark matter were set using data from 14 of the Ge detectors, giving an upper limit of $3.8 \times 10^{-44} \text{ cm}^2$ for a WIMP of mass $70 \text{ GeV}/c^2$ [134]. Two dark matter searches were presented for data from the Si detectors; the first was set on 55.9 kg days of data collected from 2006 to 2007 and observed no excess over the expected background of 1.1 events. An upper limit of $1.7 \times 10^{-41} \text{ cm}^2$ for a $10 \text{ GeV}/c^2$ mass WIMP [135]. However, in the second search, a blind analysis of 140.2 kg days of data taken from 2007 to 2008, three candidate events were observed at recoil energies of 8.2, 9.5, and 12.3 keV [133]. These events represented a statistically significant excess over the expected backgrounds of < 0.13 neutron events, $0.41^{+0.2}_{-0.08}(\text{stat.})^{+0.28}_{-0.24}(\text{syst.})$ misidentified surface ERs, and < 0.08 events from ^{206}Pb recoils from ^{210}Po decays. If interpreted as a dark matter signal, the excess is most consistent with a WIMP mass of $8.6 \text{ GeV}/c^2$ and WIMP–nucleon cross section of $1.9 \times 10^{-41} \text{ cm}^2$. This potential discovery was in tension with results from XENON10 [136, 137] and XENON100 [138] at the time of publication and

was later excluded by multiple experiments, including the next generation of CDMS.

2.4.2 SuperCDMS Soudan

SuperCDMS, the next generation of CDMS, also operated at SUL [139]. The CDMS-II low background apparatus of a cryostat surrounded by a passive shield and outer muon veto was reused, but SuperCDMS boasted updated detector designs called interleaved Z-sensitive Ionization and Phonon (iZIP), operating 15 Ge detectors with a ~ 10 kg payload. The reduction from 30 to 15 detectors represents a change in the readout electronics – the new detectors have twice as many channels. The detectors were arranged into five stacks, or “towers” of three detectors.

Surface events in the CDMS detectors are prone to suffering from reduced ionization collection [140], resulting in a reduced yield. This was problematic for ERs in particular as these events may mimic the signature of nuclear recoils, the signal region for WIMP searches. The detectors were upgraded over the original ZIP design for better volume fiducialization, allowing surface events to be discriminated. Figure 2.11 uses data collected by an iZIP detector to demonstrate how the charge measurement can be used to reject surface events, and also demonstrates how the phonon and ionization measurements can be used to separate NR and ER events, as well as the importance of energy resolution for low-energy yield discrimination.

An additional mode of detector operation was also explored, referred to as the CDMS low ionization threshold experiment (CDMSlite) [81]. In this mode, the iZIP detectors were operated at a significantly higher bias voltage, dramatically reducing the energy threshold at the cost of recoil type discrimination.

Periods of stable data taking are referred to as *runs*; these are generally split up by interruptions in the cryogenics, such as for routine maintenance. The run numbering for SuperCDMS continued from the days of CDMS, with Run 133 being the first science run of SuperCDMS Soudan, occurring from early 2012 to mid 2013. Run 134 started in mid

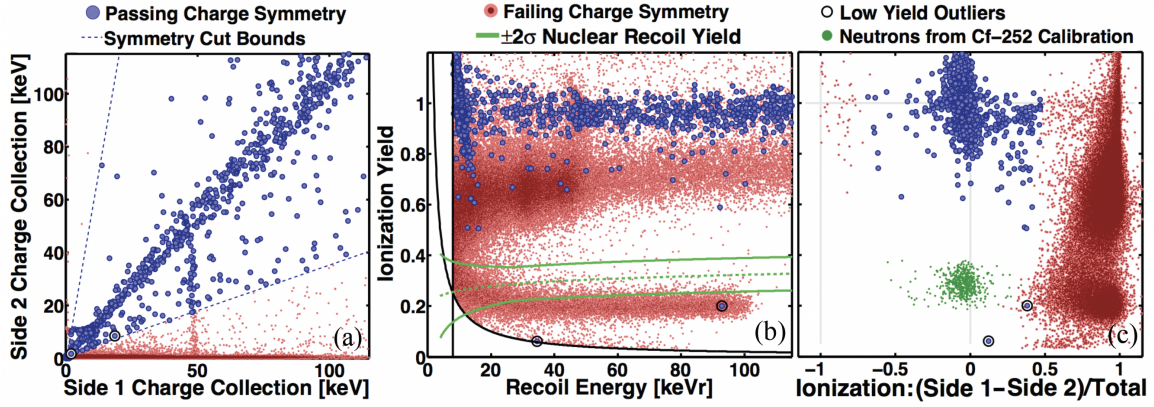


Figure 2.11: Data from an iZIP detector with a ^{210}Pb source facing Side 1, from Ref. [140]. (a) Charge collected on Side 2 vs. Side 1 of the detector. The dashed lines indicate cut lines where events with asymmetric signal (red) are removed. (b) Ionization yield vs. recoil energy. The colour scheme is the same as the previous figure. The green band indicates the region into which 95% of the bulk nuclear recoils should fall (referred to as the $\pm 2\sigma$ nuclear recoil band) determined from neutron measurement. Note the lowered yield in both the ER and NR band for surface events (which fail the charge symmetry cut). One can also observe that the larger the ionization and phonon resolution, the more difficult it is to resolve the ER and NR bands. (c) Yield vs. normalized difference in collected charge between Side 1 and Side 2. The data from (a) and (b) are shown in the plot together with NRs from a ^{252}Cf source (green). Bulk events show a symmetric ionization response. Bulk ERs, NRs, and surface events can all be separated. The two circled events in each panel are low yield outliers.

2013 and ended in mid 2014, and the final run before decommissioning was Run 135, which ran from late 2014 to mid 2015. There were three runs¹ in the CDMSlite operational mode which occurred during Runs 133 to 135, named Runs 1, 2 and 3. CDMSlite Run 1 occurred during Run 133 of SuperCDMS Soudan, Run 2 during R134–135, and Run 3 during R135. The individual blocks of data taken during runs are referred to as *series*.

Shielding

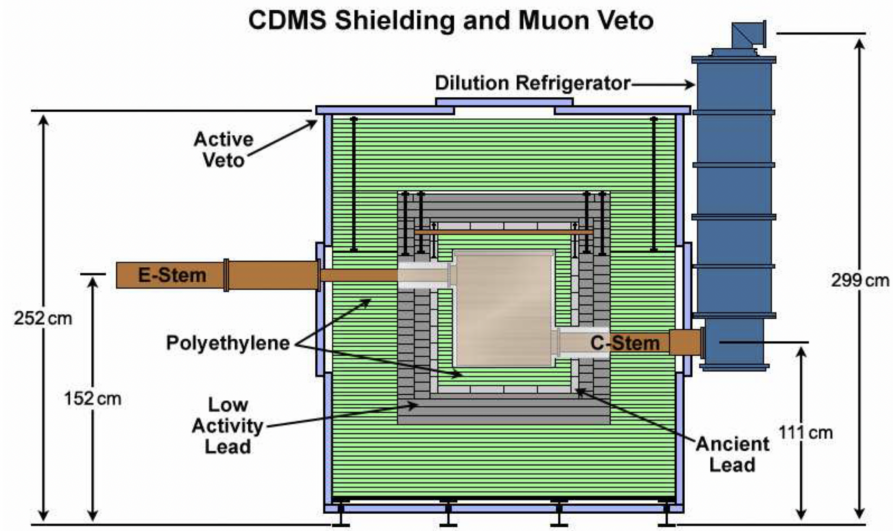
The SuperCDMS shielding, consisting of multiple layers of active and passive shielding to reduce the remaining background from cosmogenic radiation as well as radiogenic neutrons

¹In this case *run* does not refer to a period of stable data taking, but instead a particular detector setup.

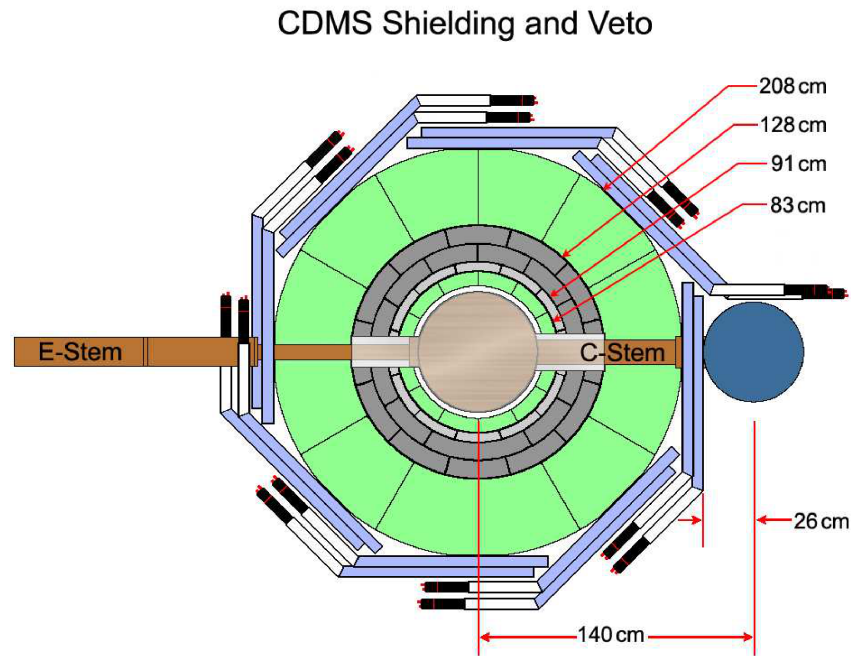
and gammas, was the same as for CDMS-II [132]. Figure 2.12 shows the various layers of shielding and the components of the cryostat.

The outermost layer of shielding consisted of an active veto constructed from 40 overlapping scintillator panels to prevent line of sight of muons to the detectors. This layer was 5 cm thick and able to tag muons with 99.7% efficiency. The active shielding completely surrounded the layers of passive shielding and the internal cold volume containing the detectors, referred to as the icebox. The passive shielding surrounded the icebox, which was connected to a commercial dilution refrigerator via a set of concentric copper tubes acting as cold finger, called the “cold stem” or “C-stem”. The outermost layer of passive shielding was a 40 cm layer of polyethylene, a hydrogenous material used for neutron moderation. Two layers of lead were used to attenuate gammas. Lead is typically contaminated with radiogenically produced ^{210}Pb , which beta decays with a half-life of 22.3 years and can produce significant bremsstrahlung. A thick 18 cm layer of outer lead and a thinner 4.5 cm inner layer of ancient lead were used to minimize the backgrounds closer to the detectors, as the ancient lead was depleted of ^{210}Pb . An inner 10 cm layer of polyethylene was used between the lead and icebox as an additional neutron moderator.

The final layer of shielding was a 2 mm thick μ -metal shield at room temperature to protect the SQUID-based readout electronics and the phonon sensors themselves from the effects of static magnetic fields. The volume between the μ -metal and the icebox was purged with boil-off nitrogen from a liquid nitrogen dewar in order to reduce backgrounds from radon daughters. The icebox itself consisted of six concentric cans constructed from oxygen-free high thermal conductivity (OFHC) copper with low levels of radioactivity, and provided an additional ~ 1.9 cm of gamma shielding.



(a) Side View



(b) Top View

Figure 2.12: Schematics of the shielding scheme from CDMS-II and SuperCDMS Soudan. The dilution refrigerator (dark blue) is connected via the C-stem to the icebox (beige), which is surrounded by inner polyethylene (green), low activity lead (light grey), regular lead (dark grey), and an additional layer of outer polyethylene. The whole passive shielding scheme is surrounded by the active scintillator veto (light blue) with connected photomultiplier tubes (white/black). Figures from Ref. [141].

Calibration and Neutralization

Calibration using gamma and broad spectrum neutron sources was vital for determining the detector response to ER and NR events, as well as to calibrate the energy scales and measure detector stability over time.

^{133}Ba decays via electron capture with a 10.5 year half-life to ^{133}Cs , producing electrons, X-rays, and gammas. The decay includes several characteristic gamma lines, the most prominent being at 356.0 keV, with additional prominent lines at 302.8, 883.8, and 276.4 keV [142]. This was selected for SuperCDMS gamma calibrations for the 356 keV gamma, which is energetic enough to penetrate the cans of the cryostat and illuminate the volume of the detectors roughly homogeneously. This is in contrast to the produced electrons and X-rays, which have a low probability to reach far outside of the source encapsulation. The 356 keV gammas are also low enough in energy to be close to the desired region of interest. Two ^{133}Ba sources were deployed during gamma calibration series, placed at the edge of the cryostat via source tubes installed along the C-stem and E-stem (which provides a path for the readout cabling to reach room temperature). Gamma calibrations were used for energy scale calibration and stability monitoring, with an average of five ^{133}Ba calibration series of several hours taken per week [143].

Neutron calibration data were used to determine the NR band, the WIMP signal region. Neutron calibrations were performed a few times per year with a single ^{252}Cf neutron source alternated between the two source tubes. This isotope decays primarily via α emission, but also has a $\sim 3\%$ probability to undergo spontaneous fission. 3–4 neutrons per decay are produced in this process, with a most probable energy of 1 MeV and maximum energy of 10 MeV. Exposure to the neutron source was minimized because neutron capture on Ge produces ^{71}Ge , a radioactive isotope which decays via electron capture with an 11.43 day half-life [144], producing a relatively short lived background. However, this process can prove useful. The subsequent X-rays or Auger electron cascades from K-, L-, and M- shell capture at 10.37, 1.31, and 0.160 keV, respectively, can be used for low energy calibration. Unlike

the radioactive daughter isotope ^{68}Ga from cosmogenically produced ^{68}Ge , the daughter isotope ^{71}Ga is stable.

It was essential to keep the detectors neutralized in order to achieve full collection of the charge signal. The neutralized state is lost when enough charge carriers are trapped by impurities or defects in the crystal that the electric field configuration is distorted and charge signals are subsequently reduced. The length of a given series was dependent on how long the detectors could be kept in a neutralized state, which was typically on the order of ~ 3 hours. Following each series, the detectors were flashed with infrared (IR) photons from LEDs mounted inside of the detector housings to release the trapped charges, which required a 10 min cooldown before the next series could commence [143].

Results

The data acquired from the SuperCDMS experiment at Soudan have been used for extensive analyses including WIMP searches. A low mass WIMP search was presented using the initial data from Run 133, setting world leading constraints for SI WIMP-nucleon interactions for WIMP masses of $\sim 4\text{--}6\text{ GeV}/c^2$ [145]. This result disfavours a WIMP explanation for the excess previously seen by CoGeNT, which also uses Ge detectors [90]. Figure 2.13 shows the published limit with previously published allowed regions and exclusion limits.

The majority of the WIMP search data from Soudan, with an exposure of 1690 kg d, were later used in a blind WIMP search [139]. One candidate event was observed, consistent with the expected backgrounds. The analysis set an exclusion limit on the SI WIMP-nucleon cross section of $1.4 \times 10^{-44}\text{ cm}^2$ at a $46\text{ GeV}/c^2$ WIMP mass, setting the strongest constraints for WIMP-Ge nucleus interactions for masses of $13\text{--}127\text{ GeV}/c^2$ at the time of publication. Figure 2.14 shows the published constraint with other contemporary constraints.

Multiple studies of detector operation at high voltage in the CDMSlite mode were used to set world leading SI nucleon-coupled dark matter limits. Run 1 had a small exposure of 6.3 kg days but achieved a 160 eV energy threshold for ERs and constrained new SI

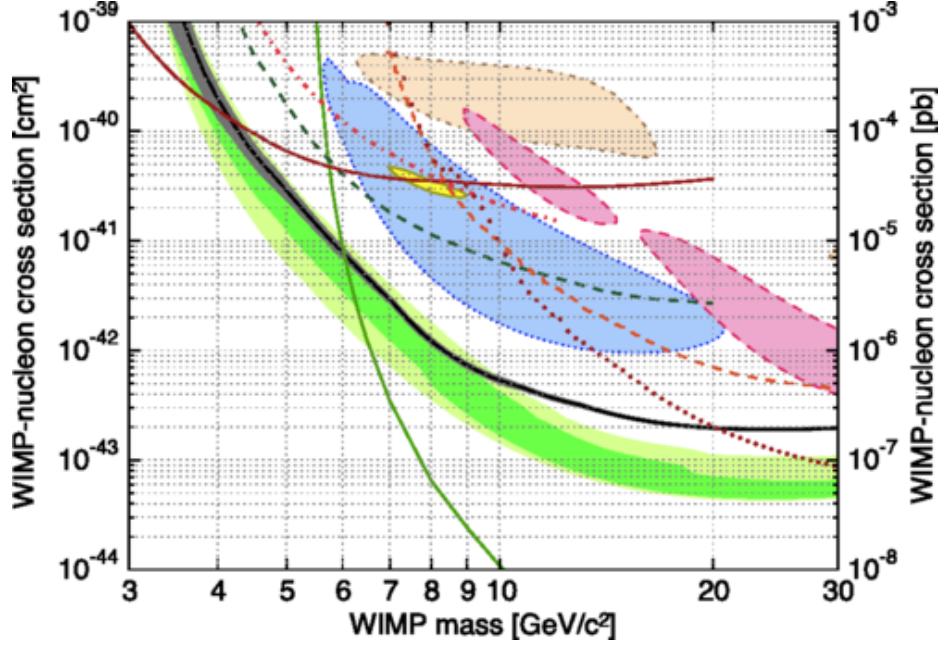


Figure 2.13: 90% upper confidence limit (solid black) on the SI nucleon-coupled WIMP dark matter cross section from the SuperCDMS Soudan low mass WIMP search, from Ref. [145]. Also shown are the expected sensitivity bands prior to unblinding at 68% (dark green) and 95% (light green) CL. Allowed regions from CDMS-II [133] (blue, 90% CL), CoGeNT [90] (yellow, 90% CL), CRESST-II [146] (pink, 95% CL), and DAMA/LIBRA [147] (tan, 90% CL) are shown. Exclusion limits include CDMS-II Ge [134] (dotted dark red), CDMS-II Ge low threshold [148] (dashed-dotted red), CDMSlite Run 1 [81] (solid dark red), LUX [149] (solid green), XENON10 S2 only [136, 137] (dashed dark green), and EDELWEISS low-threshold [150] (dashed orange).

WIMP-nucleon parameter space below $6 \text{ GeV}/c^2$ [81]. Run 2 had a significantly larger exposure of 70 kg days and achieved a 56 eV threshold for ERs, leading to the exclusion of new parameter space for WIMP masses between $1.6\text{--}5.5 \text{ GeV}/c^2$ [152, 160]. The final run, Run 3, used various improved analysis techniques over the previous runs including data salting, background modelling, and a profile likelihood method on 36 kg days of data [121]. This analysis improved upon the previous CDMSlite Run 2 limit, but did not constrain new parameter space as DarkSide-50 had already published a stronger limit [105]. Figure 2.15 shows the results from CDMSlite Runs 2 and 3 with other constraints from the time of publication.

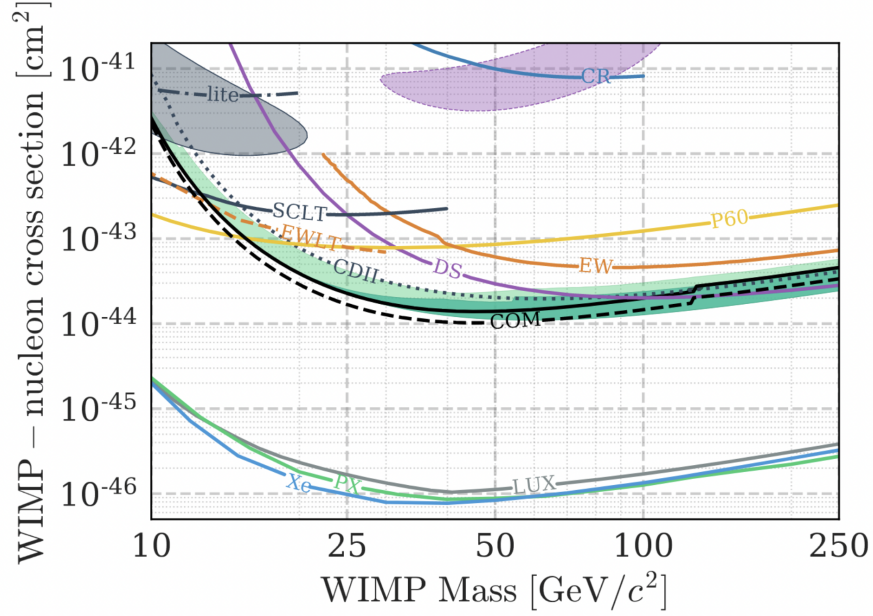


Figure 2.14: 90% upper confidence limit (solid black) on the SI nucleon-coupled WIMP dark matter cross section using the majority of SuperCDMS Soudan data (high threshold), from Ref [139]. Allowed regions from CDMS-II [133] (grey, 90% CL) and DAMA/LIBRA [147] (purple, 90% CL) are shown. 90% CL exclusion limits from CRESST (CR) [151], CDMSlite Run 2 (lite) [152], EDELWEISS (EW) [153], the SuperCDMS Soudan low threshold (SCLT) from Figure 2.13 [145], DarkSide (DS) [154], PICO-60 (P60) [155], EDELWEISS low mass (EWLT) [150], CDMS II Ge alone (CDII) [156] and combined with the high threshold result (COM), PandaX-II (PX) [157], LUX (LUX) [158], and XENON1T (Xe)[159].

Several of these data sets were also used for various dark matter searches beyond the standard nucleon-coupled WIMP paradigm. A combined analysis of CDMSlite and iZIP data was performed to set constraints on dark absorption channels, leading to world leading limits for ALP with masses from 40 to 200 eV/ c^2 , and dark photons with masses from 40 to 80 eV/ c^2 [162]. This work is the subject of Chapter 4 of this thesis. An inelastic dark matter search using bremsstrahlung and Migdal interactions was performed using a profile likelihood method on CDMSlite Run 3 data [163]. These channels are not traditionally considered, yet provide sensitivity to much lower WIMP masses. The analysis excluded new parameter space for WIMP masses as low as 0.03 GeV/ c^2 via the Migdal channel.

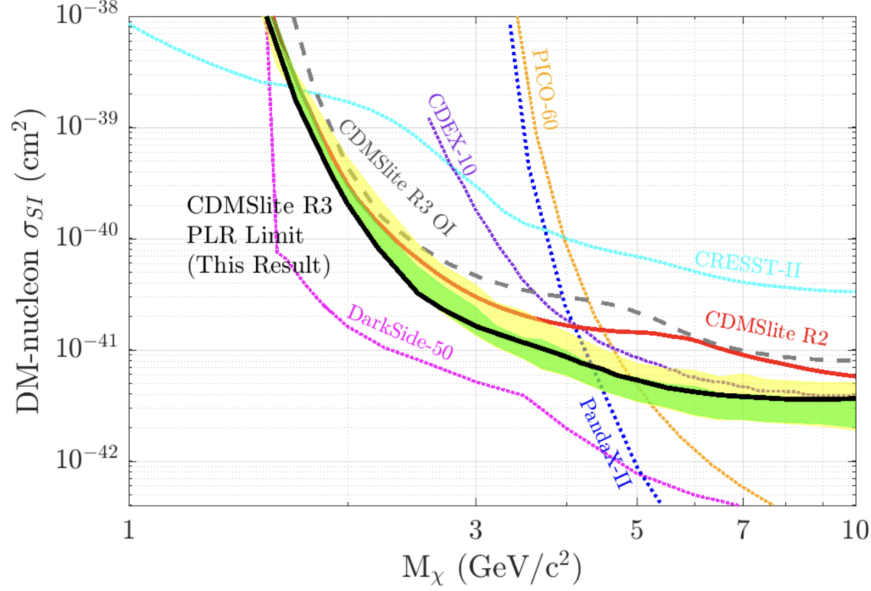


Figure 2.15: The 90% CL profile likelihood ratio (PLR) limit from CDMSlite Run 3 (black solid) with $\pm 1\sigma$ and $\pm 2\sigma$ sensitivity bands (green and yellow shaded, respectively), from Ref. [121]. The CDMSlite Run 3 (grey dashed) and CDMSlite Run 2 [152] (red solid) optimum interval (OI) limits are also shown. Other constraints include DarkSide-50 [105], PandaX-II [161], PICO-60 [155], CRESST-II [151], and CDEX-10 [86].

Other notable results from the SuperCDMS experiment at SUL include searches for Lightly Ionizing Particle (LIP) [164, 165], a study of energy loss due to Frenkel defects [166], Effective Field Theory (EFT) analyses [167, 168], as well as measurements of cosmogenic production rates [169] and the ionization yield [170] in Ge.

2.4.3 The Next Generation at SNOLAB

SuperCDMS SNOLAB is the next generation of the experiment, currently under construction at SNOLAB [171], a laboratory located in an active nickel mine near Sudbury, Ontario, Canada. The lab is located 2 km underground, providing a 6010 m water equivalent overburden [172], a factor of 3 improvement over the SUL overburden. Figure 2.16 shows the relative overburden depth of several underground laboratories and the corresponding muon flux. The muon flux is suppressed by over two orders of magnitude compared to Soudan.

The entire laboratory is operated as a class 2000 cleanroom [172], and is the deepest clean lab in the world at the time of writing.

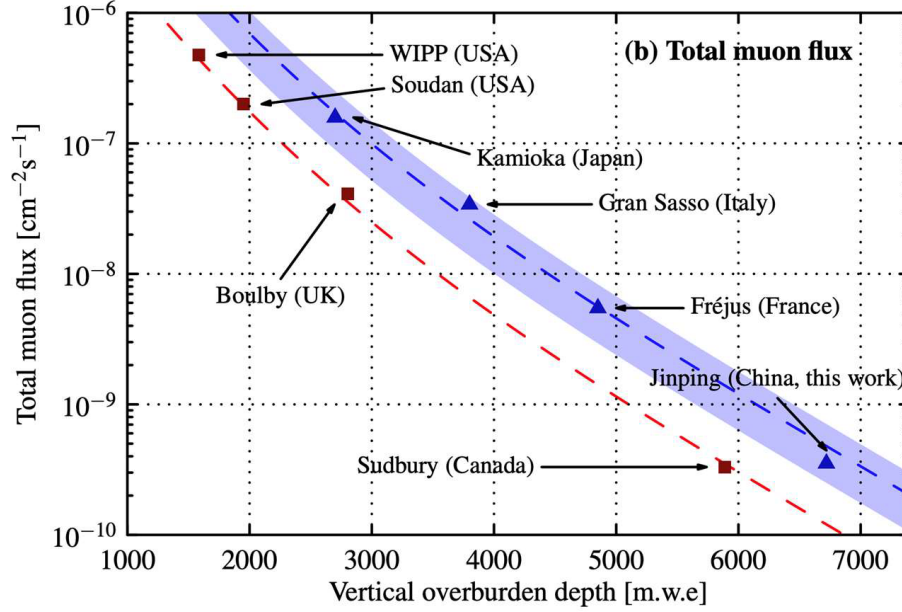


Figure 2.16: SNOLAB depth and muon flux compared to other underground laboratories, including SUL. From Ref. [173].

SuperCDMS SNOLAB will feature entirely new infrastructure, including the cryostat and shielding, and the overall background levels will be reduced compared to SuperCDMS at Soudan. As at SNOLAB, a dilution refrigerator will be used to achieve the operational temperatures. The expected operational 15–30 mK temperature at SNOLAB is colder than the Soudan operational temperature. The cold region of the experiment housing the detectors is referred to as the SNOBOX. Inside, six cylindrical copper cans will be suspended by Kevlar ropes in a setup similar to Soudan, though the SNOLAB cans are larger.

The detectors have been upgraded over those operated at Soudan, with larger volume and improved design. An additional detector type, referred to as high voltage (HV), motivated by the operation of iZIP detectors in the higher voltage CDMSlite mode will also be deployed. Section 3.4.2 in the following Chapter describes the detector designs in detail.

Shielding

There are multiple layers of shielding to maintain a low background environment, including 40 cm of inner polyethylene for neutron absorption and shielding, 20 cm of lead for gamma shielding (an innermost 1 cm layer of ultra low background lead, middle 9 cm layer of low background lead, and 10 cm outer layer of regular lead), a thin metal shield to block radon diffusion into the inner shielding layers, and a volume purged with boil-off nitrogen gas to reduce overall radon levels and therefore reduce backgrounds from radon daughters [171]. The outermost shield consists of polyethylene and 60 cm of modular water tanks to shield from the cavern neutron flux. There is no active muon veto due to the significantly lower muon flux at SNOLAB. Like at Soudan, a μ -metal shield will protect the SQUID readout electronics and detector phonon sensors from external magnetic fields.

The shielding is penetrated in two locations in a manner similar to the apparatus at Soudan, the cryogenics stem (C-stem), connecting the SNOBOX cans to the different thermal stages of the dilution refrigerator, and the electronics stem (E-stem) which guides the wiring from the SNOBOX to the electronics tank (E-tank). The E-tank is important as a vacuum bulkhead and acts as the interface for additional cooling for the 50 K stage of the dilution refrigerator, intercepting a large fraction of the heat load coming down the wiring. The experiment sits on a seismic platform designed to isolate the detectors from seismic events. This is particularly important because SNOLAB is located in an active mine where seismic events are much more frequent than in other underground laboratories (due to rock bursts and blasting). Figure 2.17 shows a schematic of the SuperCDMS SNOLAB cryostat and shielding scheme.

Calibration

A calibration scheme using gamma and broad spectrum neutron sources similar to SuperCDMS Soudan will be used to calibrate the detector response [174].

Neutron calibrations will be used to measure the ionization yield and establish a signal

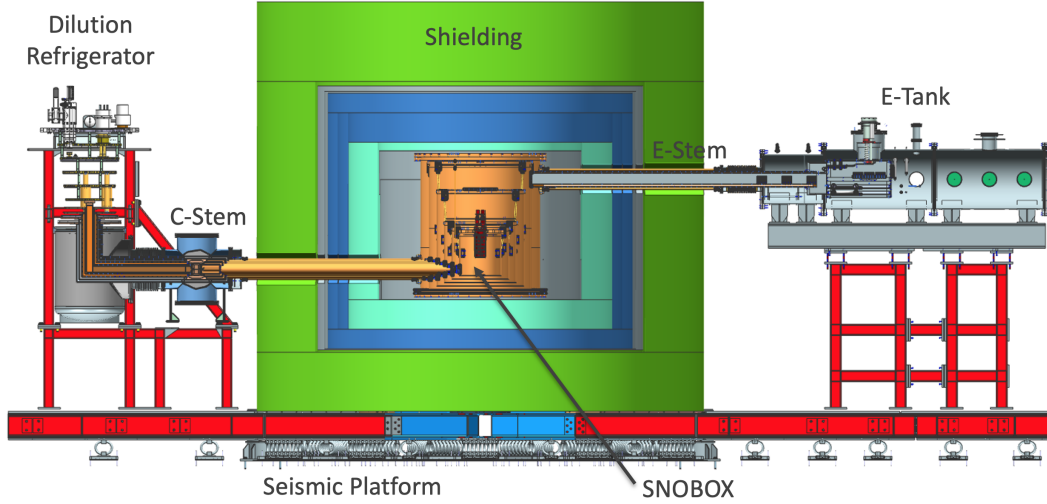


Figure 2.17: Schematic of the SuperCDMS SNOLAB cryostat and shielding scheme. The SNOBOX housing the detector towers is cooled by a dilution refrigerator via the C-stem. The detector readout cables pass through the E-stem to the E-tank, which forms the vacuum bulkhead. The SNOBOX rests on a seismic platform designed to isolate the apparatus from seismic events. Several layers of shielding include outer water tanks and polyethylene, a gamma shield, and an inner polyethylene layer.

region for nucleon-coupled dark matter searches in iZIP detectors. Neutron calibration data will also be used to measure the acceptance of the detectors to NRs. The HV detectors cannot separate NRs on an event by event basis due to the loss of yield discrimination, but the production of ^{71}Ge via neutron capture in the Ge detectors will allow for the calibration of the detector response at low energies using the resulting 10.37, 1.31, and 0.16 keV K-, L-, and M- shell capture lines. This will be the primary method of calibrating the low energy Ge detector response as low energy gammas are unable to penetrate the detector. The calibration lines will serve as a background, but should decay to pre-calibration levels in a few weeks due to the relatively short half-life of ^{71}Ge . There are no such lines in Si, which will make the calibration of the Si detectors more challenging. Broad spectrum ^{252}Cf neutron sources will be manually deployed via small tubes along the E-stem and C-stem to

positions outside of the outer vacuum can of cryostat. Simulations demonstrate that $5\,\mu\text{Ci}$ sources will provide a rate of 5 Hz in the deployed position, which will not overwhelm the data acquisition system [174]. Exposures will occur roughly 2–4 times a year for periods of 8–24 hours.

Frequent gamma calibrations will be used to monitor the stability of the phonon and ionization gains. SuperCDMS SNOLAB will deploy ^{133}Ba gamma sources. The system will be fully automated, able to move sources from a shielded container outside of the passive shield to the top and bottom of the outer vacuum can via a tube system which passes along the E-stem. The energy may be shared between neighbouring detectors, which may facilitate low energy calibrations of HV detectors. Other aspects of the detector response may also be studied, including the gamma rejection capability in the iZIPs, correcting position dependent response, measuring fiducial volume, and monitoring the stability of the energy scale calibration with environmental changes. Calibration techniques using the low-energy structure of the Compton spectrum will be explored: the Compton spectrum exhibits step-like structures at the electron binding energies because the detector collects a minimum of the binding energy of any freed electron [121]. These structures have been previously observed in Si [175], and the K-shell step has been observed in data from SuperCDMS Soudan [176]. With high statistics, these features could be used to calibrate the low energy scale.

2.4.4 SuperCDMS Backgrounds

In this section I describe backgrounds relevant to the SuperCDMS experiment. Most of the backgrounds are also relevant to SuperCDMS at Soudan [132], though some are only a concern at SNOLAB. The shielding around the SuperCDMS cryostat in both experiments is designed to block almost all external radiation; the remaining dominant background sources are from contamination of the detectors, shielding, and other parts of the experimental apparatus.

SuperCDMS categorizes backgrounds into bulk and surface backgrounds. The former produce interactions throughout the detector crystal and the latter result in energy depositions primarily close to the detector surfaces. Ref. [171] summarizes the expected backgrounds for SuperCDMS SNOLAB, which are also described here.

Radioactive impurities within the crystal will be the dominant background for the HV detectors, and also a concern for ERDM searches with iZIP detectors. Many of these backgrounds may be attributed to exposure of materials to cosmic ray secondaries during fabrication and testing on the Earth's surface. Tritium (^3H) is a cosmogenically produced beta emitter and decays with a long half-life of 12.3 years and a beta spectrum endpoint of 18.6 keV. It is a spallation product from interactions of high energy cosmic ray secondaries in the detector volume, and will be a significant background in both Ge and Si detectors. The Si detectors will also be contaminated with naturally occurring ^{32}Si , an isotope produced via spallation of cosmic ray secondaries on atmospheric argon. The concentration in the detector is dependent on the silicon used for fabrication, and is fixed once the crystal is grown. With a 153 year half-life, ^{32}Si is long lived, and it decays via beta decay with an endpoint of 227.2 keV. The daughter isotope ^{32}P is also a beta-emitter, decaying with a half-life of 14.3 days and a beta endpoint of 1.71 MeV. Other beta emitting cosmogenic isotopes may be present, but with negligible contributions to the energy region of interest. The final cosmogenically produced background of concern is activation lines in the Ge detectors. Cosmic ray secondaries produce a variety of isotopes which decay via electron capture, producing characteristic lines in the measured energy spectrum. We consider eight isotopes which are long lived enough to present a background of concern including ^{68}Ge and its ^{68}Ga daughter, ^{73}As , ^{65}Zn , ^{57}Co , ^{55}Fe , ^{54}Mn , and ^{49}V . Each isotope may decay via electron capture from the K-, L-, or M-shell, resulting in a spectral peak at each binding energy. There are no such backgrounds in Si. Care is being taken to reduce detector contamination by limiting the time that the detectors are on the surface, but without a full underground life cycle from crystal growth to deployment in the experiment, cosmogenic activation is

inevitable.

Materials other than the detector crystals also experience cosmogenic activation when exposed to cosmic ray secondaries, resulting in the production of long lived isotopes. Of particular concern is copper, which is used in the fabrication of the detector housing and towers, as well as the nested cryostat cans. Material contamination may also be introduced during the manufacturing process. ^{238}U and ^{232}Th are both unstable isotopes which are present in many materials, albeit in low concentrations. Both have a chain of radioactive decay products. Other isotopes that have been considered are ^{40}K , which is long lived and naturally present in various materials, and ^{60}Co , which is cosmogenically produced. Neither have daughter radioisotopes.

Radioisotopes accumulate on surfaces exposed to air containing dust and radon. We distinguish between surfaces which have line of sight to the detector and those which do not; I discuss the former shortly. Dust contains high concentrations of ^{238}U , ^{232}Th , and ^{40}K , and daughters from ^{222}Rn decay may implant shallowly in the surface of an exposed material. Radon daughter implantation leads to a build up of long lived ^{210}Pb , which has a short decay chain with roughly constant X-ray, beta, and alpha emissions. The betas are high energy and typically produce bremsstrahlung leading to a wide gamma spectrum.

There are various backgrounds originating from the cavern environment including naturally occurring radioactivity that results in gammas and neutrons capable of passing through the shielding and reaching the detectors. The experimental cavern is surrounded by Norite rock covered in a layer of shotcrete, and the cavern floor is concrete. Gamma backgrounds from ^{238}U and ^{232}Th as well as ^{40}K are accounted for, simulating their emission spectra and propagation through the shielding to determine the spectra expected in the detectors. The flux was estimated using assays of rocks from the SNOLAB ladder labs. Results showed that this background is expected to be subdominant to the other backgrounds. We also model the neutron background as two components: neutrons from the spontaneous fission of ^{238}U and neutrons from (α, n) reactions in the rock. Finally, radon decays in the mine

air are assumed to create a negligible contribution to the gamma background because of the radon purge surrounding the gamma shield.

Cosmic rays may produce neutrons via spallation, or interact directly in the detectors, where the neutrons would be the dominant concern; however, due to the low muon rate achieved by SNOLAB's 2km of rock overburden, this background is negligible compared to other sources, and a muon veto was deemed unnecessary. At Soudan, neutrons from muons interacting in the polyethylene shield are vetoed by the scintillator active veto with high efficiency, but neutrons produced by muon interactions in the cavern walls can have energies above 50 MeV. Fast neutrons such as these can penetrate the polyethylene shield and interact within the experimental apparatus, producing low energy secondary neutrons.

The coherent neutrino background from solar neutrino interactions via coherent elastic neutrino-nucleus scattering ($\text{CE}\nu\text{NS}$) will not be a background for the first science run of SuperCDMS SNOLAB, but will be a limiting background for future upgrade scenarios and other low mass dark matter searches. The fundamentally limiting background for future SuperCDMS upgrades will be neutrinos from the decay of ^8B at the end of the pp-III solar reaction chain [171].

The second main category of backgrounds are those which produce energy depositions near the surfaces of the detector. These are generally related to exposure of the detectors and housings to ^{222}Rn and its daughters during fabrication. The radon daughters ^{218}Po , ^{214}Pb , and ^{214}Bi can plate out on the exposed surfaces, with a decay chain leading to ^{210}Pb , which then alpha decays. If the alpha is emitted out of the material the nuclear recoil of the ^{210}Pb daughter can cause it to implant in the surface. The subsequent ^{210}Pb chain produces a variety of non penetrating radiation, though secondary bremsstrahlung can create penetrating gammas. ^{210}Pb produces low energy betas and X-rays, resulting in surface ER events. The result is a beta energy spectrum with an endpoint of 63 keV and various Auger electrons and spectral lines. The ^{210}Bi daughter undergoes beta decay with

a 1.16 MeV endpoint, also giving rise to a near-surface ER background and the aforementioned bremsstrahlung. Finally, the ^{210}Po daughter emits a very energetic alpha, which is typically out of the signal region². However, the recoiling ^{206}Pb nucleus has enough energy to potentially create a NR in the signal region: if the ^{210}Po decays on the detector surface and the ^{206}Pb nuclear recoil is directed towards the detector, then all 103 keV of the recoil energy will be deposited in the detector. If the decay occurs instead on a nearby surface, such as the detector housing, the energy is degraded because of the implantation depth of the ^{210}Po . This results in a continuum of energies up to 103 keV from this background source. An additional two background sources were identified, but not considered, in Ref. [171]: ^{14}C and ^{39}Ar also present due to detector fabrication processes and passivation with citric acid of copper surfaces. Both undergo beta decay with 163 keV and 565 keV endpoints, respectively. These backgrounds have since been taken into account in Ref. [177], which is the subject of Ch. 5 of this work.

2.4.5 The Cryogenic Underground TEst Facility

The CUTE facility is located underground at SNOLAB in the same experimental hall as SuperCDMS, as shown in Figure 2.18. It was constructed as a low background facility for testing SuperCDMS SNOLAB detectors before operation in the main experiment [178].

Motivating a Low Background Test Facility

As we have seen from the discussion of the backgrounds observed at Soudan and expected at SNOLAB, cosmogenically produced backgrounds such as tritium are a significant background for detectors constructed and tested on Earth's surface. Minimizing surface exposure during the fabrication and testing process for the SuperCDMS SNOLAB detectors is vital for ensuring that we have low enough backgrounds to perform world leading direct detection searches.

²If the emitter is implanted in the detector housing, the alpha could lose energy before hitting the detector and end up in the signal region.

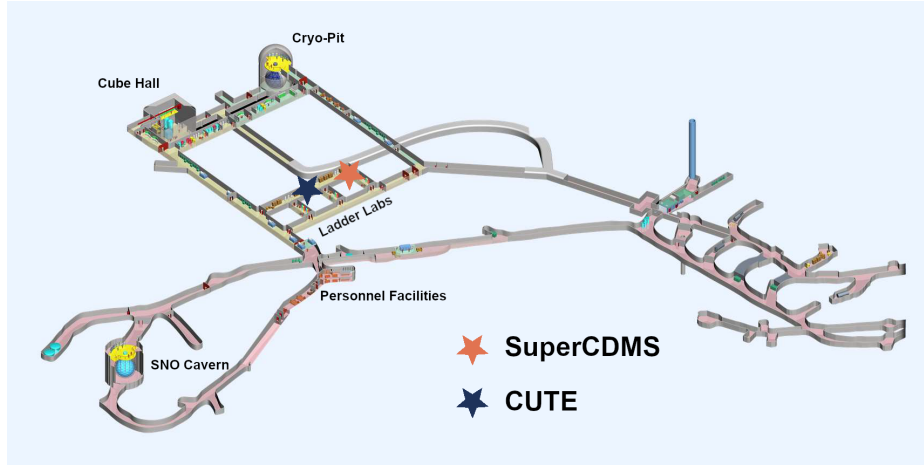


Figure 2.18: SNOLAB layout with CUTE location next to SuperCDMS SNOLAB in the ladder lab indicated. Adapted from Ref. [179].

Most SuperCDMS test facilities are surface, or near-surface, facilities subject to high rates of cosmogenic backgrounds. For most R&D programs this is acceptable, but for the testing of SuperCDMS SNOLAB detectors it would be ideal to perform the tests underground where cosmogenic activation is suppressed. Testing the detectors is impractical at SuperCDMS SNOLAB because the process of a payload turnover (warming up, changing the payload, and cooling back down) would be a several month endeavour. The CUTE facility is capable of rapid payload changes and cooldowns on the order of a couple of weeks. Additionally, CUTE provides the opportunity to troubleshoot and problem solve long before the main experiment is ready to operate detectors. Anything learned at the CUTE facility does not have to be learned at SuperCDMS when we could otherwise be taking dark matter search data.

The CUTE facility was designed to be a low background test facility for the SuperCDMS SNOLAB detectors. While the backgrounds at CUTE are not as low as expected at SuperCDMS SNOLAB due to less stringent requirements on the construction materials, there is still potential to set competitive dark matter limits using SuperCDMS detectors before the main experiment begins commissioning. The SuperCDMS readout electronics and data acquisition (DAQ) system also have the opportunity to be tested. R&D devices can also be

operated in the low background environment for characterization and dark matter searches. Following the SuperCDMS testing program, CUTE will act as a general low background test facility where other experiments can apply for facility time.

The facility is comprised of several systems including a dry dilution refrigerator, vibration mitigation system, and calibration systems. A schematic of the facility is shown in Figure 2.19. The most striking feature is the large 3.7 m diameter, 3.2 m tall water tank with a shielded hollow where the cryostat sits during facility operation. The experimental payload is housed inside of the cryostat. There is a class 500 cleanroom in which the experimental payload can be installed in the cryostat, which is moved from the cleanroom into the drywell via a monorail crane system. The drywell and cryostat can be accessed by personnel via the deck, which is accessible by stairs. Additional systems include the gas handling system which contains the necessary apparatus for operating the dilution refrigerator, as well as computers for controlling the DAQ and dilution refrigerator.

We will now explore the facility in some detail. Theses by J. Corbett [181], R. Germond [180], M. Ghaith [182], and R. Underwood [183] provide excellent overviews of the facility and many of its subsystems. Underwood’s thesis also provides results from the first dark matter search at CUTE using a ~ 10 g Si detector with an athermal phonon sensor.

Dilution Refrigerator and Cryostat

The cryogenic temperatures required for operation are achieved using a dry dilution refrigerator from CryoConcept, providing a base temperature of around 12–13 mK. There are six isolated thermal stages including room temperature, two precooling stages at ~ 50 K and ~ 4 K, the still stage (~ 700 – 800 mK), heat exchanger stage (~ 70 mK, also referred to as the cold plate stage), and the mixing chamber stage at base temperature (~ 13 mK). Copper cans on the three warmest stages reduce the radiative heat load from each stage to the next. This separation of stages is critical because the heating from blackbody radiation is proportional to T^4 , where T is the temperature. These stages are also designed as an

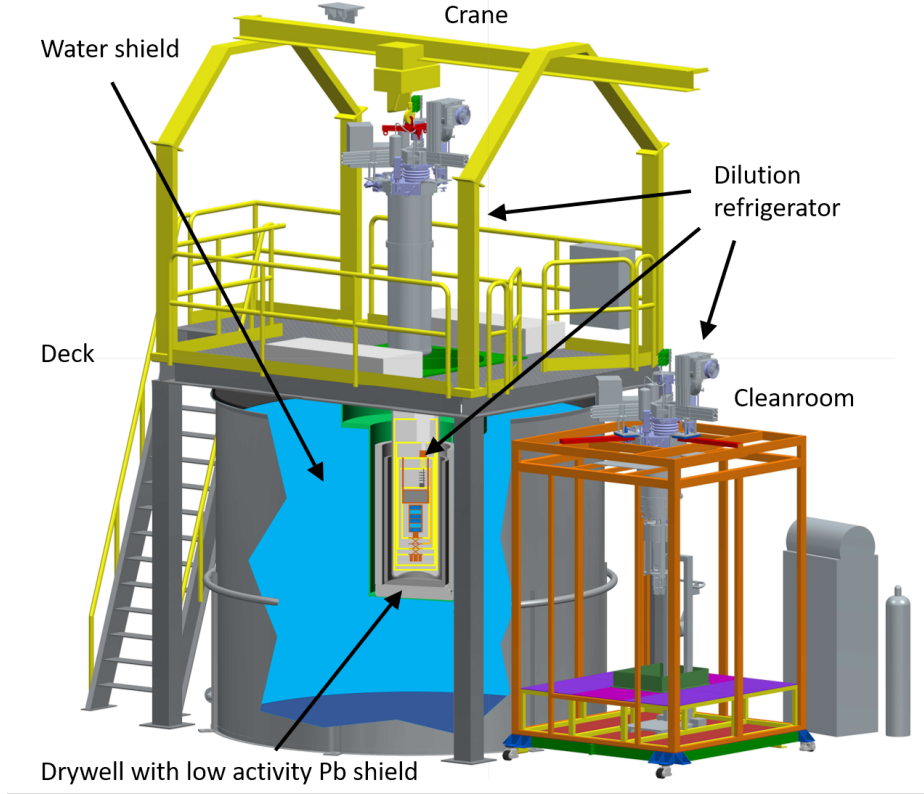


Figure 2.19: Schematic of the CUTE facility, from Ref. [180]. The detector payload is mounted in a dilution refrigerator which sits in the shielded drywell of a large water tank during facility operation. A monorail crane mounted above the water tank moves the dilution refrigerator between the drywell and a cleanroom where the payload can be accessed and changed. The drywell can be accessed from a deck above the water tank, which is accessible via stairs.

optical barrier to prevent IR photons from reaching the detector payload. These photons can liberate charge carriers in the detectors, which imposes a much stricter regime than the requirement to reach the low operating temperature.

The thermal isolation is partially achieved by a vacuum environment to prevent gas exchange from warming the inner stages. A fourth can called the Outer Vacuum Can (OVC) holds the necessary vacuum pressure. The low pressure environment is achieved in the OVC using a turbo pump backed by a scroll pump, reaching $10^{-4} - 10^{-5}$ mbar pressure. Once the cryostat gets below about 30 K, cryopumping reduces the pressure further to $\sim 10^{-7}$ mbar.

A two-stage pulse tube cooler (the warmer stage is at ~ 50 K, the colder at ~ 3 K) is used to precool to cryogenic temperatures via the nearly adiabatic expansion and compression of ^4He gas [184]. Pulse tube cryocoolers reduce the need for liquid cryogens in dilution refrigerators. However, they introduce vibrational noise, which is reduced in CUTE by mechanically decoupling the pulse tube from the cryostat. The decoupling and vibration mitigation is discussed later in this section.

The mK temperatures of dilution refrigerators are reliant on the properties of the two helium isotopes ^3He and ^4He [184]. When a mixture of ^3He and ^4He is cooled below ~ 0.87 K, it separates into two phases: a phase dilute in ^3He (roughly 6.6% concentration at very low temperature) in superfluid ^4He and a phase concentrated in almost pure ^3He . The concentrated phase sits on top of the dilute phase because of the lower ^3He density. The binding energy of ^3He is higher in ^4He than in ^3He , which causes ^3He to preferentially diffuse from the concentrate phase into the dilute phase if there are vacant energy states. This mixing is endothermic because the enthalpy of ^3He is higher in the dilute phase than in the concentrate phase. In a dilution refrigerator, this process occurs in the *mixing chamber*, which sits on the base temperature stage.

This process cannot occur when the ^3He concentration reaches 6.6%, so a mechanism to remove ^3He from the dilute phase is required. A tube extends from the bottom of the mixing chamber (the dilute phase) and connects to the still stage. ^3He evaporates out of the ^4He , which remains superfluid, and is recirculated back through the dilution refrigerator. This forms an osmotic pressure gradient, pumping more ^3He out of the dilute phase and allowing the mixing process to continue. The circulating ^3He is passed through a cold trap to remove any impurities, then is cooled again via heat exchangers.

In the CUTE facility, the cryostat is designed so that the experimental payload is connected below the mixing chamber stage via copper rods. The facility is capable of housing an entire SuperCDMS detector tower.

Shielding

The 2 km rock overburden at SNOLAB provides significant shielding from cosmic rays, though as we have previously discussed, there is still a small muon flux which can create neutron backgrounds in the facility. CUTE has several layers of passive shielding designed to mitigate neutron and gamma backgrounds. As a test facility, the shielding is designed so that the payload can be removed and exchanged as rapidly as possible. This is in stark contrast to the shielding for SuperCDMS SNOLAB, which needs to be dismantled and painstakingly reassembled in a time intensive procedure to access the payload.

To an external observer, the most prominent feature of the CUTE facility is the large 3.7 m diameter water tank which moderates neutron backgrounds and removes a large fraction of gamma radiation backgrounds. The cryostat sits within a stainless steel hollow, termed the ‘drywell’, within the water tank as shown in Figure 2.20. As the name suggests, this area is designed to separate the cryostat and other important experimental apparatus from the water. Around the cryostat are two nested layers of lead shielding, termed the outer and inner lead shields, to shield from gamma backgrounds. The outer lead cylinder of low activity lead is 8.65 cm thick with a ~ 15 cm bottom and moderates the gammas. The inner layer of lead is thinner at 2 cm thick at the sides and bottom, and is very low activity lead and essentially provides shielding to the bremsstrahlung from the outer lead. This solution is more cost effective than an entire shield constructed from the lower activity lead.

As there is a gap in these lead shielding layers to allow the cryostat to be inserted and removed, there is a 13 cm thick lead shield mounted inside of the cryostat, termed the internal lead shield. The shield is surrounded in copper with thickness of 1.5 cm. This shielding assembly sits directly above the payload to block gammas coming in from the top.

The final layer of neutron moderation for shielding of the cryostat from almost all angles is a 20 cm thick polyethylene slab which sits over the drywell. This shielding layer is mounted on wheels for quick removal to access the cryostat, and can be locked into place for safety.

The nested copper cans of the cryostat provide further layers of shielding, mostly against radiation from the warmer stages of the fridge. Finally, there is a 1.6 mm thick μ -metal shield between the inner and outer lead layers for magnetic field shielding. The shield is constructed from two overlapping cylinders with ~ 40 cm of overlap. Figure 2.20 is a schematic of the CUTE fridge and shielding systems.

The shielding was designed to reduce backgrounds to below 3 events/keV/kg/day below 1 keV according to initial simulations [185], though there were some changes to the facility from what is described in this reference. The dominant backgrounds are from the cavern walls, the external lead shield, and the cryostat vacuum can. Note in particular that the lead is low background, but not ancient lead, which is higher activity than the lead used for SuperCDMS. The background rate above 10 keV was found to be below 7 events/keV/kg/day with the expectation that the Compton component of this background will be roughly constant down to below 1 keV [183].

Radon buildup in the drywell is mitigated via purge gas from SNOLAB's compressed surface air supply. The purge gas is pumped into the bottom of the drywell below the OVC, where it displaces air with high radon concentration. The constant flow out of the drywell ensures that high radon air cannot flow back into the drywell where the ^{222}Rn daughters could plate out and produce backgrounds.

Vibration Mitigation

The dry dilution refrigerator produces significant vibrational noise because of the pulse tube cooler. Many detectors are highly sensitive to vibrational noise and electromagnetic interference, and this kind of noise environment would be detrimental. In particular, the pulse tube noise is dominant at low frequency, overlapping largely with the detector response to particle interactions, and would introduce a significant reduction in sensitivity for dark matter searches as well as attempts to understand the low energy behaviour of detectors. For this reason, the upper part of the refrigerator, including the pulse tube, turbo pump,

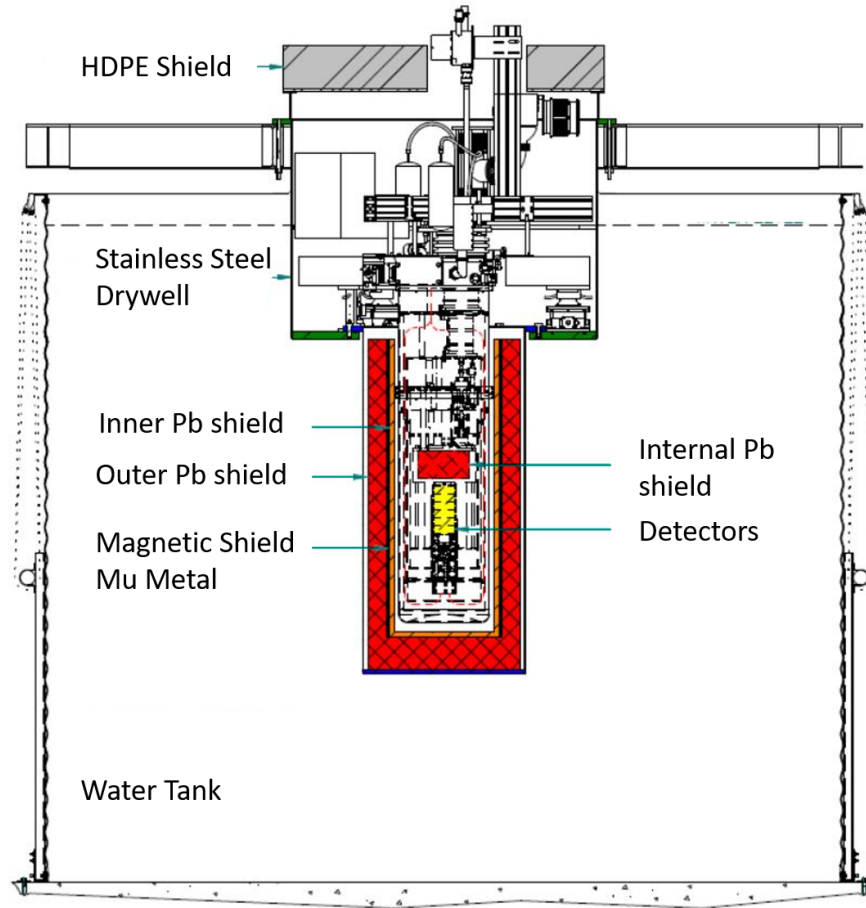


Figure 2.20: Schematic of the CUTE facility with SuperCDMS tower arrangement from Ref. [178]. The different shielding systems are indicated.

and various electronics systems, are designed to be vibrationally isolated from the lower part of the fridge which holds the experimental payload.

The vibrational decoupling is achieved by soft bellows, with no rigid connections between the 3 K and 50 K stages of the pulse tube and the respective stages of the cryostat. Thermal contact between the pulse tube and the cryostat is maintained via circulating mixture. The circulation loop is under vacuum, resulting in a net upward force due to the external lab pressure. Due to the change of the lab pressure over time, the lower part of the fridge sits on a suspension frame with three actively controlled dampers. The dampers are necessary

to decouple vibrations from the drywell and deck. The three dampers rise and lower independently to ensure that there is no tilt to the fridge, and to maintain a roughly constant gap between the upper and lower sections of the fridge. Varying this gap size changes the thermal contact between the stages, and can affect the cooling performance. The dampers and suspension frame are referred to as the suspension system, which can be seen in cryostat schematic in Figure 2.20.

Calibration

As part of the detector testing capabilities, the CUTE facility must be able to calibrate detectors. The CUTE facility has a gamma calibration system and is in the process of developing and installing a neutron calibration system.

Like SuperCDMS, a ^{133}Ba calibration source is used for gamma calibrations. Recall that the main feature of this isotope is a prominent 356 keV gamma. The encapsulated source is stored in a lead shielded box above the drywell when not in use, and is deployed by an automated system via a copper tube into the space between the cryostat and inner lead shielding. The source is attached to a beaded chain and deployed via a stepper motor. It hangs freely beyond the guiding copper tube shortly after being guided between the two layers. The position can be freely selected in the z-direction as the system monitors the position of the source. Figure 2.21 shows a schematic of the gamma calibration system.

A ^{252}Cf neutron source is currently being designed and developed for the CUTE facility. SNOLAB has heavy requirements for the movement of neutron sources through the lab; it was decided that a source would be kept in place at the CUTE facility so that it could be deployed rapidly. It is essential that the gammas from the source are shielded and that the overall background rate cannot have a significant impact at the neighbouring experiments, which include SuperCDMS SNOLAB and PICO, or on CUTE itself. Thus the source will be housed in a storage box inside of the water tank in order to prevent backgrounds both in CUTE and the neighbouring experiments. Figure 2.22 shows a schematic of the neutron

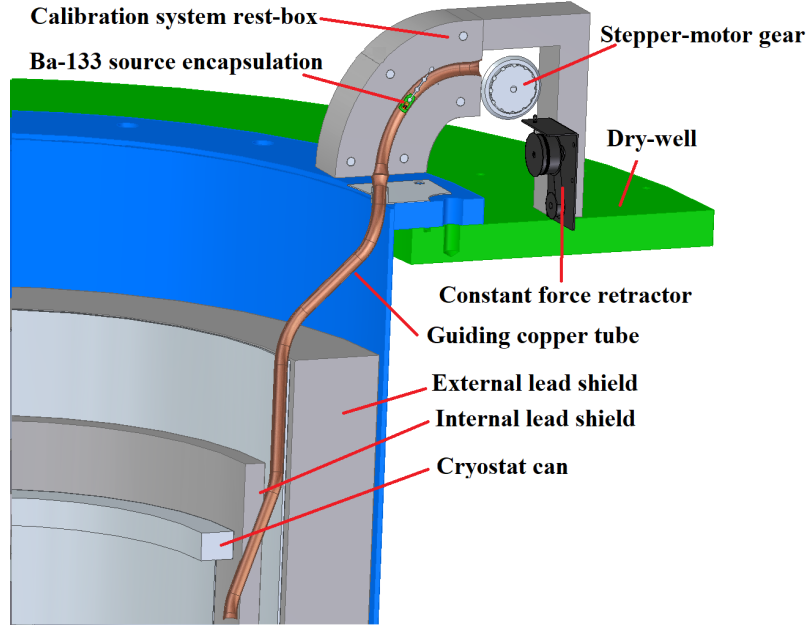


Figure 2.21: Schematic of the CUTE ^{133}Ba calibration system from Ref. [186]. The source remains in a shielded lead rest box when not in use, and is deployed on a beaded chain via a stepper motor in between the outer vacuum can and the inner lead shield. It is guided into place by a copper tube, but hangs freely once in between the two layers.

calibration system.

Similar to the gamma source, the neutron source will be attached to a beaded chain. This will run through a source tube which loops through the water tank and around the drywell. The source activity is chosen to have a goal neutron rate of 6 Hz in the detectors when the source is in the deployed position [181]. Much of the design of the ^{252}Cf source was the work of J. Corbett's thesis [181].

Now that we have motivated the search for particle dark matter, examined several candidates to search for, and introduced SuperCDMS and the CUTE facility, the next chapter will focus on the physics of the SuperCDMS detectors and the specific designs that make them excellent devices to search for dark matter.

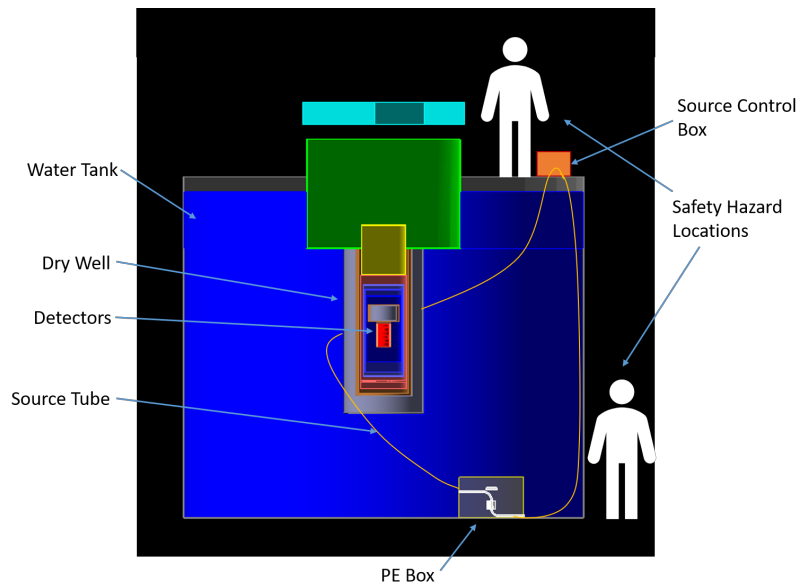


Figure 2.22: Schematic of the not-yet-implemented CUTE neutron calibration system from Ref. [181]. The source remains in a shielded polyethylene rest box in the water tank when not in use, shielding both CUTE and the neighbouring experiments. The source is deployed via a beaded chain through the source tube, which wraps around the drywell. The design ensures that the dose rate for any personnel in the regions of highest dose rate are well below any safety concerns.

Chapter 3

Detector Principles

In this chapter I provide an overview of the principles behind cryogenic semiconductor detectors like those used by SuperCDMS for dark matter searches. In Section 3.1 I cover the basic principles of semiconductor detectors and the production of charge and phonon signals. In Section 3.2 I go into more detail about phonon physics as well as the phonon measurement, and in Section 3.3 I do the same for the charge measurement. Finally, in Section 3.4 I go into specific detail about SuperCDMS detector designs.

3.1 Semiconductor Detectors

Semiconductors have a wide variety of applications, including as particle detectors. When operated at cryogenic temperatures, they make particularly interesting detectors for dark matter searches. These typically consist of a volume of target material in which the dark matter (or other particle of interest) will interact, and sensors to measure the heat and/or charge signals from the interaction. The unique properties of semiconductor detectors and their readout sensors can lead to very low energy thresholds, as well as energy resolution, compared to other dark matter detectors. Semiconductor properties are material dependent; SuperCDMS uses germanium and silicon, both of which are commonly used materials for particle detectors.

In the ground state of a semiconductor, the highest energy electrons reside in the valence

band, which is separated from a set of higher energy states comprising the conduction band by the material-dependent band gap energy, E_{gap} . Example values of E_{gap} are 0.75 eV for Ge [187] and 1.2 eV for Si [188], and are typically $\mathcal{O}(\text{eV})$ scale, though some materials currently being explored may have even lower band gap energies. This is significantly lower than the energy required to produce scintillation or ionization in other common targets for dark matter detection, such as liquid xenon. Excitations promote electrons in the valence band to states in the conduction band. For the energies involved in semiconductor detectors, the electrons can be promoted between essentially any pair of occupied and unoccupied states. Importantly, electrons cannot exist in energy states in the band gap. Electrons occupy energy states with temperature (T) dependence according to the Fermi-Dirac distribution

$$f(E) = \frac{1}{e^{(E-E_f)/k_B T} + 1}, \quad (3.1)$$

where k_B is the Boltzmann constant and E_F is the Fermi level, which corresponds to the energy state which has a 50% probability of being occupied when in thermal equilibrium. E_F lies between the valence and conduction band for intrinsic (undoped) semiconductors, thus electrons in semiconductors mostly occupy states in the valence band. However, at high enough temperatures, some electrons can occupy the conduction band.

At cryogenic temperatures, electrons mostly occupy the valence band, and only move into the conduction band if there is an energy deposition with energy surpassing E_{gap} . This makes the operation of semiconductor detectors at cryogenic temperatures ideal.

If an electron is removed from the valence band, a quasiparticle with a net positive charge is created; the quasiparticle is referred to as a *hole*. While these holes are not real particles, they do occupy energy states. In fact, for every electron in the conduction band, there is a hole in the valence band and vice versa. The promotion of an electron from the valence band to the conduction band correspondingly results in a hole moving from the conduction band to the valence band. This paired nature of the electron and the hole means that we

may consider them as electron-hole pairs (e^-h^+).

Particle interactions can create e^-h^+ pairs either by depositing energy to the nucleus (in a NR) or to electrons (in an ER). The initially freed electrons can liberate other electrons if they have sufficient momentum, resulting in a total number of freed e^-h^+ pairs proportional to the deposited energy. The average energy required to liberate an e^-h^+ pair is roughly three times larger than the band gap due to energy loss in the production of phonons.

Electrons in the conduction band can move freely in the crystal, as can the holes in the valence band via *apparent motion*: electrons move to fill the hole, creating a new hole in the process. By applying an (ideally) uniform electric field across the crystal, liberated electrons and holes drift to opposite sides of the detector. By instrumenting the surfaces with electrodes, the charge signal can be measured. The charge signal readout is described further in Section 3.3.

The propagation of electrons and holes differs as holes have an isotropic effective mass, and electrons have an anisotropic effective mass. Hole propagation mostly follows the electric field lines, whereas electron propagation is oblique to the field lines [189], an effect that is more pronounced in Ge than Si. In addition, this means that the charge sensors which measure the holes have more accurate information on the initial position of the interaction.

As alluded to in the discussion of the energy required to produce a single e^-h^+ pair, not all of the energy from an interaction results in the production of e^-h^+ pairs. The remaining energy is deposited into the phonon system. Before we discuss the details of the ionization measurement, the next section covers the phonon measurement.

3.2 Phonon Signals

In our discussion of phonon signals, it is important to recognize the different types of phonons and how they are produced. In addition, phonons are classified in two branches, acoustic and optical. Acoustic phonons correspond to coherent atomic vibrations; for optical phonons the atoms in a basic lattice cell are vibrating with opposite phase. Optical phonons are

higher energy and rapidly downconvert to lower energy acoustic phonons. Phonons are also categorized by the atomic displacement relative to the direction of their momentum, in either longitudinal or transverse modes.

Thermal phonons arise naturally from the temperature of the crystal. The thermal number density is described by the Bose-Einstein distribution for phonons with energy $\hbar\omega$:

$$\langle n \rangle = \frac{1}{e^{\hbar\omega/k_B T} - 1}. \quad (3.2)$$

In the Debye approximation, the mean phonon energy is around $\hbar\omega = 2.7k_B T$ in the zero-temperature limit [190, 191]. In the operational temperature range of SuperCDMS, $\mathcal{O}(10)$ mK, this corresponds to thermal phonon energies of order to 1 to 10 μeV . The sensors are not designed to detect these low energy phonons, a point which will be elaborated upon in Section 3.2.1.

Along with the promotion of electrons into the conduction band, phonons are also produced in particle interactions; these phonons are referred to as prompt phonons. The amount of phonons produced in a given interaction is proportional to the energy deposited. These phonons are *athermal*, or out of equilibrium, and are much more energetic than the thermal phonons as they are produced near the Debye energy [192].

These primary phonons downconvert from the optical branch to the acoustic branch via anharmonic decay, in which a single phonon splits into two phonons of lower frequency. This process conserves both energy and momentum when considering the recoil of the lattice. Alternatively, the phonons can also undergo elastic isotopic scattering, a result of an inhomogeneous crystal lattice due to the natural abundance of different isotopes and from other defects. In this process, the direction of the phonon is changed, but not the energy. The rates of these processes are both highly frequency dependent; the decay rate goes as $\tau_D \propto 1/\omega^5$, while the scattering rate goes as $\tau_S \propto 1/\omega^4$ [193]. These rates are typically $\mathcal{O}(\mu\text{s})$ or less [193]. Thus, the high frequency primary phonons quickly undergo scatterings

and decays, and have short mean-free-paths. As the phonons undergo anharmonic decay (resulting in lower energy phonons), the mean-free-path becomes larger than the size of the crystal, and these phonons are considered *ballistic*, propagating in straight lines at the speed of sound in the crystal until they reach a surface. If incident on an uninstrumented surface, they are reflected elastically; if incident on detector sensors, they have a high probability to be absorbed.

Additional phonons are produced after the interaction as charge carriers drift in the electric field, called Neganov-Trofimov-Luke (NTL) phonons [194, 195]. The charges carriers are accelerated by the field, but rapidly reach a terminal velocity due to the presence of the crystal lattice and the excess energy gained from further movement across the potential is dissipated in the form of phonons. The two charges in single e^-h^+ pair drift from the interaction site to the opposite detector surfaces; combined, they traverse the entire electric potential. The total energy, eV_b , where V_b is the detector bias voltage, is split between the NTL phonons and the kinetic energy of the charge carriers.

Finally, phonons are also produced in the form of *recombination* phonons. The charge sensors measure image charges (as described in Section 3.3). The charge carriers recombine in the electrodes, releasing the band gap energy originally expended to produce them in the form of recombination phonons. These high energy phonons, being produced near the surface, quickly interact with the surface and convert to ballistic phonons. Due to recombination, the recoil energy is contained in the phonon signal. However, it is possible for charges to become trapped near the surface, leading to a reduction in the amount of recombination phonons.

For a single particle interaction, the total phonon energy measured by the detector, E_t , with initial recoil energy E_r is the sum of the energy from the prompt, recombination, and

NTL phonons:

$$E_t = E_{prompt} + E_{recomb} + E_{NTL} \quad (3.3)$$

$$= (E_r - n_{eh}E_{gap}) + (n_{eh}E_{gap} + E_{KE}) + (n_{eh}eV_b - E_{KE}) \quad (3.4)$$

$$= E_r + n_{eh}eV_b, \quad (3.5)$$

where n_{eh} is the number of e^-h^+ pairs produced in the interaction, E_{gap} is the band gap energy, and E_{KE} is the terminal kinetic energy of the charge carriers. The inclusion of the NTL phonons demonstrates how a small signal can be dramatically amplified with large bias voltages. This was the motivation behind the CDMSlite operational mode of SuperCDMS detectors at Soudan (see Section 3.4.1) as well as the development of dedicated detectors designed to be operated at high voltage for SuperCDMS SNOLAB.

The number of electron-hole pairs produced in a given interaction is proportional to the recoil energy. It is subject to statistical fluctuation, with a variance parameterized by the Fano factor, F [196], where $F = 1$ describes behaviour similar to a Poisson distribution [197]. If the energy resolution of a detector is sufficiently small, individual e^-h^+ pairs can be observed as discrete peaks. Alternatively, if the resolution is poor, the distribution appears smooth, and the width from the e^-h^+ pair production statistics can be incorporated into the resolution measurement.

The mean number of electron-hole pairs produced for a given energy deposition, E_r , is

$$\langle n_{eh} \rangle = Y(E_r) \frac{E_r}{\epsilon_{eh}}, \quad (3.6)$$

where $Y(E_r)$ is the ionization yield describing the efficiency of e^-h^+ pair production and ϵ_{eh} is the average energy required to produce a single e^-h^+ pair. This is roughly 3.0 eV in Ge and 3.8 eV in Si.

The yield function $Y(E_r)$ is dependent on the interaction type, as NRs are less efficient at charge production than ERs. For ER interactions $Y(E_r) = 1$. In contrast, $Y(E_r)$ is less

than one and is dependent on energy for NR interactions. The average total phonon energy can be rewritten using equation 3.6:

$$E_t = E_r \left(1 + Y(E_r) \frac{eV_b}{\epsilon_{eh}} \right) \quad (3.7)$$

The yield function is discussed further in Section 3.3.1 where we take a look at different models and measurements of the yield.

3.2.1 QETs

Measurement of the phonon signal in SuperCDMS detectors relies on Transition Edge Sensors (TESs) to measure the (athermal) phonons before they thermalize [192]. A TES is made of a superconducting material, and the resistance steeply varies as a function of temperature near the critical temperature, T_c . Below T_c , the resistance is zero and the material is superconducting. Figure 3.1 shows how the resistance varies with temperature in a general form, indicating the narrow transition region.

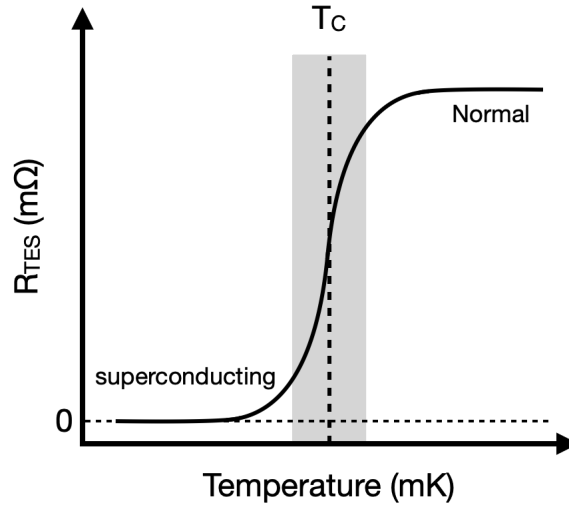


Figure 3.1: Cartoon of a typical TES resistance vs. temperature curve. Below the critical temperature, T_c , the TES is superconducting. Around T_c , the resistance sharply increases for small increases in temperature. Above the transition region, the TES is said to be in the regime of normal conduction.

The TES is cooled to a temperature below T_c , and a current is driven through it using a load resistance, which increases the TES temperature due to self heating. The bias is chosen such that the TES is in the superconducting transition region. By voltage biasing, the TES is subject to negative ElectroThermal Feedback (ETF) as long as the temperature of the TES is well above the crystal temperature. The increased resistance through the TES leads to reduced current and thus Joule heating ($P_J = V_{TES}^2/R_{TES}$, where V_{TES} is the voltage across the TES and R_{TES} is the TES resistance): the temperature of the TES quickly decreases, which then increases the Joule heating in a self-regulating manner.

TESs are not used in isolation to instrument SuperCDMS detectors. The number of phonons collected is dependent on the surface area of the TESs, but increasing the surface area also increases the heat capacity, and thus degrades the energy resolution of the sensor. Instead, the flat surfaces of the detectors are instrumented with Quasiparticle-trap-assisted Electrothermal-feedback Transition-edge-sensors (QETs). These consist of photolithographically patterned aluminium fins connected to tungsten strips with a layer of overlap called the ‘bilayer’. The large Al fin dramatically increases the surface area without increasing the heat capacity of the W TES.

The temperature of the detector is held below the critical temperature of both materials. Al has a significantly higher T_c (1.2 K) than W (varies depending on the crystal structure of the W, typically designed to be $\mathcal{O}(10 - 100 \text{ mK})$), so the Al fins are always superconducting. The W TES is biased to the transition point.

A phonon from the detector bulk with energy exceeding twice the superconducting gap energy of Al, $2\Delta_{Al} = 340 \mu\text{eV}$, incident on the detector surface is either reflected or absorbed by an Al fin. Once in the fin it can break Cooper pairs, creating quasiparticles in the Al. Alternatively, the phonon can downconvert into lower energy phonons; these can also break Cooper pairs if their energy is high enough. Phonons with too low energy to break Cooper pairs may be lost to the measurement unless they are absorbed by the TES directly. Note that thermal phonons (with energy $\mathcal{O}(\mu\text{eV})$ at these cryogenic temperatures) do not have

sufficient energy to break Cooper pairs in the Al and overlap region, and thus the detectors are not sensitive to thermal phonons unless they are directly absorbed in the TES.

As the T_c of Al exceeds that of the W strip, the superconducting gap energy is also higher for Al, and the overlapping bilayer forms a region of intermediate gap energy, i.e. $2\Delta_W < 2\Delta_{Bi} < 2\Delta_{Al}$. The quasiparticles formed in the Al diffuse into the bilayer. Here they can shed phonons, which create more quasiparticles or are lost, or undergo electron-electron scattering. Through these processes, the energy of the quasiparticles decreases. Once this happens, they become trapped in the bilayer and cannot reenter the Al. The quasiparticles diffuse into the W, where they continue losing energy, and become trapped due to an even smaller gap energy. The W TES is heated in the process, causing a change in resistance which can be measured. Figure 3.2 shows the process of the formation and diffusion of quasiparticles.

The readout of the TES signal is dependent on DC SQUIDS, which are low impedance, low noise devices which act as very sensitive magnetometers. Figure 3.3 shows a simplified schematic of the TES readout circuit. The TES is voltage biased by a voltage and bias resistor, drawing a current I_b . The voltage bias across the TES is produced by R_{sh} : as long as this resistor is much smaller than the TES resistance, most of the current goes through the shunt resistor, defining the voltage drop. Significant changes in the current in the TES branch then correspond only to marginal relative changes in shunt branch of the circuit; therefore, the bias voltage is essentially constant. The absorption of phonons from an interaction in the detector heats the TES, increasing R_{TES} and decreasing I_b . This also changes the current through an inductor L_i , which is coupled to a SQUID array. The SQUID array is biased by voltage V_{sq} and draws current I_{sq} . The change in flux through the SQUID from the change in current through L_i causes a change in the SQUID impedance and current. The change in the voltage drop across the SQUID is sensed by an amplifier. The negative input is referenced to the SQUID, and the output is coupled back to the SQUID via an inductor L_{fb} . For Soudan electronics, $L_{fb} = 0.1L_i$. For SNOLAB electronics, the

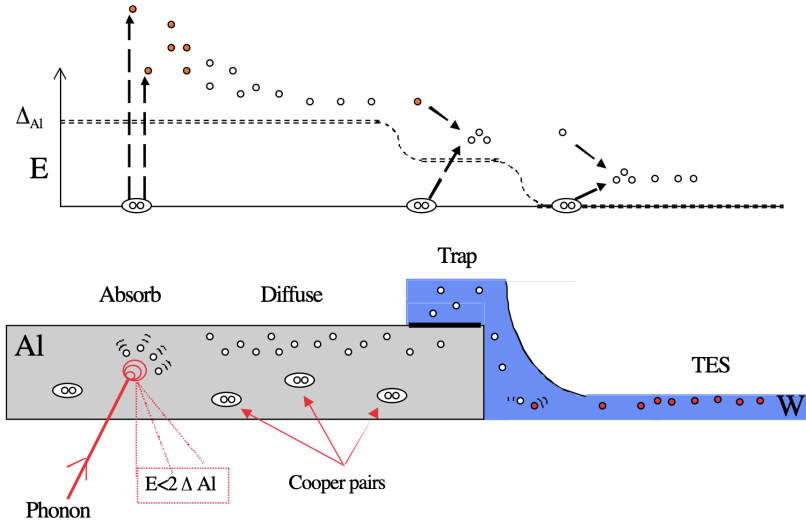


Figure 3.2: Schematic of quasiparticle trapping in a QET. Phonons from the substrate enter the superconducting aluminium fins (grey). If their energy exceeds $2\Delta_{Al}$, they can break apart Cooper pairs in the Al (paired circles), forming quasiparticles (free circles). The quasiparticles diffuse into the Al/W overlap, shedding energy as they do so. The quasiparticles become trapped, unable to travel back into the Al region with higher gap energy as shown in the energy schematic. They further diffuse into the W TES (blue), depositing their energy. The TES is heated in this process. Figure from Ref. [198].

inductor ratio is $L_i/L_{fb} = 2.4$. The output from the amplifier reaches a voltage V_o such that the current through L_{fb} compensates the change in flux through the SQUID caused by L_i . The TES current is thus converted into a voltage with an amplification factor of $R_{fb}L_i/L_{fb}$. For Soudan electronics, $R_{fb} = 1 \text{ k}\Omega$ [199], and $5 \text{ k}\Omega$ for the new SNOLAB electronics [174].

3.2.2 TES Dynamics

The following section closely follows derivations of TES response outlined in Ref. [200] and further described in theses by M. Pyle [201], S. Watkins [202], and N. Kurinsky [203].

The TES bias circuit from Figure 3.3 can be represented by the Thévenin equivalent voltage-biased circuit as shown in Figure 3.4, where we also account for any parasitic resistance, R_p . This circuit consists of a voltage $V_b = I_b R_{sh}$ with a series load resistor $R_L = R_{sh} + R_p$.

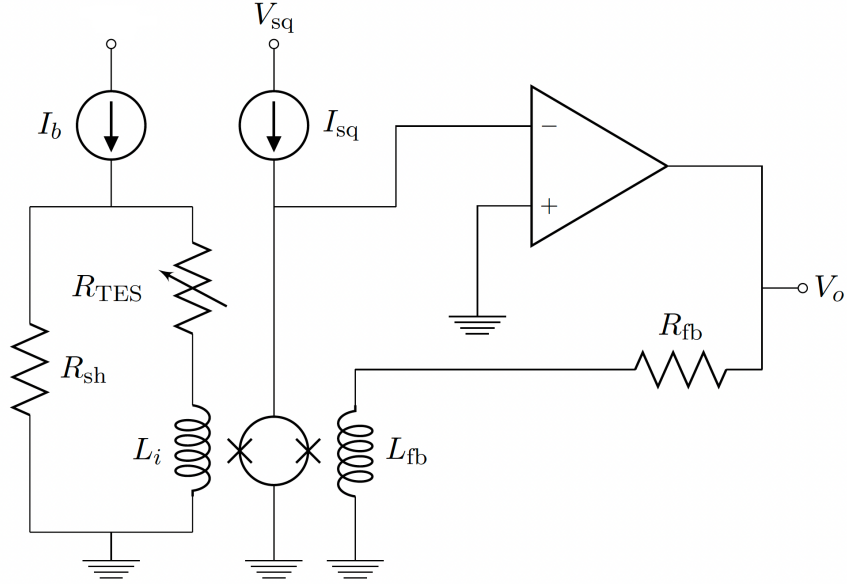


Figure 3.3: Simplified schematic of the TES readout circuit. The definitions of the bias current I_b , SQUID current I_{sq} , shunt resistance R_{sh} , TES resistance R_{TES} , feedback resistance R_{fb} , inductances L_i and L_{fb} , and voltages V_{sq} and V_o are given in the main text. Figure from Ref. [143].

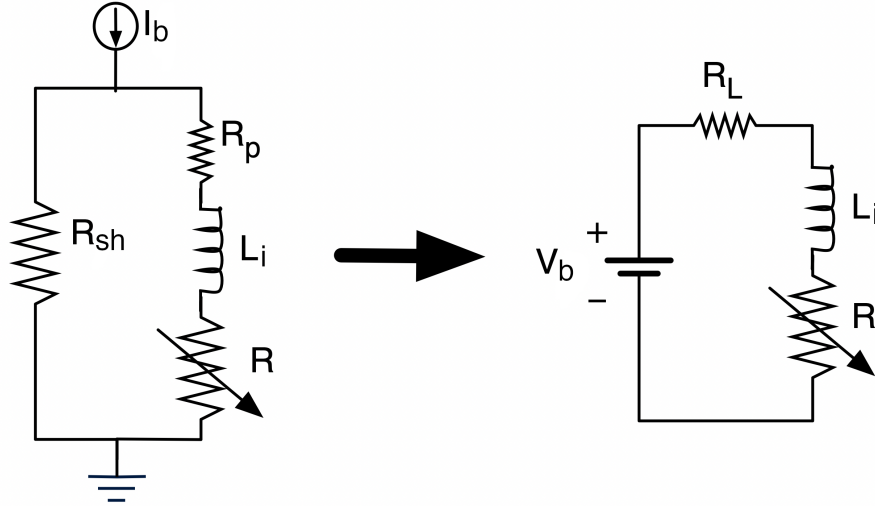


Figure 3.4: Conversion of the TES circuit (left) to the Thévenin equivalent voltage-biased circuit (right). This circuit includes a voltage $V_b = I_b R_{sh}$ with a series load resistor $R_L = R_{sh} + R_p$. Figure adapted from Ref. [201].

Two differential equations govern the response of the TES; we shall see that they are coupled because the TES resistance is dependent on both temperature and current. Given the circuit in Figure 3.4 (right), we can write an electrical differential equation which governs the time evolution of the current:

$$L \frac{dI}{dt} = V_b - IR_L - IR(T, I), \quad (3.8)$$

where L is the inductance, V_b is the bias voltage, and I is the electrical current through the TES. $R(T, I)$ is the resistance of the TES, a function of I and the TES temperature, T .

Similarly, given that the TES is cooled by a bath with temperature T_{bath} , the circuit is described by the thermal differential equation which governs the evolution of the TES temperature over time:

$$C \frac{dT}{dt} = -P_{bath} + P_J + P, \quad (3.9)$$

where C is the total heat capacity of the TES and absorber, P_{bath} is the power flowing from the TES to the bath, P_J is the Joule heating power, and P is a signal power (which may be from noise, phonons, etc.). We assume that P_{bath} follows a power law,

$$P_{bath} = K (T^n - T_{bath}^n), \quad (3.10)$$

where K and n are constants determined by thermal link to the heat bath. Typically $n = 5$ for SuperCDMS detectors. For a simple metal, $C \propto T$, though the proportionality constant is impacted during transition due to the latent heat of breaking Cooper pairs.

IV Curves

Before continuing to understand the TES response to an input signal, let us first consider its DC characteristics. Using Eq 3.8 and setting the time derivative to zero gives

$$V_b = I (R_L + R(T, I)). \quad (3.11)$$

Let us consider two extreme cases of the TES resistance, (1) when it has zero resistance in the *superconducting regime* and the temperature of the TES is well below T_c , and (2) when it is in the *normal regime* above the critical temperature. For metals at cryogenic temperatures the normal resistance is approximately constant.

When the TES is in the superconducting regime, $R_{TES} = 0$ and Eq. 3.11 simplifies to

$$V_b = I_b R_{sh} = I R_L, \quad (3.12)$$

where we have recalled that $V_b = I_b R_{sh}$. This gives us a simple linear relationship between the bias current and the TES current in the superconducting regime. The values of R_{sh} and I_b should be well known; we directly control the bias current and we measure the shunt resistance. Therefore, by measuring the TES current output and comparing it to the bias current, we can readily extract the load resistance R_L and subsequently the parasitic resistance R_p .

In the normal regime the TES now has non-zero (and approximately constant in the temperature range of concern) resistance R_n . Eq. 3.11 becomes

$$V_b = I(R_L + R_n), \quad (3.13)$$

giving a linear relationship between TES current and bias voltage in the normal regime as well.

We can measure the DC characteristics of a TES by sweeping through bias voltage

and measuring the TES current at each bias voltage. From the TES readout circuit, there is an offset in the measured output, which must be accounted for such that the linear superconducting and normal intercepts are zero. Figure 3.5 shows an example of such an IV curve where we plot the TES current vs. the bias voltage. From the slopes of the normal and superconducting regimes, we can determine R_L , R_p , and R_n using Equations 3.12 and 3.13. In the Figure we can clearly see the superconducting region where the TES current increases linearly from zero as the bias is increased. Eventually, a critical current is reached and the TES jumps from superconducting to nonzero resistance. We refer to this behaviour as ‘snapping’. The bias current required for snapping is dependent on the temperature of the TES. When increasing the bias from superconducting, there is no Joule heating and the temperature of the TES is $\sim T_{bath}$. A large current will be required to snap the TES (alternatively, if the TES is normal or in transition and the bias voltage is being dropped, the snapping point is much lower because the TES temperature larger than the bath temperature due to self heating). As we continue increasing the bias, the TES current response becomes linear again, indicating that the TES is now in the normal regime. The Figure also provides insight into a method of measuring the TES T_c : in the normal and superconducting resistance regimes the slope of the response is positive, while in the transition regime the slope is negative. By applying a low amplitude square wave (or other repeating signal) to the TES and slowly changing the TES temperature, one can observe a phase shift in the signal as the TES enters the transition regime.

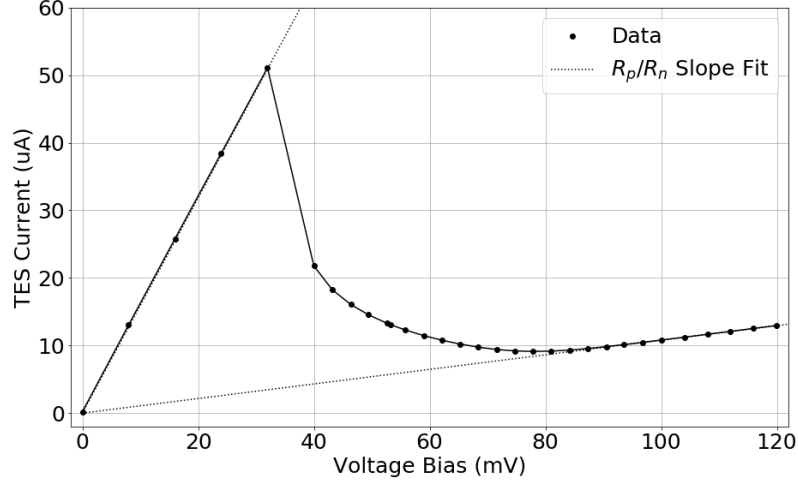


Figure 3.5: Example IV curve for a CPD detector. Fit lines to the superconducting and normal regimes are shown, from which the R_p and R_n values may be extracted.

Similarly, we can produce R_{TES} vs. V_b (RV) and P_0 vs. V_b (PV) curves as shown in Figure 3.6 using the following relationships:

$$R_{TES} = \frac{V_b}{I} - R_L \quad (3.14)$$

$$P_0 = IV_b - I^2 R_L. \quad (3.15)$$

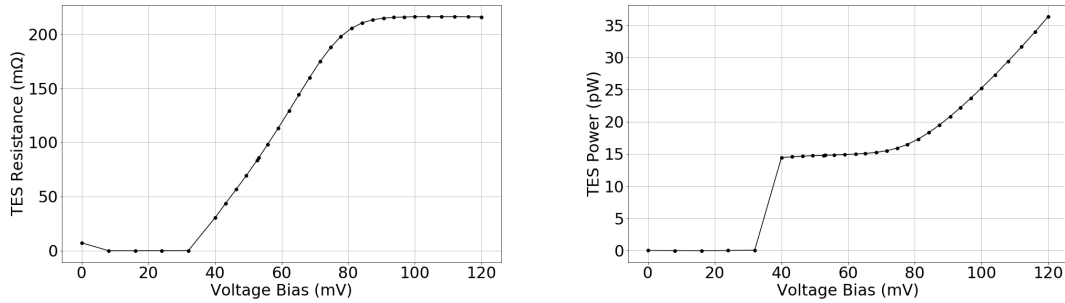


Figure 3.6: Examples of TES RV (left) and PV (right) curves for a CPD detector.

From this Figures, we can see the constant TES resistance in both the superconducting

and normal regimes, as well as the approximately constant bias power during transition.

TES Signal Response

Now we return to the response of the TES to a signal. There are several complicated nonlinear terms in the coupled differential equations which we can simplify in the small-signal limit around the equilibrium values of the resistance, temperature, and current: R_0 , T_0 , and I_0 . For small signals, the bath power can be expanded around T_0 as

$$P_{bath} \approx P_{bath,0} + G\delta T, \quad (3.16)$$

where $G = dP_{bath}/dT = nKT^{n-1}$ is the thermal conductance from the TES to the bath and $\delta T = T - T_0$. The equilibrium bath power is $P_{bath,0} = P_{J_0} + P_0$, where $P_{J_0} = I_0^2 R_0$ is the steady-state Joule power and P_0 is the steady-state signal power.

Expanding the TES resistance similarly around R_0 , T_0 , and I_0 gives

$$R(T, I) \approx R_0 + \left. \frac{\partial R}{\partial T} \right|_{I_0} \delta T + \left. \frac{\partial R}{\partial I} \right|_{T_0} \delta I, \quad (3.17)$$

where $\delta I = I - I_0$.

It is convenient to define a unitless temperature sensitivity

$$\alpha = \left. \frac{\partial \log R}{\partial \log T} \right|_{I_0} = \frac{T_0}{R_0} \left. \frac{\partial R}{\partial T} \right|_{I_0}, \quad (3.18)$$

and the current sensitivity

$$\beta = \left. \frac{\partial \log R}{\partial \log I} \right|_{T_0} = \frac{I_0}{R_0} \left. \frac{\partial R}{\partial I} \right|_{T_0}. \quad (3.19)$$

These parameters give an intuitive measure of the sensitivity of the TES resistance to changes in temperature or current. If $\alpha(\beta) = 100$ then a 1% change in the temperature (current) of the TES leads to a 100% change in the resistance. Eq. 3.17 can subsequently

be rewritten as

$$R(T, I) \approx R_0 + \alpha \frac{R_0}{T_0} \delta T + \beta \frac{R_0}{I_0} \delta I. \quad (3.20)$$

Also expanding the Joule power around R_0 , T_0 , and I_0 gives

$$P_J = I^2 R \approx P_{J_0} + 2I_0 R_0 \delta I + \alpha \frac{P_{J_0}}{T_0} \delta T + \beta \frac{P_{J_0}}{I_0} \delta I. \quad (3.21)$$

Substituting our expanded terms for the bath power and Joule power into Eq. 3.9 and rearranging terms, we have

$$C \frac{dT}{dt} = -P_{\text{bath},0} + P_{J_0} + \left(\beta \frac{P_{J_0}}{I_0} + 2I_0 R_0 \right) \delta I + G \left(\frac{P_{J_0} \alpha}{GT_0} - 1 \right) \delta T + P. \quad (3.22)$$

The term $P_{J_0} \alpha / GT_0$ appears frequently enough that Irwin defines the unitless low frequency loop gain [200]

$$\mathcal{L} \equiv \frac{P_{J_0} \alpha}{GT_0}, \quad (3.23)$$

which can be thought of as a comparison of the feedback abilities of the changes in Joule heating and the changes in thermal heat flow to the bath. Systems with large $\mathcal{L}$ will have temperature excursions damped almost exclusively by changes in Joule heating as a result of changes in the TES resistance.

Finally, the natural thermal time constant is also defined

$$\tau \equiv \frac{C}{G}, \quad (3.24)$$

and will be one of several time constants that we will have to keep in mind for understanding the TES response.

With all of these definitions in hand, Equations 3.8 and 3.9 can be rewritten for small signals as

$$\frac{d\delta I}{dt} = -\frac{R_L + R_0(1+\beta)}{L}\delta I - \frac{\mathcal{L}G}{I_0 L}\delta T + \frac{\delta V}{L}, \quad (3.25)$$

$$\frac{d\delta T}{dt} = \frac{I_0 R_0(2+\beta)}{C}\delta I - \frac{(1-\mathcal{L})}{\tau}\delta T + \frac{\delta P}{C}. \quad (3.26)$$

Equations 3.25 and 3.26 can be represented in matrix format as

$$\frac{d}{dt} \begin{bmatrix} \delta I \\ \delta T \end{bmatrix} = - \begin{bmatrix} \frac{R_L + R_0(1+\beta)}{L} & \frac{\mathcal{L}G}{I_0 L} \\ -\frac{I_0 R_0(2+\beta)}{C} & \frac{(1-\mathcal{L})}{\tau} \end{bmatrix} \begin{bmatrix} \delta I \\ \delta T \end{bmatrix} + \begin{bmatrix} \frac{\delta V}{L} \\ \frac{\delta P}{C} \end{bmatrix}. \quad (3.27)$$

We can solve the homogenous equation where $\delta P = \delta V = 0$ for the eigenvalues $\lambda_{\pm}$ and eigenvectors $\Lambda_{\pm}$ using the general solution

$$\begin{bmatrix} \delta I \\ \delta T \end{bmatrix} = A_+ \exp(-t\lambda_+) \Lambda_+ + A_- \exp(-t\lambda_-) \Lambda_-, \quad (3.28)$$

where we define $\tau_{\pm} = 1/\lambda_{\pm}$ as the characteristic time constants of the system. The eigenvectors are

$$\Lambda_{\pm} = \begin{bmatrix} \frac{1-\mathcal{L}-\lambda_{\pm}\tau}{2+\beta} \frac{G}{I_0 R_0} \\ 1 \end{bmatrix}, \quad (3.29)$$

and the corresponding eigenvalues are

$$\lambda_{\pm} \equiv \frac{1}{\tau_{\pm}} = \frac{1}{2} \left[\frac{1}{\tau_{el}} + \frac{1}{\tau_I} \pm \sqrt{\left(\frac{1}{\tau_{el}} - \frac{1}{\tau_I} \right)^2 - 4 \frac{\mathcal{L}R_0(2+\beta)}{L\tau}} \right], \quad (3.30)$$

where we have defined two new time constants

$$\tau_{el} \equiv \frac{L}{R_L + R_0(1 + \beta)}, \quad (3.31)$$

$$\tau_I \equiv \frac{\tau}{1 - \mathcal{L}}. \quad (3.32)$$

The first, τ_{el} , is the electronic time constant of the system, and τ_I is the constant-current time constant. When $\mathcal{L} > 1$, τ_I takes negative values. This is not concerning when the TES is voltage biased as τ_I is not an intrinsic time constant (as the current is not constant). As an aside, when current-biased, τ_I is an intrinsic time constant, and negative values (when $\mathcal{L} > 1$) represent thermal runaway from positive ETF. This places severe constraints on $\mathcal{L}$ and demonstrates that voltage-biasing is generally a superior option. As previously mentioned, SuperCDMS operates TESs in voltage-biased mode.

It is interesting to consider the relationship between the small-signal parameters and the physical time constants $\tau_{\pm}$ in various limiting cases. In the low inductance limit (as L goes to 0 and $\tau_+ \ll \tau_-$) we have

$$\tau_{el} \equiv \lim_{L \rightarrow 0} \tau_+ = \frac{L}{R_L + R_0(1 + \beta)}, \quad (3.33)$$

$$\tau_{eff} \equiv \lim_{L \rightarrow 0} \tau_- = \tau \frac{1 + \beta + R_L/R_0}{1 + \beta + R_L/R_0 + (1 - R_L/R_0)\mathcal{L}}. \quad (3.34)$$

Here τ_{el} is the electrical time constant that we have just defined, and τ_{eff} is the zero-inductance effective thermal time constant. If we consider τ_{eff} in the limit as $\beta \rightarrow 0$ and assume that the TES resistance is significantly higher than the load resistance, we have

$$\lim_{\substack{\beta \rightarrow 0 \\ R_L \ll R_0}} \tau_{eff} = \frac{\tau}{1 + \mathcal{L}}. \quad (3.35)$$

This implies that increasing the loop gain $\mathcal{L}$ can lead to faster time constants, which is desirable for improving the energy resolution of our detectors. This will be seen when we

explore the baseline energy resolution in further detail in Section 3.2.3.

With this, let us return to our general solution and determine the response in the ideal operational case where $\delta V = 0$. Phonons with energy E couple in to the system with a delta function response

$$\delta P(t) = E\delta(t). \quad (3.36)$$

We assume initial conditions $\delta I(0) = 0$ and $\delta T(0) = 0$ and solve for $A_{\pm}$ to determine the response to an instantaneous power signal. For a signal power $\delta P(t)$, we have the current response

$$\delta I(t) = \frac{(\tau_I/\tau_+ - 1)(\tau_I/\tau_- - 1)}{\tau_I^2(1/\tau_+ + 1/\tau_-)} \frac{E}{(2 + \beta)I_0R_0} \left(e^{-t/\tau_+} - e^{-t/\tau_-} \right). \quad (3.37)$$

It is also of interest to consider the frequency response of the TES. As described in Ref. [202], we can switch to frequency space using a Fourier transform, and Equations 3.25 and 3.26 become

$$i\omega\delta I = -\frac{R_L + R_0(1 + \beta)}{L}\delta I - \frac{\mathcal{L}G}{I_0L}\delta T + \frac{\delta V}{L}, \quad (3.38)$$

$$i\omega\delta T = \frac{I_0R_0(2 + \beta)}{C}\delta I - \frac{(1 - \mathcal{L})}{\tau}\delta T + \frac{\delta P}{C}. \quad (3.39)$$

Similar to the procedure we outlined in the time domain, Equations 3.38 and 3.39 can be represented in matrix format as

$$\begin{bmatrix} i\omega + \frac{R_L + R_0(1 + \beta)}{L} & \frac{\mathcal{L}G}{I_0L} \\ -\frac{I_0R_0(2 + \beta)}{C} & i\omega + \frac{(1 - \mathcal{L})}{\tau} \end{bmatrix} \begin{bmatrix} \delta I \\ \delta T \end{bmatrix} = \begin{bmatrix} \frac{\delta V}{L} \\ \frac{\delta P}{C} \end{bmatrix}. \quad (3.40)$$

Defining the matrix as M , we can describe the current and temperature responses to small power or voltage excitations by multiplying by the inverse, M^{-1} :

$$\begin{bmatrix} \delta I \\ \delta T \end{bmatrix} = M^{-1} \begin{bmatrix} \frac{\delta V}{L} \\ \frac{\delta P}{C} \end{bmatrix}. \quad (3.41)$$

The complex admittance (and its reciprocal the complex impedance) is defined by a single matrix element

$$\partial I / \partial V \equiv \delta I / \delta V = M_{1,1}^{-1} / L, \quad (3.42)$$

which can be written as

$$\frac{\partial I}{\partial V}(\omega) = \frac{1}{R_L + i\omega L + R_0(1 + \beta) + \frac{R_0 \mathcal{L}}{1 - \mathcal{L}} \frac{2 + \beta}{1 + i\omega\tau/(1 - \mathcal{L})}} = \frac{1}{Z_{circ}(\omega)}. \quad (3.43)$$

For brevity and later use, we have defined the circuit complex impedance

$$Z_{circ}(\omega) \equiv R_L + i\omega L + Z_{TES}(\omega), \quad (3.44)$$

where we have also defined the TES complex impedance

$$Z_{TES}(\omega) \equiv R_0(1 + \beta) + \frac{R_0 \mathcal{L}}{1 - \mathcal{L}} \frac{2 + \beta}{1 + i\omega\tau/(1 - \mathcal{L})}. \quad (3.45)$$

The power-to-current transfer function is similarly determined by setting $\delta V = 0$ and examining the response to a power signal δP . This too is determined by a single matrix element

$$\frac{\partial I}{\partial P} \equiv \frac{\delta I}{\delta P} = \frac{M_{12}^{-1}}{C}. \quad (3.46)$$

The power-to-current transfer function is strongly related to the complex impedance;

in fact, once the complex impedance is determined, we can readily calculate the power-to-current transfer function.

$$\frac{\partial I}{\partial P}(\omega) = -\frac{1}{Z_{circ}(\omega)I_0} \frac{Z_{TES}(\omega) - R_0(1 + \beta)}{R_0(2 + \beta)} \quad (3.47)$$

We cannot directly measure the power-to-current transfer function with our detectors, but the relationships show that we can use voltage impulses to instead measure the complex admittance and TES parameters, and through this calculate $\partial I/\partial P$ using Equations 3.43 and 3.47.

As an aside that will be useful for us later in Section 3.2.3 when we contemplate TES noise, we come back to our previously defined TES complex impedance, $Z_{TES}(\omega)$, and now consider that for ETF we have $\delta P = -I_0\delta V$. We can write the TES complex admittance in terms of the circuit complex admittance and the power-to-current transfer function

$$\frac{dI}{dV_{TES}}(\omega) \equiv \frac{1}{Z_{TES}(\omega)} = M_{1,1}^{-1}/L - \frac{I_0}{C}M_{1,2}^{-1} = \frac{\partial I}{\partial V} - I_0\frac{\partial I}{\partial P}. \quad (3.48)$$

This will allow us to convert the TES Johnson noise from voltage noise into current noise (see Section 3.2.3).

The Block Model

Thus far, the derivations provided have been under the assumption of the most basic model of the thermal circuit: a single heat capacity, or block, connected to the thermal bath with one thermal conductance. However, real detectors may have additional complexities, and two-block and even three-block models can be considered. Each block describes a separate thermal body. These models are described in some detail in Ref. [204] and a derivation of a two-block model can be found in N. Kurinsky's thesis [203].

Here we consider only a two-block model, which introduces a second thermal differential equation to represent the additional thermal body. The primary effect that we are concerned

with is the introduction of an additional time constant. Figure 3.7 shows a possible two-block model with an additional heat capacitance connected between the TES and the bath. The TES and extra heat capacity are denoted C_{tes} and C_1 , respectively. Ref. [204] describes this configuration as the ‘intermediate’ block model. The heat conductance from the TES to the intermediate-block is $g_{tes,1}$ and there is an additional conductance $g_{1,b}$ between the second block and the bath. In this model the temperature of the additional block is not equal to the temperature of the TES. This additional thermal block could be the absorber, which is nominally assumed to be at the same temperature as the bath in the single block model.

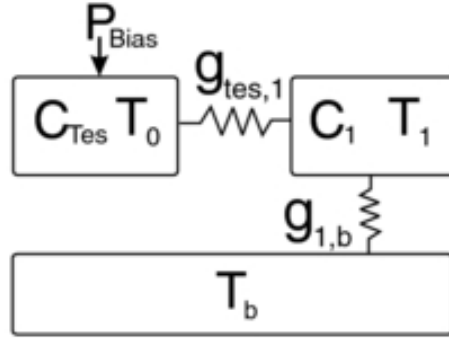


Figure 3.7: Two block TES model where there is an additional heat capacity connected between the TES and the bath, from Ref. [204]. This is specifically referred to as the ‘intermediate’ block model. The second thermal body could be the absorber crystal, for example.

The TES complex impedance in this model (neglecting the impedance of the rest of the circuit) becomes

$$Z_{TES,2}(\omega) = R_0(1 + \beta) + \frac{\mathcal{L}_I}{1 - \mathcal{L}_I} \frac{R_0(2 + \beta)}{1 + \frac{i\omega\tau_I}{1 - \mathcal{L}_I} - \frac{g_{tes,1}(T_1)}{(g_{tes,1}(T_1) + g_{1,b})(1 - \mathcal{L}_I)} / (1 + i\omega\tau_3)}, \quad (3.49)$$

where the *effective loop gain* $\mathcal{L}_I$ in this case differs from the loop gain of the one block

model, $\mathcal{L}_I = P_0\alpha/(g_{tes,1}(T_0)T_0)$. We have defined two new time constants,

$$\tau_I = \frac{C_{tes}}{g_{tes,1}(T_0)(1 - \mathcal{L}_I)}, \quad (3.50)$$

$$\tau_3 = \frac{C_1}{g_{tes,1}(T_1) + g_{1,b}}. \quad (3.51)$$

In the limit that $g_{1,b} \gg g_{tes,1}$, the extra block becomes indistinguishable and we arrive back at the impedance for the one block model with heat capacity C_{tes} . This is the limit that is typically assumed for SuperCDMS detectors.

If we consider the complex impedance of the whole circuit, we can write this in the form

$$Z_{circ,2}(\omega) = A(1 + i\omega\tau_{el}) + \frac{B}{1 + i\omega\tau_I + C/(1 + i\omega\tau_3)}, \quad (3.52)$$

where A , B , and C are constants. Having three time constants, this *3-pole model* may be fit to complex impedance measurements and the Irwin parameters may be determined from the fit parameters $(A, B, C, \tau_1, \tau_2, \tau_3)$.

For the remainder of this chapter, we will use our simple one block model, but we will return to the two-block/3-pole model in later chapters.

Absorbed Energy in the High Loop Gain Limit

In the high loop gain limit, or strong feedback limit, the circuit response is fast enough that the temperature change is negligible. The square-wave power going to the bath does not vary, and thus the power entering the TES approaches the reduction in Joule power due to ETF. Furthermore, we recall that $\tau_- \propto \mathcal{L}$, and thus when $\mathcal{L}$ is large, the TES response time is faster.

Consider a TES operating in the high $\mathcal{L}$ limit as it absorbs some energy E , increasing the resistance and changing the current flowing through the TES. The change in Joule power is equal in magnitude and opposite in sign to the power absorbed by the TES, $dP_{abs} = -dP_J$.

For a small change in current, $\delta I(t)$, the energy absorbed by the TES is

$$E_{abs} = - \int_0^\infty \Delta P_J dt \quad (3.53)$$

$$= - \int_0^\infty [V_{TES}(t)(I_0 + \delta I(t)) - V_{TES,0}I_0] dt. \quad (3.54)$$

For sign conventions, note that a negative bias gives negative V_b and I_0 , and positive $\delta I(t)$. A positive bias gives positive V_b and I_0 , and negative $\delta I(t)$. From our circuit in Figure 3.4, the TES voltage is

$$V_{TES}(t) = V_b - (I_0 + \delta I(t)) R_L - L \frac{d\delta I(t)}{dt}, \quad (3.55)$$

$$V_{TES,0} = V_b - I_0 R_L \quad (3.56)$$

thus we can write the absorbed energy as

$$E_{abs} = - \int_0^\infty \left[(V_b - 2I_0 R_L) \delta I(t) - \delta I(t)^2 R_L - L \delta I(t) \frac{d\delta I(t)}{dt} \right] dt. \quad (3.57)$$

Note that the last term integrates to $L \delta I(t)^2|_0^\infty = 0$, as there is no net change to the energy stored in the inductor, $LI^2/2$. Thus the total energy absorbed by the TES in the high $\mathcal{L}$ limit is

$$E_{abs} = - \int_0^\infty [(V_b - 2I_0 R_L) \delta I(t) - \delta I(t)^2 R_L] dt. \quad (3.58)$$

Measuring and Modelling TES Response

This section also closely follows the description outlined in Ref. [202], where S. Watkins discusses how groups like SuperCDMS measure the response to sine-wave or square-wave impulses. We voltage bias the TES and add a repeating signal (either square-wave or sine-wave), deconvolving the measured response to extract the measured complex admittance/complex impedance.

For a signal $V_s(t)$ with Fourier transform $\tilde{V}_s(f)$, we have (noting that $f = \omega/2\pi$) through

the convolution theorem:

$$\tilde{V}_s(f) \frac{\partial I}{\partial V}(\omega) = \tilde{I}_{TES}(f). \quad (3.59)$$

Using Eq. 3.59, we can take our measured response and solve for the complex admittance

$$\frac{\partial I}{\partial V}(\omega) = \frac{\tilde{I}_{TES}(f)}{\tilde{V}_s(f)}. \quad (3.60)$$

Square waves are better than sine waves for such an analysis because the Fourier transform of a sine wave is a Dirac delta – one would have to sweep through a variety of wave frequencies in order to measure the full frequency response. This is a time consuming and ultimately unnecessary endeavour; the Fourier transform of a square-wave is defined across many frequencies and therefore using a single square-wave measurement provides information across a large range of frequencies. This is the method used by SuperCDMS for measuring the TES response.

We can explicitly demonstrate the convenience of the square-wave signal measurement. The Fourier series for a square-wave with peak-to-peak voltage amplitude A_{sg} and signal frequency f_{sg} is given by

$$V_s(t) = A_{sg} \frac{2}{\pi} \sum_{n=1,3,5,\dots}^{\infty} \frac{\sin(2\pi n f_{sg} t)}{n}. \quad (3.61)$$

Taking the discrete Fourier transform and using the relationship to the complex admittance gives

$$\frac{\partial I}{\partial V}(\omega) = \begin{cases} \frac{\tilde{I}_{TES}(f)}{A_{sg} \frac{2}{n\pi}} & \omega = 2\pi n f_{sg} \text{ for } n = 1, 2, 3, \dots, \\ 0 & \text{otherwise.} \end{cases} \quad (3.62)$$

The QETpy package [205] was developed by S. Watkins and C. Fink for analysis of QET parameters using the procedure described above. Models for both the one block (2-pole) and two-block (3-pole) thermal models are implemented and can be used to fit to the

complex admittance measurements.

3.2.3 TES Noise and Baseline Energy Resolution

There are several noise sources of importance which we can understand and model; this will also allow us to estimate the baseline energy resolution of our detectors and its dependence on parameters that are under our control.

Thermodynamic noise sets fundamental noise limits on the noise equivalent power, and subsequently the baseline energy resolution, of the TES. These sources include Johnson (or Nyquist) noise [206, 207], which are thermodynamic fluctuations associated with electrical resistance, and Thermal Fluctuation Noise (TFN) noise, also called phonon noise. Other noise sources will further degrade the TES sensitivity, including SQUID noise.

We consider three sources of thermodynamic noise as well as the SQUID noise: Johnson noise in the passive components (i.e. the load) of the TES circuit (includes the shunt resistor as well as parasitic resistance), Johnson noise in the TES, and TFN noise across the thermal link between the TES and the bath. The noise is given as power, current, or voltage-referenced noise (S_P , S_I , or S_V) in units of W^2/Hz , A^2/Hz , or V^2/Hz , respectively.

Load Johnson Noise

In general, the voltage noise for a resistor R at temperature T is frequency independent and can be written as [200]

$$S_V = 4k_b T R. \quad (3.63)$$

The passive or load components of the TES circuit consist of the shunt resistor as well as any parasitic resistance, which can be from various sources. The voltage Johnson noise for the load components can be taken as the sum over the various components

$$S_{V_{load}} = 4k_b \sum_{load} T_i R_i. \quad (3.64)$$

We can simplify this by writing an effective load temperature R_L as in Refs. [201, 203], defined as

$$T_L = \frac{\sum T_i R_i}{R_0}. \quad (3.65)$$

We use the complex admittance of the circuit to convert the voltage noise to current noise and use the newly defined load temperature:

$$S_{I_{load}} = 4k_b T_L I_0^2 R_0 \left| \frac{\partial I}{\partial V}(\omega) \right|^2. \quad (3.66)$$

The power noise of the load resistance can also be determined using the current-to-power transfer function

$$S_{P_{load}} = \frac{4k_b T_L I_0^2 R_0}{\mathcal{L}^2} ((1 - \mathcal{L})^2 + \omega^2 \tau^2) \quad (3.67)$$

$$= \frac{4k_b T_L T_0 G}{n \mathcal{L}^2} \left(1 - \left(\frac{T_b}{T_0} \right)^n \right) ((1 - \mathcal{L})^2 + \omega^2 \tau^2), \quad (3.68)$$

where we have used that

$$I_0^2 R_0 = P_0 = \frac{G T_0}{n} \left(1 - \left(\frac{T_b}{T_0} \right)^n \right). \quad (3.69)$$

TES Johnson Noise

The TES Johnson noise is more complicated than the Johnson noise for the load components because the temperature and current dependence creates nonlinear effects. Irwin and Hilton [200] demonstrate an approximation using nonlinear Markov fluctuation-dissipation techniques for nonlinear resistors by R. L. Stratonovich [208], neglecting terms $\mathcal{O}(V^2)$:

$$S_{V_{TES}} \approx 4k_b T_0 R_0 (1 + \beta)^2. \quad (3.70)$$

This relationship is similar to the general Johnson voltage noise for a simple resistor, but we see that the TES voltage noise is increased by the current sensitivity, β . We convert

the voltage noise to current noise using the complex impedance of the TES. Recall that we showed in Eq. 3.48 that we can write this as

$$\frac{dI}{dV_{TES}} = \frac{\partial I}{\partial V}(\omega) - I_0 \frac{\partial I}{\partial P}(\omega). \quad (3.71)$$

Thus the conversion of the TES voltage noise to current noise is given by

$$S_{I_{TES}}(\omega) = 4k_b T_0 R_0 (1 + \beta)^2 \left| \frac{\partial I}{\partial V}(\omega) - I_0 \frac{\partial I}{\partial P}(\omega) \right|^2. \quad (3.72)$$

It will also be useful to consider the TES power noise, which we calculate using the current-to-power transfer function as we did with the load Johnson noise:

$$S_{P_{TES}}(\omega) = \frac{4k_b T_0 R_0 I_0^2 (1 + \beta)^2}{\mathcal{L}^2} (1 + \omega^2 \tau^2) \quad (3.73)$$

$$= \frac{4k_b T_0^2 G (1 + \beta)^2}{n \mathcal{L}^2} \left(1 - \left(\frac{T_{bath}}{T_0} \right)^n \right) (1 + \omega^2 \tau^2). \quad (3.74)$$

Thermal Fluctuation Noise

The TFN accounting for the exchange of energy by phonons across the thermal conductance, G , is flat in power space across all frequencies

$$S_{P_{TFN}} = 4k_b T_0^2 G F_{TFN}(T_0, T_{bath}), \quad (3.75)$$

where $F_{TFN}(T_0, T_{bath})$ is a unitless function defined in Ref. [200]. The function is based on results from Ref. [209], and is dependent on the thermal conductance exponent, n , and whether the mean-free-path of the phonons is large or small compared to the length of the heat link. Explicitly, we have the following for ballistic and diffuse phonons:

$$F_{TFN}(T_0, T_{bath}) = \begin{cases} \frac{(T_{bath}/T_0)^{n+1} + 1}{2} & \text{ballistic phonons} \\ \frac{n}{2n+1} \frac{(T_{bath}/T_0)^{2n+1} - 1}{(T_{bath}/T_0)^n - 1} & \text{diffuse phonons.} \end{cases}$$

This function typically takes values from 0.5 to 1 and includes the correction to the TFN Power Spectral Density (PSD) due to nonlinear thermal conductance. The current-noise fluctuations from TFN are calculated using the current-to-power transfer function

$$S_{I_{TFN}}(\omega) = 4k_b T_0^2 G_{TFN}(T_0, T_{bath}) \left| \frac{\partial I}{\partial P}(\omega) \right|^2. \quad (3.76)$$

SQUID Noise

The noise from our readout electronics is dominated by the SQUID noise. The noise can be generalized by a simplified model of $1/f$ noise [203]

$$S_{I_{SQUID}}(\omega) = C_{SQUID} \left[1 + \left(\frac{\omega_{SQUID}}{\omega} \right)^{n_{SQUID}} \right]^2, \quad (3.77)$$

where $I_{SQUID,\infty}$ is a constant characterizing the white noise at high frequency, ω_{SQUID} determines the characteristic frequency of the $1/f$ behaviour, and n_{SQUID} is an exponent generalizing the $1/f$ behaviour.

Total Noise

We combine all of the aforementioned sources of noise to determine the total expected TES noise spectrum in transition

$$S_{I_{tot}}(\omega) = S_{I_{load}}(\omega) + S_{I_{TES}}(\omega) + S_{I_{TFN}}(\omega) + S_{I_{SQUID}}(\omega). \quad (3.78)$$

It is useful to consider the noise in the superconducting and normal regimes, in which the noise is less complex because there is no TFN noise. Note that the SQUID noise is not regime-dependent as there is no dependence on the TES itself.

In the normal regime, we assume that the temperature does not change appreciably from T_0 and use the normal complex impedance, $Z_N(\omega)$, to convert the voltage noise to current

noise. This can be written as $Z_N(\omega) = R_L + R_N + i\omega L$. In this regime, the SQUID noise dominates and can be readily modelled. The total normal current noise is

$$S_{I_{tot}}^N(\omega) = S_{I_{load}}^N(\omega) + S_{I_{TES}}^N(\omega) + S_{ISQUID}(\omega) \quad (3.79)$$

$$= \frac{4k_b T_L R_0}{|Z_N(\omega)|^2} + \frac{4k_b T_0 R_N}{|Z_N(\omega)|^2} + S_{ISQUID}(\omega). \quad (3.80)$$

In the superconducting regime, the TES Johnson noise is zero as the resistance of the TES is zero. The complex impedance in this regime is $Z_{SC}(\omega) = R_L + i\omega L$, and we can write the total noise as

$$S_{I_{tot}}^{SC}(\omega) = S_{I_{load}}^{SC}(\omega) + S_{ISQUID}(\omega) \quad (3.81)$$

$$= \frac{4k_b T_L R_L}{|Z_{SC}(\omega)|^2} + S_{ISQUID}(\omega). \quad (3.82)$$

By measuring and modelling the noise in the normal and superconducting regimes, we can determine the SQUID noise parameters and also determine the effective load temperature, T_L . This workflow is implemented by S. Watkins and C. Fink in the aforementioned QETpy package.

Baseline Energy Resolution

The baseline energy resolution of the TES is the expected standard deviation of zero-energy fluctuations; we have previously alluded to the fact that this is dependent on τ_- . The expected baseline energy resolution can be estimated from the TES power noise. We have shown above the expression for the total current noise, which we can convert to the TES power noise by using the power-to-current transfer function

$$S_{P_{tot}}(\omega) = S_{I_{tot}} \left| \frac{\partial I}{\partial P}(\omega) \right|^{-1}. \quad (3.83)$$

For a Dirac delta energy deposit, the commonly derived expression for the energy resolution is

$$\sigma_E = \left[\int_0^\infty \frac{d\omega}{2\pi} \frac{4}{S_P(\omega)} \right]^{-1/2}. \quad (3.84)$$

However, SuperCDMS detectors are not in the Dirac delta energy deposit regime because of the QETs, which have a characteristic collection time of athermal phonons, τ_{ph} . This complicates the calculation of the energy resolution. In addition, the collection of phonons is imperfect, and some of the energy is lost to the measurement. This effect is parameterized by the phonon-collection efficiency, ϵ_{ph} , and is detector dependent.

Assuming a single phonon-collection time, we can write the signal model in the frequency domain as

$$p(\omega) = \frac{\epsilon_{ph}}{1 + i\omega\tau_{ph}}. \quad (3.85)$$

In this case, the baseline energy resolution can be written in terms of $p(\omega)$:

$$\sigma_E = \left[\int_0^\infty \frac{d\omega}{2\pi} \frac{4 |p(\omega)|^2}{S_P(\omega)} \right]^{-1/2}. \quad (3.86)$$

We will calculate the expected energy resolution for SuperCDMS TES and QET design. We ignore the SQUID noise, which is not intrinsic to the TES design. Grouping all of the thermal fluctuation power noise (this includes Johnson noise), we can combine terms and write

$$\begin{aligned}
S_{P_{tot}}(\omega) &= S_{P_{load}}(\omega) + S_{P_{TES}}(\omega) + S_{P_{TFN}}(\omega) \\
&= \left(\left[\frac{T_L}{T_c} \left(1 + \frac{\omega^2 \tau^2}{(1 - \mathcal{L})^2} \right) + \frac{(1 + \beta)^2 (1 + \omega^2 \tau^2)}{\mathcal{L}^2} \right] \frac{\left(1 - \left(\frac{T_b}{T_c} \right)^n \right)}{n} \right. \\
&\quad \left. + F_{TFN} \right)
\end{aligned} \tag{3.87}$$

where we have written $F_{TFN} = F_{TFN}(T_c, T_{bath})$ for brevity in this rather lengthy equation. Note that we are also assuming that $T_0 \approx T_c$. We can further rewrite this in a more palatable form which will also be easier to integrate in Eq. 3.84 as

$$S_{P_{tot}} = 4k_b T_c^2 G \left[F_{TFN} + \left(\frac{T_L}{T_c} + \frac{(1 + \beta)^2}{\mathcal{L}^2} \right) \frac{\left(1 - \left(\frac{T_b}{T_c} \right)^n \right)}{n} \right] (1 + \tau_*^2 \omega^2) \tag{3.88}$$

where we have defined a new TES time constant

$$\tau_*^2 = \frac{\frac{T_L}{T_c(1 - \mathcal{L})^2} + \frac{(1 + \beta)^2}{\mathcal{L}^2}}{\frac{n F_{TFN}}{\left(1 - \left(\frac{T_b}{T_c} \right)^n \right)} + \frac{T_L}{T_c} + \frac{(1 + \beta)^2}{\mathcal{L}^2}} \tau^2. \tag{3.89}$$

With this, the energy resolution can be analytically determined

$$\sigma_E^2 = \frac{4k_b T_c^2 G}{\epsilon_{ph}^2} \left[F_{TFN} + \left(\frac{T_L}{T_c} + \frac{(1 + \beta)^2}{\mathcal{L}^2} \right) \frac{\left(1 - \left(\frac{T_b}{T_c} \right)^n \right)}{n} \right] (\tau_* + \tau_{ph}). \tag{3.90}$$

It is interesting to examine the dependence of the baseline energy resolution on T_c . We note that

$$\sigma_E \propto \sqrt{T_c^2 G} \propto \sqrt{T_c^{n+1}} = T_c^3, \tag{3.91}$$

where we have used that typically $n = 5$. We see a very strong dependence on the T_c of the TES. It is of great interest to develop and fabricate detectors with low T_c .

From this result we can see that there are three main parameters under our control that we may adjust to improve the energy resolution: T_c , ϵ_{ph} , and τ_{ph} . The critical temperature is material dependent; SuperCDMS W films are a mixture of two crystalline phases of W, α and β . These have T_c values of 15 mK and ~ 1 K, thus the T_c of the mixture can be highly customizable. The phonon-collection time is related to physical parameters

$$\tau_{ph} \propto \frac{1}{f_{Al} p_{abs}} \frac{h_{crystal}}{v_{ph}}, \quad (3.92)$$

where f_{Al} is the fractional coverage of the detector surface by the aluminium absorber fins, p_{abs} is the probability that a phonon incident on the aluminium is absorbed, $h_{crystal}$ is the thickness of the crystal, and v_{ph} is the phonon velocity in the substrate. Therefore the phonon-collection fall-time is dependent on the detector dimensions and coverage by the Al fins. Note that τ_{ph} in Si is less than that in Ge because of both the higher sound speed and higher p_{abs} due to better impedance matching at the Si/Al boundary. This drives the difference in energy resolution between SuperCDMS SNOLAB Si and Ge detectors.

The phonon-collection efficiency ϵ_{ph} is also dependent on the physical design of the QETs. It can be maximized by matching the Al fin length with the quasiparticle diffusion length as well as increasing the amount of Al/W overlap, which ensures that the quasiparticles make it to the TES. However, there are several loss mechanisms which limit the phonon-collection efficiency [210]. M. Pyle outlines many of these loss mechanisms in his thesis [201] and categorizes them into several stages. He also provides estimates of the efficiencies for several of these stages.

The first stage of loss is from the downconversion of the initial interaction phonons into ballistic phonons which may not be energetic enough to break Cooper pairs in the Al fins. Sub gap phonons with energy $< 2\Delta_{Al}$ will instead eventually thermalize in the crystal, their energy being lost to the measurement. Pyle estimates this efficiency to be 98% for Si and 94% for Ge. The efficiency is lower in Ge because of the lower phase velocity, and therefore a larger percentage of sub gap energy states. This property is fundamental to the crystal,

and cannot be changed for a given material.

There can be additional losses from passive components made from Al on the detector surface, all of which collect and sink athermal phonons. This efficiency can be estimated as the ratio of Al in the fins to that in the passive components, and depends highly on the detector design. The amount of passive Al has been dramatically reduced from CDMS II to more recent detectors, increasing the efficiency (41% – 58% in CDMS II vs. 78% SuperCDMS Soudan [201]).

The next stage of loss occurs once phonons have entered the Al fins and create a cascade of quasiparticle production in the Al. Phonon–electron and electron–phonon processes dominate over electron–electron processes in the superconductor. The initial quasiparticles scatter and generate phonons and more quasiparticles at the superconducting gap energy of Al. Phonons produced with energy $< 2\Delta_{Al}$ are emitted into the substrate, and are lost to the measurement. Phonons with energy $> 2\Delta_{Al}$ will eventually break another Cooper pair. Therefore, if all of the electronic kinetic energy goes into producing a single phonon, the cascade of quasiparticle production is very efficient. However, if the energy goes into producing many sub gap phonons, the process is inefficient. Estimates based on the phonon density of states put this efficiency at 55% [201].

There is a further reduction in efficiency due to quasiparticle recombination and local trapping as the quasiparticles in the Al diffuse towards the W. The Al quasiparticles relax to the bandgap edge, and then diffuse randomly until either recombining into Cooper pairs, trapping in a localized region with lower T_c , or making their way into the W. The trapping rates depend on the quality of the Al film, and must be experimentally determined.

Finally, there is inefficiency from the transmission coefficient from Al to W. If there is a large impedance between the two materials, i.e. the Al and W are not well connected, T_c varies sharply with position and a high energy quasiparticle can readily transfer energy to other electronic excitations. However, if the materials are well connected, T_c varies slowly and a large fraction of the energy will end up as sub gap phonons. These immediately

migrate back to the crystal and are lost to the measurement. This leads to two competing requirements of the impedance: the Al/W coupling must be balanced such that it is large enough that the transmission of quasiparticles is large, but small enough so that the W cascade is efficient and the energy loss from sub gap phonons is minimized.

3.3 Charge Signals

Having spent considerable time discussing phonon signals and their measurement, we now turn our attention to the other important signal for SuperCDMS detectors: ionization. In this section, we cover the charge readout, processes which can interfere with proper charge propagation, and a discussion of the yield function first mentioned in Section 3.2.

Note that not all SuperCDMS detectors are instrumented for charge readout, which we will see when we look at the specific detector designs in Section 3.4. Detectors designed for charge readout are instrumented with charge electrodes on the surface. As described in Section 3.2, an applied detector bias voltage drifts electrons and holes to opposite sides of the detector. Charges liberated in an interaction in the detector will drift and create image charges in the surface electrodes. The induced current produced by a charge q moving with velocity $\mathbf{v}$ in an electric field $\mathbf{E}$ is given by the Shockley-Ramo theorem [211, 212]

$$\mathbf{I} = q\mathbf{E} \cdot \mathbf{v}. \quad (3.93)$$

It is the induced image charge that is read out, not the charges produced in the interaction, but the measured signal is proportional to the amount of charge liberated in the initial interaction. There are several mechanisms which complicate the charge carrier propagation and can result in either deficient or excess charge measurement.

When an interaction occurs near the electrodes, some charges may drift to the grounded phonon sensors instead of the opposite face electrode due to the electric field configuration near the surface. When this occurs, the event cannot be properly characterized due to the

asymmetric charge collection. Depositing a thin layer of amorphous silicon between each electrode and the detector surface significantly reduces this back-diffusion, though there is still a $\sim 10\mu\text{m}$ ‘dead layer’ in which events still have incomplete charge collection [132]. However, this also provides a way of identifying these surface events. Many interactions at the detector surfaces are from radioactive contaminants implanted on the detector surface, and these may be identified and rejected. As previously mentioned, this is particularly important because the reduced charge signal amplitude may lead to the misidentification of surface ER events as NR events.

There are processes which may either increase or decrease the number of charges propagating across the detector. In impact ionization (II), the drifting charges liberate loosely bound charges, increasing the number of charges traversing the detector. In this case, the measured energy increases due to the additional charge carriers. Alternatively, the measured energy may decrease if the propagating charge falls into a charge vacancy in the crystal, reducing the total number of charges. This is a process called charge trapping (CT). Figure 3.8 shows a schematic of both processes compared to ‘normal’ charge propagation of a single e^-h^+ pair. The processes of II and CT result in non-quantized energy signals. For detectors with single e^-h^+ pair energy resolution, these events fill in regions between the e^-h^+ pair peaks.

Another effect that can impact the number of charge carriers propagating in the crystal is charge leakage in the presence of a high electric field [214]. One type of leakage is bulk leakage, where impurities in the crystal result in loosely bound charges, which may be freed by sub-gap IR photons. The total NTL amplification of such a liberated charge is dependent on the initial position; the measured energy is thus a uniform distribution between zero and full NTL amplification, assuming that the impurities are uniformly distributed in the detector. Alternatively, surface leakage occurs when charges trapped near the surface are liberated (charges are more likely to trap near surfaces due to a higher number of defects) or charges are injected from the fabrication contacts. These charges undergo full NTL

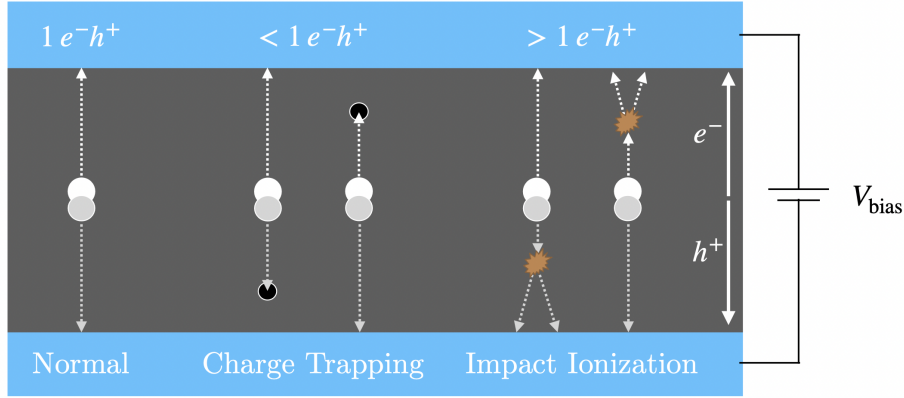


Figure 3.8: Diagram of how II and CT events increase/reduce the total number of charge carriers. From Ref. [213]

amplification (and thus appear as a quantized peak in devices with single e^-h^+ pair energy resolution).

The charge signal is read out using a Junction gate Field Effect Transistor (JFET) (for Soudan electronics) or High Electron Mobility Transistor (HEMT) (for SNOLAB electronics) charge amplifier circuit. A simplified schematic for this readout circuit is shown in Figure 3.9. The detector is represented by a capacitance C_d . The charge induced in the circuit by drifting charges from an interaction in the detector is rapidly collected onto a feedback capacitor, C_f , causing a voltage spike at the output, V_o .

The capacitor drains through a feedback resistor, R_f with time constant $C_f R_f$. A large bias resistor, R_b , between the bias source (providing the bias voltage V_b) and the detector prevents the induced signal current from draining into the bias source. The feedback loop is also protected from the biasing source by a coupling capacitor, C_c . Any stray capacitance in the circuit is represented by C_s .

With this, we can now consider the shape of the response to a charge signal. At SuperCDMS operation voltages the charges fully traverse the detector and induce a signal on the electrodes in less than $1.7 \mu\text{s}$ [215]. The rise time is set by the amplifier and is typically around $1 \mu\text{s}$ [216]. During the quick rising edge, $V_o = Q/C_f$. The decay time is set by the

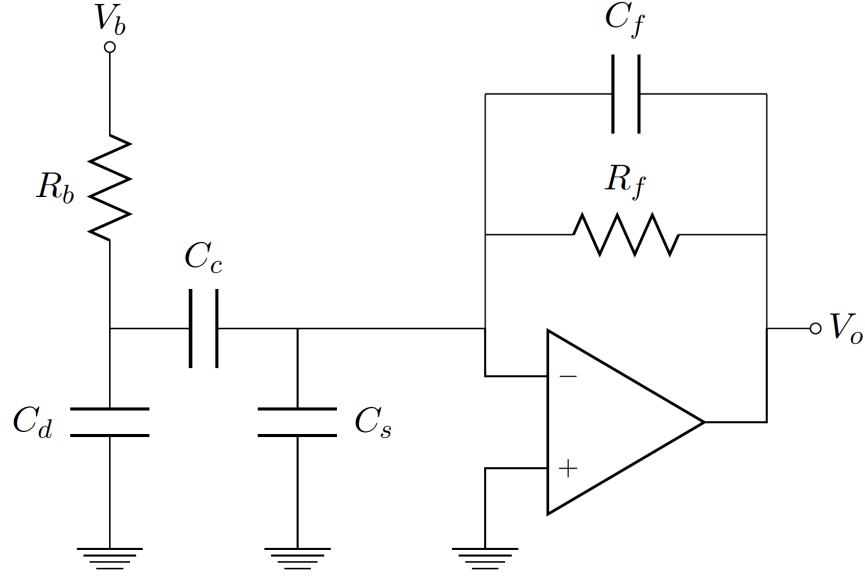


Figure 3.9: Schematic of the charge readout circuit for SuperCDMS detectors at Soudan. From Ref. [143].

circuit. The amplitude of the peak is proportional to the induced charge, and therefore the number of e^-h^+ pairs produced.

Once we have measured the charge signal and under the assumption that we have 100% charge collection efficiency, we can calculate the energy of the event. We call this measured energy the charge energy, E_Q . This is related to the number of e^-h^+ pairs, n_{eh} , and the average energy required to produce an e^-h^+ pair for an ER, ϵ_{eh} , as

$$E_Q = n_{eh}\epsilon_{eh}. \quad (3.94)$$

Here we assume 100% charge collection efficiency. For completely collected ERs (assuming no II or CT), $E_Q = E_r$; for NRs, $E_Q = Y(E_r)E_r$. The yield is vital for discrimination between ERs and NRs.

3.3.1 Yield Model

We have brought up the yield function $Y(E_r)$ several times at this point without describing its form. In this section I will describe the most prevalent theoretical model, as well as historical and recent measurements of the yield in both silicon and germanium.

The main theoretical description of the yield function is given by Lindhard et al. [217–219], though here we give a slightly simpler form as in Ref. [22]. For a nucleus with atomic number Z and atomic mass A , the yield is modelled as

$$Y_{Lindhard}(E_r, k) = \frac{kg(\varepsilon)}{1 + kg(\varepsilon)} \quad (3.95)$$

where we have defined several parameters for convenience:

$$\varepsilon = 11.5E_r[\text{keV}]Z^{-7/3}, \quad (3.96)$$

$$k = 0.133Z^{2/3}A^{-1/2}, \quad (3.97)$$

$$g(\varepsilon) = 3\varepsilon^{0.15} + 0.7\varepsilon^{0.6} + \varepsilon. \quad (3.98)$$

Note that I’ve defined the yield also as a function of k , as this is sometimes treated as a free parameter. The Lindhard model is considered standard at high recoil energy, but is not expected to be valid at low recoil energies. Several extensions to the model have been put forward such as those in Refs. [220, 221]; a short summary of these models is provided by N. Mast in his thesis [215]. A more recent model by Sarkis et al. [222] builds upon the Lindhard model, taking into account the atomic binding energy. This closely follows the Lindhard model at high recoil energies, but imposes a sharp cutoff at twice the binding energy. The authors claim that for Ge their model is a reasonable approximation even close to the cutoff value, but provide a caveat that the Si model is reasonable only for nuclear recoil energies above 500 eV. Their numerical model uses only the electronic stopping power, which is a function of k and ε , and the binding energy as fit parameters

and was fit to available measurements with good agreement.

In Figures 3.10 and 3.11 I provide a summary of historical and recent measurements of the ionization yield for silicon and germanium, respectively. These are compared to the theoretical Lindhard model, as well as the Sarkis model.

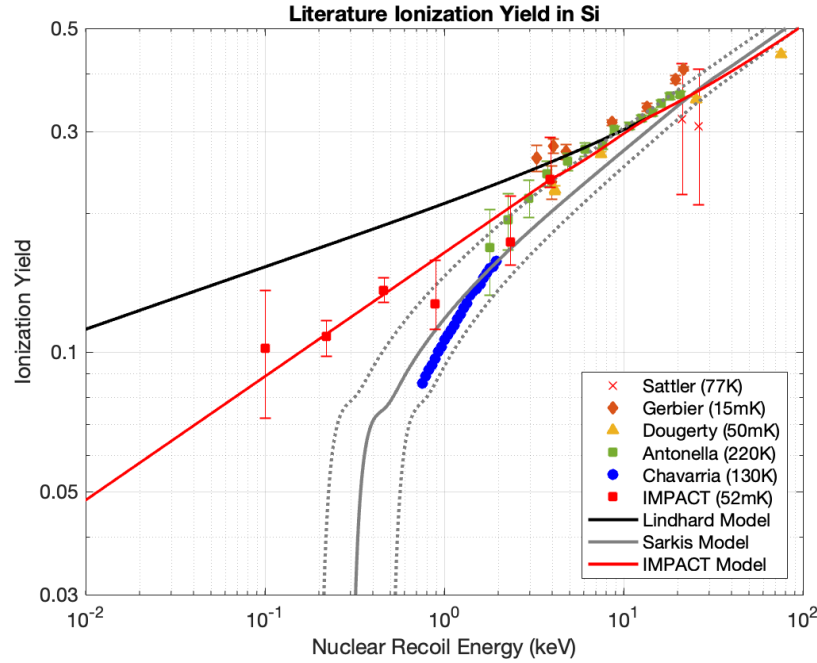


Figure 3.10: Summary of literature values for the ionization yield in silicon. Measurements are from Sattler et al. [223], Gerbier et al. [224], Antonella [225], Dougerty et al. [226], Chavarria et al. [227], and Ionization Measurement with Phonons At Cryogenic Temperatures (IMPACT) [228] (data and model), in order of publication date. The detector temperature is given for each measurement. Lindhard [217–219] theory curve and the Sarkis [222] model are shown for comparison. The dotted lines indicate the 1σ uncertainty band on the Sarkis model.

Early measurements of the yield in Si at keV–MeV nuclear recoil energy scales e.g. by Sattler et al. [223] and Gerbier et al. [224] agree well with Lindhard theory. However, a measurement by Dougerty et al. [226] in 1992 showed the first hints of discrepancy with a deviation from theory at the lowest recoil energy of ~ 4.2 keV with yield lower than predicted

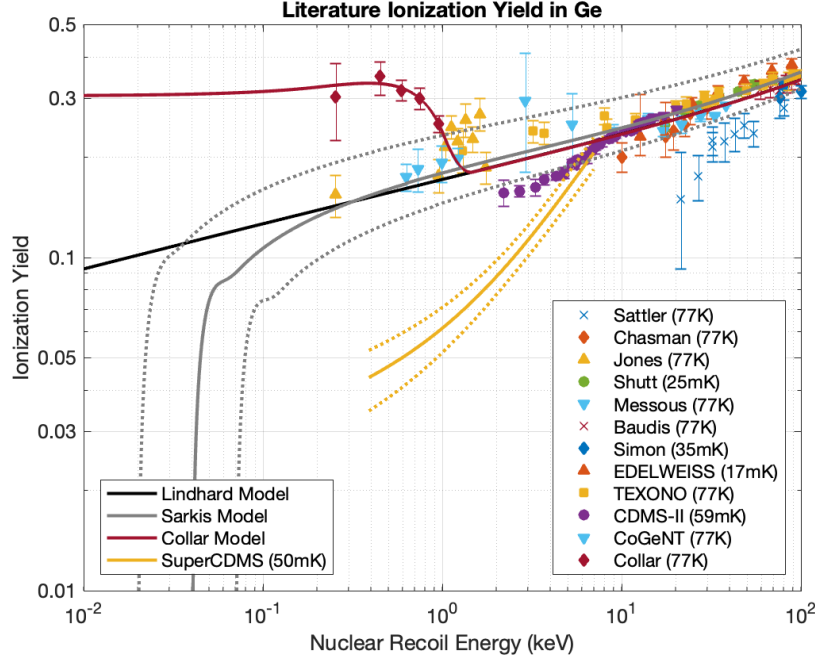


Figure 3.11: Summary of literature values for the ionization yield in germanium. Measurements are from Sattler et al. [223], Chasman et al. [229], Jones et al. [230], Shutt et al. [231], Messous et al. [232], Baudis et al. [233], Simon et al. [234], EDELWEISS [235], TEXONO [236], CDMS-II [148], CoGeNT [90], Collar et al. [237], and the SuperCDMS photoneutron result [170]. Lindhard [217–219] theory curve and the Sarkis [222] model are shown for comparison. The dotted lines indicate the 1σ uncertainty band on the Sarkis model (grey) and the SuperCDMS photoneutron result (yellow).

by Lindhard theory. The trend was confirmed and shown at even lower recoil energies in more recent measurements by Chavarria et al. [227] in 2016 and Antonella [225] in 2017. A more recent measurement by SuperCDMS IMPACT [228] pushes down to sub-keV nuclear recoil energies, continuing to show deviation from Lindhard while remaining consistent with other measurements, though showing some tension with Chavarria et al. Before IMPACT, the measurements were very compatible with the Sarkis model, which was fit to the data from Chavarria et al., Antonella, and others. There is some tension between the Sarkis model and IMPACT, however, as the IMPACT result implies that the binding energy cutoff is at a lower energy.

The measurement history is a little more extensive for the ionization yield of Ge. As with Si, early measurements showed good agreement with Lindhard at high nuclear recoil energies. Measurements as low as 10 keV were performed by Sattler et al. [223] and Chasman et al. [229] in the 1960's, and Jones et al. [230] pushed below 1 keV in the 1970's. Further measurements by Shutt et al. [231], Messous et al. [232], Baudis et al. [233], Simon et al. [234], EDELWEISS [235], TEXONO [236], and CoGeNT [90] continued to show consistency with Lindhard, likely in part due to the much lower binding energy in Ge. CDMS-II [148] showed a slightly decreasing trend in the yield, though possibly still consistent with both Sarkis and Lindhard. Two diverging measurements have been made at lower energies: Collar [237], which actually shows an increased yield below 1 keV, and the SuperCDMS photoneutron result [170], which shows a deviation in the opposite direction, with a decrease in the measured yield. No experiment has reached the cut off from the Sarkis model.

There is a general lack of agreement between theory and measurement at low recoil energy, and even some inconsistencies between measurements. This is particularly concerning in the ongoing search for dark matter: the push towards lower mass dark matter candidates has necessitated a corresponding push towards sub-keV recoil energies. The NR ionization yield can dramatically impact the shape of the background and signal spectra in the region of interest and thus the current situation imparts substantial uncertainty on measurements. However, the push towards lower mass dark matter will likely be concurrent with improvements in yield measurements.

3.4 SuperCDMS Detectors

With the principles of SuperCDMS detector operation in hand, we will now turn our attention to specific detector designs implemented by SuperCDMS. I provide an overview of detectors used in the previous generation at Soudan and how they motivated the detector designs for SuperCDMS SNOLAB, as well as different R&D devices SuperCDMS has been

studying in the interim.

3.4.1 SuperCDMS Soudan Detectors

Since operation at Soudan, a mainstay of SuperCDMS detector technology has been the iZIP detector. The name stems from the ‘interleaved’ design, which alternates ionization and phonon sensors with ~ 1 mm spacing between the channels. At Soudan, only Ge iZIP detectors were used. These are cylindrical crystals with 76 mm diameter, 25 mm height, and 600 g mass. The sensors are photolithographically patterned on the flat faces of the detector. As shown in Figure 3.12, the phonon and charge channels are interleaved and form a meandering pattern.

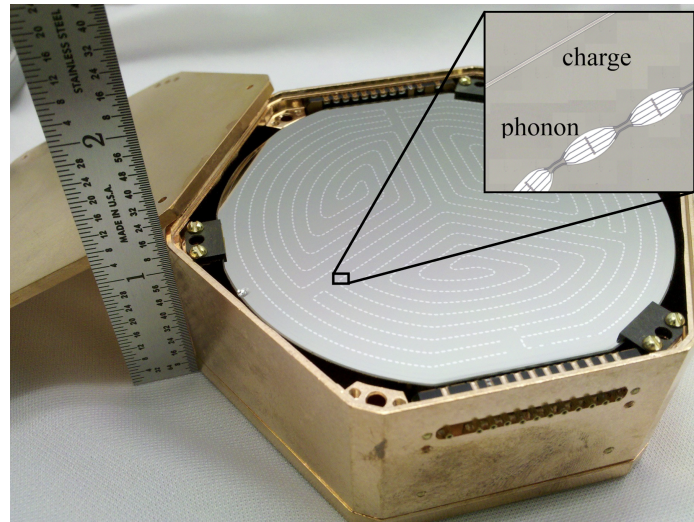


Figure 3.12: An iZIP detector in its housing, adapted from Ref. [140]. The phonon channels are clearly visible and form a meandering pattern; the charge channels are not visible but are interleaved with the phonon channels. The inset panel shows a close up view of each type of channel.

The sensors are divided into distinct channels. There are two ionization channels on each side, consisting of a circular central, or inner, sensor as well as an outer ring. There are also four phonon channels on each side: an outer channel sharing the same area as the outer charge channel, and three even inner wedges sharing the same area as the inner charge

channel. The top and bottom faces are offset by 60° for better position information. Figure 3.13 shows the channel layout and naming convention.

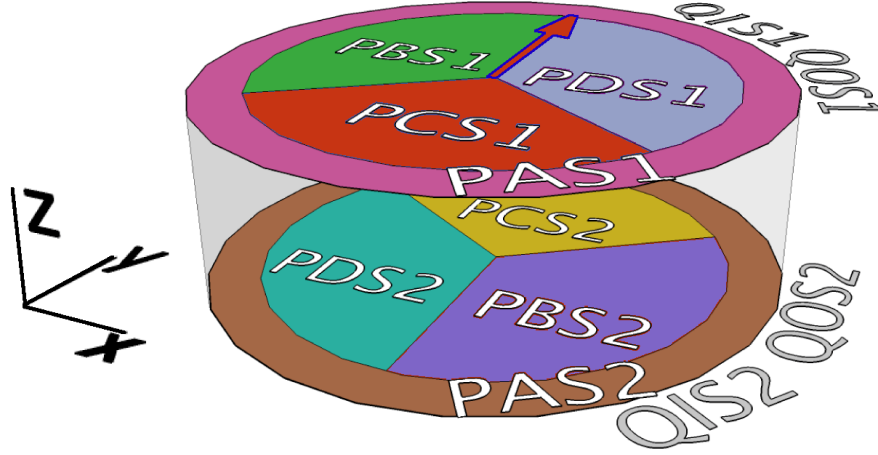


Figure 3.13: Channel map of a Soudan iZIP detector. The four phonon channels are labelled A-D and identified by the side, e.g. PAS1 is phonon channel A on side ‘1’, the top of the detector. On a given side the outer charge channel (QOS1/QOS2) overlaps with the outer phonon channel (PAS1/PAS2), while each inner charge channel shares the same area as the three inner phonon channels.

Across the detector, a symmetric ± 2 V bias is applied, where the ionization channels are biased and the phonon channels are grounded. This creates a field structure ideal for rejecting surface events: charges that would normally drift to the electrode on the opposite surface are instead drifted to the grounded phonon sensors and the charge signals on each side are largely asymmetric [140]. This effect is demonstrated in Figure 2.11 in an earlier discussion. The bias voltage is limited so that the energy in NTL phonons is roughly equivalent to the recoil energy. This allows the iZIPs to achieve yield discrimination by comparing the phonon measurement to the ionization measurement: ERs and NRs can be separated by the difference in their yield. The success of discrimination at low energies is dependent on the resolution. The ionization resolution typically dominates the phonon resolution, and thus limits the precision of the yield discrimination and how low in energy this can be applied.

There were 15 detectors operated at Soudan, arranged into five towers with three detectors apiece. The tower layout and the detector arrangement are shown in Figure 3.14. More generally, the detectors are labelled by their tower number and position within the tower, ordered from top to bottom. For example, T2Z3 is the detector at the bottom of the stack in tower 2, which corresponds to detector 1106 in the Figure.

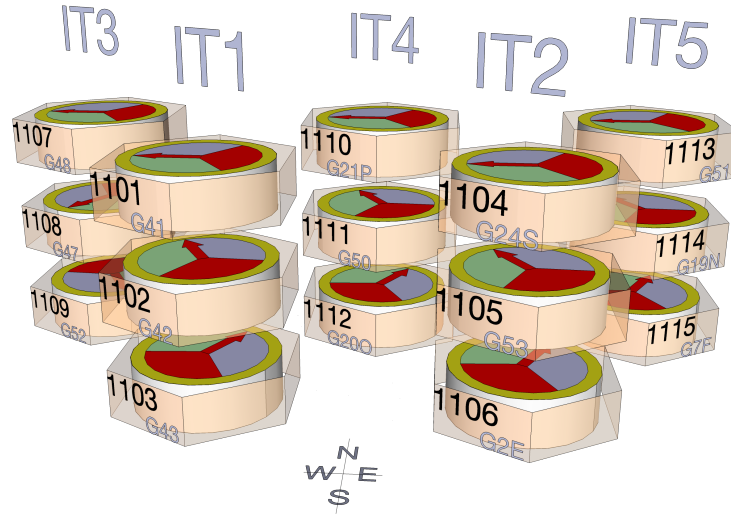


Figure 3.14: Layout of the five towers at SuperCDMS Soudan. The grey labels above each tower identify the tower number and the grey labels on each detector identify the crystal. Each detector is also given a unique number.

An important lesson was learned at Soudan by operating some of the detectors in an alternative mode in which a much higher bias voltage of ~ 70 V was applied to one side of the detector while the other side was grounded. Only the grounded phonon channels were read out. This operational mode is referred to as CDMSlite.

This mode takes advantage of the NTL effect, effectively converting the charge signal into a much larger phonon signal. A 70 V potential across a Ge detector corresponds to an amplification factor of ~ 24.3 . The ionization measurement is thus redundant, and the ionization sensors are not read out in this mode. The yield based discrimination is lost, but

one advantage for NR dark matter searches is that the ER background is suppressed relative to NR background because the ER background is spread out and ‘stretched’ relative to the NR background on account of the greater ionization yield.

The main advantage to this operational mode is significantly improved energy resolution and threshold. A threshold of 56 eV_{ee} ¹ and 70 eV_{ee} was achieved in CDMSlite Run 2 [152, 160] and Run 3 [121], respectively, and both runs achieved $\sim 10 \text{ eV}_{ee}$ baseline energy resolution. This allowed the setting of limits on WIMP dark matter at masses well below what was achievable in the standard iZIP operational mode. Figure 3.15 shows the low energy spectrum from CDMSlite Run 2.

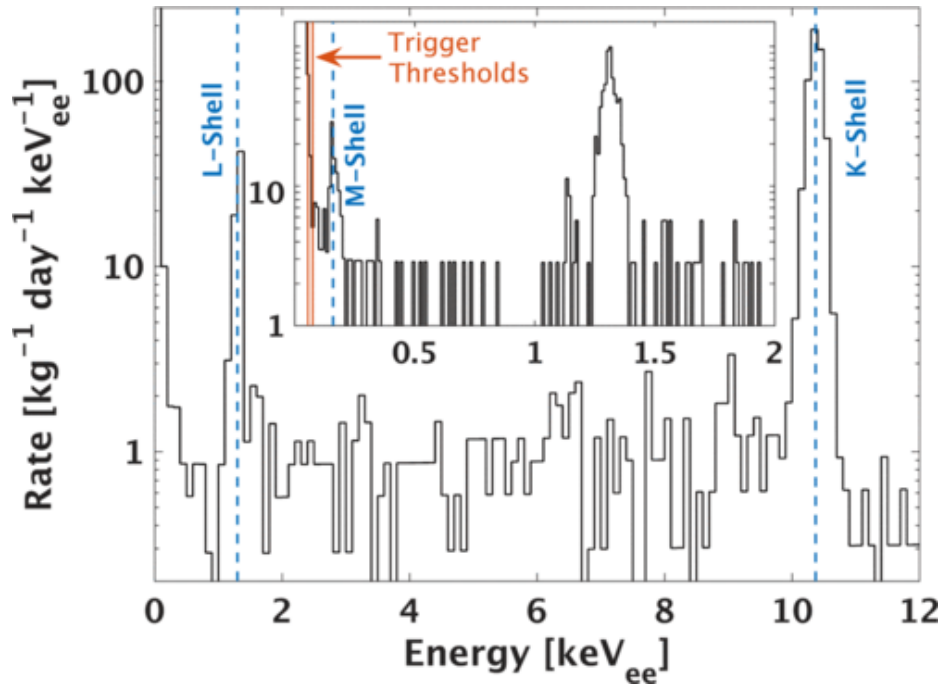


Figure 3.15: CDMSlite Run 2 event energy spectrum from Ref. [152]. The impressive baseline energy resolution and threshold means that the 160 eV M-shell peak from Ge electron capture is clearly resolvable. The L-shell and K-shell are also indicated.

One complication with the CDMSlite operation mode results from the non-uniformity of

¹Recall that the eV_{ee} is the electron-equivalent energy scale, which assumes an ER interaction.

the electric field across the detector. Calculations of the electric field configuration show that the field is distorted at high radius, and that charge carriers originating in this region will not experience full NTL amplification. This results in improper event energy reconstruction. Therefore it is essential in this operational mode to remove high radius events. Figure 3.16 shows the difference in electric potential experienced by charge carriers at high radius based on their initial position in the detector for CDMSlite Run 3. A radial parameter was constructed for each run by comparing the signals of the outer channel with the inner channels (see Ref. [183] for more details on how this analysis is performed).

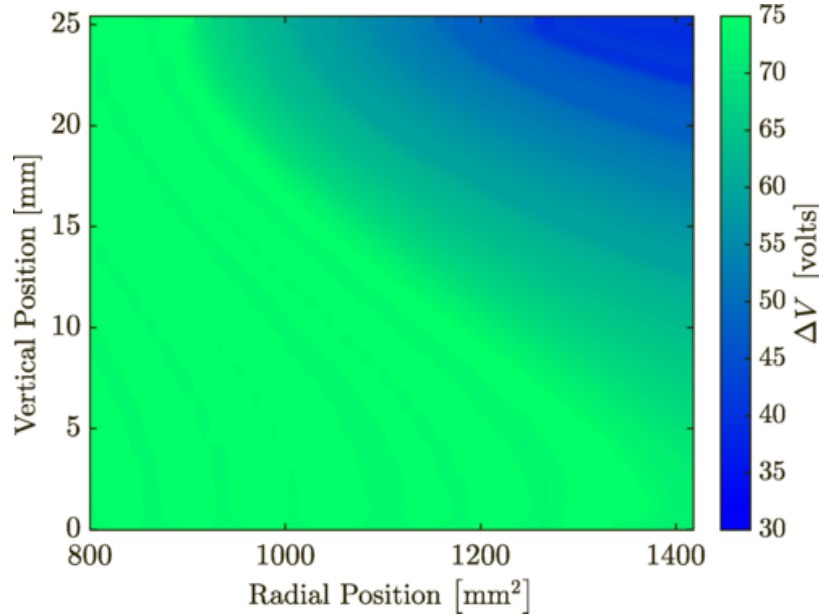


Figure 3.16: Calculated voltage bias map at high radius for the electric field configuration used in the CDMSlite operational mode. The difference in electric potential, ΔV between the final collection points of the charge carriers and the initial position is shown. A bias of +75 V is assumed at the top of the crystal and the bottom is grounded. Charge carriers in this region experience reduced bias voltage and subsequently reduced NTL amplification. From Ref. [121].

3.4.2 SuperCDMS SNOLAB Detectors

Motivated by the success of the CDMSlite operational mode at Soudan and the complementarity of the iZIP detectors, new designs have been developed for SuperCDMS SNOLAB [171]. Both Si and Ge detectors will be used for parameter space complementarity: the kinematics of WIMP scattering mean that higher atomic mass Ge is more sensitive to lower cross sections, while Si is more sensitive to lower mass WIMPs. The detectors are larger than those used at Soudan, with 100 mm diameter and 33.3 mm height, and a mass of 1.39(0.61) kg for Ge(Si).

Updated versions of iZIP-style detectors will also be used, with a 6(8) V symmetric bias voltage for Ge(Si). These detectors will have yield based event discrimination, and will provide sensitivity to 5–10 GeV WIMP masses.

A new detector design directly motivated by CDMSlite has been developed, the HV detector. This is designed to be operated at a symmetric 100 V bias voltage to take advantage of NTL amplification. These detectors will be sensitive to sub-GeV dark matter as well as LDM and bosonic dark matter like dark photons and ALPs. They are not instrumented with any ionization sensors, and are entirely optimized for phonon readout, with significantly higher Al coverage than the iZIP detectors (35% vs. 4%). For both detector types, the QET design has been optimized to double the quasiparticle collection efficiency compared to CDMS II [174].

For SuperCDMS SNOLAB, it was clear that designing detectors with as much position sensitivity as possible was vital. Both detector designs have updated sensor patterns compared to the Soudan design. The HV detectors have 6 phonon channels and no ionization sensors. The phonon channels are of equal area and divided between an inner circle, two outer rings, and three wedges in the intermediate space. Both faces are instrumented for two-sided readout, and the bottom face is offset by 60°. The lack of yield discrimination means that these detectors must rely on phonon position sensitivity to reject surface backgrounds and events at high radius which suffer from reduced NTL gain.

The new iZIP designs also have 6 phonon channels with a central disk, an outer ring, and four intermediate channels between the core and ring. Both sides are instrumented, and in this case the bottom face is offset by 45° from the top. There are two ionization channels on each side. The outer channel shares the same area as the outer phonon channel, and the inner overlaps the remaining five phonon channels. Figure 3.17 shows the channel layout for both iZIP and HV style detectors.

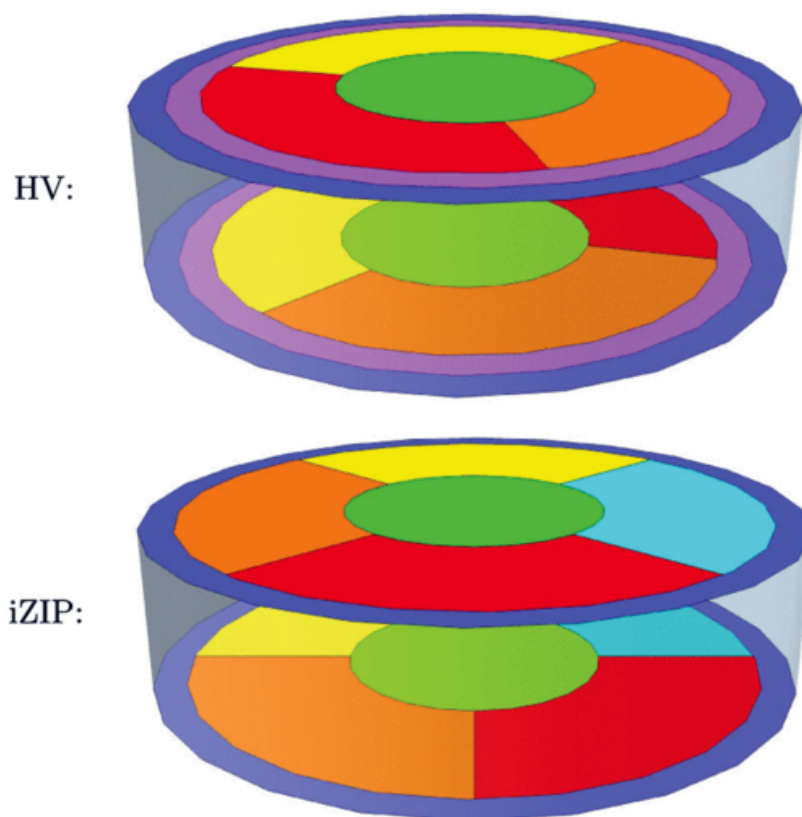


Figure 3.17: SuperCDMS SNOLAB detector channel layouts for the HV (top) and iZIP (bottom) detectors [171]. The HV detectors are instrumented only with phonon sensors. The iZIP outer ionization channel shares the same area as the outer phonon channel.

The channel layouts for both detector styles were selected from several options designed to provide optimum position sensitivity, as outlined in Ref. [203]. The channel layout gives

radial position information, and z-position information is given by the difference in measurement between the two sides.

It is vital that we reduce both the phonon and ionization resolution; this allows both detector styles to push down to lower thresholds, and means that the HV detectors will have better position sensitivity. The JFETs in the charge amplifier circuit are being replaced with lower noise HEMTs for the iZIP ionization readout. As discussed in Section 3.2.3, reducing the T_c of the TESs leads to a significant improvement in baseline phonon energy resolution.

With the HV detectors, leakage current will be a significant concern as this could potentially lead to significant degradation of the phonon energy resolution. Another interesting aspect of the HV detector design is that NR/ER discrimination may be partially recoverable if the energy resolution is fine enough. ERs appear in distinct peaks based on integer numbers of electron-hole pairs, whereas NRs have more phonon energy per e^-h^+ pair and can end up between the peaks. This resolution will not be achieved for SuperCDMS SNOLAB, but this possibility can be explored with future upgrade scenarios.

A payload of four detector towers with six detectors each will be deployed in the initial run at SNOLAB. Towers 1 (T1) and 4 (T4) will be dedicated iZIP towers, with 6 Ge iZIPs in T1 and 4 Ge and 2 Si iZIPs in T4. Towers 2 (T2) and 3 (T3) will be dedicated HV towers, with 4 Ge and 2 Si HV detectors apiece. Table 3.1 summarizes the detector payload as well as the resolution and voltage bias parameters for each kind of detector. Design goals for the resolution parameters were originally given in Ref. [171]; however, they have since been updated in Ref. [177] based on scaling laws from characterized R&D devices.

The current SuperCDMS cryostat design is capable of housing a total of 7 towers, a reduction from the originally planned 31 tower capacity. However, the cryogenic system is still capable of providing cooling power for a 31 tower system, thus there is potential in the future to significantly upgrade the SuperCDMS SNOLAB tower capacity.

	iZIP		HV	
	Ge	Si	Ge	Si
Voltage bias [V]	6	8	100	100
Number of detectors	10	2	8	4
Total exposure [kg·yr]	45	3.9	36	7.8
Phonon resolution [eV _t]	33	19	34	13
Ionization resolution [eV _{ee}]	160	180	—	—

Table 3.1: Summary of expected detector parameters for SuperCDMS SNOLAB. Exposures assume 4 years of operation with 80% duty cycle. Resolution estimates are updated values from Ref. [177].

3.4.3 R&D Detectors

SuperCDMS has multiple avenues of R&D for detector improvement, and operates several surface facilities for detector characterization, which has less stringent background requirements than dark matter searches. In this section I will describe two different types of detectors used for both characterization and dark matter searches.

HVeV detectors are small $1 \times 1 \times 0.4 \text{ cm}^3$ high purity Si crystals. One side is instrumented with QETs and the other side is held at a high bias voltage $\mathcal{O}(100 \text{ V})$. These devices are capable of single charge resolution; this combined with the applied high voltage lends the detector the name ‘HV + eV’. The QET readout is typically divided into two readout channels with the same area. Figure 3.18 shows a HVeV detector in its housing.

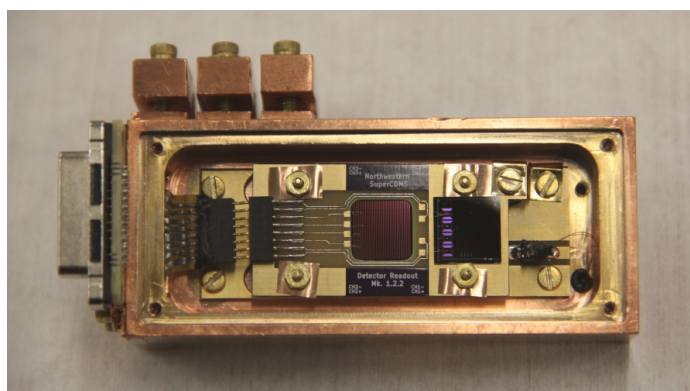


Figure 3.18: Photograph of a HVeV detector used in HVeV Run 2, from Ref. [238].

These detectors are typically operated at surface facilities for characterization studies as well as dark matter searches as in Refs. [239, 240]. Thanks to the single-charge sensitivity, competitive constraints were placed on low mass (electron-interacting) dark matter, and dark absorption channels with just 0.49 g days and 1.2 g days exposure, respectively. A 3 eV energy resolution (at 100 V, this corresponds to $\sim 3\%$ resolution on an e^-h^+ pair) was achieved in the second HVeV run. Figure 3.19 shows a comparison between the measured energy spectra for Runs 1 and 2. The bias voltage in Run 1 was -140 V compared to 100 V in Run 2, thus the Run 1 data are rescaled for a direct comparison.

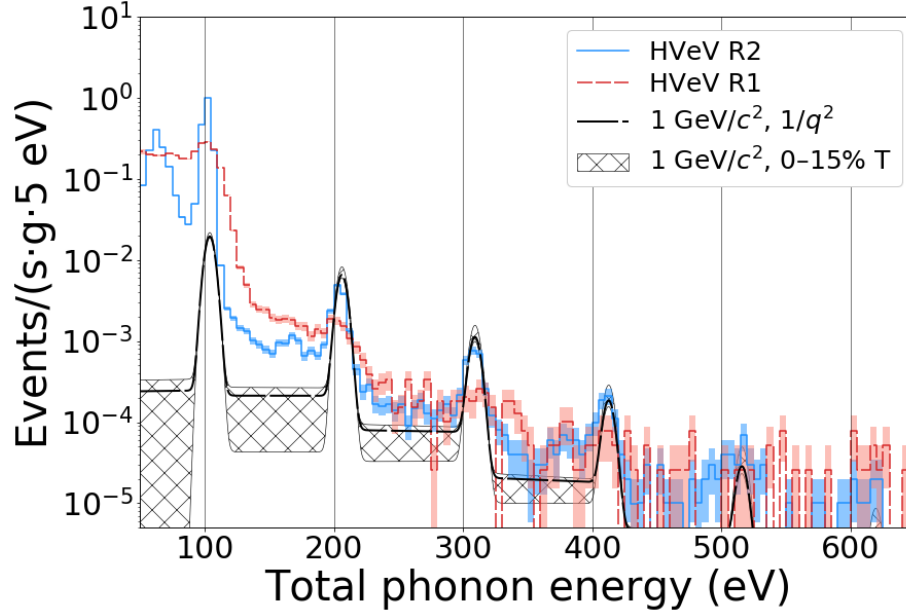


Figure 3.19: Comparison between the measured energy spectrum in HVeV Run 1 [239] and Run 2 [240]. The Run 1 data are rescaled to match the Run 2 100 V bias. The single e^-h^+ pair resolution is clearly visible with peaks at integer multiples of 100 eV as expected for a 100 V bias. Also shown is the signal model for a 1 GeV/ c^2 electron-interacting dark matter candidate with $F_{DM}(q) \propto 1/q^2$ (see Section 2.2.2) with II probability of 2% and CT probability of 11%. A trapping range of 0–15% is also shown. Figure from Ref. [238].

Thanks to their operation at high voltage, the HVeV detectors are also used for studying II and CT probabilities. Photons from a laser brought in via an optical fibre are used to

calibrate the detectors. The model from Ref. [203] was used to fit the calibration data and to determine a $11 \pm 3\%$ CT probability and $2_{-2}^{+3}\%$ II probability for HVeV Run 2 [240]. These devices are also being used to study leakage background from operation at high voltage and for the IMPACT yield measurements, for which they were designed.

SuperCDMS has also operated athermal phonon detectors referred to as Cryogenic PhotoDetectors (CPDs), developed by the CPD Collaboration and first characterized at SLAC [241]. These are 1 mm thick Si wafers with 3.81 cm radius and 10.6 g mass, instrumented on a single side with QETs organized into a single readout channel. This leads to minimal position dependence and a fast phonon-collection time, and therefore low energy resolution. The other side is not instrumented. One such device was characterized and has a measured baseline resolution of 3.86 ± 0.04 (stat.) $_{-0.00}^{+0.19}$ (syst.) eV [241]. The low resolution is also due in part to the low T_c of 41.5 mK.

The low energy threshold means that CPD devices are excellent devices to study low mass dark matter via nucleon interactions. 9.9 g days of data were acquired with a CPD in a surface facility at the SLAC National Accelerator Laboratory and used to set competitive limits on SI WIMP interactions in Ref. [242]. An excess event rate was observed at low energy above the expected flat Compton background, as shown in Figure 3.20. Other experiments have also seen excesses at low energy [85, 243–245], including the HVeV detectors, motivating further studies of the low energy backgrounds. In the case of CPD detectors, the excess is possibly due to the presence of stress fractures.

The same device was also operated in an underground facility and used to set constraints on nucleon-coupled dark matter with (marginal) improvement to the cross-section sensitivity [183]. A second version of the CPD detector, CPDv2, with similar dimensions and improved QET mask was fabricated and operated for a dark matter search at the University of Massachusetts Amherst [180].

With R&D devices such as the HVeV and CPD detectors, SuperCDMS continues to produce competitive dark matter results while SuperCDMS SNOLAB is under construction.

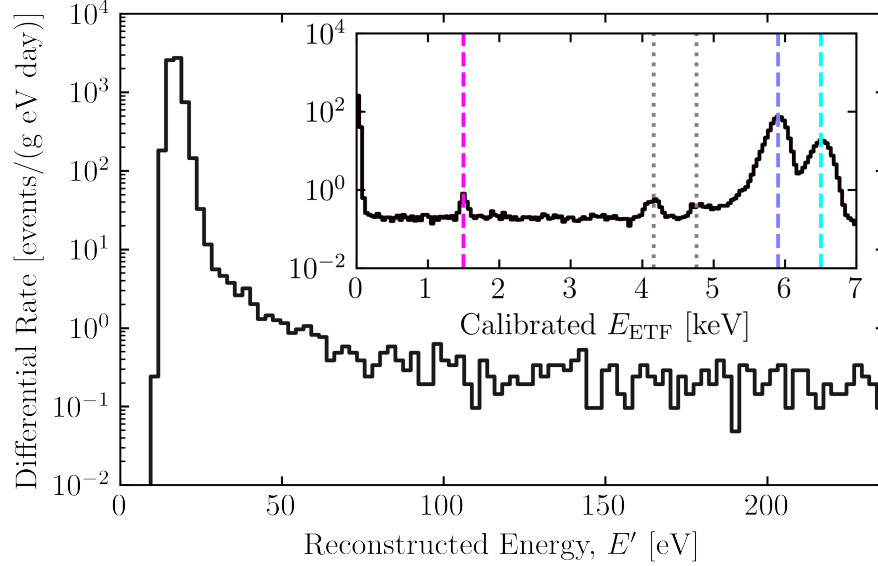


Figure 3.20: Measured energy spectrum for the CPD dark matter search, showing the rising rate at low energy. The inset plot shows peaks used for energy calibration, including Al-fluorescence (pink), ^{55}Fe K- α (blue) and K- β (cyan) at 5.9 keV and 6.5 keV, respectively. The grey dotted lines correspond to Si escape peaks. Figure from Ref. [242]

The success of these detectors also provides a path forward for future upgrade scenarios and the search for dark matter beyond SuperCDMS SNOLAB. In the meantime, the next chapter presents a dark matter search using data from SuperCDMS Soudan.

Chapter 4

Dark Absorption at SuperCDMS Soudan

In this chapter, I describe an analysis of iZIP data from Run 133 at SuperCDMS Soudan to set limits on the coupling strength of dark photons and axion-like particles published in Ref. [162]. This was the first time that SuperCDMS data were used to set constraints on ERDM candidates.

As previous searches had focused on WIMPs, the data selection criteria from Ref. [145] were optimized with nuclear recoils in mind. Here I describe how the data selection criteria were adapted for electron recoils, the determination of efficiency and energy resolution models, as well as the limit setting method of a simple counting experiment. The final limit was determined as a combined analysis with data from CDMSlite Runs 2 and 3 to produce a total ‘SuperCDMS Soudan’ limit on dark absorption of dark photons and axion-like particles. The CDMSlite-specific analysis was performed by R. Germond and is described in detail in his thesis [180].

We obtain constraints on the ALP axioelectric coupling g_{ae} and the dark photon kinetic mixing ϵ using a Poisson counting method, using simple Poisson statistics to determine the upper limit at 90% confidence level on the number of events within a signal window. No background subtraction is performed: it is assumed that all events could be signal, setting a conservative upper limit. This method requires relating a measured event rate in a region of interest to a dark matter interaction rate, for which we must know the efficiencies of our

event selection criteria. The dark absorption signal is a gaussian at the mass-equivalent energy of the dark matter candidate with a width given by the energy resolution of the detector. In order to set a limit, we require that the energy resolution be characterized. The energy resolution is also what sets our region of interest, which is a 1σ signal window around the mass-equivalent energy.

4.1 Event Selection and Signal Efficiency

In this section, I describe the data quality criteria, or cuts, used for this analysis. This analysis uses data from SuperCDMS Soudan Run 133, which were previously used for the SuperCDMS low-mass WIMP search [145]. Seven iZIP detectors with the lowest trigger thresholds were selected for that analysis, with data acquired between October 2012 and July 2013. Here the same live-time selection is used, excluding periods directly following ^{252}Cf neutron calibrations. I exclude from this search one of the seven detectors which showed evidence of incomplete charge collection on one side, as well as two detectors with shorts on one or more of the phonon/charge readout channels. The four remaining detectors are listed in Table 4.1 with their exposures.

	T1Z1	T2Z1	T2Z2	T4Z3
Exposure (kg·days)	80.2	82.9	80.9	83.8

Table 4.1: The four selected iZIPs and their exposures used for the dark absorption analysis.

Energy Estimators

Before discussing the details of the analysis, I will provide an overview of the different energy estimators used throughout the analysis. The phonon and charge signals are processed using an optimal filter. An optimal filter fits a template to the measured trace in frequency space, accounting for the measured noise spectrum, by weighting the different frequencies according

to the ratio of their prevalence in the template and the noise. It can be shown that this leads to the most accurate reconstruction of the pulse amplitude if the shape of the template is the same as the shape of the measured pulse (hence the name). More details on the optimal filter and different implementations can be found in Appendix A. The energy scales are calibrated using the ^{133}Ba ER and ^{252}Cf calibration data.

The charge energy estimators include:

$Q_{inner,N}$: The optimal filter amplitude of the inner charge channel on side $N \in \{1, 2\}$.

$Q_{outer,N}$: The optimal filter amplitude of the outer charge channel on side $N \in \{1, 2\}$.

$Q_{inner,mean}$: The mean of $Q_{inner,1}$ and $Q_{inner,2}$. This was used as the charge energy estimator in Ref. [145].

Q_N : The sum of $Q_{inner,N}$ and $Q_{outer,N}$ for side $N \in \{1, 2\}$.

The phonon energy estimators are:

P_{total} : The Non-Stationary Optimal Filter (NSOF)¹ phonon energy estimator on the sum of the phonon channels. This energy estimator has the best energy resolution. It measures the total phonon energy, that is, the sum of the phonon production from the primary interaction and the NTL phonons. The ratio of the two components is fixed for bulk events, but differs for surface events with reduced NTL phonon production, so it cannot be used as a proxy of the energy of the primary interaction when surface events are included. The energy units on this parameter are typically given with a subscript t , e.g. keV_t .

P_{recoil} : This parameter is derived from the total phonon energy estimator P_{total} by subtracting the NTL phonon contribution, which is calculated based on the measured charge signal and the applied bias voltage: $P_{recoil} = P_{total} - (Q_1 + Q_2)V_b/\epsilon$, where V_b is the

¹A modified version of the optimal filter accounting for the specific pulse shape variations observed in iZIP detectors; for details see Appendix A

detector bias voltage and ϵ is the average energy required to produce an e^-h^+ pair. It has worse energy resolution than P_{total} because of the dependence on the ionization signal, which has inherently larger resolution than the phonon signal. However, in contrast to P_{total} it can be used to measure the recoil energy across event populations that include surface events.

4.1.1 Data Quality Criteria

Events used for the limit analysis must fulfill a set of data quality criteria, or cuts, including a trigger requirement that the detector in question issued a trigger, while no other detector did (single-scatter cut), a muon veto cut ensuring that the event is not coincident with a signal in the muon veto detector, and a fiducial volume (FV) cut. The single-scatter requirement is motivated by the dark matter interaction model; while many backgrounds have a signature of multiple energy depositions (e.g. gammas), dark absorption is a point-like process.

The trigger, single-scatter, and muon veto cuts are identical to those described in Ref. [145], and have a combined efficiency of approximately 95% for each detector, with strong energy dependence near the trigger threshold. For this analysis we imposed a lower threshold of 2 keV, higher than the trigger threshold, chosen such that there is only a slight energy dependence over the analysis range.

In the low-mass WIMP search, an additional boosted decision tree (BDT) cut was trained on nuclear recoils from ^{252}Cf data and implemented to select for WIMP-like events. This was essential as the dominant backgrounds are from ERs. With an ER signal as the focus of the dark absorption analysis, no such cut is necessary. In the following Section, adaptations to the FV cut are described.

4.1.2 Fiducial Volume Cuts

The fiducial volume cuts are used to reject ^{210}Pb surface-event background as well as remove events near the detector surfaces which may have reduced NTL gain from the distortion in electric field in these regions. The FV cuts take advantage of the detector charge channel design, described in Section 3.4.1. Bulk events are selected to have symmetric charge measurements between the two sides of the detector, and to have the majority of the charge collected on the inner channel on both sides. In the low-mass WIMP search [145], these cuts were developed for NRs using the charge signal. Their efficiencies were determined using ^{252}Cf calibration data, a good proxy for NRDM, and were found to be energy dependent. However, the behaviour of charge signals is very different between nuclear and electron recoils, and the efficiency as determined for the WIMP search is inappropriate for this ERDM search.

The signal of interest is a point-like, or single scatter, ER interaction, and the signal efficiency should be determined using the same type of events. The first instinct might be to use the ^{133}Ba electron recoil calibration data; however, these and many of ER backgrounds are gamma interactions, which tend to scatter multiple times in the detector and are not a good proxy for point-like interactions. Instead, we have a single identifiable sample of monoenergetic single scatter events at $E_K = 10.37\text{ keV}$ from Ge K-shell capture events.

This sample of events is monoenergetic, thus cannot be used to determine the energy dependence of the cut efficiency. Therefore, I define a new FV cut, dependent only on the charge signal distribution and referred to as the charge fiducial volume (QFV) cut, in a manner designed to be energy independent. The cut consists of two separate cuts: a radial charge cut removing events near the cylindrical surface, and a charge symmetry cut removing events near the flat surfaces. I will describe the definition of each cut, and then the calculation of the cut efficiency using the Ge K-shell events in the remainder of this section.

Radial Charge Cut

The radial charge cut is composed of separate selection criteria on each side of the detector, though the cuts are determined in the same manner for both sides. As a reminder, the goal of this cut is to remove events at high radius close to the sidewalls. The radial position of an event in the detector can be roughly estimated by comparing the amount of charge collected on the outer vs. inner charge channels on each side (note that we cannot actually calculate the physical position of an event, only compare relative to other events).

The newly defined radial charge cut removes events with charge collected in the outer electrodes exceeding a certain fraction of the charge collected on the inner electrode, which should be energy independent for point-like events.

The relationship between the outer and inner charge signals can be seen in Figure 4.1 for one of the sides of a detector, T2Z2. The distribution of energy between the two charge channels can be seen very clearly for the 10.37 keV line: bulk events are collected almost entirely by the inner charge channel and appear on the x-axis at 10.37 keV. The distribution is broadened by the noise on each channel. There is an additional distribution of 10.37 keV events on the y-axis with full charge collection by the outer charge channel. These correspond to events at high radius. Finally, there is a band with negative slope connecting the two distributions with partial energy collection on each channel.

To define the cut for a given side N ($N = 1, 2$), the zero-energy resolution σ_0 of the outer charge channel is measured for events with $Q_{inner,N}$ between 12–30 keV. The radial charge cut is then defined such that the ratio of charge collected on the outer to the inner charge channel does not exceed $3\sigma_0/10.37$ keV.

This cut definition was chosen to ensure that the majority of the 10.37 keV distribution with full charge collection on the inner channel is kept. Figure 4.1 shows the definition of this new cut line. The distribution is similar for the other detectors used in the analysis.

The definition of the cut should be energy independent. However, due to the inherent noise in the charge measurement, at lower energies even events with low collected charge

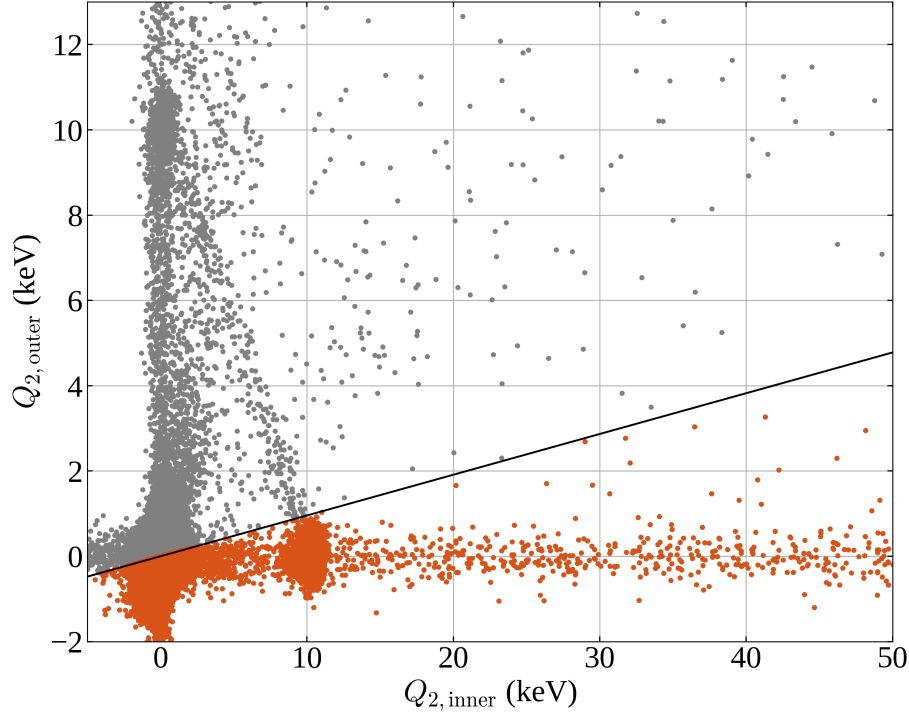


Figure 4.1: Signal amplitude in the outer ($Q_{2,outer}$) vs. inner ($Q_{2,inner}$) charge channels for side 2 of detector T2Z2, from Ref. [162]. The cut line (black) removing events with an outer signal above a fraction of the inner signal is shown. Events passing the cut are shown in orange. The 10.37 keV line is prominent.

on the outer electrode will be reconstructed outside of the cut limit and will fail the cut, reducing the efficiency of the cut at low energies. This can be seen in Figure 4.1, where the cut partition extends into the noise distribution. This effect is quantified in Section 4.1.3 by modelling the signal distribution between the inner and outer electrodes using the Ge K-shell events.

Charge Symmetry Cut

The charge symmetry cut is designed to remove events near the flat faces of the detector. The electric field configuration of the interleaved design means that events near these surfaces have reduced or no charge signal on the opposite side, while events occurring in the detector bulk are expected to have symmetric charge collection on the sensors on both sides of the

detector. Surface events can be rejected by requiring that the signal amplitude is similar between the two sides, hence the name “charge symmetry” cut.

In Ref. [145], due to the radial charge cut removing all events with significant outer charge signal, the charge symmetry cut was defined using only the signals on the inner charge channels. With the new radial cut definition from 4.1.2, the total charge signal may include contribution from the outer electrodes. Thus I redefine the charge symmetry cut for this analysis using the total charge signal on each side.

I define a cut parameter as the ratio of the difference between the charge collected on each side to the sum of the total charge collected on both sides:

$$\frac{Q_1 - Q_2}{Q_1 + Q_2} \quad (4.1)$$

This distribution of this parameter at a given recoil energy is gaussian about 0 for bulk events. We expect that the distribution will widen for lower energies as the noise starts to dominate, and narrow towards higher energies. The mean μ and resolution σ of this distribution are measured in 1 keV bins from 1 – 9 keV, using P_{recoil} as an energy estimator. Additionally, the mean μ_K and resolution σ_K are measured for the K -shell distribution at 10.37 keV by selecting events with P_{recoil} between 9 – 11.8 keV. The means and resolutions are then used to fit a model to the upper and lower edge of the distribution: $\mu \pm 3\sigma$. The model is defined as a function recoil energy P_{recoil} :

$$f(P_{recoil}) = \begin{cases} a(e^{-bP_{recoil}} - e^{-10.37b}) + c & P_{recoil} < 10.37 \text{ keV} \\ c & P_{recoil} \geq 10.37 \text{ keV} \end{cases}$$

where $c = \mu_K \pm 3\sigma_K$. The form of the model ensures continuity at 10.37 keV. Above this energy, the charge symmetry cut is defined such that the cut parameter is less than $3\sigma_K$; due to the narrowing distribution the efficiency will slightly increase, so the assumption of constant efficiency above 10.37 keV is conservative. The cut definition can be seen in Figure

4.2.

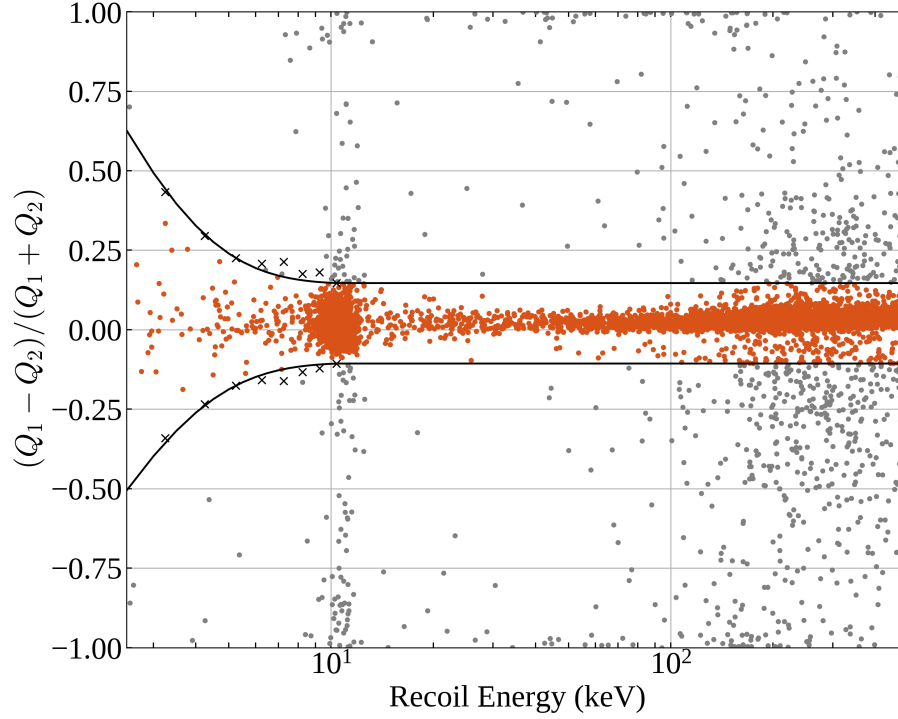


Figure 4.2: The charge symmetry cut parameter vs. recoil energy for detector T2Z2, from Ref. [162]. Bulk events are localized in a narrow band around zero in this parameter, widening towards lower energies. A Gaussian is fit to the band in 1 keV bins at low energy; the 3σ positions are indicated (black x's). The cut line (black lines) is an exponential fit to these points below 10.37 keV and constant at higher energies. Events passing (orange) and failing (grey) the cut are indicated.

Features of note include the prominent 10.37 keV distribution, as well as a broadening of the bulk (symmetric) event band at low recoil energies as we just described. This Figure is also slightly deceptive at higher energies because of the logarithmic scale and increased event density – the band appears to widen again at higher recoil energies, which would invalidate our claim that the assumption of constant efficiency is conservative. However, measuring the width of the distribution as a function of energy demonstrates that the band continues to narrow as energy increases, substantiating the claim.

4.1.3 QFV Cut Efficiency

I determine the efficiency of the QFV cuts using the passage fraction of the Ge K-shell electron capture events at 10.37 keV. ^{71}Ge is produced by neutron activation during ^{252}Cf calibrations, and decays with a half-life of 11.43 days [144]. We can take advantage of this short half-life and use it to estimate the number of ^{71}Ge events in a given data set as long as the data are taken over a long enough period in which the decay would be evident. In contrast to the dark matter analysis, for this calculation I include the time period immediately following the ^{252}Cf calibration period as this bolsters the ^{71}Ge statistics.

Event Selection All events with P_{recoil} in a window of $10.37 \pm 4\sigma_{\text{precoil}tNF}$ (where $\sigma_{\text{precoil}tNF}$ is the measured resolution of the 10.37 keV peak in P_{recoil}) are selected to ensure that all ^{71}Ge events are selected while excluding as much background as possible. The parameter P_{recoil} is a good energy estimator, though with poor energy resolution. It provides a better selection criterion here than P_{total} because events with incomplete charge collection are better reconstructed, whereas P_{total} would be an underestimate for events outside of the FV, and we must ensure that we are considering these outer events in the efficiency calculation. Otherwise, we overestimate the efficiency. The assumption that no Ge K-shell events fall outside of the selected region of P_{recoil} is qualitatively confirmed by examining the event time distribution of the region below the $10.37 - 4\sigma$ lower bound. The spectrum is flat, indicating a negligible ^{71}Ge contribution.

Our event selection is divided into two bins: *passing* events pass both the QFV cuts and have P_{total} scaled to keV from keV_t^2 in the range of $10.37 \text{ keV} \pm \sigma_{\text{pt}NF}$, where $\sigma_{\text{pt}NF}$ is the measured resolution of the 10.37 keV peak in P_{total} . This is the same window used for the definition of the limit setting method at each mass in Section 4.4.1. All other events are in the *failing* events bin.

²Recall that the total phonon energy includes the NTL phonon contribution. We rescale from keV_t to keV using Equation 3.7 under the assumption that the events are all ERs.

Likelihood Fit to Event Time A likelihood fit is performed to the time of the events selected as described in the previous section. The ^{71}Ge K-shell events are modelled by a decaying exponential with half-life of 11.43 days, and a flat background is used to account for everything else. Note that in the defined energy region there are events from tritium beta decay, as well as possible ^{65}Zn electron capture events at 8.98 keV. However, these have much longer half lives of 12.32 years and 243.94 days [144], respectively. Thus these backgrounds are relatively flat in time compared to ^{71}Ge . We also note that there are 10.37 keV decays from cosmogenic ^{68}Ge , but the 270.3 day half-life means that this background will also appear flat on these time scales.

The fraction of events that pass the livetime cuts for each data series as a weight factor in the probability distribution functions in the likelihood fit. This weight factor, $f(t)$ is a function of event time t . For N events with event times $t_1, t_2, \dots, t_N$, the probability distribution functions are $Ae^{-\lambda t_i} f(t_i)$ for ^{71}Ge (where λ is the ^{71}Ge decay rate) and $Bf(t_i)$ for the flat background (i.e. everything other than ^{71}Ge), respectively. The likelihood function to be maximized is

$$L(r) = \prod_{i=1}^N \left[rAe^{-\lambda t_i} + (1-r)B \right] f(t_i), \quad (4.2)$$

where $0 \leq r \leq 1$ is the fraction of ^{71}Ge events. Thus rN is the estimated number of ^{71}Ge events and $(1-r)N$ is the estimated number of background events in a given bin.

Uncertainties are determined at the 1σ level by drawing random values of r and calculating the likelihood function for this value. Another random number, Z , is drawn between 0 and 1. If the ratio $L(r)/L(r_{ml}) > Z$, where r_{ml} is the value of r maximizing the likelihood function, then the value of r is accepted. A number n is drawn from a Poisson distribution with $\lambda = N - rn$ is then the assumed number of ^{71}Ge events. This process is repeated, resulting in a distribution for the number of ^{71}Ge events from which the 1σ confidence interval is determined.

There were three periods of ^{252}Cf activation in SuperCDMS Soudan Run 133, each of

which effectively resets the rate of ^{71}Ge decays. Thus there are three different time periods over which the likelihood fit is performed, each following an activation period. Figure 4.3 shows the event time distribution with the likelihood fit for both the passing and failing event bins.

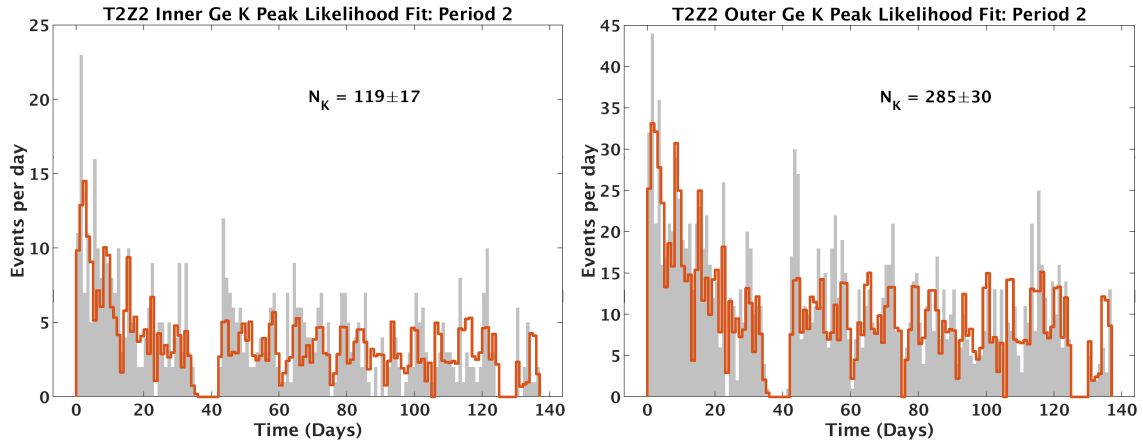


Figure 4.3: T2Z2 event time spectrum for the second post-Cf data period (Period 2) for passing (left) and failing (right) event bins. All events have P_{recoil} within $4\sigma_{precoil tNF}$ of 10.37 keV, where $\sigma_{precoil tNF}$ is the measured resolution of the 10.37 keV peak in P_{recoil} . Passing events pass both QFV cuts and additionally have P_{total} within $\pm 1\sigma_{ptNF}$ of 10.37 keV (σ_{ptNF} is the measured resolution of the 10.37 keV peak in P_{total}). The likelihood fit to the spectrum is shown in orange, where the model is an exponential decaying with the ^{71}Ge half-life plus a flat background, scaled by the fractional live-time. The fit estimate of the number of ^{71}Ge K-shell events, N_K is also shown.

The combined QFV and signal window efficiency at 10.37 keV is then the fraction of K-shell events in the ‘passing’ bin for all three periods over all K-shell events (‘passing’ plus ‘failing’ bins) in all three periods. Table 4.2 shows the number of events in each event bin for each time period, as well as each detector. The calculated QFV-signal window efficiencies for each period, as well as the total QFV-signal window efficiency, are also shown.

		T1Z1	Z2Z1	Z2Z2	T4Z3
Period 1	N_K Pass	42±14	76±13	63±22	73±12
	N_K Fail	153±27	184±25	64±30	148±24
	Efficiency	(21.7±6.5)%	(29.2±4.4)%	(49.3±14.4)%	(33.1±5.1)%
Period 2	N_K Pass	101±15	98±13	119±17	155±16
	N_K Fail	185±25	219±22	285±30	320±25
	Efficiency	(35.2±4.6)%	(31.0±3.5)%	(29.4±3.7)%	(32.5±2.8)%
Period 3	N_K Pass	125±17	114±13	137±17	96±12
	N_K Fail	263±29	188±22	213±28	240±25
	Efficiency	(32.3±3.8)%	(37.8±3.9)%	(39.1±4.3)%	(28.7±3.4)%
Total QFV Efficiency		(30.8±2.7)%	(32.8±2.3)%	(36.1±3.2)%	(31.4±2.0)%

Table 4.2: The number of ^{71}Ge K-shell events, N_K , in each bin for each of the three different post-Cf time periods. The combined QFV and signal window efficiency for each time period is shown separately, as well as the total QFV efficiency. As the efficiency also takes into account the $\pm 1\sigma$ signal window used in the limit setting process, the QFV efficiency alone can be found by multiplying by a factor of $1/0.68$.

Radial Charge Cut Efficiency Low Energy Correction Factor While the two components of the QFV cut are designed to have constant efficiency as a function of energy, it is clear from Figure 4.1 that this is not the case at low energies for the radial charge cut. Events at low energy with low outer charge signal may still be reconstructed outside of the cut selection due to the inherent noise in the charge signal measurement. As mentioned in Section 4.1.2, this effect is modelled and taken into account in the final QFV efficiency.

To do this, I consider only the radial charge cut on side 1 of the detectors. This is sufficient because there are very few events that are removed by the side 2 cut that are not removed by the side 1 cut, and attempting to consider the effects of both cuts simultaneously would be non trivial. The ratio of outer charge to total charge on side 1, i.e. $Q_{1,outer}/Q_1$, is

modelled for the K peak events as a gaussian at 0 and 1 with a linear background between as shown in Figure 4.4. As the K peak region also consists of non-K-shell events, I do a background subtraction using the background rate in the region above the K peak, which is also indicated in the Figure. A gaussian centred at 0 plus a linear background is fit to the distribution from -0.2 to 0.8, then a gaussian centred at 1 is added with amplitude such that applying the radial charge cut to the distribution results in the correct passage fraction (the area of the model probability distribution function (PDF) below the cut location is equal to the passage fraction).

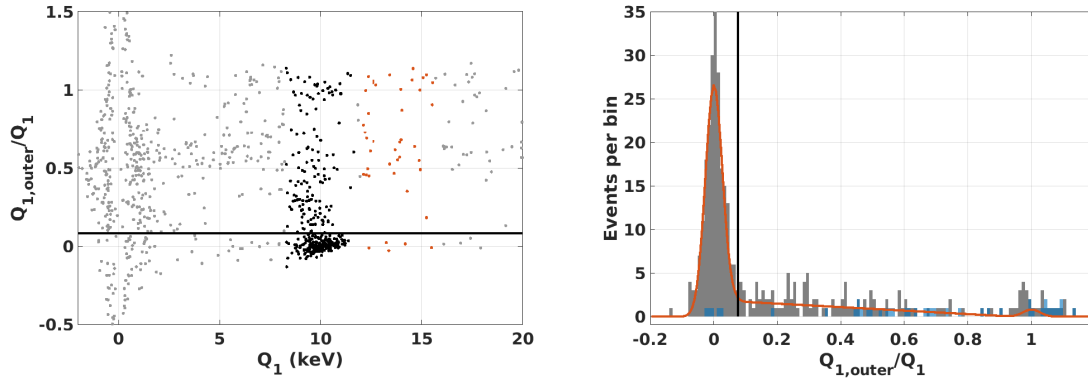


Figure 4.4: (Left) The T2Z2 outer to total charge ratio vs. total charge for the side 1 channels. The K peak events (black) and the events above the K peak used for background subtraction (orange) are indicated. The radial charge cut (black line) is also shown. (Right) The K peak (grey solid) and background above the K peak (blue solid) event distributions from the left figure histogrammed in the ratio of outer to total charge on side 1 for T2Z2. The model (orange line) is fit to the background subtracted distribution from -0.2 to 0.8, with an additional gaussian centred at 1 with amplitude such that the area of the model PDF below the cut location (black line) is equal to the cut passage fraction.

The model is then deconvolved by the zero-energy resolution of the ratio $Q_{1,outer}/Q_1$ using events at 10.37 keV in the inner charge channel, and then convolved with the resolution of the same quantity at lower energies. The distribution widens while the cut value remains constant. The fractional area of the model passing the cut at each energy below 10.37 keV is then taken as a correction factor to the QFV efficiency at that energy. This results in a

total QFV efficiency that decreases with energy below 10.37 keV.

Total Cut and Signal Window Efficiency The final cut efficiency is a combination of the efficiencies of all selection criteria, including the new QFV cuts, as well as the size of the signal window for the limit setting process. Figure 4.5 shows the final efficiency for detector T2Z2 as a function of energy with 1σ uncertainty band. The uncertainty is dominated by the statistical uncertainty on the QFV cuts from the likelihood fits. Based on how we defined the QFV cuts, we expect that the efficiency is conservative at energies above 10.37 keV.

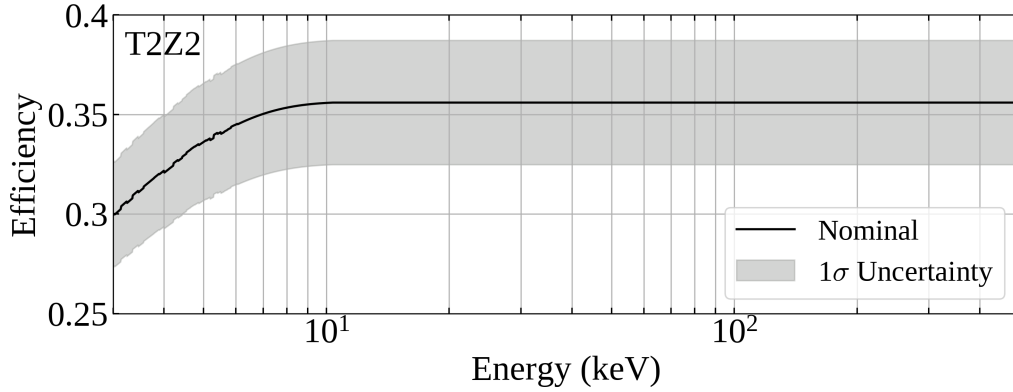


Figure 4.5: The total combined efficiency with signal window size for detector T2Z2 for the iZIP dark absorption analysis, from Ref. [162]. The nominal value (solid line) and 1σ uncertainty (grey band) are shown.

A Note on the Assumption of Point Like Events For this analysis we have made the assumption that the events are point-like³, depositing all of their energy within the detector in a localized range. Under these assumptions, the cut efficiency should be conservatively constant to higher energies. We take a moment here to analysis whether or not the assumption of point-like energy depositions is appropriate.

The average distance travelled by an electron can be determined from the stopping

³Technically we assume that the events are accurately represented by the 10 keV electron capture (EC) events, which are close to point-like.

power. Here I use data from National Institute of Standards and Technology (NIST) [246] of the electron range as a function of energy in Ge, as shown in Figure 4.6. The maximum energy deposition that we consider for the dark matter search is from a 500 keV electron, which has a range of 0.511 mm. This is non-negligible on the scale of the detector and such an event could not be considered truly point-like.

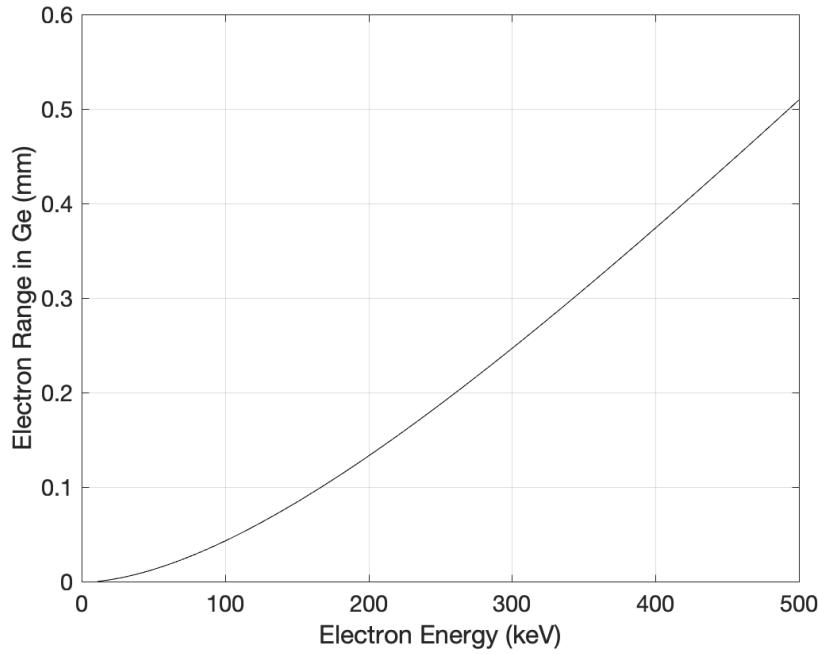


Figure 4.6: The average range of an electron in Ge, calculated from the stopping power from NIST ESTAR [246]. A Ge density $\rho = 5.323 \text{ g/cm}^3$ is assumed.

Here we attempt to estimate the impact of the electron range on the FV by assuming that the FV is composed of a distinct physical boundary (in truth, the FV). The definition of the FV results in less than roughly 8–14% of the event energy being deposited outside of the FV for passing events⁴. This is both cut and detector dependent, but for simplicity we assume that the energy deposition outside of the FV cannot surpass 12%. This assumed value turns

⁴The radial charge cut requires that less than $\sim 8\%$ of the energy is collected by the outer channel and the charge symmetry cut requires that the difference in charge collected between the two sides of the detector is less than $\sim 14\%$ of the total charge collected. This varies slightly for each detector.

out to have little impact on the final result. For a 500 keV electron, this corresponds to 60 keV. A 60 keV electron has a range of 0.019 mm in Ge; therefore, a 500 keV electron that begins in the FV must travel a distance of at least $r - s_0 = 0.482$ mm within the FV to be kept by the QFV selection.

For the following we assume that the path of an electron is a straight line. Consider an electron that starts with shortest distance x to the FV boundary as shown in Figure 4.7a. We can calculate the maximum angle α_0 between the electron trajectory and the shortest path to the FV boundary at which the electron can be emitted while still being rejected by the FV cut:

$$\cos\alpha_0 = \frac{x}{r - s_0}. \quad (4.3)$$

This geometry is also shown in Figure 4.7a. The fraction f of events emitted with angle $\alpha < \alpha_0$ is given by the solid angle

$$f = \frac{1}{4\pi} \int_0^{2\pi} \int_0^{\alpha_0} \sin\theta d\theta d\phi = \frac{1 - \cos\alpha_0}{2} = \frac{1}{2} \left(1 - \frac{x}{r - s_0} \right). \quad (4.4)$$

We exploit the vertical symmetry of the detector and calculate the loss in the top half of the detector to simplify the geometry. The calculation is dependent on the assumption of the physical geometry of the FV cut, which is not well understood. We assume that the FV is cylindrical and symmetric within the detector volume, having radius R_{fv} and height H_{fv} .

We estimate the effective loss in the FV efficiency by integrating the fraction of events kept over the FV

$$V_{eff} = \int_0^{r-s_0} (1 - f) dV + V_r, \quad (4.5)$$

where $V_r = \pi(R_{fv} - (r - s_0))^2(H_{fv}/2 - (r - s_0))$ is the volume in which a starting electron cannot travel far enough outside of the FV to be rejected by the cut regardless of emission

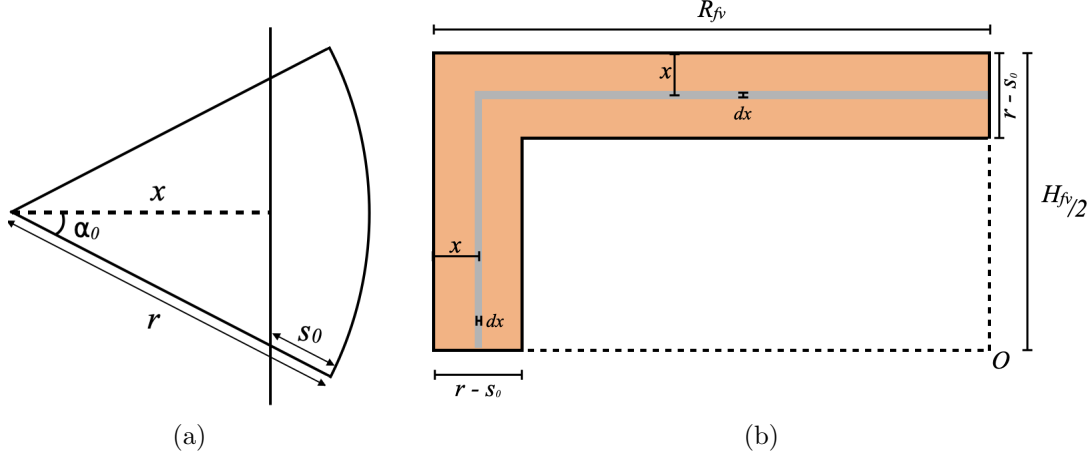


Figure 4.7: (left) Geometry of the electron and FV boundary, where x is the shortest distance to the FV boundary, α_0 is the maximum emission angle of the electron track relative to x while still being removed by the FV cut, r is the total electron range, and s_0 is the minimum distance the electron must travel outside of the FV to be rejected. (right) The geometry of the fractional volume element dV (grey) for electrons a distance x from the FV boundary. The area shown represents the cylindrical FV region of the detector in cross section view, where O represents the centre of the detector. R_{fv} is the radius of the FV and $H_{fv}/2$ is the height. We use the symmetry of the detector in the vertical direction and consider only half of the geometry. The orange area shows the area within the FV in which an emitted electron could potentially be rejected by the FV cut based on the range of the electron r and the minimum distance that the electron must travel outside of the FV to be rejected, s_0 .

angle, and the fractional volume element dV is

$$dV = 2\pi(R_{fv} - x)(H_{fv}/2 - x)dx + \pi(R_{fv} - x)^2dx. \quad (4.6)$$

The integrated loss is the relative change in the fiducial volume $(V_{fv} - V_{eff})/V_{fv}$, where $V_{fv} = \pi R_{fv}^2 H_{fv}/2$. Assuming conservatively a QFV efficiency of 45% (the lowest measured efficiency, 30.8%/0.68 accounting for the 1σ window), we calculate the loss for all possible dimensions of the FV (the radius and height of the FV have a constrained relationship to ensure that the ratio of the FV to the total detector volume is 45%). We find a loss on the order of 1.9–2.7%, with the largest loss occurring for the extreme case of a z-only FV

cut (i.e. no radial cut). More reasonable FV dimensions put the loss closer to 2%. This is negligible in comparison to the uncertainty from the efficiency calculation. In addition, there will also be a few events starting outside the FV that deposit most of their energy inside, compensating for some of the loss. Finally, I assume that most electrons don't go in a straight line, so the effective distance to consider is shorter and thus the loss is smaller than calculated here.

4.2 Energy Calibration and Resolution Model

With the event selection criteria fully defined and the efficiency characterized, we now consider the energy estimator to be used for the limit setting process. As previously mentioned, we use P_{total} as this has the best energy resolution and is a good energy estimator for bulk events. However, this must be calibrated from total phonon energy into recoil energy, and nonlinear effects at high energies should be taken into account. We also must characterize the energy resolution of each detector as part of the limit setting process. Both of these tasks are accomplished by modelling the energy and width of various calibration peaks throughout different data types.

4.2.1 Energy Resolution Model

The resolution of the energy parameter, P_{total} , is modelled using the same energy dependent model as described in the CDMSlite Run 2 analysis [160]. The total energy resolution is written as the sum of three different components:

$$\sigma_T^2 = \sigma_0^2 + \sigma_F^2 + \sigma_{PD}^2, \quad (4.7)$$

where σ_0 is the baseline energy resolution, σ_F is an energy dependent term taking into account the variation in the number of produced charge carriers (the number of e^-h^+ pairs is discrete, but the baseline resolution is too poor to distinguish individual pairs – we can

parameterize the Fano statistics with an energy dependent broadening of the resolution), and σ_{PD} is a term accounting for position dependence, which is also energy dependent.

This energy dependent model can be parameterized as a function of the electron-equivalent recoil energy ($E_{ptNF,scaled}$) determined by rescaling the measured total phonon energy P_{total} to recoil energy:

$$\sigma_T^2(E_{ptNF,scaled}) = \sigma_0^2 + F\varepsilon E_{ptNF,scaled} + (AE_{ptNF,scaled})^2, \quad (4.8)$$

where F is the Fano factor, ε is the average energy needed to produce an electron hole pair in germanium, and A is a term quantifying the position dependence.

Peak Energy and Resolution Measurements This model is fit to the measured resolution of five different peaks in the low background and calibration data sets. The mean and resolution of each peak in the different data sets are determined using a maximum likelihood method. The peak energies, their sources, data sets in which they are found, and the models used to determine the peak parameters are summarized:

- 0 keV (baseline noise): measured using random triggers passing quality cuts in low background data. The phonon energy estimator for these events uses the OF0 algorithm discussed in Appendix A. The distribution is modelled by a single Gaussian.
- 10.37 keV Ge K-shell electron capture peak: low background data immediately following ^{252}Cf calibration periods to increase the signal to background ratio. Modelled by three Gaussians, one at each of the K-shell electron capture energies of Ge, Ga, and Zn (10.37 keV, 9.8 keV, and 8.98 keV, respectively) plus a flat background.
- 66.7 keV: during ^{252}Cf calibration, a prompt gamma ray is emitted after neutron inelastic scattering on ^{73}Ge . This peak is therefore found in californium calibration data. Modelled by a Gaussian plus a linear background.

- 356 keV: the most prominent gamma from ^{133}Ba decay, found in ^{133}Ba calibration data. Modelled by a Gaussian plus a step function (convolved with the energy resolution) plus an exponential background. This is not a physically motivated model, but gives a reasonably good description of the data.
- 511 keV: annihilation radiation from positron emission, e.g. from cosmogenic ^{68}Ga decays. Low background data immediately following ^{252}Cf calibration periods are used to bolster statistics as it was observed that the peak was more prominent in these data. Modelled by a Gaussian plus a linear background.

The P_{total} energy estimator is used for each dataset other than the baseline noise, which uses ptOF0 (see Appendix A) to avoid the positive amplitude bias inherent in allowing a search window for the pulse start time. Figure 4.8 shows the fit results for each calibration peak for T2Z2.

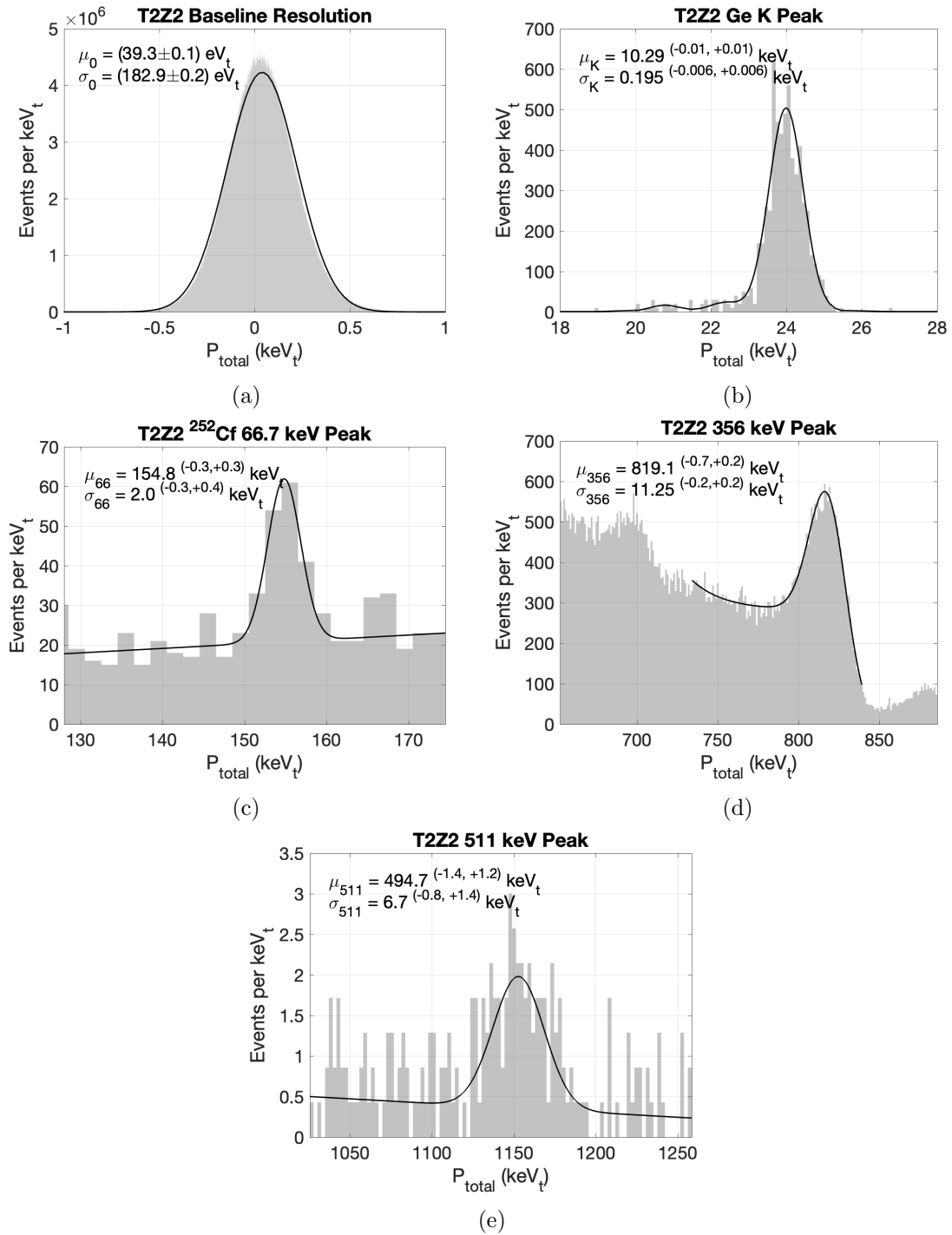


Figure 4.8: Maximum likelihood fit results to each calibration peak for detector T2Z2. The data (grey) are fit by models (black line) described in the body text. The relevant gaussian parameters for the calibration peak are shown.

The total phonon energy estimator P_{total} must be converted to recoil energy. As we saw in Eq. 3.3, we can write the total phonon energy E_{ptNF} as the sum of the recoil energy and the energy in NTL phonons

$$E_{ptNF} = E_{recoil} + E_{NTL}. \quad (4.9)$$

The energy in the NTL phonons is proportional to the recoil energy, and is dependent on the applied bias voltage across the detector. For the iZIP detectors in this run, the bias voltage was 4 V (± 2 V on each face). The total recoil energy can be expressed as a function of the applied voltage, V , and the average energy needed to produce a single electron hole pair, ε (3 eV in germanium):

$$E_{ptNF} = E_{recoil} \left(1 + \frac{eV}{\varepsilon} \right). \quad (4.10)$$

Thus the total phonon energy parameter can be converted to recoil energy with a simple rescaling. However, there are nonlinear effects in the energy scale at high energy, resulting in a reduction in total phonon energy. I use the measured peak energies in P_{total} as a function of the true peak energy to model the non linearity as a quadratic effect, and rescale the P_{total} parameter to recoil energy accounting for the nonlinearity. Thus the P_{total} to recoil energy calibration is energy dependent. Figure 4.9 compares the nonlinearity of the energy scale when simply converting P_{total} to recoil energy using Equation 4.9 to a quadratic scaling.

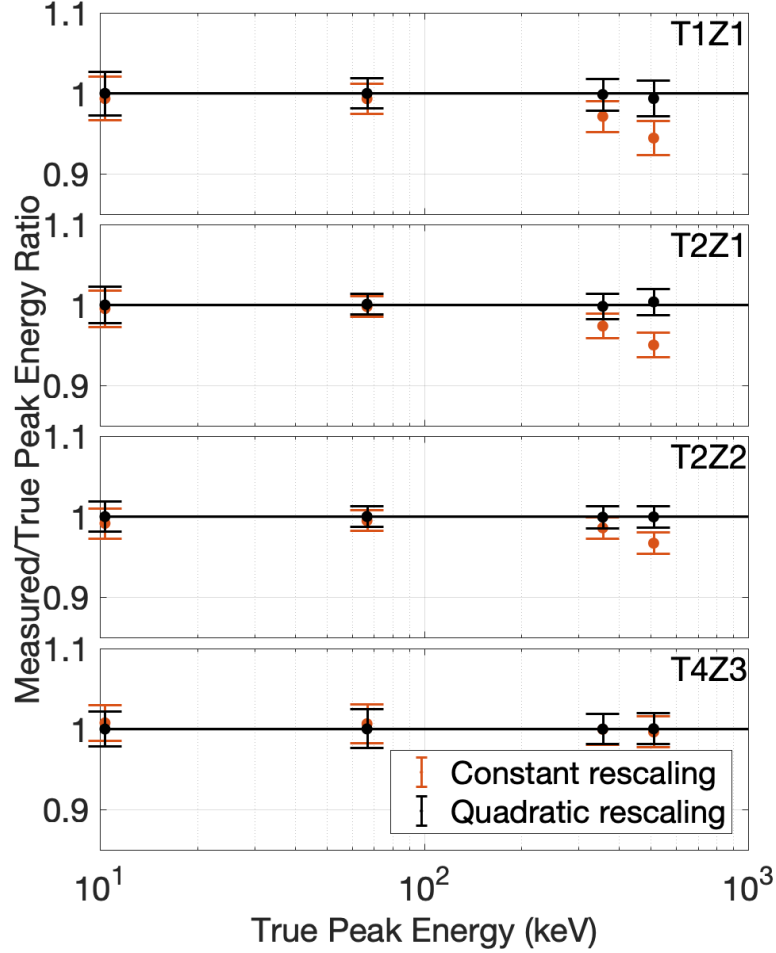


Figure 4.9: The measured/true peak energy as a function of true peak energy (except the baseline noise peak) for each detector, demonstrating the non-linearity (except for T4Z3, which shows linear behaviour) of the energy scale with no correction applied (orange) vs. the energy scaled by a quadratic fit to the five peaks (black).

The measured peak widths are converted from total phonon energy to calibrated recoil energy. Eq. 4.8 is fit to the measured peak widths as a function of true peak energy, weighting each point by the respective uncertainty. The $1\text{-}\sigma$ uncertainty band is also determined. Table 4.3 shows the resulting fit parameters for each detector, and Figure 4.10 shows the resolution model and uncertainty band for T2Z2.

	σ_0 (eV)	$F\varepsilon$ (eV)	$A (\times 10^{-3})$
T1Z1	100.5 ± 3.4	2.7 ± 0.5	19.5 ± 0.1
T2Z1	69.9 ± 4.6	2.4 ± 1.1	15.4 ± 0.3
T2Z2	79.0 ± 2.9	1.2 ± 0.3	13.6 ± 0.1
T4Z3	80.2 ± 5.4	0.6 ± 1.2	19.0 ± 0.4

Table 4.3: Fitted resolution model parameters and their uncertainties for the four iZIP detectors included in the dark absorption analysis.

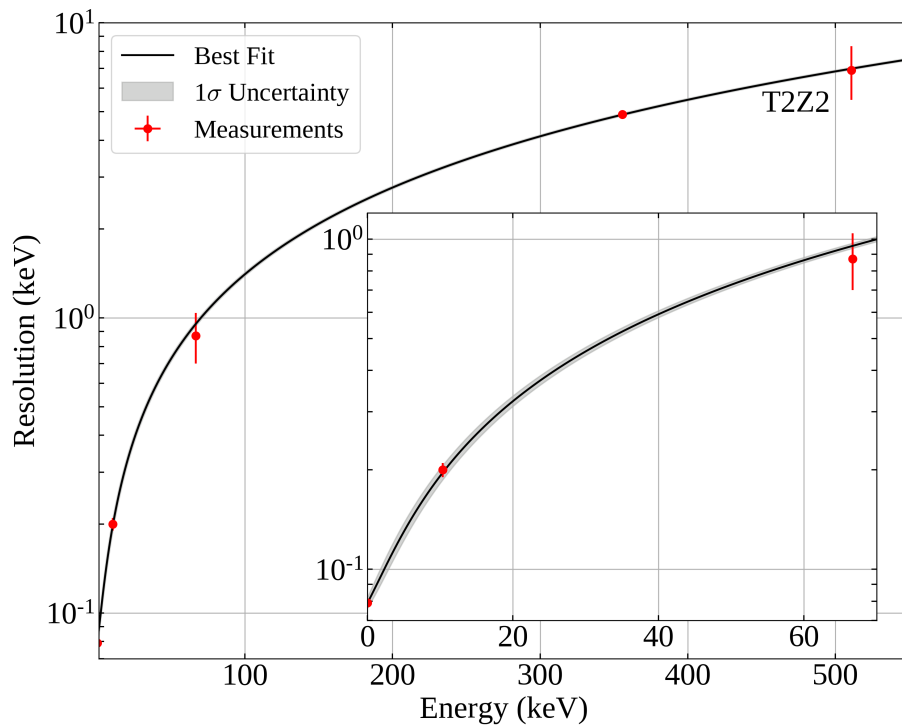


Figure 4.10: The final fitted resolution model as a function of energy for detector T2Z2. The measured peak resolutions (red points) are used to determine the best fit line (solid line) and 1- σ uncertainty band. The inset plot shows the same model between 0 and 70 keV. From Ref. [162]

The total calibrated energy spectra are shown in Figure 4.11 for each detector used in the dark absorption search. The effects of the live time cuts, quality cuts, and the new QFV cuts are shown sequentially.

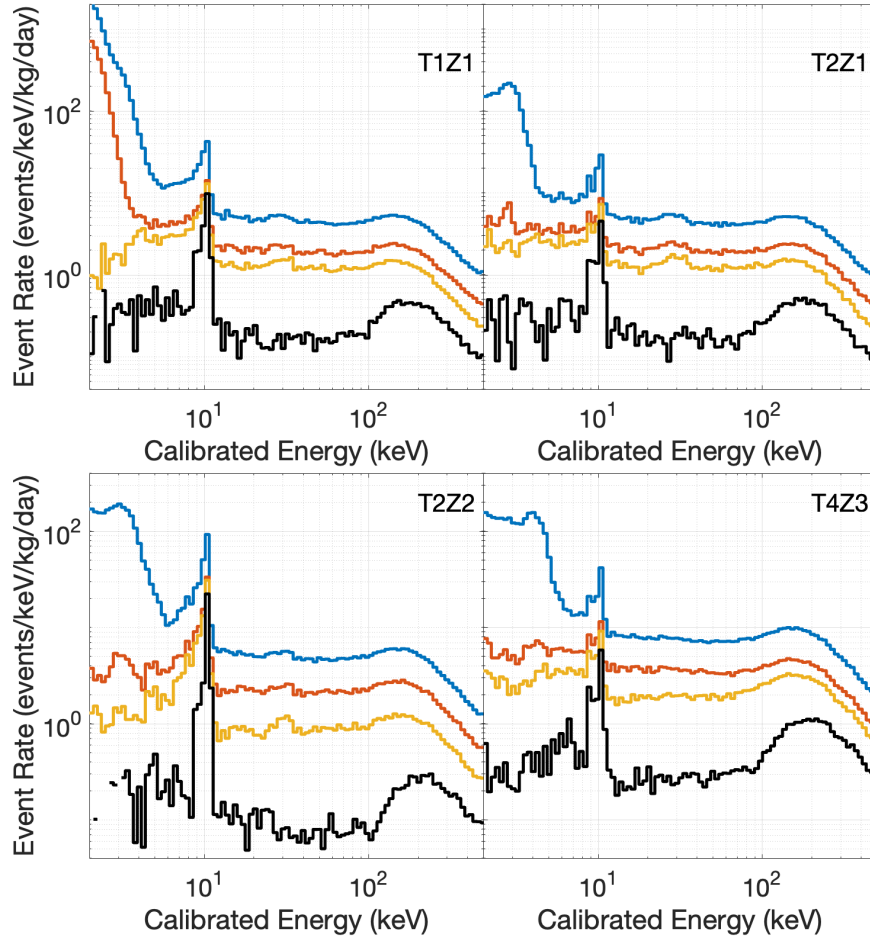


Figure 4.11: The calibrated energy spectra for each of the four detectors used in the dark absorption search, sequentially demonstrating the effects of different cuts. All physics triggers are shown in blue, while the effects of live time cuts (orange), and all non-QFV cuts (yellow) are compared. The final spectra with all cuts including the new QFV cuts are shown in black.

4.3 CDMSlite Data

The CDMSlite data used for this dark absorption search are from Run 2 and Run 3. Run 2 data were acquired in 2014 from February to November, and Run 3 was divided into two sub-runs, Run 3-1 and Run 3-2, with data acquired in 2015 from February to March and from April to May, respectively. Table 4.4 summarizes the exposure for each run. For this analysis, we consider Run 2, 3-1, and 3-2 as separate data sets.

	Run 2	Run 3-1	Run 3-2
Exposure (kg·days)	70.1	31.5	29.4

Table 4.4: The exposure of CDMSlite Run 2, Run 3-1, and Run 3-2.

The CDMSlite energy selection criteria and signal efficiencies up to the analysis range of 25 keV were already determined in prior analyses. In the original WIMP search for CDMSlite Run 2, the efficiency was only determined up to 2 keV [152]. However, it was extended up to 30 keV in a later analysis of backgrounds [169] by comparing ERs from ^{133}Ba calibration data to simulated spectra (not a single scatter background, but we assume that the CDMSlite analysis range is low enough in energy to ignore multiple scatters). Saturation effects at the highest energies were modelled. The CDMSlite Run 3 WIMP search implemented a PLR method to measure the backgrounds at high energy and constrain them at low energy, and therefore the efficiency had already been calculated up to 25 keV [121] in a similar manner to Run 2. Figure 4.12 shows the efficiencies for each run.

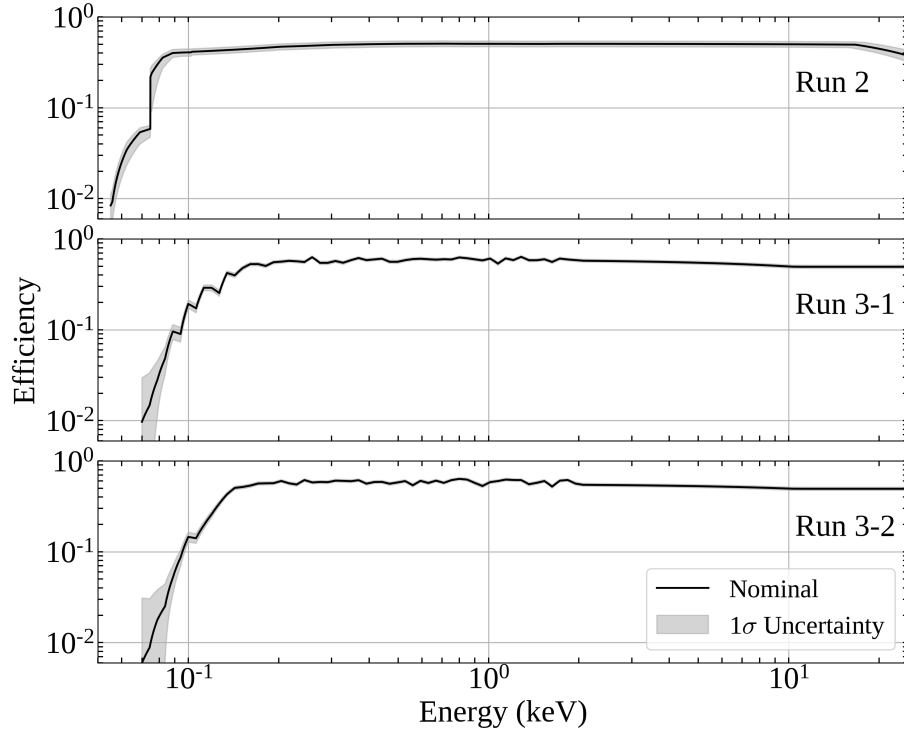


Figure 4.12: The energy dependent signal efficiency for CDMSlite Run 2 (top), Run 3-1 (middle), and Run 3-2 (bottom) up to 25 keV, the upper limit of the CDMSlite analysis range. The nominal efficiencies (black) are shown with their 1σ uncertainty bands (grey shaded). From Ref. [162].

Energy resolution models were also determined for the CDMSlite Run 2 and Run 3 WIMP search analyses [121, 152]. The model in Eq. 4.8 was fit to the measured widths of the K-, L-, and M-shell electron capture peaks. The fit parameters for each run are given in Table 4.5, and the models are shown in Figure 4.13.

	σ_0 (eV)	B (eV)	A ($\times 10^{-3}$)
Run 2	9.26 ± 0.11	0.64 ± 0.11	5.68 ± 0.94
Run 3-1	9.87 ± 0.04	0.87 ± 0.12	4.94 ± 1.27
Run 3-2	12.7 ± 0.04	0.80 ± 0.12	5.49 ± 1.13

Table 4.5: The energy resolution model parameters for CDMSlite Run 2, Run 3-1, and Run 3-2, from Refs. [121, 152].

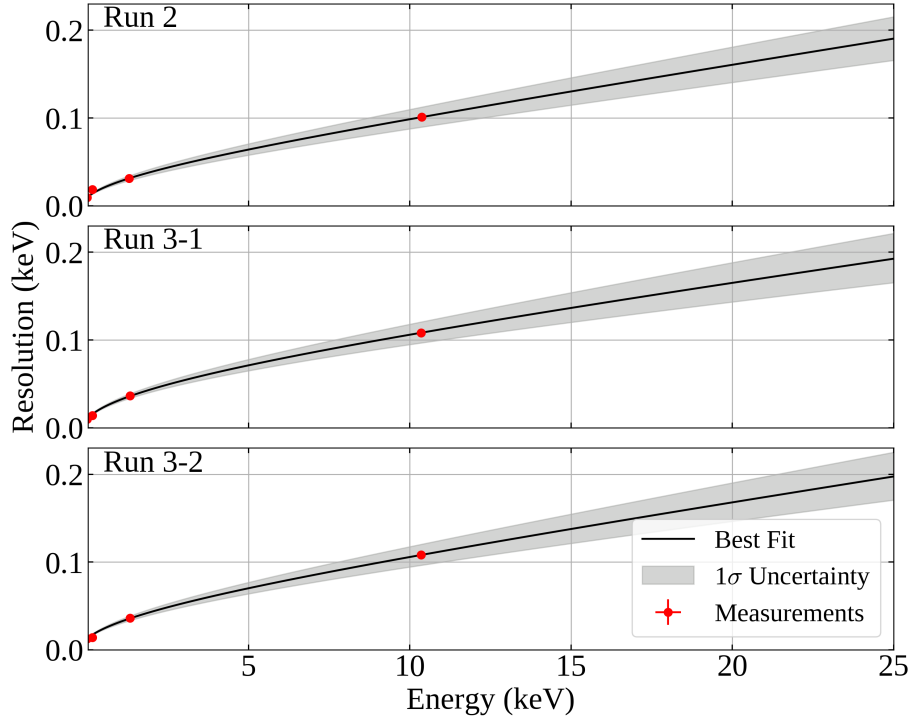


Figure 4.13: The energy resolution models for CDMSlite Run 2 (top), Run 3-1 (middle), and Run 3-2 (bottom) up to 25 keV. The best fit curves (black) and their 1σ uncertainty bands (grey shaded) are shown with the measured widths (red points with uncertainty bars) of the three Ge EC peaks. From Ref. [162].

4.4 Limit Setting

Altogether there are seven different data sets, each with their own resolution model and signal efficiency: four iZIP detectors and three CDMSlite data periods. For each dataset, upper limits on the event rate are calculated at 90% CL using a Poisson counting method and converted to upper limits on each dark matter coupling parameter. We combine the datasets in a manner that avoids selection bias in order to produce a total combined limit for SuperCDMS Soudan. As there is no background modelling or subtraction, only an upper limit on the signal strength for each dark matter model is extracted.

4.4.1 Limit Setting Method

The signal model is the same for both dark photons and ALPs, a gaussian centred at the dark matter mass-equivalent energy with a width determined by the energy dependent resolution model at that same mass-equivalent energy.

The analysis mass range for the iZIP detectors is from $3 \text{ keV}/c^2$, sufficiently above the trigger threshold (around 1 keV), to $500 \text{ keV}/c^2$, just below the highest energy peak (511 keV) used for energy calibration and resolution measurement. The analysis range for R. Germond's CDMSlite analysis is from $40 \text{ eV}/c^2$ to $25 \text{ keV}/c^2$, resulting in an overlap with this iZIP analysis between $3 \text{ keV}/c^2 - 25 \text{ keV}/c^2$. The $25 \text{ keV}/c^2$ analysis threshold is chosen to avoid saturation effects caused by the high NTL amplification.

For each mass, an upper limit on the observed signal rate is first calculated by counting the number of events with recoil energy (calibrated and scaled P_{total}) within a window about the mass-equivalent energy. A Poisson upper limit on the number of events is calculated and converted to a rate by dividing by the efficiency-corrected exposure, taking into account the window size – this is implicitly done in the calculation of the iZIP QFV efficiency, but must be accounted for in the CDMSlite datasets. The window size chosen is a compromise between maximizing the signal efficiency with a large window and also maximizing the signal to background ratio with a smaller window. A 1σ window about the mass-equivalent energy is chosen, where σ is the energy resolution determined from the resolution at the mass-equivalent energy. The window near the trigger threshold is more complicated for CDMSlite, as described in Refs. [162, 180]. For the iZIP analysis, this complication is avoided by cutting off the low energy region where the signal efficiency changes dramatically. Pushing this analysis to lower energies is not necessary as the sensitivity of the CDMSlite data here is superior anyway.

The upper limits on the signal event rates for each mass are calculated separately for each detector. A 1σ uncertainty band is also calculated. The upper (lower) bound is generally determined by using the upper (lower) limit on the resolution and the lower (upper) limit

on the efficiency. To see this, a larger energy resolution increases the signal window size, increasing the number of events, leading to a higher limit on the event rate (and subsequently an higher (worse) bound on the coupling constant). A lower efficiency has the same effect. The dominant uncertainty on the iZIP limits is the statistical uncertainty from the likelihood fits. Similarly, the efficiency dominates the uncertainty on the CDMSlite limits.

To combine the different datasets, we wanted a method that was not unfairly weakened by detectors with particularly high background rate, but that also minimizes selection bias from just picking the detectors with the lowest rate at each mass. We follow the following procedure for each dark matter mass:

- Select the dataset with the lowest observed upper limit on the rate,
- discard any dataset where the difference between the observed rate and the lowest rate exceeds three times the uncertainty on the difference (3σ),
- include the remaining datasets in the combined limit calculation,
- sum the number of events in the selected datasets and calculate the upper 90% limit on this number,
- use the total efficiency-weighted exposure to determine the 90% upper limit on the combined event rate.

A Monte Carlo simulation was performed, showing that the bias introduced by discarding datasets in this manner is less than 1% if the background rate that all detectors are exposed to is the same. Since the background rates are different, the actual bias is even less.

The upper limit on the rate is thus calculated for each individual data set, as well as for all data sets combined. In addition, combined limits are calculated for each of the two data types (iZIP and CDMSlite), using the same selection method for the subsets of data. Upper and lower 1σ uncertainty bands are also calculated using the uncertainty on the resolution and efficiency.

The rate limits are converted to limits on the dark photon kinetic mixing constant ε and the ALP coupling constant g_{ae} using Equations 2.37 and 2.32, respectively. As the lowest mass considered in these analyses was $40 \text{ eV}/c^2$, in-medium effects did not need to be considered for the dark photon kinetic mixing. Therefore, in both calculations the photoelectric cross section is used. An extensive literature search was performed by M. Wilson, who developed a nominal cross section as well as a conservative cross section based on the available data. These photoelectric cross sections are shown in Figure 4.14. The main limit and the lower uncertainty band use the nominal photoelectric cross section, and the upper bound of the uncertainty band uses the conservative photoelectric cross section.

4.4.2 Results and Discussion

Using the limit setting method described above, the seven datasets consisting of four iZIP detectors and three CDMSlite runs are used to calculate three separate limits. Figure 4.15 shows the calculated combined 90% CL upper limits on the event rates for SuperCDMS Soudan, as well as the individual iZIP and CDMSlite rates. Inclusion of the iZIP data degrades the combined rate in the overlap region due to the higher background rates, likely due to these data being collected 1–2 years before the CDMSlite Runs when backgrounds from cosmogenic activation were higher. The combined rate is lower than the CDMSlite rate in the region of the 10.37 keV Ge K-shell peak, where the rate is run-dependent due to differences in Cf activation periods.

The resulting limits on the coupling parameters for dark photons and ALPs are shown in Figures 4.16 and 4.17, respectively. Constraints from other direct detection experiments are also shown for a view of the current status of the parameter space. At the time of publication, the SuperCDMS Soudan constraint was the strongest laboratory constraint on ALPs in the $(40\text{--}186) \text{ eV}/c^2$ mass range. The limit on dark photon kinetic mixing was similarly world leading or highly competitive in the same range. The sensitivity in this mass range is provided entirely by the CDMSlite data.

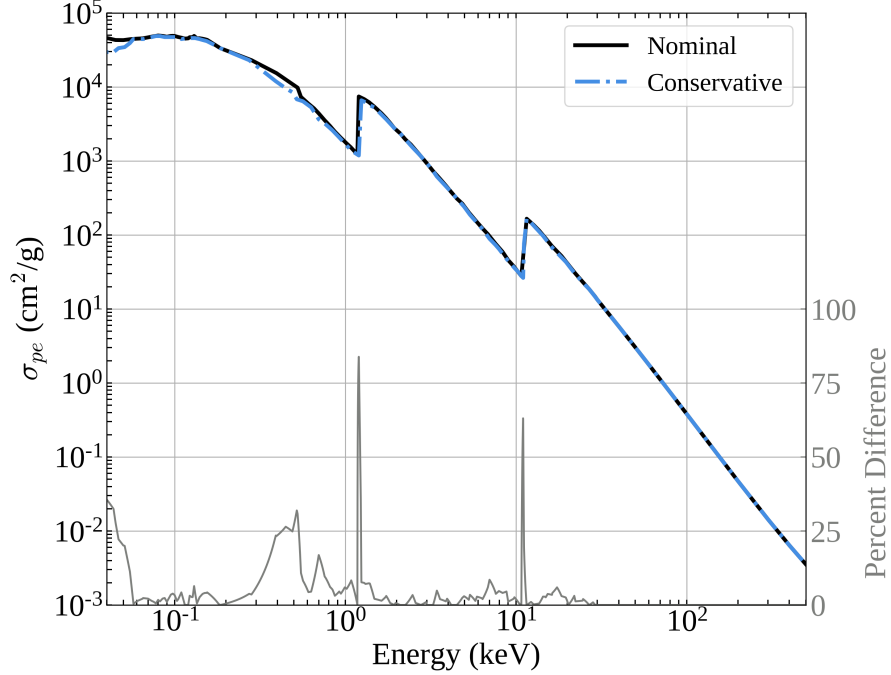


Figure 4.14: The photoelectric cross section $\sigma_{p.e}$ for Ge as a function of photon energy in units of cm^2/g . The solid black line is the nominal cross section, compiled by M. Wilson as described in Ref. [213], following the approach taken by Hochberg et al. in Ref. [71] and using data from [247–250]. The dashed blue line is the conservative cross section, determined using the smallest values of $\sigma_{p.e}$ found in literature. The grey line is the percent difference between the nominal and conservative photoelectric cross sections. The two sharp peaks in the percent difference near the atomic binding energies are artifacts of the finite size of the energy steps of the data in the literature. Figure from Ref. [162].

There is a vast area of parameter space consistent with dark photons constituting all of the CDM [261], with the upper bound given by the grey transparent region in Figure 4.16. Direct detection searches are now sensitive enough to probe this parameter space. The constraint above $\sim 20 \text{ keV}/c^2$ comes from the $A' \rightarrow 3\gamma$ process. Dark photons with mass below twice the mass of the electron can only decay via an electron loop into three photons. The rate of this process is highly mass dependent: $\Gamma \propto m_{A'}^9$. While the process is heavily suppressed for low mass dark photons, it is relevant for the parameter space that direct detection experiments are currently able to probe. The constraint is imposed

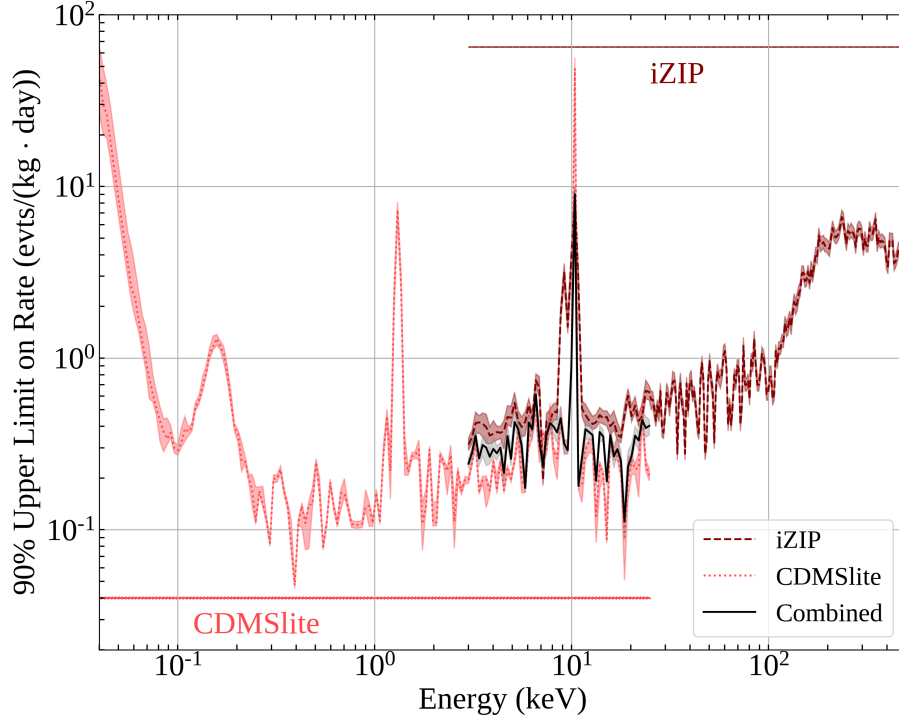


Figure 4.15: Calculated upper limits on rates from combining data from the CDMSlite runs (light red dotted line), the iZIP detectors (dark red dashed line), and all data sets (black solid line, shown only in the overlap region). The CDMSlite analysis range is from 40 eV – 25 keV while the iZIP analysis range extends from 3 – 500 keV; both ranges are indicated. The shaded region around each line corresponds to the uncertainty on the rate limits. The elevation in the limits near 10 keV, 1 keV, 160 eV and in iZIP data around 9 keV results from the higher measured rates due to the Ge K-, L- and M-captures and the K-capture of ^{65}Zn respectively. Figure from Ref [162].

by the requirement that the population of photons from this decay process is smaller than the measured diffuse X-ray background [261]. Masses below $\sim 20 \text{ keV}/c^2$ are more strongly constrained by the lower bound on the effective number of relativistic neutrino species, N_{eff} , which is well constrained from BBN and CMB measurements. Dark photon dark matter can dump energy into the photon bath, leading to a temperature increase and subsequent decrease in N_{eff} [261]. The constraint on ϵ shown in Figure 4.16 is set by the lower bound on N_{eff} of 2.39 at 95% CL [268].

The strongest overall direct detection constraints for dark photons come from the LXe

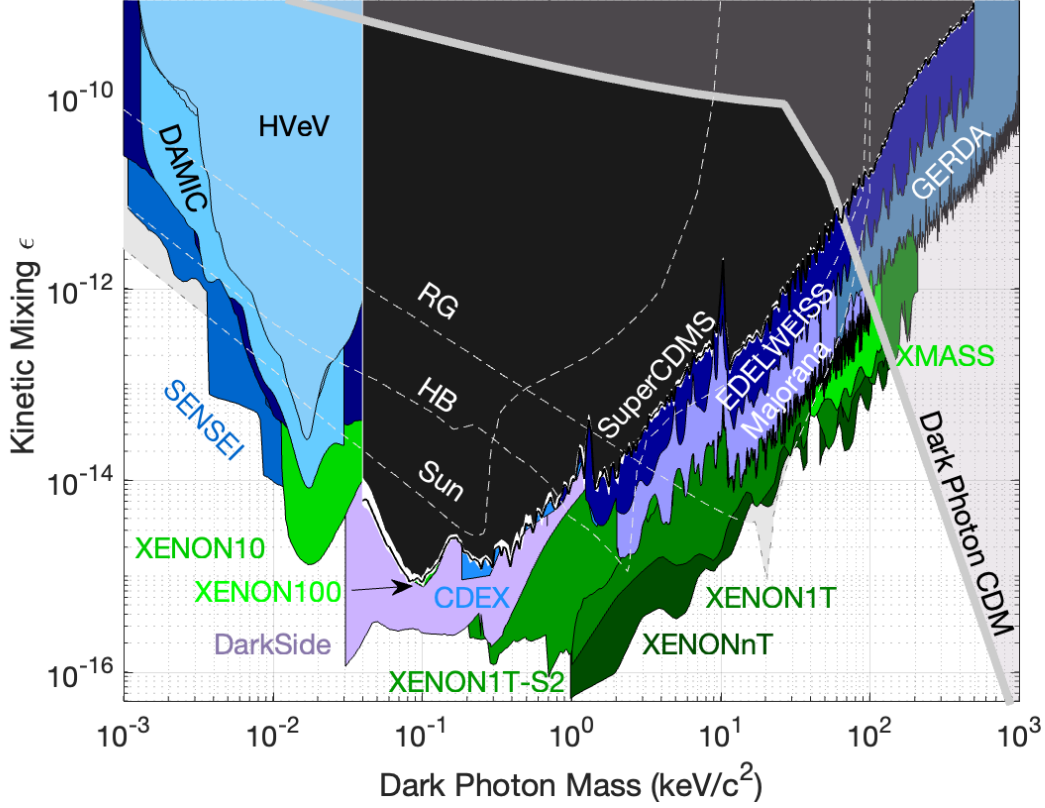


Figure 4.16: SuperCDMS Soudan upper limit (solid black) with uncertainty band (shaded white) on the dark photon’s kinetic mixing. Also shown are limits set by other direct-detection experiments, including EDELWEISS-III [83] and EDELWEISS-SubGeV [84], GERDA [251], CDEX [252], DAMIC [253], SENSEI [254], the SuperCDMS HVeV device [239, 240], An *et al.*’s analysis of XENON10 and XENON100 [255], Hochberg *et al.*’s analysis of XENON100 [71], XENON1T [256, 257], XENONnT [258], XMASS [259], and DarkSide-50 [260], which was published after our results. The dashed lines show limits set from anomalous energy loss mechanisms in the Sun, horizontal branch stars (HB), and red giants (RG) from [255]. These constraints have almost been entirely surpassed by direct detection constraints, with the remaining constraints shown in shaded grey. The region below the dark photon CDM boundary indicates the parameter space where dark photons could constitute all of the dark matter [261].

experiments, particularly XENON1T and XENONnT. At the time of writing, XENONnT has the lowest overall constraint on ϵ of 5.3×10^{-18} at $m_{A'} \approx 1 \text{ keV}/c^2$ [258], exceeding

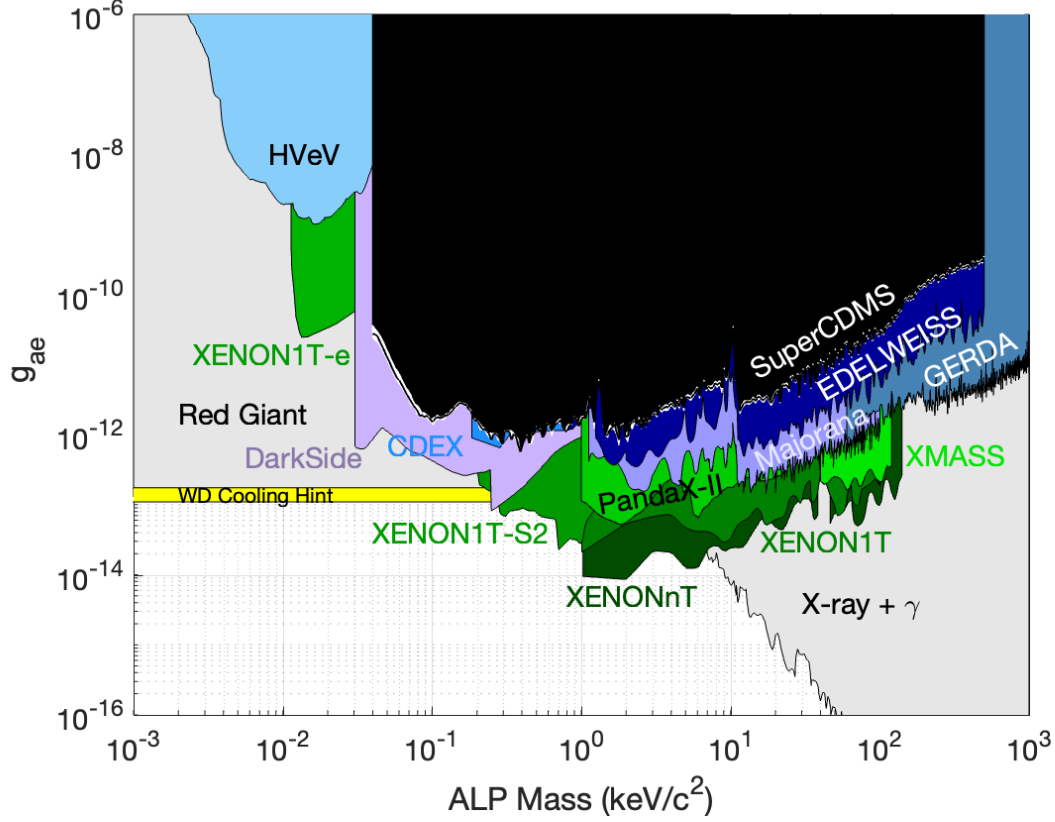


Figure 4.17: SuperCDMS Soudan upper limit (solid black) with uncertainty band (shaded white) on the axioelectric coupling constant g_{ae} . Also shown are limits set by other direct-detection experiments including: EDELWEISS-III [83], Majorana [262], GERDA [251], CDEX [252], PandaX-II [101], XENON100 [263], XENON1T [256, 257, 264], XMASS [259], DarkSide-50 [260], which was published after our results, and SuperCDMS HVeV [240]. The grey shaded regions are excluded by the observed cooling of red giants (RG) [265] and by X-ray and gamma constraints on the lifetime of the process $a \rightarrow 2\gamma$ [266]. The parameter space consistent with a hint from white dwarf (WD) stars [267] is also shown (yellow).

the prior XENON1T constraint [256]. XENON1T has been able to improve its low mass reach through the so-called S2-only analysis, and holds the strongest constraints in the 0.2–1 keV/ c^2 mass range [257] by exploiting the lower threshold of the S2 signal compared to the S1 signal (see Sec. 2.3.1). The LAr experiment DarkSide-50 holds the strongest constraint from 30–200 eV/ c^2 [260]. Other notable experiments with weaker constraints in this mass

range include CDEX [252], XENON100 [255], and XMASS [259]. The strongest constraint for masses 12–300 eV comes from an analysis [255] of XENON10 data with single-electron sensitivity [136].

Silicon detectors dominate the low mass region with the strongest constraints below 12 eV/ c^2 set by SENSEI [254]. SuperCDMS HVeV set strong constraints in Run 1 [239] and Run 2 [240] with the small devices in surface facilities, and DAMIC [253] has also set strong constraints in the sub-100 eV/ c^2 mass range.

Though SuperCDMS Soudan was never competitive in the keV/ c^2 mass range, it is still interesting to compare to the other Ge semiconductor experiments, which are unable to set constraints as competitive as XENON. EDELWEISS-III [83] dominates the SuperCDMS Soudan limit above 1 keV, though a more recent (2020) result from the EDELWEISS-subGeV program [84] is competitive with DAMIC. SuperCDMS Soudan was able to take advantage of the CDMSlite operational mode and probe lower masses; the original EDELWEISS-III limit only reached as low as ~ 1 keV/ c^2 . GERDA has set the strongest Ge constraint above ~ 100 keV and strongest overall constraint above 210 keV/ c^2 where the XENON analyses end. The Majorana demonstrator published dark photon constraints from 1.1 to 100 keV/ c^2 with better sensitivity than the other Ge experiments [262].

Stellar constraints are set from anomalous energy loss mechanisms from the sun, horizontal branch stars, and red giants [255]. These astrophysical constraints have been almost entirely surpassed by direct detection with the exception of the Sun constraint below around 3.6 eV/ c^2 and a small part of the red giant constraint around 20 keV/ c^2 .

There are several astrophysical constraints on ALP dark matter, including from the luminosity of red giants, requiring $g_{ae} < 1.3 \times 10^{-13}$ [265] (grey shaded region at low mass in Figure 4.17). More stringent constraints are imposed at higher masses from constraints on the process $a \rightarrow 2\gamma$; ALPs may decay to two photons via an off-shell electron loop, but a viable ALP dark matter candidate must have a life-time longer than the age of the Universe. X-ray and gamma searches from the Milky Way and M1 galaxy constrain this

decay, and thus also g_{ae} [266]. This constraint corresponds to the grey shaded region above $\sim 7 \text{ keV}/c^2$ in Figure 4.17.

A hint from the cooling of white dwarfs has indicated potential parameter space for ALP dark matter, giving a target in the g_{ae} -mass plane [267]. This is represented by the yellow band in Figure 4.17. XENON experiments have already probed this band for $m_a > 200 \text{ eV}/c^2$, but at lower masses the band remains a target.

The current status of direct detection constraints for ALP dark matter is similar to that for dark photons. At the time of the original publication, SuperCDMS Soudan had the best direct detection constraint on g_{ae} from 40–200 eV/c^2 . A more recent measurement by DarkSide-50 has since surpassed this limit and probed masses as low as 30 eV/c^2 [260]. SuperCDMS HVeV Run 2 [240] has the strongest constraints for masses below 11 eV, while XENON1T-e, an analysis using S2 signal to reach single electron sensitivity, provides the strongest constraint in the range 11–30 eV/c^2 [264]. SENSEI and DAMIC do not provide analyses on the axioelectric coupling, but based on their dark photon sensitivity their constraints would surpass HVeV. Similarly, based on the dark photon constraints, the XENON10 constraint would be stronger than the XENON1T-e constraint.

4.4.3 Enhancing Sensitivity with the Inclusion of Dark Compton Scattering

Dark Compton scattering has typically been neglected in absorption searches, this search included. In Ref. [72], Hochberg et al. demonstrate that considering dark Compton scattering can enhance the discovery potential at higher masses. They perform a conservative recalculation of limits from SuperCDMS Soudan, EDELWEISS III, and GERDA, shown in Figure 4.18.

For intermediate-sized detectors like those used by SuperCDMS, EDELWEISS, and GERDA, the SM photon may escape the detector before depositing all of its energy, as described in Section 2.2.3. Simulations taking into account the detector geometry would be

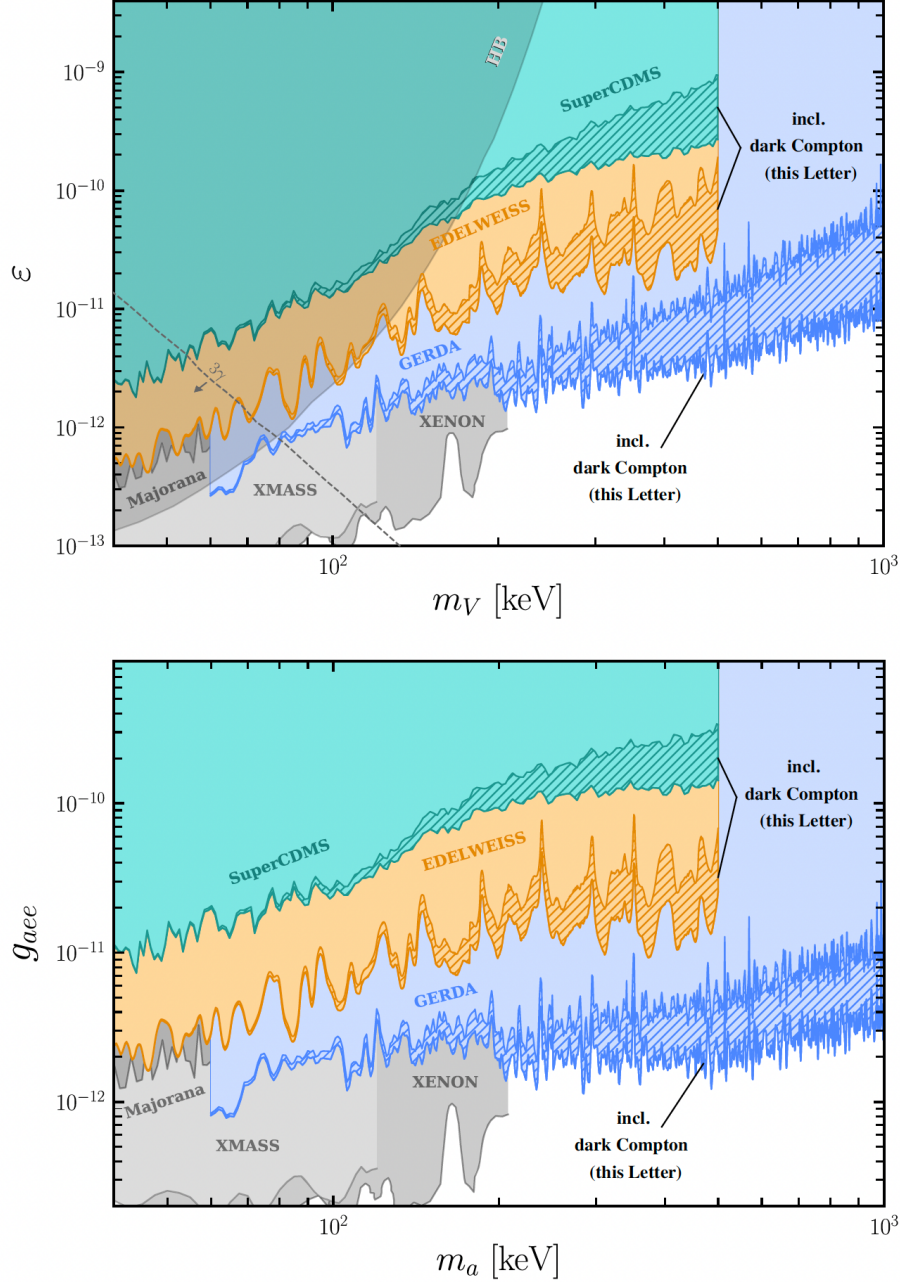


Figure 4.18: 90% CL constraints on dark photon kinetic mixing (top) and axioelectric coupling to electrons (bottom) and the impact of dark Compton scattering (hatched) from Ref. [72]. Note that the 2017 constraints from the Majorana demonstrator [269] shown are earlier results than the 2022 shown in Figures 4.16 and 4.17, which have improved sensitivity [262].

required to produce accurate signal models. However, the recalculated limits use a back-of-the-envelope calculation based on the probability of observing a photon at a depth x in the detector material,

$$P(x) = e^{-x/\lambda}, \quad (4.11)$$

where λ is the energy and material dependent attenuation length.

Here I provide a more detailed re-analysis of the SuperCDMS data, producing simulations of the interactions of the SM photon in order to calculate a more accurate limit. Due to the more complicated signal model, the new limits are calculated using the OI method.

Data Selection

Dark Compton scattering has little impact on limits below around $100 \text{ keV}/c^2$ dark matter mass as seen in Figure 4.18, thus complicating the analysis by including the CDMSlite data sets is unnecessary. Additionally, for most of the mass range of interest, only one detector is selected by the selection method described earlier in the chapter. T2Z2 has a significantly lower event rate at high energies than the rest of the detectors, and for much of the high mass range is the only detector contributing towards the limit.

Therefore, for this extended analysis I consider only data from T2Z2. I include the energy dependent resolution model and the efficiency calculated for this detector. Note that the QFV efficiency described earlier included the $\pm 1\sigma$ signal window size in its calculation. Since I will perform an OI analysis, the limitation to a 1σ signal window is unnecessary and the QFV efficiency will be adjusted accordingly (i.e. the T2Z2 QFV efficiency above 10.37 keV with the signal window imposed is 36.1% , so here I will be using $36.1\%/0.68 = 53.1\%$ instead).

Gamma Simulations

The energy split between the recoil and the produced gamma is essentially fixed, thus we can simulate only the gammas and trivially combine this with the initial energy deposit. I produce simulations of monoenergetic gammas distributed evenly in a SuperCDMS Soudan iZIP detector using the GEANT4-based SuperCDMS simulation software, SuperSim. This publicly available package defines the detector geometry and any experimental apparatus and simulates the behaviour of source particles in the detector and surrounding geometry. For a summary of the package, see Refs. [171, 270]. I simulate 10^6 photons for photon energies from 10 – 500 keV in steps of 1 keV. Below 10 keV, it is very unlikely that the photon can escape the detector (e.g. around 0.17% of 10 keV photons may escape without considering the FV cuts. With the FV cuts, the value is more like 0.03%). For each photon, I take the total deposited energy and also calculate energy-weighted x , y , and z coordinates. For example, for a given photon, the energy-weighted x coordinate is

$$X_{weight} = \frac{x_I T_C + \sum_i x_i E_i}{E_t + T_C} \quad (4.12)$$

where x_I is the position of the dark Compton interaction (equal to the initial position of the photon), T_C is the initial electron recoil energy from the dark Compton scatter, i is an index running over the number of steps for the given photon, x_i is the initial position of the particle in step i , E_i is the energy deposited in step i , and E_t is the total energy deposited by the photon. These coordinates are important for defining a fiducial volume cut to approximate the effect of the QFV cuts.

Ideally, we would have a calibration source with the same behaviour as the expected signal to determine the efficiency of the QFV cuts on this signal. While we have a ^{133}Ba gamma source for ER calibration, this is a source external to the detector and many of the events may be more impacted by edge-effects. Instead, I attempt to make a physical FV cut on the simulated data based on what the QFV cut is attempting to achieve.

I define a sub-volume of the detector with fractional volume (compared to the original detector volume) equal to the QFV efficiency at 10.37 keV (the QFV cuts were designed conservatively to have constant efficiency above 10.37 keV for single-scatter events). First, let's consider what the impact of the charge symmetry cut is. This cut is designed to remove events which have asymmetric charge collection on the two sides of the detector, effectively removing events which start in the region of distorted electric field close to the detector faces. Figure 4.19 shows the electric field lines for a SuperCDMS Soudan iZIP detector. The field becomes significantly distorted around 1.5 mm, thus I remove 2 mm from both the top and bottom surfaces to be conservative.

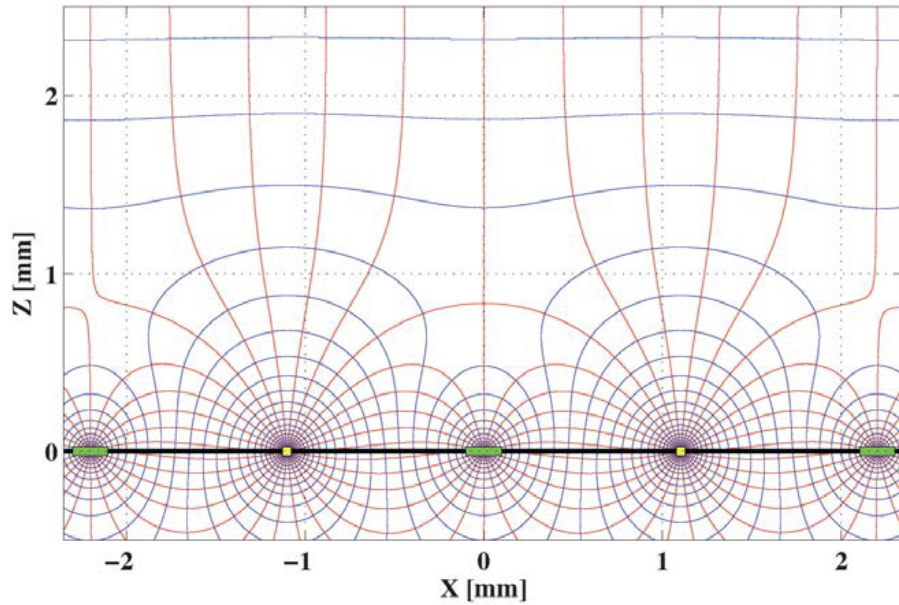


Figure 4.19: Electric field line distribution for a SuperCDMS Soudan iZIP detector near the bottom detector face, from Ref. [140]. This motivates a cut removing 2 mm from the top and bottom surfaces of the detector.

To define the radial physical FV cut on the simulated data, I then calculate the radius required to make the ratio of the sub-volume to the total detector volume equal to the QFV efficiency, which is 53.1% for T2Z2. The resulting radius of the sub-volume is 30.24 mm. To check if this is an appropriate radial cut location, I compare the sub-volume to the location

of the outer charge channel, as the radial charge cut is designed to remove events which have a significant portion of their total signal deposited in this channel. Figure 4.20 shows the cut location and the defined sub-volume compared to the outer charge channel boundary. The FV radius is smaller than the channel boundary, which is not surprising given oblique electron propagation and that we restrict the signal deposited on the outer channel.

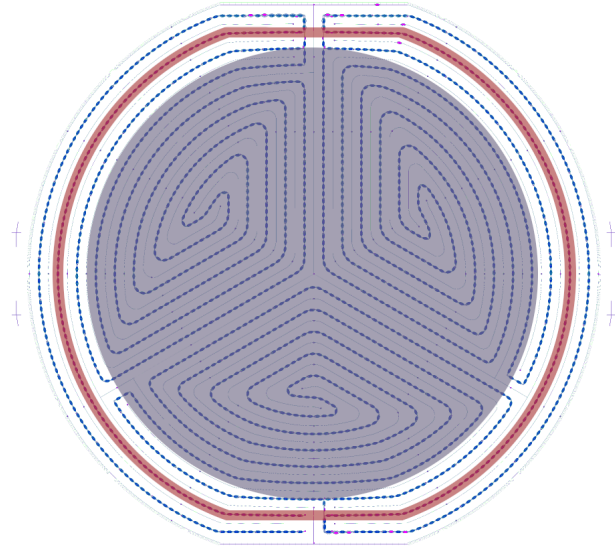


Figure 4.20: View of the iZIP phonon and charge channel layout with the outer charge channel boundary (red, shaded: assumed to be the phonon rail between the outer charge rail and outermost inner charge rail) compared to the physical radial FV defined for the simulated gammas (purple, shaded).

The physical volume cut used for the simulated data is

$$X_{weight}^2 + Y_{weight}^2 < (30.24 \text{ mm})^2 \quad (4.13)$$

$$2 \text{ mm} < Z_{weight} < 23.1 \text{ mm} \quad (4.14)$$

Figure 4.21 shows the simulated spectra for two different gamma energies, 20 keV and 300 keV, and the impact of the FV cut. The pre-cut spectra both consist of a main peak

at the gamma energy, indicating full energy deposition in the detector, a peak at 0 keV indicating that the gamma fully escapes with no energy deposition, and a distribution in between, corresponding to gammas which escape the detector but deposit some fraction of their energy first. For the lower energy 20 keV gamma, most events deposit fully in the detector, with a small fraction fully escaping. There are few events which have partial energy deposition. In contrast, the higher energy 300 keV gamma has a lower fraction of events in the full deposition peak and a larger fraction in the escape peak. The partial deposition Compton spectrum is well populated.

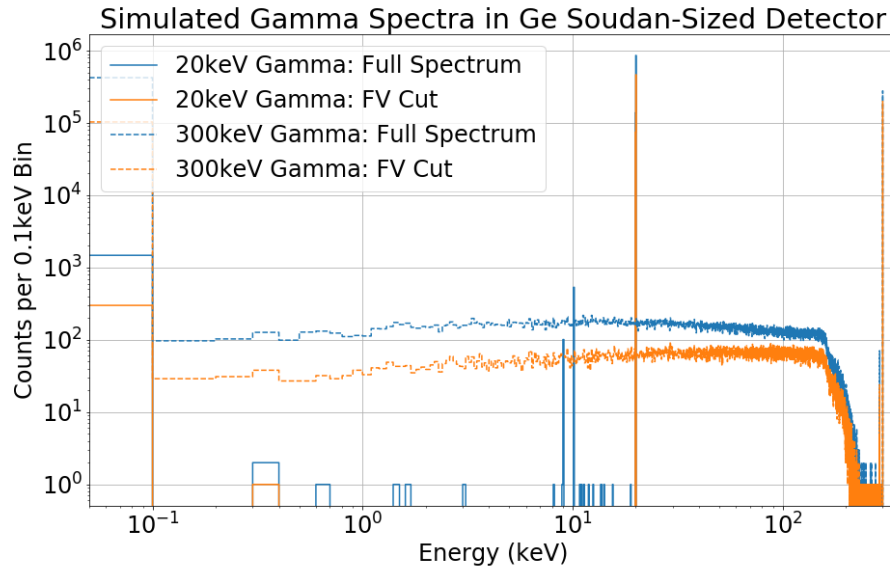


Figure 4.21: Simulated spectra for a 20 keV gamma (solid) and a 300 keV gamma (dashed) with 10^6 simulated photons each. The total spectra (blue) are compared to the impact of the FV cut (orange).

Total Combined Spectra

The total energy deposited in the detector in a dark Compton scattering event is the energy of the initial electron recoil and the energy deposited by the SM photon, which is given by the simulation. In the mass range of interest, the photon energy and electron recoil energies

are fixed. Thus the total spectrum is just the simulated gamma spectrum shifted by the energy of the electron recoil, smeared by the energy dependent detector energy resolution. The dark absorption signal shape is the same as before, a Dirac delta at the mass-equivalent energy smeared by the resolution. The combined signal from both processes is used for the limit setting, noting that the rates of the absorption and Compton processes are slightly different.

The final spectra for the limit calculation have the energy dependent efficiency applied. For the dark absorption process, the signal is a gaussian at the mass-equivalent energy with width given by the energy resolution. This is multiplied by the signal efficiency (with QFV efficiency applied and the factor accounting for the signal window removed). For the dark Compton spectrum I use the simulated photon spectrum with the FV cut applied, effectively including the FV cut efficiency. I then multiply this by the signal efficiency of the remaining quality cuts. The rate of the dark Compton process is properly taken into account for the normalization of the spectrum. The spectra for the two interaction channels are then combined. Figure 4.22 shows the combined spectra for several masses for ALPs. The dark photon spectra are similar, with slightly different ratios between the absorption and dark Compton rates.

Impact on Limits

I use the OI method [271, 272] rather than Poisson for the limit calculation as the signal shape is no longer a simple gaussian. This method is a way to look for a signal in the presence of an unknown background. It searches for an interval, called the optimum interval, that most strongly indicates that a proposed cross section would be too high. A summary of the OI method can be found in the appendices of Ref. [183]. As a preliminary check, I calculated the OI limit for the absorption channel alone and found a result that closely matched the Poisson limit.

Figure 4.23 shows the resulting limits for ALPs and dark photons. The improvement

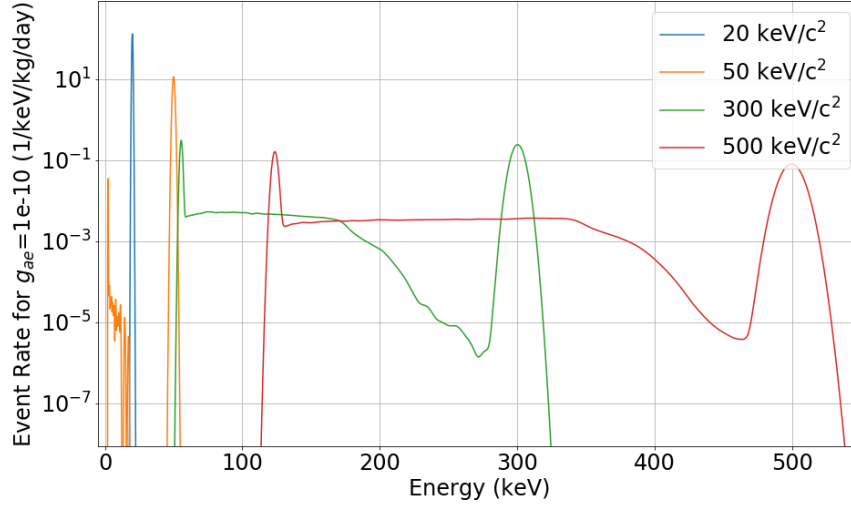


Figure 4.22: The final ALP combined dark Compton and dark absorption spectra for 20, 50, 300, and 500 keV/c^2 ALPs with efficiency applied. The rate of the 20 keV/c^2 gamma escape is too low to be seen on this scale.

above around 100 keV/c^2 over just the absorption channel alone is evident, and the limits are similar to those calculated by Hochberg et al. in Ref. [72].

At higher masses, the produced photon becomes more likely to completely escape the detector, particularly in the outer regions of the detector. This produces the peak at lower energy in the spectrum, and around 215 keV/c^2 dark matter mass the escape peak starts to overtake the rate of the full absorption peak. This combined with the overall slightly lower event rate below 100 keV (where the escape peak tends to lie for masses considered by this analysis) means that the OI method selects the escape peak as the optimum interval. Figure 4.24 demonstrates this effect, comparing low and high mass dark photons and the optimum interval found by the OI method.

The impact of this effect is somewhat weakened by the FV cut, which reduces the relative amplitude of the escape peak to the full deposition peak. Due to the necessity of the QFV cuts to remove surface backgrounds and events with incomplete charge collection, we cannot take the full advantage of this effect but I show it here to highlight that if we can build

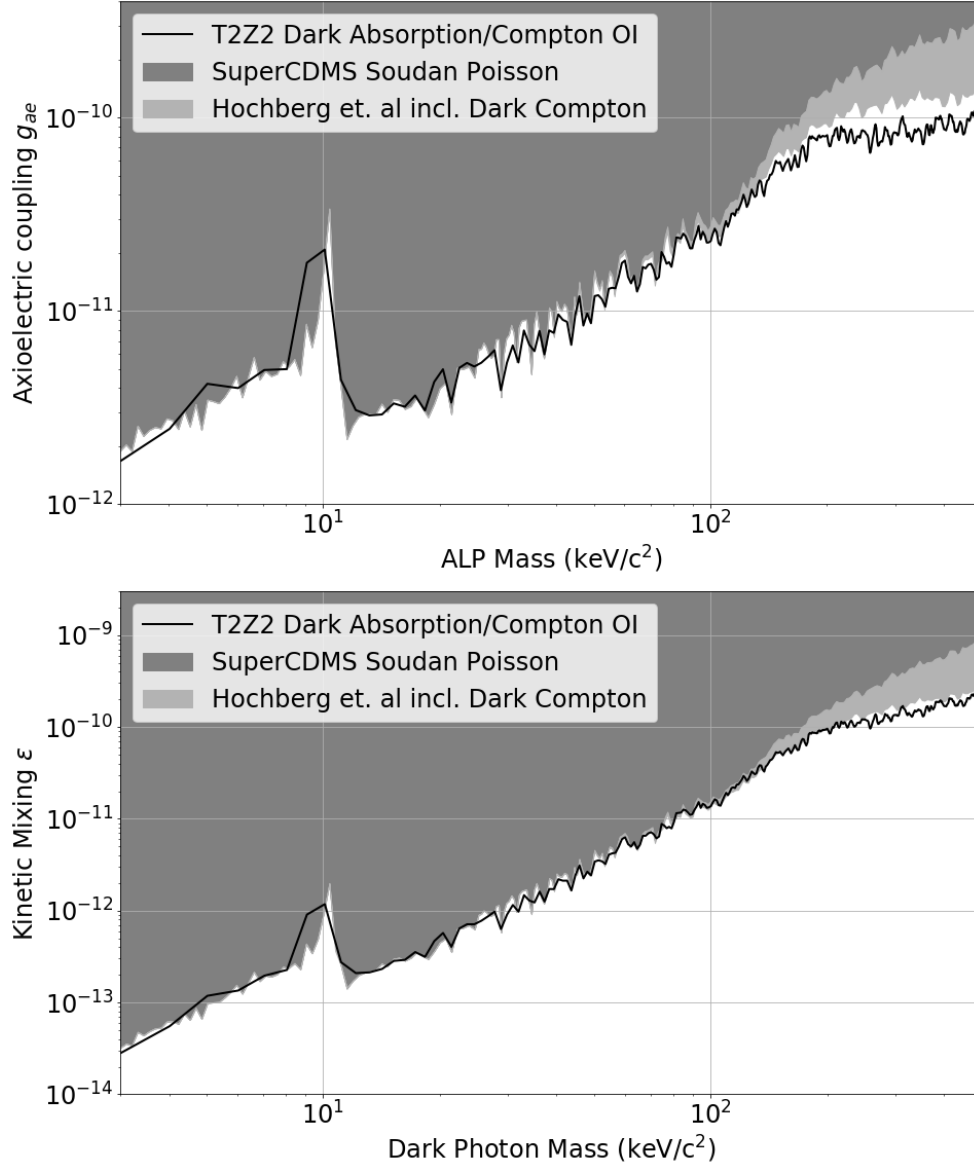


Figure 4.23: SuperCDMS Soudan T2Z2 upper 90% CL OI limit on the ALP axioelectric coupling g_{ae} (top) and the dark photon's kinetic mixing constant (bottom) considering both dark absorption and dark Compton interactions (black, solid). The limit is compared to the combined SuperCDMS Soudan Poisson limit (dark grey shaded) described earlier in the Chapter as well as the back-of-the-envelope calculation by Hochberg et al. [72] (light grey shaded). The difference at low masses is from using only T2Z2 data as well as the different limit setting process.

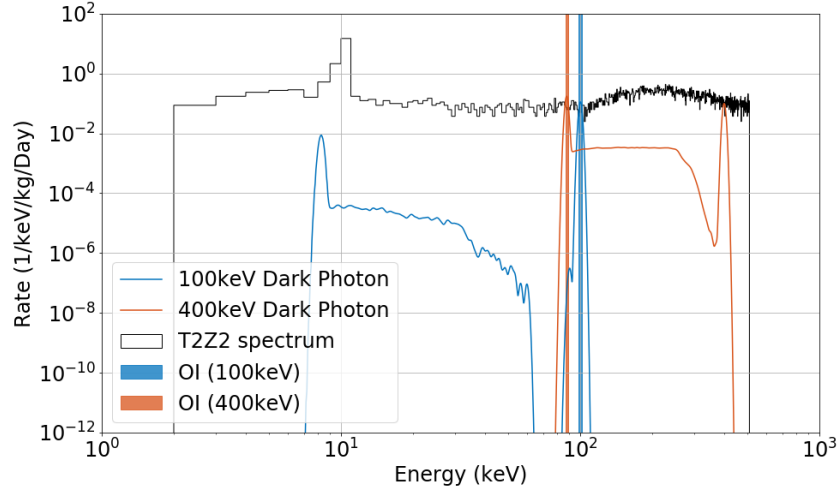


Figure 4.24: Comparison of the optimum interval for a $100 \text{ keV}/c^2$ (blue) vs. $400 \text{ keV}/c^2$ (orange) mass dark photon. The measured T2Z2 energy spectrum is shown (black) with the combined dark absorption and dark Compton signal spectrum with the FV cut applied (blue/orange). The signal rate is determined using the OI method, with the optimum interval indicated (blue/orange shaded).

detectors with smaller regions for edge effects or use analysis techniques to keep the outer volume of the detector, it may be possible to use this effect to our advantage.

Chapter 5

ERDM Long Term Planning Sensitivity Projections

Sensitivity projections are an important part of demonstrating the expected capabilities of direct detection dark matter experiments. In 2016, SuperCDMS presented forecasts for the expected sensitivity of SuperCDMS SNOLAB to WIMP-nucleon couplings [171]. These projections were calculated using internal Matlab-based sensitivity code. At the time, no ERDM projections were produced.

In this chapter I describe various detector and background upgrade scenarios using SuperCDMS technology designed to exploit the future potential of the SuperCDMS SNOLAB facility. In Ref. [177] we explore these different scenarios and their potential science reach, providing sensitivity projections for SI WIMP-nucleon interactions, dark photons, ALPs, and dark photon-mediated LDM.

While producing forecasts for different upgrade scenarios, we also updated the SI WIMP projections for SuperCDMS SNOLAB from Ref. [171] and provided new ERDM projections. We now account for the new cryostat geometry, which has been reduced from 31 to 7 tower capacity as described in Section 3.4.2, and include ionization leakage backgrounds, discretized ionization production, ^{14}C and ^{39}Ar surface backgrounds, and corrections to the ionization collection model. We also account for an increase in the neutron background from the mu-metal shield, improve the assumed ionization yield function based on recent measurements, use an improved energy resolution scaling model, and improved estimates of

the detector parameters. Additionally, we now provide all sensitivities using a PLR method rather than the OI method for limit setting. Projections are provided at the 90% CL. A number of experiments (~ 100) are drawn from a background model, and the PLR limit is calculated for each. The median value is taken as the limit for each dark matter mass.

The sensitivity projections described in this section were accomplished by the SuperCDMS forecasting team, including O. Wen, H. Coombes, T. Reynolds, and myself, and was and organized by S. Golwala. I was responsible for the implementation of ERDM models and the calculation of their projections. The focus of this chapter will be on the ERDM results; the NRDM projections can be found in Ref. [177].

In Section 5.1 I give an overview of the sensitivity code, the backgrounds, how the detector response is applied to a recoil spectrum, the ERDM models implemented, and the limit setting procedure. In Section 5.2 I show ERDM forecasts for SuperCDMS SNOLAB. In Section 5.3, I provide an overview of the different upgrade scenarios considered for the future of SuperCDMS at SNOLAB, and in Section 5.4 the resulting forecasts are shown.

5.1 Sensitivity Code

The forecasting process using the Matlab-based sensitivity code is implemented as follows. We identify and model all expected backgrounds as a function of recoil energy. The signal spectra for a given dark matter candidate are also determined for a range of masses to be tested. These background and signal spectra are converted into functions of total phonon energy (in units of counts/keVt/kg/day), and we account for the detector response.

Two different approaches are taken for the treatment of ionization production. NTL amplification with HV detectors can allow us to reach single electron hole pair sensitivity, necessitating that we account for the quantized nature of ionization. This is the first approach. In the second approach, which is used for detectors with poorer energy resolution, we treat ionization as a continuous distribution with statistical fluctuations that are assumed to be negligible. I refer to these two approaches as the discrete ionization code and

the continuous ionization code, respectively. The former is used for HV detectors while the latter is used for lower voltage detectors with poor energy resolution such as the iZIP-style, as well as 0V detectors where ionization production is not relevant.

In this section I describe the sensitivity code and forecasting method in detail. I follow a slightly more in depth description which can be found in T. Reynold's thesis [273], though there have been a few changes or updates which I identify along the way.

5.1.1 Backgrounds

The expected backgrounds for SuperCDMS SNOLAB have already been summarized in Chapter 2.4. In this section we discuss how the backgrounds are modelled in the sensitivity code. The rate and shape of each background is determined as a function of recoil energy, with the rates being provided in counts/keVr/kg/day. The rates can be rescaled as desired, either to explore the impact of a given background, or to calculate the sensitivity for a different background scenario.

Many of the backgrounds are taken from the internal SuperCDMS BackgroundExplorer tool, which collates GEANT4 simulations. Relevant backgrounds have additionally been updated to the new 7-tower-cryostat geometry.

The beta-decaying detector contaminants ^3H , ^{32}Si , and its daughter ^{32}P are parameterized as a function of the electron kinetic energy E_e and beta endpoint Q [274]

$$\frac{dR}{dE_R} = R_{tot} \sqrt{E_e^2 + 2E_e m_e c^2} (Q - E_e^2) (E_e + m_e c^2) F(Z, E_e), \quad (5.1)$$

where R_{tot} is the total rate, m_e is the electron mass, Z is the atomic number, and $F(Z, E_e)$ is the Fermi function. Because of the relatively low energy endpoints of the relevant isotopes¹, we use the non-relativistic approximation

$$F(Z, E_e) = \frac{2\pi\eta}{1 - e^{-2\pi\eta}}, \quad (5.2)$$

¹Note that the endpoint of ^{32}P is 1.711 MeV, thus the non-relativistic approximation starts to break down.

where

$$\eta = \frac{\alpha Z(E_e + m_e c^2)}{p_e c}. \quad (5.3)$$

Here α is fine structure constant, and p_e is electron momentum. The rate R_{tot} is assumed to consist of a rate from cosmogenic production R_{prod} (which is affected by the time spent above ground t_{prod} and time spent underground t_{ug}) and another from preexisting contamination R_{cont} :

$$R_{tot} = R_{prod} \left(1 - e^{-t_{prod}\lambda}\right) e^{-t_{ug}\lambda} + R_{cont}, \quad (5.4)$$

where $\lambda = \ln 2/t_{1/2}$ is the isotope lifetime. We take $t_{ug} = 1$ yr and $t_{prod} = 60$ d. For ^3H we assume a production rate of 90 atoms/kg/day in Ge and 125 atoms/kg/day in Si [171]. Isotopes like ^3H are assumed to be expelled from the detector crystal at the time of pulling, but ^{32}Si remains in the Si crystal. The ^{32}Si contamination R_{cont} is taken as 80 decays/kg/day [275]. We assume a contamination of 1.7 times this value as a conservative goal. The same rate is assumed for the daughter radioisotope ^{32}P . Note that R_{prod} for ^{32}Si (and ^{32}P) and R_{cont} for ^3H are assumed to be 0.

The ^{14}C and ^{39}Ar surface backgrounds are modelled by the same beta spectrum, though their rates are cut in half to account for their surface nature – only half of the decays will be directed towards the detector. ^{14}C is present due to the passivation of the copper housing with citric acid, leaving a few angstrom layer of carbon. A total rate of 5 nBq/cm² on all detector surfaces is conservatively assumed based on the surface density of citric acid and the natural abundance of ^{14}C . ^{39}Ar is associated with the argon-ion sputtering method use to deposit the sensor material on the detector surfaces. The ^{39}Ar rate is based on the abundance in commercial Ar gas. With a rough estimate that 1% of the ^{39}Ar ends up in the TES films, we obtain a rate of 1 nBq/cm² on the top and bottom faces of the detectors.

The cosmogenic lines, which are a Ge-only background, are modelled as delta functions with energies at the electron capture K-, L-, M-, and N- shells. The peak heights are based

on measurements from CoGeNT [89], where we rescale to an above ground production time $t_{prod} = 60$ d, the same exposure time as we used for ^3H . Note that no cool-off time is taken into account; in reality, there will be several months between the detectors arriving underground and being ready for dark matter data taking, thus the rates assumed for these isotopes are conservative. The heights of the lower energy L-, M-, and N-shell peaks are scaled based on the branching ratios shown in Table 5.1, which also shows the energy of each peak. We consider the isotopes $^{68}\text{Ge}^2$, ^{68}Ga , ^{65}Zn , $^{73/74}\text{As}$, $^{56/57/58}\text{Co}$, ^{55}Fe , ^{54}Mn , and ^{49}V .

Isotope	Half Life	K (keV)	L_1 (keV)	M_1 (keV)	N_1 (keV)
^{68}Ge	270.8 d	10.367	1.299	0.1595	0.0015
		(86.406%)	(11.496%)	(1.962%)	(0.136%)
$^{68}\text{Ga}^*$	67.6 m	9.659	1.1962	0.1398	0.001
		(88.473%)	(9.798%)	(1.618%)	0.112%
^{65}Zn	244.26 d	8.979	1.0967	0.1225	0.001
		(88.53%)	(9.77%)	(1.60%)	(1.01%)
$^{73/74}\text{As}$	17.77 d/80.3 d	11.103	1.4146	0.1801	0.001
		(87.60%)	(10.45%)	(1.79%)	(0.158%)
$^{56/57/58}\text{Co}$	77.27 d/271.8 d	7.112	0.8446	0.0913	0.001
	/70.86 d	(88.75%)	(9.63%)	(1.6%)	(0.073%)
^{55}Fe	2.73 yr	6.539	0.7691	0.082.3	0.001
		(88.53%)	(9.83%)	(1.57%)	(0.065%)
^{54}Mn	312.3 d	5.989	0.6960	0.0741	0.001
		(88.96%)	(9.48%)	(1.50%)	(0.05%)
^{49}V	330 d	4.966	0.5609	0.0587	0.001
		(89.28%)	(9.26%)	(1.43%)	(0.034%)

Table 5.1: Energies of the K, L, M, and N-shell electron capture peaks for the cosmogenically produced isotopes in Ge [276]. Half lives and the electron capture probabilities for each shell (in %) are also shown. Electron capture probabilities are calculated according to Ref. [277]. * ^{68}Ga is a product of ^{68}Ge .

The shapes of the cavern neutron spectra are simulated using Source4C and propagated through the experiment materials to produce the expected spectra reaching the detectors. The neutron flux is normalized to 4000 neutrons/day/m² based on Ref. [278]. Cosmogenic

²The rate of ^{68}Ge is non-trivial; unlike the other isotopes, the contamination is not removed by the crystal growing process.

neutrons are modelled as a flat spectrum with rate determined from simulations of muons with angular and energy distribution appropriate for SNOLAB [171].

Surface backgrounds from ^{222}Rn daughters are determined based on a simulation [279] which replicated results from SuperCDMS Soudan where a tower was exposed to a Si wafer with ^{210}Pb implanted to test the surface electron rejection abilities of the iZIP detectors [140]. The simulation results are normalized to a goal of 25 nBq/cm² on the detector surfaces.

Coherent neutrino scattering is expected to be the limiting background for future direct detection experiments. The differential cross section for a neutrino of energy E_ν interacting coherently with a nucleus of mass M is [280]

$$\frac{d\sigma}{dE_R} = \frac{G_F^2 M}{4\pi} \left(1 - \frac{ME_R}{2E_\nu^2}\right) Q_W^2 F^2(E_R), \quad (5.5)$$

where E_R is the recoil energy, G_F is the weak Fermi constant, Q_W is the weak hypercharge, and $F^2(E_R)$ is the Helm form factor [281]. The cross section is convolved with the neutrino flux of different neutrino sources, including solar neutrinos, atmospheric neutrino, and neutrinos from the diffuse supernovae background.

A background that we have neglected to mention thus far is coherent photon scattering. This background was not initially considered in Ref. [171], but as A. Robinson points out in Ref. [282], this process would be a significant background for low threshold detectors looking for rare signals on the eV scale. The rate is shown to be similar to the Compton rate in the 10–100 eV range for Ge, and rises dramatically below 1 eV. This background is modelled analytically based on the cross section presented in Ref. [282], as shown in Figure 5.1. We assume the same rough energy dependence of the rate and normalize the rate to match the Compton background rate at low energies. A proper inclusion of this background would require simulations, but this background is not included in GEANT4.

The final background of note is the newly added leakage background for the HV detectors. This background is modelled as three components based on previous experimental

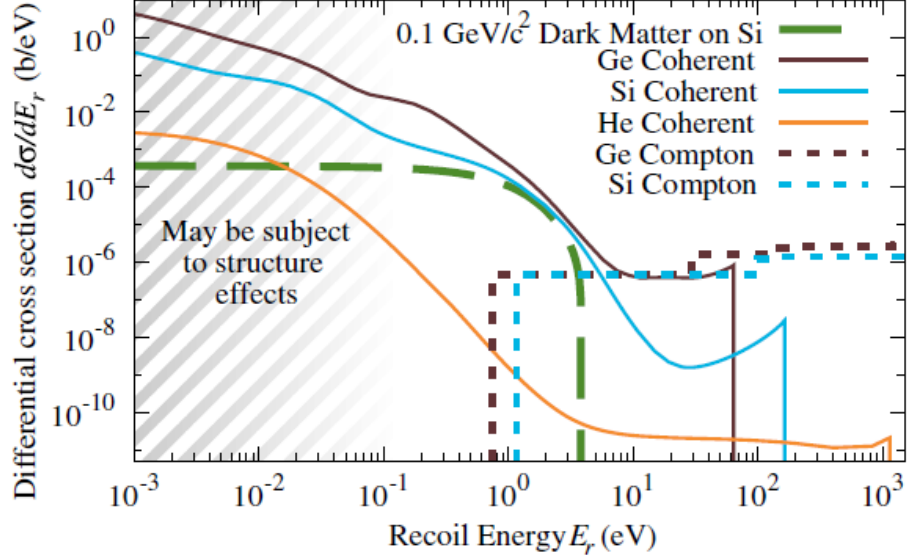


Figure 5.1: The coherent photon scattering rate for different materials compared to the Compton background rate. From Ref. [282].

results. The first two components are Poisson bulk and surface leakage, where the rate R_j of observing a pileup of j leakage events is

$$R_j = R_{tot} \frac{(R_{tot} t_{sep})^{j-1}}{(j-1)!} e^{-R_{tot} t_{sep}}, \quad (5.6)$$

where R_{tot} is the total leakage rate; t_{sep} is the separation time, which can be thought of as the amount of time required to distinguish two separate events as distinct. The calculation of t_{sep} is covered later in this section. The PDF of bulk leakage is a uniform distribution in energy, while surface leakage is a delta function at integer numbers of e^-h^+ pairs. The final leakage component is non-Poissonian surface leakage.

We have not yet measured ionization leakage in large detectors, so we use scaling based on smaller detectors to determine the expected leakage rate. Currently we only include Poisson surface leakage in our projections because we have not seen evidence for the other two components³. Therefore, the effect of the leakage background, barring CT and II is to

³Non-Poissonian surface leakage was initially modelled because of results from HVEV Run 1 [239]; however, it was later determined that this background was from UV/optical photons.

produce an increased rate at integer multiples of eV_b based on the amount of pileup.

5.1.2 Spectra

Once the background and signal spectra have been determined in recoil energy (in units of counts/keVr/kg/day), the next step is to model the detector response. It is most convenient to work in total phonon energy space, particularly for combining ER and NR backgrounds. Hence, the result is reported as a rate in counts/keVt/kg/day.

In the continuous code where we treat ionization as continuous, we convert a given recoil energy E_R to phonon energy using the conversion

$$P_t = E_R \left(1 + yY \frac{eV_b}{\epsilon} \right), \quad (5.7)$$

where y is an ionization collection efficiency, Y is the yield function, V_b is the bias voltage, and ϵ is the average energy required to produce an e^-h^+ pair. The only difference between this form and Eq. 3.7 is that now we have introduced the ionization collection efficiency y to account for any imperfect ionization collection. The functional form of this factor depends on the location of an event in the detector and the recoil type. For example, events in the bulk of the detector are assumed to have perfect ionization collection, while events near the sidewalls will have reduced collection efficiency. This will be elaborated upon shortly.

The yield model $Y(E_R)$ has been updated from Ref. [273], which used the Lindhard model [217–219] for Ge with a forced cut off at low energies and a piecewise model for Si based on Lindhard and Antonella data [225]. In this work, for Ge we use the nuclear recoil yield model developed by Sarkis et al. [222], which closely follows the Lindhard [217–219] model above 1 keV, but incorporates a physically motivated cut off near 40 eV. We use the recent IMPACT measurement of the nuclear recoil yield for Si [228], with no imposed cut off. The yield models are consistent between the discrete and continuous ionization codes.

Coming back to the conversion from recoil energy rates to phonon energy rates, by taking

the derivative of Eq. 5.7 with respect to E_R we obtain

$$\frac{dP_t}{dE_R} = 1 + yY \frac{eV}{\epsilon} + E_R y \frac{dY}{dE_R} \frac{eV}{\epsilon}. \quad (5.8)$$

This gives a Jacobian factor for converting the recoil energy rate into phonon energy rates:

$$\frac{dR}{dP_t} = \frac{dR/dE_R}{dP_t/dE_R}. \quad (5.9)$$

The Jacobian factor depends on the position of an event in the detector, as well as the recoil type.

The discrete ionization code similarly uses a Jacobian factor to convert recoil energy rates to total phonon energy rates. The factors are different between the continuous code and discrete code because of different partitioning, and because the discrete code additionally discretizes the charge production. For a given recoil energy, a PDF of the number of liberated e^-h^+ pairs is produced. The distribution of e^-h^+ pairs is dependent on both the Fano factor F [283] and the average number of charges produced,

$$Q_{avg} = \frac{E_R Y(E_R)}{\epsilon}. \quad (5.10)$$

We assume $F = 1$ for NRs and $F = 0.13(0.0115)$ in Ge(Si) for ERs [284]. For $F = 1$ the PDF of the e^-h^+ distribution is a Poisson distribution, while for $F < 1$ we use a binomial distribution. If the recoil energy is less than the band gap or the yield threshold, no ionization is produced.

Because the detector response is dependent on event position, in both the discrete and continuous ionization code, we partition the detector based on the radial and z position. These partitions consist of the detector bulk, high radius near the side walls, the sidewalls themselves, high- z close to the detector faces, and the top/bottom faces themselves. The HV detectors do not require the high- z partition because the electric field is uniform enough that

this area can be regarded as bulk. Table 5.2 shows how the detector volume is partitioned for each recoil type in the continuous and discrete ionization code. The sidewall and top/bottom face partitions are based on the surface area of the different regions of the detector.

Detector region	NRs Cont.	NRs Disc.	ERs Cont. (Ge/Si)	ERs Disc.
Bulk	0.85	0.9	0.5/0.675	0.9
High z	0.05	—	0.056/0.075	—
High r	0.1	0.1	0.444/0.25	0.1
Sidewalls	A_r/A_t	A_r/A_t	A_r/A_t	A_r/A_t
Top/bottom faces	A_z/A_t	A_z/A_t	A_z/A_t	A_z/A_t

Table 5.2: Fraction of the detector volume (first three rows) and surface area (last two rows) for each partition for ERs and NRs in the continuous (Cont.) and discrete (Disc.) ionization code. A_z is the surface area of the side walls, A_r is the surface area of the faces, and A_t is the total detector surface area. The HV detectors do not use a high z-partition because the electric field is uniform enough that this volume can be regarded as bulk. There is no region of overlap between the high-z and high-r regions in the continuous code for simplicity. This table is reproduced and updated from Ref. [273].

In the discrete code, several assumptions are made about charge propagation. Charges from events in the detector bulk and at high radius are assumed to travel straight up and down, ignoring the oblique propagation of electrons. That is, the initial and final radial position of an event is the same. Furthermore, bulk events are assumed to experience full charge collection (barring the effects of CT and II) corresponding to an ionization collection efficiency $y = 1$. In contrast, high radius events can experience reduced NTL amplification.

An electric field simulation demonstrated that events receive full NTL amplification for radius $r < 45$ mm in a SuperCDMS SNOLAB HV detector [273]. The simulation motivated the use of PDFs to model the ionization collection efficiency as a function of z position in the detector. At high radius we model the ionization collection efficiency by a linear PDF, while at the sidewalls for surface backgrounds the PDF is flat. Table 5.3 shows the assumed

ionization collection efficiencies for the discrete and continuous ionization code.

Detector region	NRs Cont. (Ge/Si)	NRs Disc.	ERs Cont. (Ge/Si)	ERs Disc.
Bulk	1	1	1	1
High z	1	—	1	—
High r	0.75/0.9	Linear PDF	0.75/0.9	Linear PDF
Sidewalls	0.488/0.585	Flat PDF	0.525/0.585	Flat PDF
Top/bottom faces	0.65	1	0.7/0.65	1

Table 5.3: Ionization collection efficiencies for the different background partitions in the continuous (Cont.) and discrete (Disc.) sensitivity code. More complicated models of charge propagation are assumed in the discrete code. This table is reproduced and updated from Ref. [273]. Note that the given ionization collection efficiencies for continuous code are for the iZIP detectors. For HV-style detectors, all ionization collection efficiencies are assumed to be 1 in this code, which is an oversimplification. In general, the continuous code is not used for HV detectors.

The final effect on charge propagation that we consider here is II and CT; both processes alter the number of free charges propagating in the detector and therefore the energy in NTL phonons of an event. Recall that CT reduces the overall number of charge carriers as they become trapped in impurities and defects. In contrast, II can liberate charges from such defects. These processes are only included in the discrete ionization code.

The discrete ionization code assumes that II and CT behaviour, including the occurrence probability, P_{II} and P_{CT} for II and CT, respectively, is the same for electron and for holes. In reality, measurements using the HVeV detectors have demonstrated that this is inaccurate; however, a more complex model of II and CT has yet to be implemented in the sensitivity code.

We assume that the probabilities of each process are constant throughout the detector. When a charge carrier undergoes II or CT, the NTL phonon energy of the charge carrier gained or lost, respectively, can take any value up to eV_b . Consider a number n of e^-h^+ pairs subject to either II or CT (affecting only one of the charge carriers). The PDF for the

NTL energy gained or lost (respectively), u , in units of eV_b is n -th order convolution of the uniform PDF with itself:

$$P_n(u) = \frac{1}{2(n-1)!} \sum_{k=0}^n (-1)^k \binom{n}{k} (u-k)^{n-1} \text{sgn}(u-k). \quad (5.11)$$

We can write the total PDF for the CT and II processes individually by summing over $P_n(u)$ and taking into account the probability of the process, giving

$$P_{tot}^{CT}(E_r + N_{eh}eV_b - ueV_b) = \sum_{j=1}^{N_{eh}} (P_{CT})^j P_j(u), \quad (5.12)$$

$$P_{tot}^{II}(E_r + N_{eh}eV_b + ueV_b) = \sum_{j=1}^{N_{eh}} (P_{II})^j P_j(u). \quad (5.13)$$

The total PDF for the combined processes is the sum of the individual processes. In the sensitivity code, we only model the first order processes. That is, we do not consider additional CT or II on liberated charges and only one charge carrier from a pair may be affected. Additionally, we limit the process to 10 CTs or IIs as the probability from further summands is negligible for the probabilities considered (for large numbers of charge carriers this assumption starts to break down). We treat surface II, where a single charge worth of NTL gain is produced as the charge traverses the entire detector, as a separate process with its own probability, P_{SI} .

Above a certain energy, which is detector dependent, we use an approximation to reduce computation time. The distribution $P_n(u)$ approaches a Gaussian as $n \rightarrow \infty$; therefore, for large n we assume that given N_{eh} created charges in an interaction, on average a fraction P_{CT} will suffer CT with an average NTL phonon energy loss of $eV_b/2$. Similarly, on average a fraction P_{II} will suffer II with an average gain in phonon energy of $eV_b/2$. The total phonon energy is then approximated as a shift based on these probabilities

$$P_t = E_R + N_{eh}eV_b \left[1 - \frac{P_{CT}}{2} + \frac{P_{II}}{2} + P_{SI} \right]. \quad (5.14)$$

For the SuperCDMS SNOLAB detectors this approximation starts around 0.5 keV_t , where the energy resolution is large enough that the approximation is valid. However, the energy at which the approximation starts is in part chosen based on the phonon resolution, which makes the choice questionable for detectors with low phonon resolution. Work is ongoing to implement a more sophisticated model of CT and II and to mitigate the issue with the approximation.

5.1.3 Detector Energy Resolution

Many aspects of the detector response are highly dependent on the energy resolution. In Ref. [171] the energy resolutions assumed for the forecasts were based on the goal performance of the detectors. We have discussed the dependence of the phonon energy resolution on detector parameters previously in Chapter 3.2.3.

The sensitivity code makes use of scaling laws and known parameters of two different detectors in order to determine how new detectors will behave. The first detector is G48, a Ge iZIP detector. The performance of this detector is described in detail in M. Pyle's thesis [201]. The second detector is a Si CPD named PD2, whose operation at CUTE is described in Chapter 6. The detector parameters relevant for the scaling laws are shown in Table 5.4. We have already seen τ_{ph} , the phonon-collection time, and τ_{eff} , the ETF timescale, and made mention of many of the other parameters in earlier chapters. The remaining parameters will be discussed further in the text.

Property	PD2	G48
Radius (cm)	3.81	3.81
Height (cm)	0.1	2.54
Phonon scattering length ℓ_{scat} (cm)	0.19	3.05
Critical temperature (mK)	40	—
Fraction Al coverage f_{Al}	0.025	0.039
Al to W Transmission $t_{W,Al}$	1×10^{-4}	—
τ_{eff} (μs)	60	—
τ_{ph} (μs)	17	767
σ_{pt} (eVt)	4	—
τ_{pos} (μs)	3.9	50
Phonon scattering time (μs)	—	7.7
f_{xyz}	—	7.5

Table 5.4: Properties of detectors G48 and PD2 used for the scaling of detector parameters in the SuperCDMS sensitivity package. Unused parameters are indicated by —. Table reproduced from Ref. [273].

We consider two different regimes for the phonon energy resolution. The first is the position-optimized regime, where we want to keep as much position information is the signal as possible, for example for phonon fiducialization. A large fractional Al coverage f_{Al} is required to minimize the phonon-collection time, absorbing the phonons before they bounce around and too much of their position information is lost. The HV detectors fall into this regime.

The second regime is the energy-optimized regime, into which the iZIP detectors and 0V detectors fall. In this regime, σ_{pt} is minimized without consideration for the loss of position dependence. Setting $\tau_{eff} = \tau_{ph}$ gives the optimum trade off in TES TFN, which increases with f_{Al} but is reduced by bandwidth matching of the phonon signal to the TES ETF timescale [201]. We determine the fractional Al coverage needed to satisfy this condition.

First, the scattering length for a phonon is given by the Sabine equation [285]

$$\ell_{scat} = 4 \frac{V_{tot}}{A_{tot}}, \quad (5.15)$$

where A_{tot} and V_{tot} are the total surface area and volume of the detector, respectively. Using the scaling $\tau_{eff} \propto T_c^3$ we can calculate τ_{eff} for our detector as

$$\tau_{eff} = \tau_{eff}^{PD2} \left(\frac{T_c^{PD2}}{T_c} \right)^3. \quad (5.16)$$

The fractional Al coverage f_{Al} required is thus

$$f_{Al} = \frac{\tau_{ph}^{(PD2)}}{\tau_{ph}} \frac{\ell_{scat}}{\ell_{scat}^{(PD2)}} f_{Al}^{(PD2)} f_{mat1}, \quad (5.17)$$

where f_{mat1} is a material dependent constant equal to 1 for Si and 4.5 for Ge. This factor accounts for the slower speed of sound and lower phonon absorption probability by the sensors in Ge.

In the position-optimized regime we have fixed f_{Al} . We cannot assume that $\tau_{eff} = \tau_{ph}$, but we can calculate the phonon time scale based on a given f_{Al} as

$$\tau_{ph} = \tau_{ph}^{(PD2)} \frac{\ell_{scat}}{\ell_{scat}^{(PD2)}} \frac{f_{Al}^{(PD2)}}{f_{Al}} f_{mat1}. \quad (5.18)$$

With the relevant parameters determined in either the energy-optimized or position-optimized scenario, we can calculate the baseline phonon energy resolution as [201]:

$$\sigma_{pt} = \sigma_{pt}^{(PD2)} \left(\frac{T_c^{PD2}}{T_c} \right)^3 \frac{t_{W,Al}^{(PD2)}}{t_{W,Al}} \sqrt{\frac{f_{Al} A_{tot}}{f_{Al}^{PD2} A_{tot}^{(PD2)}}} \sqrt{\frac{\tau_{ph} + \tau_{eff}}{\tau_{ph}^{(PD2)} + \tau_{eff}^{(PD2)}}}, \quad (5.19)$$

where $t_{W,Al}$ is the W-to-Al transmission probability. The expected phonon energy resolutions for the SuperCDMS SNOLAB detectors given in Table 3.1 were calculated according to this formalism.

In order to calculate the resolution of the position dependent signal, we need to know the position dependent time scale. This is dependent on the phonon scattering time

$$t_{scat} = \frac{\ell_{scat}}{v_s}, \quad (5.20)$$

where v_s is the speed of sound in the material. The position dependent time scale is

$$\tau_{pos} = \max \left(t_{scat}, \tau_{pos}^{(G48)} \frac{f_{Al}^{G48}}{f_{Al}} \frac{t_{scat}}{t_{scat}^{(G48)}} f_{mat2} \right). \quad (5.21)$$

We set a maximum on τ_{pos} of t_{scat} because the position dependent signal cannot be collected faster than the time it takes for a phonon to scatter once. We also define f_{mat2} as another material dependent constant equal to 1 for Ge, and $f_{mat2} = 1/(4.5(v_{s,Si}/v_{s,Ge}))$ for Si. The resolution of the position dependent signal is calculated

$$\sigma_{pos} = \sigma_{pt} \sqrt{\frac{\tau_{pos} + \tau_{eff}}{\tau_{ph} + \tau_{eff}}}. \quad (5.22)$$

When we are looking at the radial position dependent signal, we typically only look at the outer channels. Assuming that the outer channels occupy $f_{pos} = 1/n$ of the detector area, we must incorporate a factor to account for the fact that we only see thermal fluctuation noise in 1 of n channels. The radial phonon energy resolution is thus

$$\sigma_{r,pos} = \sigma_{pos} \sqrt{f_{pos}}. \quad (5.23)$$

For example, we assume $n = 6$ for the SuperCDMS SNOLAB HV detectors, which use the 2 outer channels of 12 total channels.

Once the background and signal spectra have been determined and the energy resolution has been calculated based on the detector parameter scalings, the spectra are convolved with an energy dependent resolution model. The resolution degrades at high energy because of local saturation of the TESs and because of variations in the pulse shape which are not

accounted for by our template model. We have seen this in Chapter 4 with the iZIP phonon resolution. In the sensitivity code we use the following model for the energy resolution as a function of energy E :

$$\sigma(E) = \sqrt{\sigma_0 + f^2 E^2}, \quad (5.24)$$

where σ_0 baseline energy resolution, and f is a factor accounting for systematics on energy measurements including position dependence, similar to the CDMSlite and iZIP energy resolution model in Chapter 4. We use $f = 1\%$ for low voltage detectors (continuous code) and $f = 0.1\%$ for HV detectors, which have been updated since Ref. [273]. The latter is motivated because the NTL phonons are generated in a way that is independent of z -position, so z -dependence is reduced. There is no Fano term as this is taken into account by the nature of the charge signal production in the discrete code, but is neglected for the continuous code.

5.1.4 Cuts

All background and signal spectra have detector-dependent cuts applied to them. A 95% energy-independent cut is applied across all detector types and all spectra to mimic the effect of basic data quality cuts.

Phonon Fiducialization

We can use the position dependence in the phonon signal to remove events close to the detector surfaces, outside of the detector bulk. This is referred to as the phonon fiducial volume (PFV), as discussed in Chapter 4. A parameter that is important to the detector response is the fraction of the phonon signal expected to contain position dependent information f_{xyz} . This parameter depends on the fractional Al coverage as well as the phonon

absorption probability. Using scaling ratios, we can calculate

$$f_{xyz} = f_{xyz}^{(G48)} \frac{f_{Al}}{f_{Al}^{(G48)}} \frac{1}{f_{mat3}}, \quad (5.25)$$

where f_{mat3} is another material dependent constant based on the amount of isotropic scattering in the material. The position dependent signal is small for isotopically purer materials like Si ($f_{mat3} = 2$) compared to materials like Ge ($f_{mat3} = 1$). Here we use G48 because PD2 is a single channel detector with little information on position dependence. From G48, it is known that $\sim 50\%$ of the position dependent phonon signal reaches the outer channel for events at high radius, vs. only $\sim 25\%$ for bulk events [273].

We define P_z as the difference between the measured position dependent phonon signal on the top and bottom phonon sensors, and use this as an energy estimator for our z -dependence cuts. This is only relevant for 2-sided detectors. Similarly, we define P_r as the energy collected on the outermost channels (on both sides if relevant) to be used for defining a cut for high radius events.

In the continuous model we use a simple model for determining P_z and P_r as shown in Table 5.5 based on iZIP detectors. For surface events near the detector faces, P_z is a fraction of the total phonon energy P_t because both the NTL and recoil phonons exhibit z -dependence because all are generated near the surface. Therefore $P_z = f_{xyz} P_t$ in this region. For bulk events, only recoil energy contributes to the position dependent signal. The NTL phonons show no z -dependence, therefore $P_z = 0.75 f_{xyz} E_R$, where the factor 0.75 was chosen based on G48 data. Determining P_z at high radius is complicated because NTL phonons cannot be described as fully symmetric nor asymmetric. To be conservative, we take $P_z = 0$ in this region. The scalings for P_r were also based on an analysis of G48.

The P_z and P_r estimators in the discrete model are treated in a more sophisticated manner, which could in principle be applied to the continuous code in the future. Starting with P_z , we assume a one sided sensor with the instrumented side at $z = 0$ and the other

Estimator	Value
P_z (z surface)	$f_{xyz}P_t$
P_z (high r)	0
P_z (bulk)	$0.75f_{xyz}E_R$
P_r (bulk/high z)	$0.25(0.75)f_{xyz}P_t$
P_r (high r)	$0.5(0.75)f_{xyz}P_t$

Table 5.5: Energy estimators used for determining cuts in the continuous ionization model. The scaling factors are based on measurements from G48.

side at $z = h$. To determine the distribution of P_z as a function of the z position of an event, we integrate over the charge path for the NTL phonons because of the z -dependence of the position information:

$$P_z(z) = \frac{dP_z}{dP_t}(z)E_R + n_{eh}eV \int_0^z dz' \frac{dP_z}{dP_t}(z'). \quad (5.26)$$

A comparison of the P_z signal against the time delay in the charge signals in G48 gives us the relationship [201]

$$\frac{dP_z}{dP_t}(z) = \left(\frac{dP_z}{dP_t} \right)_{max} (1 - z), \quad (5.27)$$

where data from G48 also give us the estimate $\left(\frac{dP_z}{dP_t} \right)_{max} = f_{xyz}$. For detectors with sensors on both sides, P_z is determined for both sides individually and we take the difference as P_z .

The discrete code P_r model is defined in a similar manner, though we shall see that the r - and z -dependence complicates matters. We have

$$P_r(z, r) = \frac{dP_r}{dP_t}(z, r)E_R + n_{eh}eV \int dz' \frac{dP_r}{dP_t}(z', r). \quad (5.28)$$

In Ref. [273], $\frac{dP_r}{dP_t}(z, r)$ is parameterized as

$$\frac{dP_r}{dP_t}(z, r) = f_o - \frac{N_o}{N_i} f_i, \quad (5.29)$$

where $f_{i,o}$ is the fraction of signal on the inner/outer channel, respectively, and $N_{i,o}$ is the

number of inner/outer channels. We have redefined this metric by removing the contribution of the inner signal and focus only on the outer signal,

$$\frac{dP_r}{dP_t}(z, r) = f_o. \quad (5.30)$$

We use a coordinate system where the radial parameter is scaled to the height of the detector, h , and define r_i as the edge of the inner sensors and r_o as the total detector radius. Considering first a one sided detector, we define $z = 0$ as the instrumented face and $z = 1$ as the opposite face. The detector is broken up into five regions based on how NTL and recoil phonons from events originating in the region are shared between the channels. The definition for each partition is shown in Table 5.6, together with the assumed f_o and f_i , and a schematic is shown in Figure 5.2. The position dependent signal is assumed to be proportional to the distance of an event from the instrumented surface, $f \propto (1 - z)$. For a two-sided detector, P_r is the sum of the signals from both sides.

Region	f_i	f_o
1: $r < r_i - 1$	$A(1 - z)$	0
2: $r_i - 1 < r < r_i; z < r_i - r$	$A(1 - z)$	0
3: $r_i - 1 < r < r_i; z > r_i - r$	$\frac{A}{2}(\frac{z_b}{z} + 1 - z_b - z)$	$\frac{A}{2}(-\frac{z_b}{z} + 1 - z_b - z)$
4: $r > r_i; z > r - r_i$	$\frac{A}{2}(\frac{z_b}{z} + 1 - z_b - z)$	$\frac{A}{2}(-\frac{z_b}{z} + 1 - z_b - z)$
5: $r > r_i; z < r - r_i$	0	$A(1 - z)$

Table 5.6: Regions used to define the radial phonon fiducial volume cut for a single sided detector in the discrete ionization model, and the fraction of position-dependent signal on the inner and outer channels. The regions are dependent on the assumed model of phonon energy distribution described in the text, based on data from G48. Here $A = (dP_r/dP_z)_{max}$ and $z_b = r_i - r$. Region 1: NTL and recoil phonons entirely confined to inner sensor(s) (may not exist depending on detector geometry if $r_i < 1$). Region 2: Recoil phonons entirely confined to inner sensor(s), NTL phonons are shared. Region 3 and 4: All phonons are shared. Region 5: Recoil and NTL phonons measured by the outer sensor only. Note that this model is an oversimplification of phonon physics.

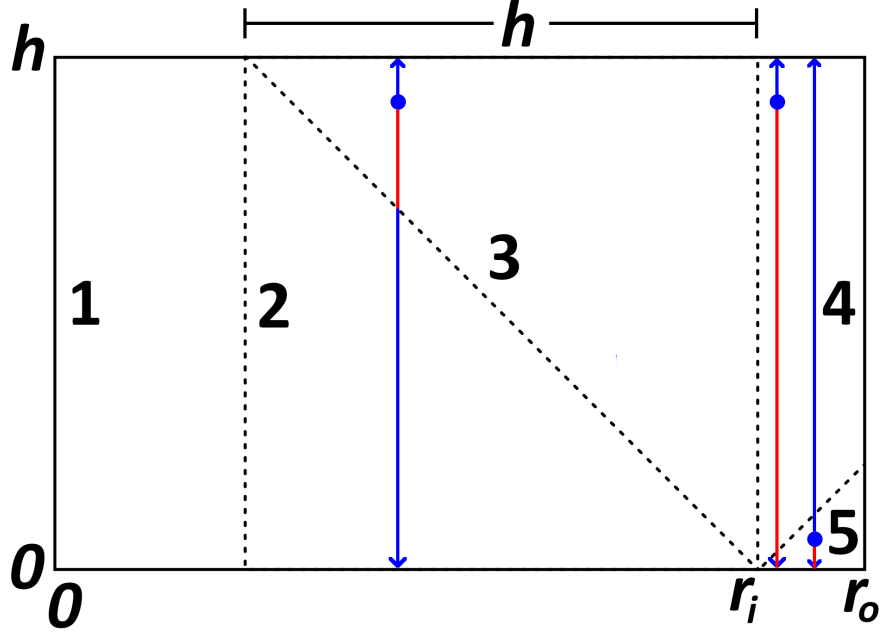


Figure 5.2: Schematic of the phonon fiducial volume regions defined for a single sided detector, where the instrumented face is on the bottom. The total detector radius r_o and the location of the edge of the inner sensors r_i are scaled based on the height of the detector h . Each region from Table 5.6 is labelled. The paths of charge carriers from events beginning in regions 3, 4, and 5 are shown as blue lines, with the event location represented by blue circles. The red lines indicate the portion of the path for which emitted NTL phonons have a position-dependent part of the signal reaching the outer channel in this model. A similar schematic can be constructed for a two sided detector.

In order to implement the phonon FV cut, we assume that P_z and P_r are distributed as Gaussians with resolutions σ_{P_z} and σ_{P_r} . The signal in either estimator is dependent on the position of the event, and also the recoil type, so we calculate the distribution for each category.

In the continuous code, we define 2σ bands for each event category, and events which deviate by 2σ from the expected bulk NR band are removed. The discrete ionization code is geared towards the high voltage detectors and we want a higher passage fraction of events at low energy, so the cut definition is modified from the continuous version. Here, a flat passage

fraction is required for bulk NR events. The chosen efficiency is 90% for z -fiducialization and 95% for radial fiducialization. Though these cuts are optimized for NRs, the passage fraction is still high for bulk ER and we do not change the cut definitions for ERDM models.

Phonon Timing Resolution and Pileup Window

With the newly implemented leakage background, it is vital for HV detectors that we reject the pileup of single carrier leakage events in order to minimize the impact of leakage on the sensitivity. Doing so reduces the event rate below the second e^-h^+ -pair peak, but cannot help with the first e^-h^+ -pair peak. The cut is based on the optimal filter timing resolution, given by

$$\sigma_t = \sqrt{\tau_{coll}\tau_{eff}} \frac{\sigma_{pt}}{P_t}. \quad (5.31)$$

This optimal filter is similar to the timing resolution for separating two pulses. This works as follows: given a two-pulse model with amplitudes E_1 and E_2 and pulse time separation δt yielding a reduced χ^2 for the optimal filter of χ_2^2 , and single phonon pulse amplitude E_0 with optimal filter χ^2 of χ_1^2 , we ask what value of δt satisfies $\Delta\chi^2 = \chi_2^2 - \chi_1^2 > 1$ ($\Delta\chi^2 > 1$ is standard model of estimating uncertainties). The result is given by

$$\sigma_{\delta t} = \sqrt{\tau_{coll}\tau_{eff}} \frac{\sigma_{pt}}{\sqrt{E_1(E_1 + E_2)}}. \quad (5.32)$$

For simplicity, we assume that the main concern is that two single pulses with energy of a single e^-h^+ pair pileup as a $2e^-h^+$ pair pulse, so $E_1 = E_2 = eV_b$. The timing resolution then reduces to

$$\sigma_{\delta t}^{1eh} = \sqrt{\tau_{coll}\tau_{eff}} \frac{\sigma_{pt}}{eV_b\sqrt{2}}. \quad (5.33)$$

The cut is implemented by discarding all events with $\Delta\chi^2 > N$, which rejects pileup pulses separated in time by more than $N\sigma_{\delta t}^{1eh}$. We use $N = 2$ for these projections.

Ionization and Yield Cuts

The following cuts are only relevant for detectors which measure charge signals like the iZIPs, and are only implemented in the continuous ionization code. We implement a QFV cut in a similar manner to the PFV cut, calculating a 2σ band for the energy estimators and comparing each event distribution to the bulk NR band. In this case, the z -energy estimator Q_z is the difference between the charge signals collected on each side, with resolution $\sqrt{2}\sigma_Q$ (σ_Q is resolution on single channel). Similarly, the radial estimator Q_r is the sum of charge collected on the outer channels. This parameter also has resolution $\sqrt{2}\sigma_Q$.

Events near the detector faces are treated as 100% collected on the respective side, so $Q_z = Q$, where Q is the total charge signal. In contrast, bulk events are given a 10% asymmetry to account for effects like CT and II. For the radial charge parameter we estimate how much of the charge signal is collected by the outer channels: events at high radius are given $Q_r = 0.6Q$, while bulk events are given $Q_r = 0.2Q$ [273]. Events near the detector faces are assumed to be entirely inner events and given $Q_r = 0$.

The final cut is a yield discrimination cut for NRs vs. ERs, which is designed to accept NRs. This cut is only appropriate for NRDM signals. The estimator is Q_m , the mean charge signal on the top and bottom sensors. This parameter is assumed to be $Q_m = 0.5Q$ for events at high z and r , and $Q_m = Q$ in the detector bulk. A 2σ window on the bulk NR events is defined and used to reject events in the other categories. This results in a high passage fraction for NRs, as well as a somewhat high passage fraction for sidewall ERs, which have reduced charge collection and mimic NR signals. All other event categories have low passage fraction except at low energies where the detector resolution makes the bands hard to distinguish. The impact of this cut is a steep drop in backgrounds for the iZIP-style detectors above a few keV_t . Ref. [273] shows examples of the various cut efficiencies for different event types.

5.1.5 Treatment of Threshold with Leakage Background

We have poor dark matter sensitivity around the first 2–3 leakage peaks at $N_{eh}eV_b$ (for $N_{eh} = 1, 2, 3, \dots$) for the HV detectors. The event rates can additionally be so high depending on the detector size and scenario that the limit setting procedures fail due to the expected number of events being too high. To mitigate this, we use a conservative way of ignoring the leakage peaks. We discard the region below the highest leakage peak for which the rate is more than 10 times the background rate at high (roughly 1 keV_{ee}) energies. This also discards the region in between the peaks with significant leakage rates (which can also have high event rates from CT and II of the leakage background), effectively raising the threshold to just above the first leakage peak for which the rate requirement is achieved. This effect will be demonstrated in Section 5.4.1 where I show the background spectra for HV detectors.

5.1.6 ERDM Signal Models

Both the discrete and continuous sensitivity code were originally developed to provide forecasts exclusively for SI WIMP-nucleon interactions. As part of my work for the forecasting effort, I adapted the forecasting code to be able to implement alternative dark matter models. Beyond the models discussed here, the code is flexible enough to be extended to include other models in the future. Models for dark photon kinetic mixing, ALP axioelectric coupling to electrons, and for dark photon-mediated LDM are implemented, and I show forecasts for each. Our forecasting efforts focus on eV-scale dark photon and ALP dark matter. We neglect the contribution of dark Compton scattering unlike in Chapter 4, which has negligible impact in this mass range.

Dark Photons

As discussed in Chapters 2 and 4, dark photons can kinetically mix with the SM photon, resulting in a coupling to electrons with strength parameterized by the kinetic mixing constant ε . The dark photon signal is an absorption peak at the mass-equivalent energy of the dark photon. In the continuous code, this is implemented as a gaussian centered at the mass-equivalent energy with width equal to the energy dependent resolution at this energy, normalized by the rate for a given ε . In the discrete code, the signal spectrum is generated in the same manner as the cosmogenic peaks, where the ionization spectrum is generated for an event with energy T_A and scaled to the appropriate rate.

Unlike in the dark absorption analysis described in Section 4.4.2, which did not consider masses low enough to require a correction for in-medium effects, here we consider masses down to the band gap energies of Ge and Si. In this mass range, we cannot neglect the in-medium effects, which are taken into account using the complex conductivity. In Figure 5.3 I show the real and imaginary parts of the complex conductivity used to derive ε as described in Section 2.2.3.

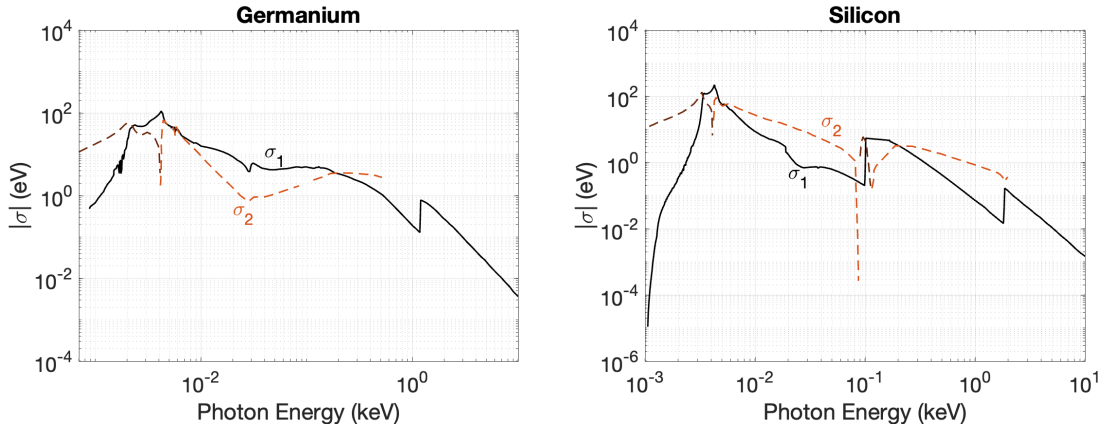


Figure 5.3: Absolute value of the real (σ_1 , black) and imaginary (σ_2 , orange) parts of the complex conductivity in germanium (left) and silicon (right). The dashed curves show the positive (negative) parts of σ_2 in a lighter (darker) colour. These data were compiled by M. Wilson as described in Ref. [213], following the approach taken by Hochberg et al. in Ref. [71] and using data from [247–250].

Axion-Like Particles

ALPs have the same expected signal shape as dark photons. Forecasts can be generated for dark photons first, and then converted to forecasts on the axioelectric coupling constant g_{ae} through a mass dependent rescaling, which also removes the in-medium effects considered for dark photons. Figure 5.4 shows the photoelectric cross section used in the sensitivity code. The curves were compiled by M. Wilson as described in his PhD. thesis [213]. The rough shapes around 2 eV and 6 eV in Ge are artifacts resulting from how different measurements in the literature are combined.

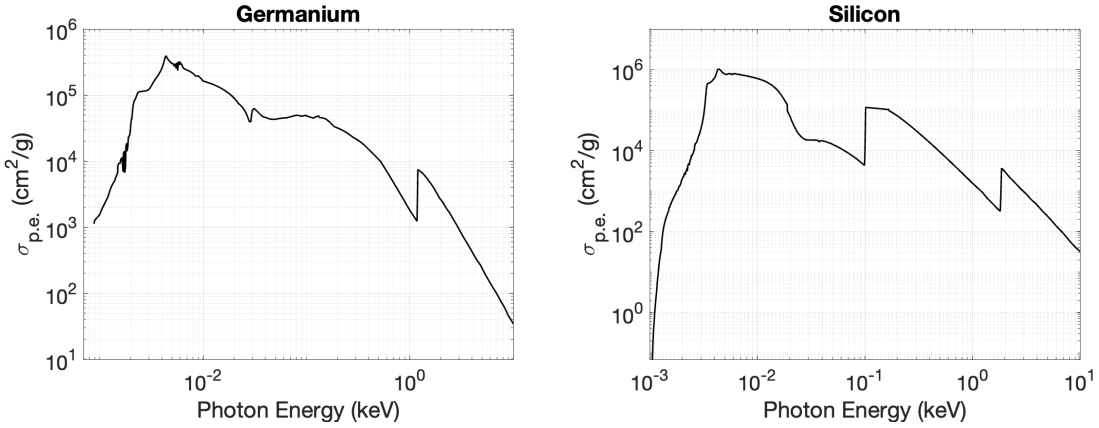


Figure 5.4: Photoelectric absorption cross section in germanium (left) and silicon (right). These data were compiled by M. Wilson as described in Ref. [213], following the approach taken by Hochberg et al. in Ref. [71] and using data from [247–250].

Light Dark Matter

We forecast for dark photon mediated couplings of LDM of MeV/c^2 scale ($1\text{--}100\text{ MeV}/c^2$). Both light and heavy mediators are considered, calculating the signal according to the formalism in Ref. [47], which we described in Section 2.2.2.

This analysis uses the publicly available electron recoil spectra, numerically calculated using the program QEdark [47, 48]. The recoil spectra are binned in E_r with bin size 0.1 eV, and are provided for masses in the range of $0.1\text{ MeV}/c^2$ to $10\text{ GeV}/c^2$. QEdark assumes a

Si bandgap of 1.1 eV, which differs from the value of 1.2 eV used in this analysis. This is taken into account, as is a difference in the dark matter density, ρ_χ . The original QEdark output assumes a value of 0.4 GeV/cm^3 , as opposed to the 0.3 GeV/cm^3 value used in the forecasting code. Figure 5.5 shows the raw recoil spectra for several masses and an assumed cross section of $\bar{\sigma}_e = 1 \times 10^{-37} \text{ cm}^2$.

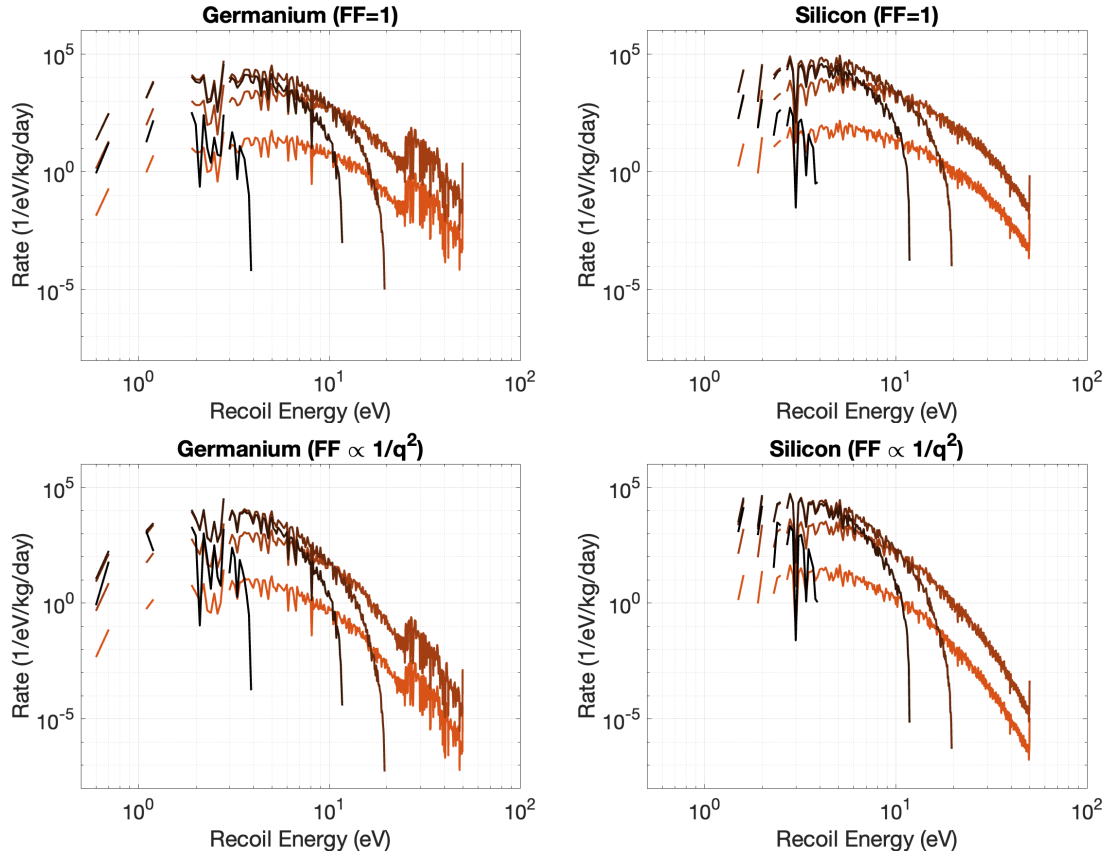


Figure 5.5: Recoil energy spectra for various LDM masses from QEdark [47, 48]. The colour indicates the dark matter mass, 1, 3, 5, 1000, and $1 \times 10^5 \text{ MeV}/c^2$ range from black to orange in colour, respectively. Gaps in the spectra are from bins where the rate is 0 and the use of a logarithmic scale.

Note in particular that at high masses the shape of the spectrum does not change, though the rate drops dramatically. The spectrum becomes insignificant above 50 eV, the maximum output bin provided by QEdark. The behaviour at the lowest energies is attributed to the

coarse 0.1 eV binning and several bins where the rate is 0. Work is ongoing to increase the bin density for future SuperCDMS analyses, as well as to make the dark matter halo parameters consistent with the values recommended in Ref. [45].

Figure 5.6 shows the parameter space currently excluded by direct detection experiments for dark photon-coupled LDM. The CCD and cryogenic semiconductor experiments dominate at the lowest masses while the LXe experiments provide the strongest overall cross section constraints. Note that XENON1T, which sets the strongest constraints from heavy mediator-coupled LDM does not set constraints for a light mediator, and SENSEI and DAMIC set the strongest constraints over the majority of the mass range of interest.

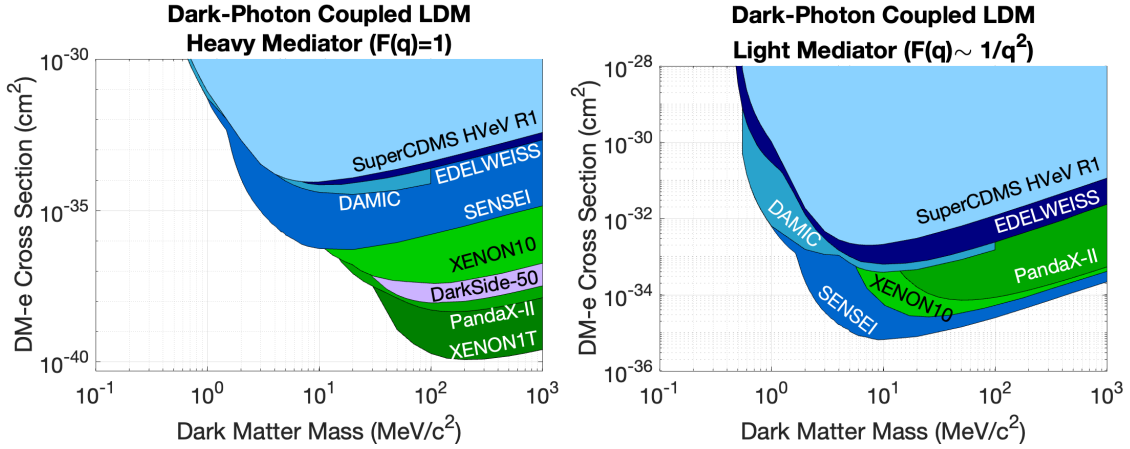


Figure 5.6: Summary of current constraints for dark-photon coupled LDM for a (left) heavy mediator and (right) light mediator. LXe constraints including XENON10 [286], XENON1T [256], and PandaX-II [100] are shown in green, the LAr experiment DarkSide-50 [126] is shown in purple, and Ge/Si experiments including SuperCDMS HVeV Run 1 [239], EDELWEISS [84], DAMIC [253], and SENSEI [115] are shown in blue. DarkSide and XENON1T did not publish constraints for a light mediator.

5.1.7 Forecast Calculation

Once the background and signal spectra have been modelled and the detector response has been applied, we can calculate the projected sensitivity. The forecasts depend on the

exposure, $\mathcal{E}$, which is the product of the length of the data taking period and the mass of the payload. For SuperCDMS SNOLAB, this data taking period is 4 years with an 80% duty cycle. We assume the same exposure for the future scenarios.

The analysis range is limited by $E_{thresh} = 7\sigma_{pt}$ as the lower threshold while the upper bound E_{upper} is dependent on the detector type. The number of expected background events is determined given the modelled backgrounds by integrating the rate of each background over the analysis range and multiplying by the experimental exposure:

$$N_b = \sum_{i=1}^{n_b} \mathcal{E} \int_{E_{thresh}}^{E_{upper}} \frac{dR_i}{dP_t} dP_t, \quad (5.34)$$

where n_b is the number of background components, and $\frac{dR_i}{dP_t}$ is the differential rate of background component i as a function of total phonon energy P_t . The number of signal events N_s for a given dark matter mass m_{DM} is determined the same way for a fixed coupling strength. A number of pseudoexperiments are generated from the background spectrum by drawing a number N_{sim} of event energies from a Poisson distribution with mean N_b .

We calculate the sensitivity using two different limit setting methods. The first is the OI method that we have seen previously in Chapter 4. This method involves no background subtraction and is a conservative method of setting a limit. The 2016 SuperCDMS SNOLAB projections [171] used this method. I do not present the OI limits in this work, as they are typically weaker than the sensitivities calculated with the following method.

The second method is the PLR method, similar to what was done for the CDMSlite Run 3 WIMP search [121]. This method takes into account backgrounds, producing stronger limits as long as the uncertainty on the background is noticeably smaller than the background itself. In this method, there are normalization parameters which are allowed to float along with the signal normalization. The backgrounds are lumped into three categories: (1) all non neutron and non-neutrino backgrounds, (2) neutron background from cavern and material contamination, and (3) the solar CE ν NS background. The normalizations of these three

backgrounds are free parameters. The likelihood function for observed data d consisting of N_{sim} events at energies $E_j \in \{E_1, E_2, \dots, E_{N_{sim}}\}$ is given by

$$\mathcal{L}(d|s, b) = P(N_{sim}|N(s, b)) \prod_{j=1}^{N_{sim}} f(E_j, |s, b) \prod_{k=1}^3 G(N_{bkg,k}|b_k), \quad (5.35)$$

where b_k is the normalization of background k , $b = \{b_k\}$ are the collective background model parameters, s represents the signal model parameters (the mass and rate normalization), and $f(E|s, b)$ is the PDF defined by summing the three background components and the signal spectra with normalizations given by s and b , and normalized to an integral of 1. $P(N_{sim}|N(s, b))$ is the Poisson probability function of observing N_{sim} events given an expected $N(s, b)$ events, and $G(N_{bkg,k}|b_k)$ is the Gaussian probability function for observing N_{obs} events given an expected number of events b_k and assumed width of 10%.

The sensitivity is calculated by minimizing the test statistic

$$\lambda(\sigma_{DM}) = \frac{\mathcal{L}(d|s, \hat{\hat{b}})}{\mathcal{L}(d|\hat{s}, \hat{\hat{b}})}, \quad (5.36)$$

where s is the signal model being tested, $\hat{s}$ and $\hat{\hat{b}}$ are the signal and background parameters which maximize the likelihood, and $\hat{\hat{b}}$ is the set of background model parameters which maximizes the likelihood when the signal model parameters are fixed to those being tested, s .

Limits are calculated at the 90% CL. Note that for dark photons and ALPs, we are setting the limit on ε^2 and g_{ae}^2 because of the square dependence in the rate (for LDM or WIMPs the limit is set directly on the relevant scattering cross section). We generate a large number of experiments, usually 50–100, and the final limit is taken as the median of the results.

5.2 SuperCDMS SNOLAB ERDM Projections

In Ref. [171] we showed sensitivity forecasts for SI WIMP dark matter for SuperCDMS SNOLAB HV and iZIP detectors, calculated using the continuous ionization code. As previously discussed, forecasts presented in that work were OI limits, whereas we present PLR limit in the future upgrade process of Ref. [177]. Additionally, we did not present ERDM limits at that point in time. For the future forecasting effort, we also updated the NRDM projections. We now include ionization leakage background for the HV detectors, which significantly impacts the low mass reach and raises the analysis threshold. The change in cryostat geometry from 31 to 7 towers (as described in Section 3.4.2) is also reflected, as well as updates to the background model, including an increase in the neutron background from the mu-metal shield. We also updated the ionization yield function, and implemented an energy dependent energy resolution model. Ref. [177] compares the originally published SuperCDMS SNOLAB OI nucleon-coupled dark matter limits with the updates mentioned above, as well as the new OI limits to those calculated using the PLR method.

Figure 5.7 shows the background spectra for the SuperCDMS SNOLAB HV detectors produced by the discrete ionization code for the forecasts. Major categories of event types are shown independently including bulk ERs, bulk NRs, etc., as well as the total overall background and leakage background. The bulk ER background dominates above around 100 eVt, below which the solar neutrino $\text{CE}\nu\text{NS}$ begins to become the dominant non-leakage background. The single e^-h^+ pair resolution is particularly evident for the Si detectors.

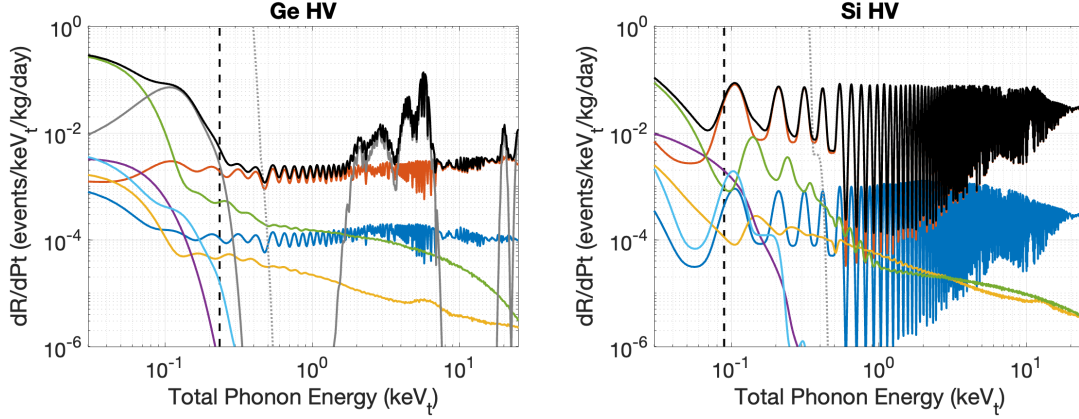


Figure 5.7: Projected background spectra for SuperCDMS SNOLAB HV detectors produced using the discrete ionization code detector response with all analysis cuts for Ge (left) and Si (right). Backgrounds include bulk ERs from Compton and cosmogenic betas (red), bulk ERs from cosmogenic EC (grey), surface ERs (dark blue), surface NRs from ^{206}Pb (purple), bulk NRs from neutrons (yellow), bulk NRs from solar neutrino $\text{CE}\nu\text{NS}$ (green), coherent photon scattering (light blue), and Poissonian surface leakage (dotted). The leakage background exceeds the plot scale below $3 - 4\text{eV}_t$. The total background (excluding leakage) is also shown (black), as is the analysis threshold at $7\sigma_{pt}$ (dashed vertical line).

Figure 5.8 shows the same total background spectra from Figure 5.7 with signal spectra for a selection of masses for each of the ERDM models. The signal spectra are scaled to the coupling strength determined by the PLR limit setting. We can see that for the LDM models that the sensitivity begins once the leakage background becomes subdominant, with the same behaviour observed for low mass ALPs and dark photons.

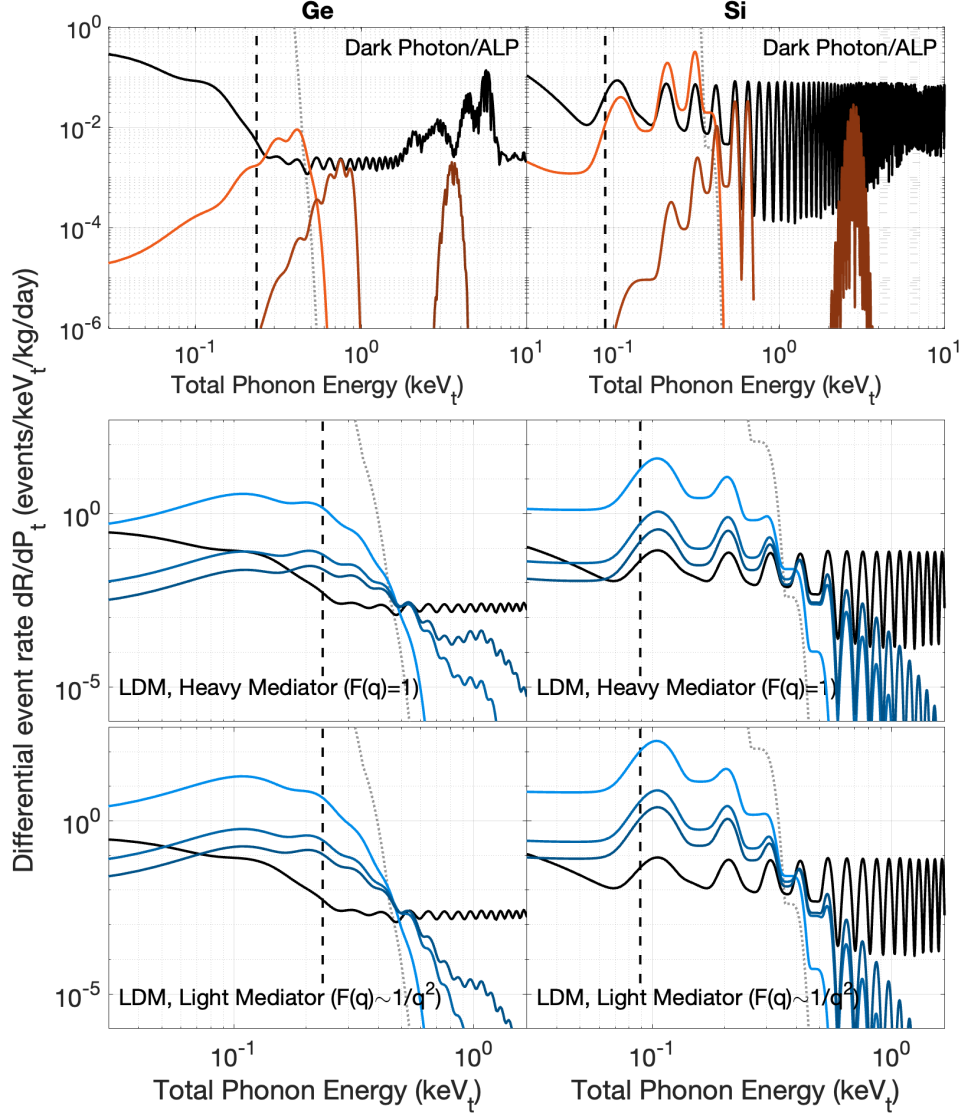


Figure 5.8: Projected total background spectra with all analysis cuts for Ge (left) and Si (right) with example signal spectra for each ERDM model. The signal spectra are scaled for coupling strength at the PLR exclusion sensitivity. The lightest colour indicates the lightest mass and the darkest colour indicates the heaviest mass in all cases. The total background (excluding leakage) is also shown (black), as is the analysis threshold at $7\sigma_{pt}$ (dashed vertical line). (top) Spectra for 10, 20, 100 eV/c^2 dark photon/ALPs dark matter (the signal spectra are the same for both candidates). (middle) Heavy mediator coupled LDM signal spectra for masses 3, 10, 1000 MeV/c^2 . Note that the total phonon energy axis is shared with the bottom plots and differs from the bosonic dark matter. (bottom) Light mediator coupled LDM signal spectra for the same masses.

In Figure 5.9 I show the forecasts for ERDM for SuperCDMS SNOLAB using the updated sensitivity code compared to current constraints from direct detection experiments. As mentioned above, there are no original OI limits for comparison. Projections are performed with and without considering leakage background (we don't expect the latter scenario to be realized, I show it only to demonstrate the impact on the sensitivity); the reduction in mass reach with leakage backgrounds is dramatically apparent.

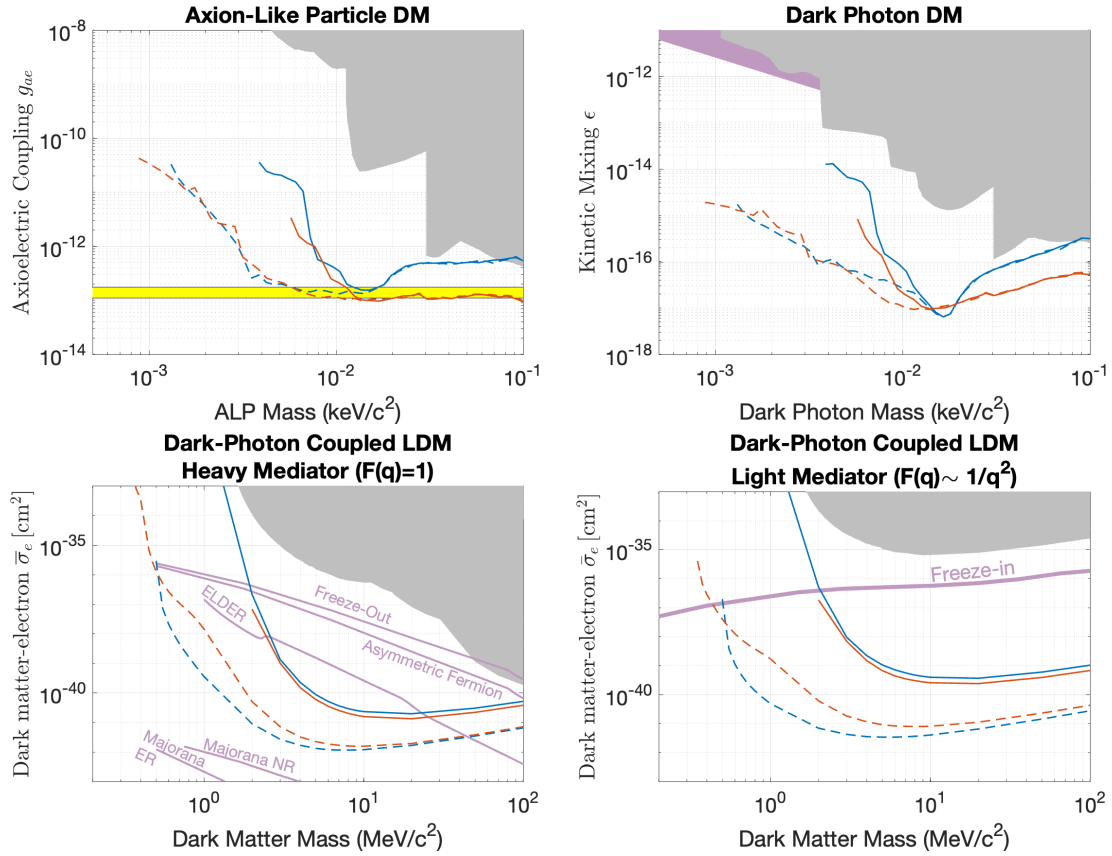


Figure 5.9: Sensitivity forecasts for each ERDM model for SuperCDMS SNOLAB HV detectors. Projections with (solid) and without (dashed) leakage background are compared for Ge (red) and Si (blue). Current constraints (grey shaded) are also shown. For ALP dark matter the parameter space indicated by a hint from stellar cooling [287] is shown in yellow. The pink shaded region for dark photons is excluded by stellar constraints [255]. For LDM, sharp theory targets from Ref. [49] are shown, assuming $M_{A'} = 3m_\chi$ where A' is the dark photon mediator and χ is the dark matter. There are several dark matter targets for the heavy mediator.

For the dark absorption models, the presence of leakage background only affects masses below $\sim 15 \text{ eV}/c^2$ because the energy deposited in the absorption process scales with mass, and the leakage background is mostly confined to the first few e^-h^+ pair peaks. In contrast, the LDM models are affected by the leakage at all masses. We saw in Figure 5.5 that the recoil energy imparted by the scattering does not tend to exceed 50 eV even at high masses, and only the overall rate of the spectrum changes as the LDM mass increases. The spectra remain at a maximum at low energies and the sensitivity is driven by the threshold, meaning that the leakage background is problematic for all LDM masses.

For ALP dark matter we can expect sensitivity down to $g_{ae} \sim 1 \times 10^{-13}$, reaching or surpassing the parameter space indicated by the hint from white dwarf cooling [287] down to $13 \text{ eV}/c^2$ with the Ge detectors, regardless of leakage backgrounds. With the expected levels of leakage background, we can still constrain a significant portion of the untouched ALP parameter space down to $4 \text{ eV}/c^2$, with possible sensitivity down to the band gap of Ge if the leakage backgrounds can be mitigated. The trend is similar for dark photons, where we can expect to constrain new parameter space from above $4 \text{ eV}/c^2$, and probing the kinetic mixing parameter on the order of $\varepsilon \sim 1 \times 10^{-17}$. Most of the coupling parameter strength for both dark matter models comes from the Ge detectors, though Si provides slightly lower mass reach when considering leakage backgrounds.

For LDM models we will be able to constrain new parameter space above $1.3 \text{ MeV}/c^2$ for heavy mediator couplings (or $0.37 \text{ MeV}/c^2$ if the leakage is negligible) and reach cross section sensitivity of $\bar{\sigma}_e \sim 1.3 \times 10^{-41} \text{ cm}^2$. We will have sensitivity to several of the sharp theory targets identified in Ref. [49] and summarized in Section 2.2.2, though we will not be able to probe the Majorana fermion dark matter scenario even with no leakage background. The behaviour is similar for the light mediator, where we can expect sensitivity down to $\bar{\sigma}_e \sim 2 \times 10^{-40} \text{ cm}^2$. The sensitivity between Si and Ge is similar when we include the leakage backgrounds, with Ge providing slightly lower cross section sensitivity and Si providing slightly lower mass reach. In contrast, without leakage we see that Si generally provides the

lowest cross section reach, except at the lowest masses where only Ge has sensitivity.

It is clear from the results of the SuperCDMS SNOLAB projections that the leakage background will be a significant concern for the sensitivity of future HV detector upgrade scenarios. We consider avenues of both reducing the leakage background and using an alternative type of detector with zero bias voltage.

5.3 SuperCDMS SNOLAB Upgrades

In this section I present the various improvements that we consider for the different upgrade scenarios of SuperCDMS SNOLAB, including background upgrades, new detector types, and upgrades to the detectors. These upgrades are discussed in detail in [177], which I follow closely here.

5.3.1 Background Upgrade Scenarios

We define three different background improvement scenarios, **Bkg 1**, **Bkg 2**, and **Bkg 3**. The **Bkg 1** scenario represents improvements which are close at hand and readily implemented, just not in time for SuperCDMS SNOLAB, whereas **Bkg 2** and **Bkg 3** represent progressively more complicated and expensive scenarios to implement. They are, however, possible through well-understood techniques.

Three backgrounds which are both significant and readily improved are addressed in the **Bkg 1** scenario: (a) around 18% of the bulk ER background is from U/Th in Kapton and Cirlex (a thicker version of Kapton made by laminating Kapton with adhesive) in flex cables and detector clamps; (b) the mu-metal shield produces a significant neutron background contributing around 35–40% of the total NR background; (c) surface backgrounds are dominated by radon daughter plateout on detector surfaces. The specific improvements for each of these backgrounds are as follows:

- (a) Remake the flex cables with lower activity Kapton, reducing the ^{238}U and ^{232}Th contamination by a factor of 100 and 6, respectively.

- (a) Replace the Cirlex detector clamps with ones fabricated from alternative materials of significantly lower contamination. This would almost completely eliminate this background.
- (b) Source a new mu-metal shield with alternative material or replace the shield with Helmholtz bucking coils.
- (c) Reduce ^{210}Pb on the detector faces from 25 nBq/cm^2 to 10 nBq/cm^2 by fabricating one (rather than two) detectors at a time, and performing crystal polishing in a lower radon environment (10 Bq/m^3). Currently a shallow underground environment is used, but this is not radon-abated. Perform detector packaging and tower assembly in a low background ($< 0.1\text{ Bq/m}^3$) cleanroom. This process currently occurs at the Stanford Radon Suppression Facility with background rates of 10 Bq/m^3 .
- (c) Reduce ^{210}Pb on the detector sidewalls from 50 nBq/cm^2 to 1 nBq/cm^2 via post-fabrication etching.

The **Bkg 2** and **Bkg 3** scenarios are much more costly, involving lowering ^{32}Si rates via sourcing or isotopic separation, as well as reducing backgrounds from cosmogenic activation of the detector and cryostat materials, and further reducing material contaminants. The **Bkg 3** scenario requires isotopic purification of silicon and an all-underground and radon-abated detector lifecycle starting from before crystal growth. All copper parts would be electroformed and machined underground. This would necessitate expensive new facilities at SNOLAB.

Through the forecasting process, we found that the background improvements in scenarios **Bkg 2** and **Bkg 3** do not provide significant additional science reach over the **Bkg 1** scenario. Instead, detector improvements can provide reach to more parameter space without incurring the significant costs of the advance background scenarios.

5.3.2 Detector Types

Detector Sizes

We considered a variety of detector sizes, motivated by the scaling of energy resolution with detector size as well as the success of the HVeV detectors in surface dark matter searches. **SNOLAB-size** is the standard $\varnothing 10 \text{ cm} \times 3.3 \text{ cm}$ cylindrical detector size designed for SuperCDMS SNOLAB as described in Section 3.4.2. We also consider two smaller detector sizes, “**10cm³**” ($3 \times 3 \times 1.2 \text{ cm}^3$), and “**1cm³**” ($1 \times 1 \times 0.4 \text{ cm}^3$), where the 1cm³ detectors are the same dimensions as the HVeV detectors. Reducing the detector size improves the phonon energy resolution; the phonon sensor area and thus total volume of the TES film are reduced, and phonon energy resolution scales with the square root of TES volume. The 10cm³ detector size was chosen as an intermediate between the SNOLAB-sized and 1cm³ detectors.

Detector Types

There are two kinds of detectors that can provide interesting sensitivity for ERDM searches. The first is **HV** as described in previous sections, which uses NTL amplification to reach low energy thresholds, but for which leakage backgrounds could be problematic. The second is **0V**, initially considered for sub-GeV NRDM searches. This consists of a simplified detector design with a single phonon sensor and no ionization electrodes or applied electric field. This maximizes the detector count (and thus the target mass), given the channel count which is constrained by the SuperCDMS SNOLAB facility. Table 5.7 summarizes the detector types, their dimensions, the number of detectors housings that can be accommodated in two SuperCDMS towers, and their raw exposures.

Forecasts were also generated by O. Wen and H. Coombes for NRDM for additional detector types, which include the standard iZIP and new phonon iZIP (piZIP) detectors. The piZIP detector uses the same electric field configuration of the iZIP, but with a new

Detector Size	Type	Dimensions	Mass (g)		Number of Detectors	Raw Exposure (kg·yr)	
			Ge	Si		Ge	Si
SNOLAB	HV	$\varnothing 10 \text{ cm} \times 3.3 \text{ cm}$	1400	610	12	54	23
10 cm ³	HV	$3 \times 3 \times 1.2 \text{ cm}^3$	57	25	36	6.6	2.9
	0V				144	26	12
1 cm ³	HV	$1 \times 1 \times 0.4 \text{ cm}^3$	2.1	0.93	36	0.48	0.21
	0V				144	1.0	0.42

Table 5.7: Masses and exposures for nominal exposure option (two Towers), assuming four years of data-taking and 80% duty cycle. Different detector types of the same size can have different channel counts, resulting in differing numbers of detectors that can be accommodated by two Towers. Adapted from Ref. [177].

phonon sensor configuration. The ionization yield is measured on an event-by-event basis by separating the NTL and primary phonon signals. Forecasts are not calculated for ERDM with these detector types as they provide little sensitivity to the low mass region of interest. They can provide some sensitivity to higher mass dark photons and ALPs, though in this mass range they would likely not be competitive with the upcoming LXe based experiments.

5.3.3 Detector Upgrade Scenarios

We consider a variety of detector upgrades that will improve the overall performance of each of the different detector types, and define three scenarios based on increasing maturity, **Det A**, **Det B**, and **Det C**. Quantitative improvements for each scenario are shown in Table 5.8 for each detector type and size.

Quantity	Detector		Detector Upgrade Scenario					
			A		B		C	
	Type	Size	Si	Ge	Si	Ge	Si	Ge
Baseline phonon energy resolution (eV)	0V	1cm ³	0.5	1.0	0.13	0.28	0.013	0.027
		10cm ³	2.6	4.7	0.7	1.5	0.07	0.14
	HV	1cm ³	0.96	1.17	0.38	0.42	0.064	0.066
		10cm ³	3.2	4.7	1.2	1.5	0.19	0.21
		SNOLAB	11.8	20.7	3.9	6.0	0.58	0.72
Systematic phonon	HV		0.001		0.001		0.001	
energy resolution	0V		0.01		0.01		0.01	
Ionization leakage (Hz/cm ²)	HV		2.5×10^{-3}		2.5×10^{-4}		2.5×10^{-5}	
II length (mm)	HV		200		400		4000	
CT length (mm)	HV		400		400		4000	

Table 5.8: **Detector upgrade scenarios.** Quantitative improvements for each detector type and size and for each scenario (**Det A**, **Det B**, and **Det C**). The iZIP and piZIP scenarios are not shown: they are not relevant for the ERDM projections as they are not sensitive in the mass range of interest. Table adapted from Ref. [177].

Phonon Energy Resolution

There are three improvements/changes considered that can improve the baseline phonon energy resolution.

Detector Size The energy resolution degrades as the square root of the total sensor area, which is affected by the detector size.

TES T_c Energy resolution scales with the cube of the TES critical temperature, T_c . SuperCDMS W films are a mixture of two crystalline phases of W, α and β , which have

T_c values of 15 mK and 1 – 4 K [288], respectively. This allows for a customizable T_c for the W film. We assume T_c values of 40, 30, and 20 mK for scenarios **Det A**, **Det B**, and **Det C**, respectively. T_c values as low as 19 mK have already been achieved for Si [177].

W/Al Transmission/Trapping When a quasiparticle created in the Al film is incident on the W/Al film interface, it has a certain probability to pass through the interface and become trapped in the W TES. We currently estimate a probability of 10^{-4} for both SuperCDMS SNOLAB and for the **Det A** scenario, and assume improvements of $10^{-3.5}$ and 10^{-3} for the **Det B** and **Det C** scenarios, respectively. Processes for this improvement have not yet been demonstrated.

As mentioned in the description of the detector response model, at high energy the position dependence of the pulse shapes becomes more important and cannot be fully corrected for in the energy estimation. We assume a systematic component of the energy resolution of $f_\sigma E_{pt}$, where f_σ is the fractional energy resolution. Currently, $f_\sigma = 0.01$ can be regularly achieved, e.g. $\sim 5 \times 10^{-3}$ in both CDMSlite Run 2 and Run 3 [121, 160], and we assume better performance for the HV detectors. We assume that this systematic remains constant with detector scenario.

Ionization Energy Resolution

The ionization energy resolution is only important for iZIP and piZIP detectors, for which ERDM limits are not calculated. Thus I will not go into detail about the improvements for these detectors, though more details can be found in Ref. [177].

Ionization Leakage

The high voltage applied across SuperCDMS HV detectors enhances the production of a leakage current of charge carriers tunnelling from the electrodes. This is a random Poissonian

process, and the resulting charges drift from one surface to the other across the full bias voltage, producing a signal of eV_b . If close enough together in time, these events can be indistinguishable, producing apparent signals of $N_{eh}eV_b$ for $N_{eh} = 1, 2, 3, \dots$. This leakage background is offset from other backgrounds, which have additional contribution from recoil energy. With fine enough energy resolution, separation between ER and leakage peaks could be achieved, which would improve the sensitivity for ERDM models significantly.

Improvements to the leakage background can come from reducing the surface injection using an insulating blocking layer like SiO_2 , or through discrimination by reducing the phonon energy resolution to separate out the leakage peaks. Additionally, by reducing the phonon timing resolution, leakage pileup events could be separated, improving the leakage background in the $N > 1$ peaks.

5.4 ERDM Forecasts for Upgrade Scenarios

In this section I show the resulting forecasts for each of the detector and background upgrade scenarios and different detector types. For ERDM we focused on the HV detectors as well as the 0V detectors with the understanding that the iZIP-style detectors would not provide the most interesting science reach. The HV detector forecasts are produced using the discrete ionization model code while the 0V forecasts are produced using the continuous ionization code.

5.4.1 Background Models

The total background spectra (all backgrounds summed) with analysis cuts applied for each detector and background scenario are shown for both Ge and Si for the 0V detectors in Figure 5.10 and the HV detectors in Figure 5.11. For each detector size there are nine scenarios.

The main difference between the three background scenarios can be seen in the level of cosmogenic tritium and ^{32}Si at high total phonon energies in Figure 5.10. Note that there

is not much difference in the background improvement between Bkg 1 and Bkg 2 for Ge, though the ^{32}Si reduction is substantial for Si. The most dramatic difference can be seen in the Bkg 3 scenario, where all cosmogenic backgrounds have been negated through an underground detector lifecycle.

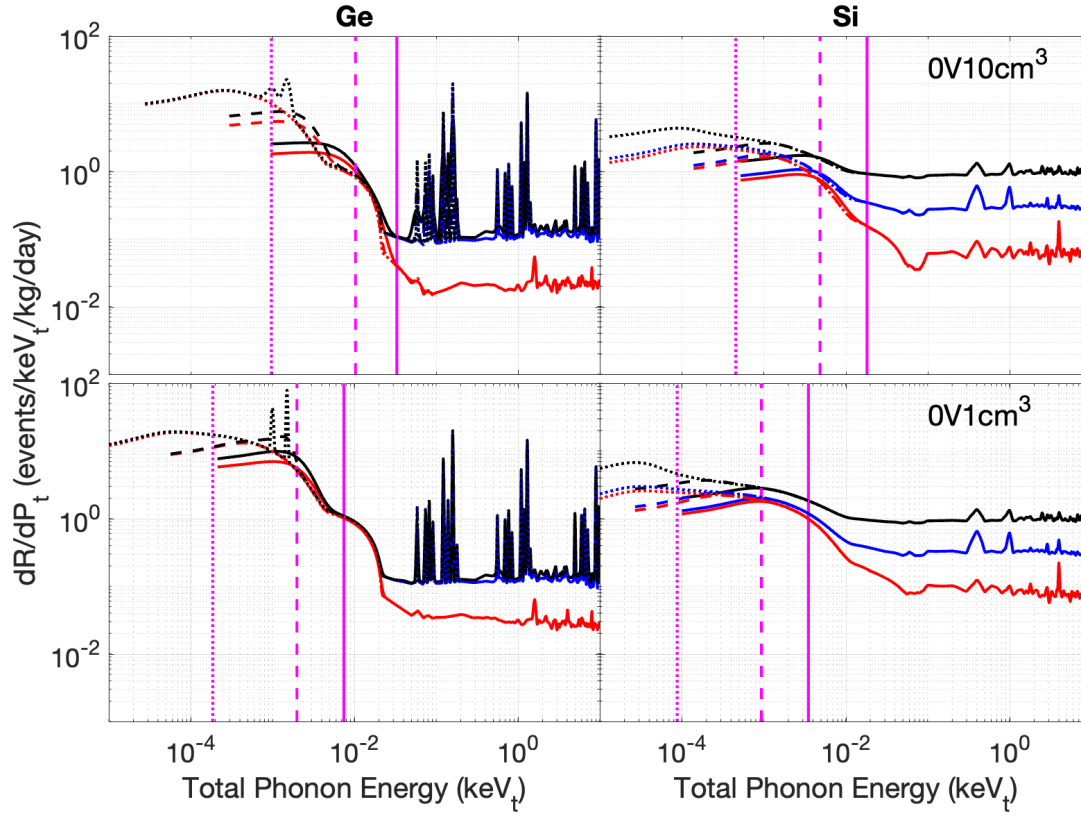


Figure 5.10: Background model spectra with analysis cuts applied for 0V 10cm³ (top) and 1cm³ (bottom) detectors for Ge (left) and Si (right), determined with continuous ionization model. Colour indicates the background scenario (black, blue, red for Bkg 1, Bkg 2, and Bkg 3, respectively), and line style indicates the detector scenario (solid, dashed, dotted for Det A, Det B, and Det C, respectively). Pink lines indicate the threshold (line style indicates the detector scenario as with the backgrounds).

The shapes of the spectra for the 0V detectors do not change dramatically at high energy (though one can observe that the cosmogenic peaks in Ge narrow with improving energy

resolution). However, there is a dramatic difference in threshold for these detectors between the detector improvement scenarios which will drive the sensitivity.

Figure 5.11 focuses on the low-energy regime for the HV detectors to highlight the quantized ionization and the leakage background. For these detectors we effectively set a higher threshold for the limit setting process based on the rate of the leakage background as described in Section 5.1. The energy range that is excluded is visualized in Figure 5.11 by a shaded bar for each detector scenario. It is the same for each background scenario, and always masks out the first leakage peak. The improved detector scenarios lower the energy at which the masked region cuts off based on the leakage rate. Also shown in Figure 5.11 are the background spectra without leakage background for comparison. The difference is particularly dramatic for the SNOLAB-sized detectors.

We note in particular the effect of the improved phonon energy resolution and reduced leakage rate with the improved detector scenarios. The resolution improvement is most obvious in the first leakage peak at eV_b (100 eV_t). Additionally, we can observe the push to slightly lower threshold in the improved detector scenarios as the effective lower analysis threshold set by the leakage background decreases. This is most obvious for the 1cm^3 detectors in the **Det C** scenario. Finally, in this Figure we can clearly see the between-peak fill in from II and CT (particularly below 200 eV_t).

We found that background improvements beyond the **Bkg 1** scenario have only marginal impact on the final forecasts. In the following Sections I show results for the **Bkg 1** scenario only, but a summary of all forecasts can be found in Appendix B. The HV 1cm^3 detectors provide weaker sensitivity than the other detectors, and are also relegated to the Appendix.

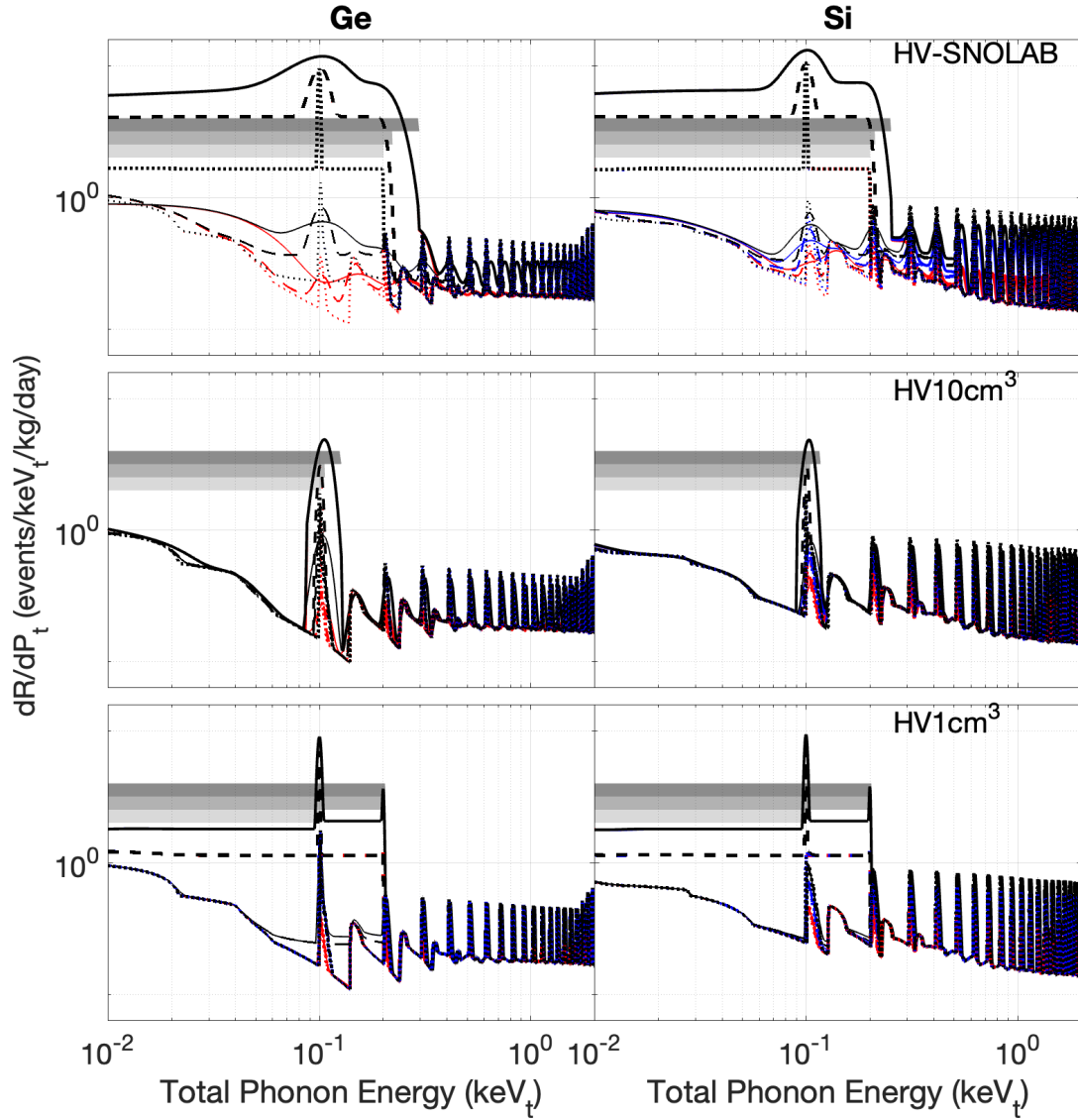


Figure 5.11: Background model spectra with analysis cuts applied for HV detectors for Ge (left) and Si (right). Colour indicates the background scenario (black, blue, red for Bkg 1, Bkg 2, and Bkg 3, respectively), and line style indicates the detector scenario (solid, dashed, dotted for Det A, Det B, and Det C, respectively). The backgrounds with no leakage are shown with thinner line width and the same colour and line styles. The effect is most obvious for the SNOLAB-size detectors as the smaller detectors are mostly hidden by the Det C scenario. The shaded regions indicate energy range removed for the limit setting process due to high leakage background, with the highest opacity indicating the Det A scenario and the lowest indicating the Det C scenario. The upper edge of the region indicates the effective analysis threshold for the given detector scenario.

5.4.2 Dark Absorption

Figure 5.12 shows the projections for 0V detectors for dark photons and ALPs for the Bkg 1 scenario. The overall trend in behaviour is the same between the two dark matter models, so I will comment on them both generally.

The dark absorption channels see significant value from the improved energy resolution of the detector scenarios for 0V detectors. As shown in Figure 5.12, we can push to very low masses with improved detector scenarios. Even with the **Det A** scenario, the 1cm^3 detectors will provide increased mass sensitivity over SuperCDMS SNOLAB, though the reach at $> 10\text{eV}/c^2$ masses is generally not as strong because of the lower exposure. The 1cm^3 detectors tend to outperform the 10cm^3 detectors because of the lower energy resolution, able to probe lower masses; however, the 10cm^3 detectors have stronger overall coupling parameter sensitivity above $10\text{eV}/c^2$ if the **Det B** scenario can be achieved. There is little benefit to the **Det C** scenario over the **Det B** scenario for the 1cm^3 detectors, while the 10cm^3 detectors would need the **Det C** scenario to achieve similar mass sensitivity to the 1cm^3 **Det B** scenario. We can probe sub-eV/ c^2 masses with the **Det B** scenario for Ge 1cm^3 detectors, or **Det C** Ge 10cm^3 detectors, and have overall coupling parameter reach of $\epsilon \sim 3 \times 10^{-18}$ and $g_{ae} \sim 6 \times 10^{-14}$ in the 10cm^3 **Det C** scenarios.

For the 1cm^3 detectors, considering a twenty times exposure **Det B** scenario (green dashed projections) can push the sensitivity past what can be achieved by the **Det C** scenario and becomes competitive with the coupling parameter reach of the 10cm^3 detectors. For ALPs in particular, the mass reach of the 1cm^3 Ge detectors would probe the parameter space hinted at by stellar measurements down to $\sim 4\text{eV}/c^2$ masses.

Figure 5.13 shows the projections for the HV detectors for dark photons and ALPs for the Bkg 1 scenario. Generally the Ge constraints are stronger than Si, other than around $15\text{eV}/c^2$. The improvement with detector scenario is not as impressive as with the 0V detectors because of the limitations of the leakage background. The **Det A** SNOLAB-sized detectors have similar sensitivity as SuperCDMS SNOLAB above $\sim 15\text{eV}/c^2$, but can

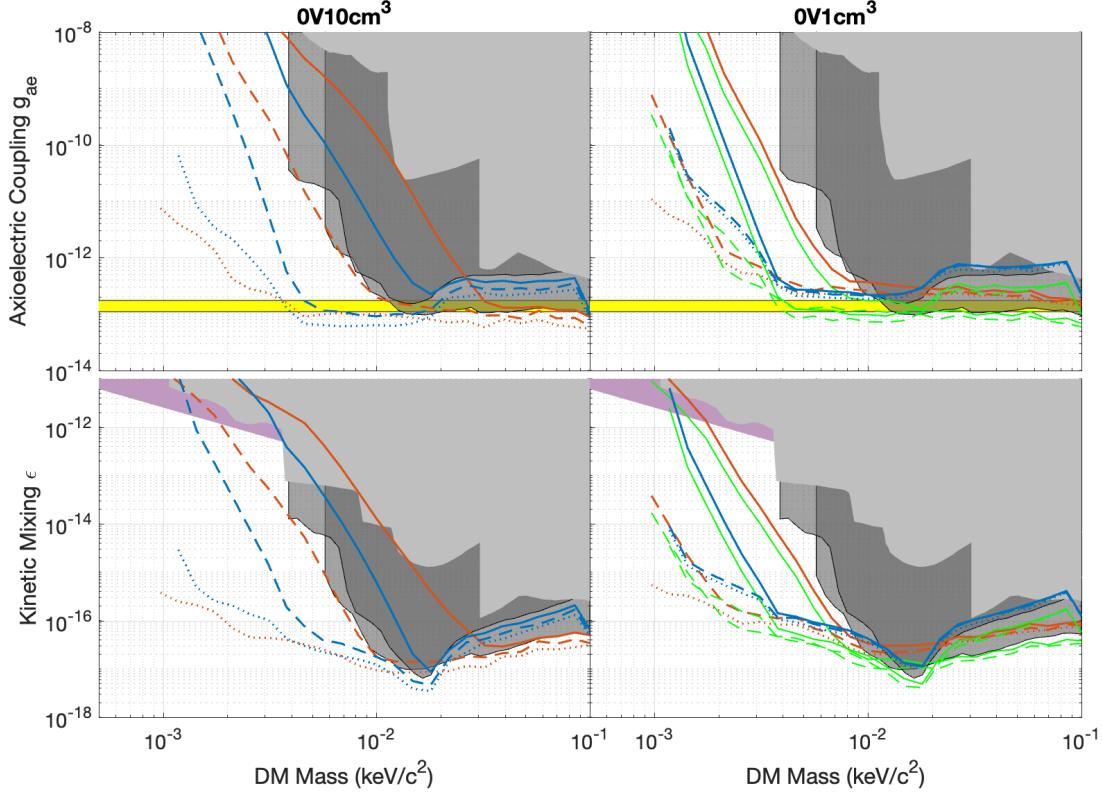


Figure 5.12: Summary of forecasts for ALP dark matter (top) and dark photon dark matter (bottom) using 0V detectors. The 0V 10cm³ (left) and 1cm³ (right) detectors are shown separately. All forecasts are the Bkg 1 scenario. The line colour indicates the material, either Ge (red) or Si (blue), and the line style indicates the detector scenario Det A, Det B, and Det C (solid, dashed, dotted). For the 1cm³ detectors, an additional twenty times exposure is shown (green) for Det A and Det B, which would require use of the entire SuperCDMS cryostat. Current constraints are shown in solid light grey and the expected sensitivity for SuperCDMS SNOLAB is shown in translucent dark grey for Ge and Si. The hinted parameter space from white dwarfs is shown in yellow for ALPs and stellar constraints are shown in pink for dark photons.

provided stronger sensitivity at lower masses because of the improved leakage background.

The HV detectors outperform the 0V detectors in an intermediate mass range ($\sim 5 - 30$ eV) even with the Det A scenario. For ALPs the SNOLAB-sized detectors can even

probe past the white dwarf hinted parameter space. However, at low masses the leakage background completely dominates even in the improved detector scenarios. The leakage background is particularly problematic for the SNOLAB sized detectors, which do not achieve the same sensitivity as the 10cm^3 detectors below $\sim 3\text{eV}/c^2$. If the leakage background can be mitigated, the HV 10cm^3 detectors gain similar sensitivity as the 0V 10cm^3 Det C scenario.

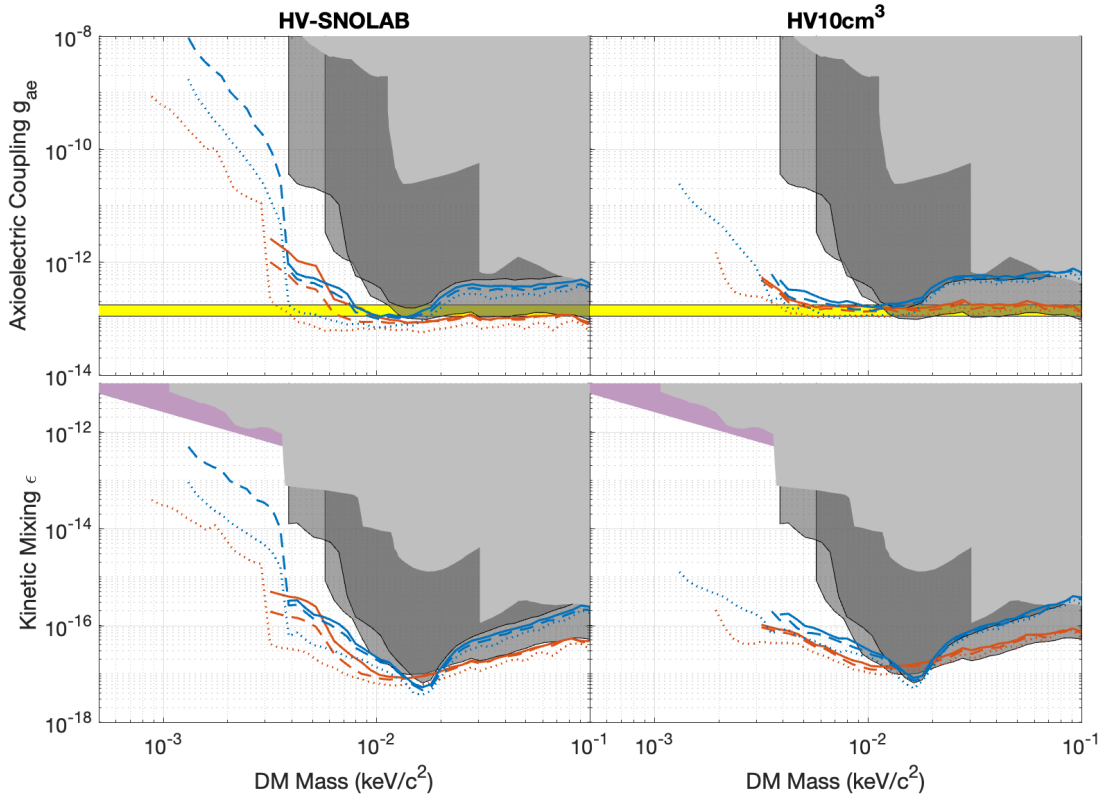


Figure 5.13: Summary of forecasts for ALP dark matter (top) and dark photon dark matter (bottom) using HV detectors. The HV SNOLAB-sized (left), 10cm^3 (right) detectors are shown separately. The line colour indicates the material, either Ge (red) or Si (blue), and the line style indicates the detector scenario **Det A**, **Det B**, and **Det C** (solid, dashed, dotted). Current constraints are shown in solid light grey and the expected sensitivity for SuperCDMS SNOLAB is shown in translucent dark grey for Ge and Si. The hinted parameter space from white dwarfs is shown in yellow for ALPs and stellar constraints for dark photons are shown in pink.

5.4.3 Light Dark Matter

Figure 5.14 shows the forecasts using 0V detectors for dark photon-mediated LDM with a heavy and light mediator. Note that the 0V 10cm³ **Det A** scenario does not have sensitivity to LDM as the threshold is not low enough. With LDM, the sensitivity is primarily threshold driven. The main sensitivity comes from the Si detectors, with the Si 0V 1cm³ **Det A** detectors surpassing the projected sensitivity of SuperCDMS SNOLAB (Ge 0V 1cm³ detectors require the **Det B** scenario to achieve the same sensitivity). Additionally, only the **Det B** scenario is required for the 1cm³ detectors. There is a slight improvement with the Ge **Det C** scenario at masses below 2 MeV/ c^2 , but otherwise the **Det C** and **Det B** 1cm³ detectors show very similar sensitivity.

The 10cm³ Si detectors can provide stronger constraints in the $> 2 \text{ MeV}/c^2$ mass range with the **Det B** scenario thanks to the larger exposure, and achieving the **Det C** scenario would provide mass reach down to 3.5 MeV/ c^2 with the Ge detectors (5 MeV/ c^2 with Si). Overall, the 10cm³ detectors provide the best cross section reach at $\sim 1 \times 10^{-42} \text{ cm}^2$ for both heavy and light mediators. Alternatively, a similar cross section reach can be achieved with a twenty times larger exposure scenario of 1cm³ detectors in the **Det B** scenario. We can probe more of the parameter space of the sharp targets from Ref. [49] with the low mass sensitivity.

Figure 5.15 shows the forecasts using HV detectors for dark photon-mediated LDM with a heavy and light mediator for the **Bkg 1** scenario. The SNOLAB-sized and 10cm³ detectors provide similarly strong overall cross section sensitivity, reaching cross sections $\sim 10^{-42} \text{ cm}^2$ for both mediators. The mass reach does not improve quite as dramatically with improving detector scenario as it does with the 0V detectors, but the overall cross section at all masses does improve.

The Ge SNOLAB-sized detectors provide the overall strongest mass reach down to 3.5 MeV/ c^2 in the **Det C** scenario, similar to the 0V detectors, though for the HV detectors the two materials provide similar overall sensitivity.

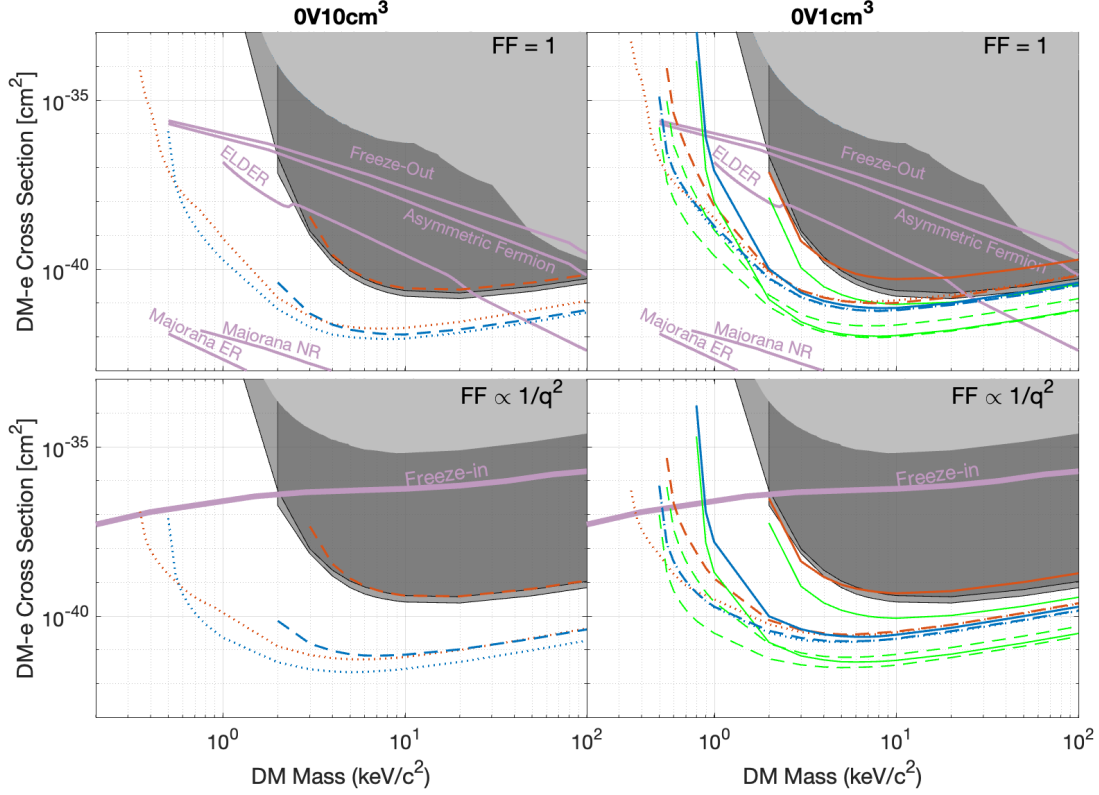


Figure 5.14: Summary of forecasts for LDM coupled to a heavy (top) and light (bottom) mediator using 0V detectors. The 0V 10cm³ (left) and 1cm³ (right) detectors are shown separately. The line colour indicates the material, either Ge (red) or Si (blue), and the line style indicates the detector scenario Det A, Det B, and Det C (solid, dashed, dotted). For the 1cm³ detectors, an additional twenty times larger exposure is shown (green) for Det A and Det B, which would require use of the entire SuperCDMS cryostat. Current constraints are shown in solid light grey and the expected sensitivity for SuperCDMS SNOLAB is shown in translucent dark grey for Ge and Si. Sharp targets from Ref. [49] are shown in pink.

5.4.4 Forecasting Conclusions

With the detector upgrades considered here, there are two very promising and complementary avenues for improved sensitivity to dark absorption and LDM channels. The main challenge for the HV detectors will be significantly reducing the leakage background, while for the 0V detectors we must push to very low energy resolutions. By pursuing both 0V and

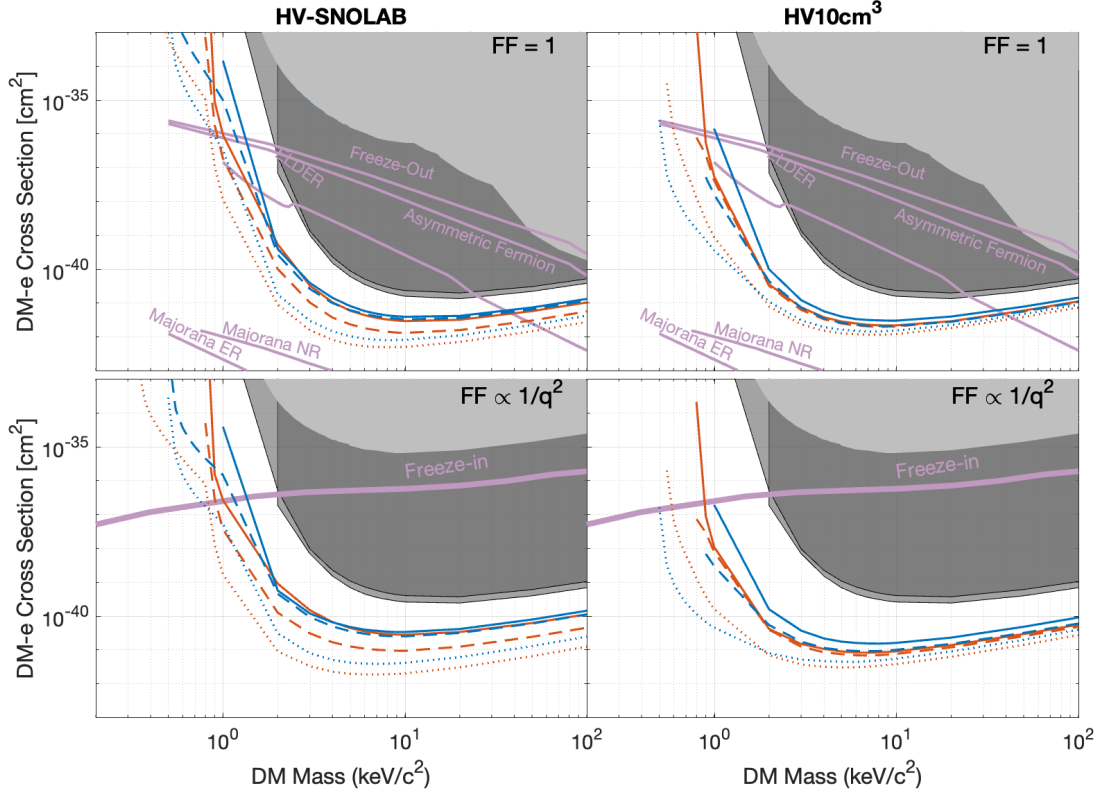


Figure 5.15: Summary of forecasts for LDM coupled to a heavy (top) and light (bottom) mediator using HV detectors. The HV SNOLAB-sized (left) and 10cm^3 (right) detectors are shown separately. The line colour indicates the material, either Ge (red) or Si (blue), and the line style indicates the detector scenario **Det A**, **Det B**, and **Det C** (solid, dashed, dotted). Projections for the **Det A** 10cm^3 scenario are not shown because these detectors do not have low enough threshold to have sensitivity to LDM in this scenario. For the 1cm^3 detectors, an additional twenty times larger exposure is shown (green) for **Det A** and **Det B**, which would require use of the entire SuperCDMS cryostat. Current constraints are shown in solid light grey and the expected sensitivity for SuperCDMS SNOLAB is shown in translucent dark grey for Ge and Si. Sharp targets from Ref. [49] are shown in pink.

HV options, we can mitigate the risk of not being able to achieve one or the other. The two programs are complimentary, with the HV detectors typically able to provide slightly better coupling sensitivity at higher masses and the 0V detectors providing slightly stronger low mass sensitivity in the most costly upgrade. Perhaps the most promising finding from this

work is that we do not need to achieve the more costly background scenarios **Bkg 2** and **Bkg 3**. We are almost entirely exposure limited in all cases, and both mass and cross section reach are more readily achievable by detector improvements and exposure increases. Figure 5.16 shows the expected reach for a selection of upgrade scenarios for each dark matter model.

For the dark absorption channels, our main area of improvement with both detector types is mass reach. The considered improvements do little to improve the sensitivity above masses of $10 \text{ eV}/c^2$, though we do see slight improvement with the HV SNOLAB-sized Ge detectors or the twenty times exposure 0V 1 cm^3 detectors. We can push down to or near to the Si band gap with 0V and HV detectors in the **Det C** scenario. For LDM the strong dependence of the sensitivity on threshold means that we can improve both mass and cross section reach with each detector upgrade scenario. Overall, the 0V detectors are very promising if the energy resolution can be reduced to the **Det B** or **Det C** values. We can achieve reach at very low masses in all of the considered ERDM parameter spaces either with the **Det C** 10 cm^3 or **Det B** 1 cm^3 twenty times exposure scenario, which would use the entire cryostat. The HV detectors can also be a promising option if the leakage background can be significantly reduced. The 10 cm^3 detectors can provide low mass reach for the dark absorption channels, and both SNOLAB-sized and 10 cm^3 detectors show promising reach for LDM even in the **Det A** scenario.

With this set of upgrades we expect to be able to constrain a large portion of parameter space beyond what is achievable with SuperCDMS SNOLAB, without the need for the most costly background upgrade scenarios. We must also consider the approach for NRDM, so we cannot necessarily fill the facility with the detectors that would give the best sensitivity to ERDM. In Ref. [177] we show that we will be able to push down into to the neutrino fog for $1 - 10 \text{ GeV}/c^2$ mass WIMPs with 10 cm^3 iZIP and piZIP detectors with a twenty times exposure scenario. The best reach for sub- GeV/c^2 NRDM comes from the 0V 1 cm^3 and 10 cm^3 detectors, particularly the Si detectors because of the improved energy resolution

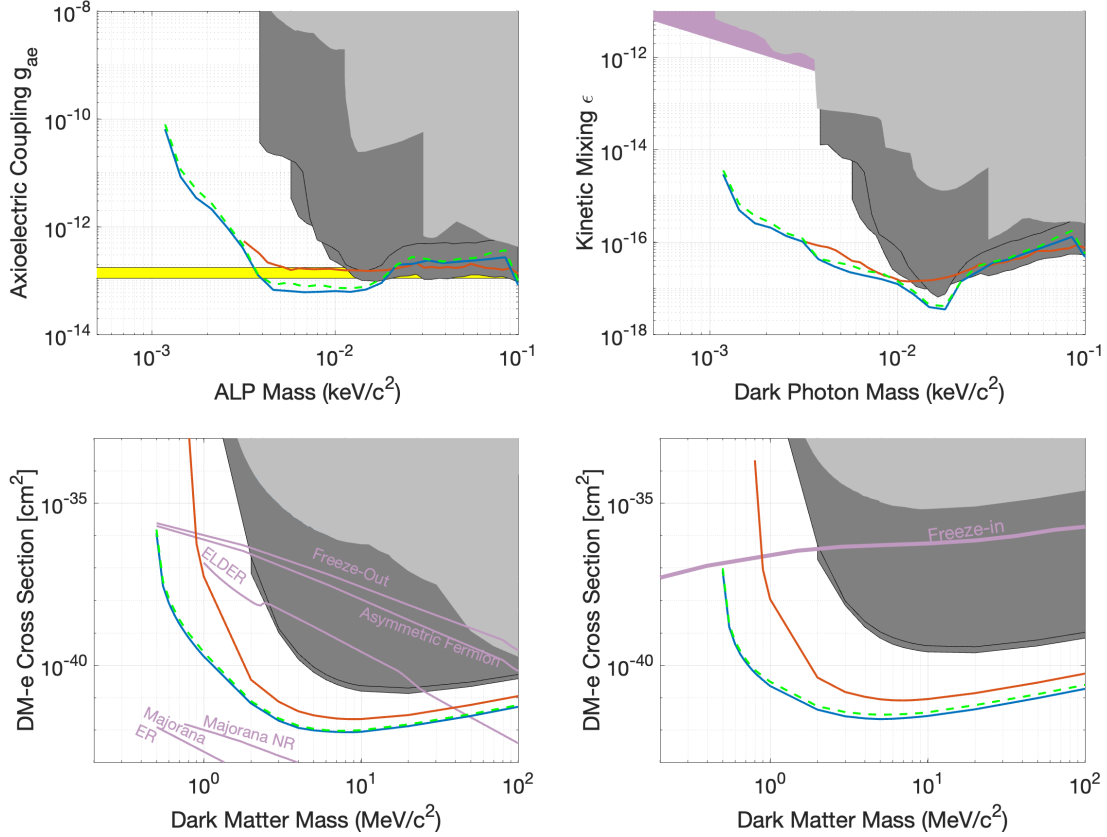


Figure 5.16: Sensitivity forecasts for each ERDM model (ALP – top left; dark photon – top right; LDM with heavy mediator – bottom left; LDM with light mediator – bottom right) showing a snapshot of the expected sensitivity with in hand technology (red, HV 10cm³ Ge detectors with **Det A** upgrade) and with significant detector performance upgrades (blue, 0V 10cm³ Si detectors with **Det C** upgrade). The latter sensitivity can also be achieved with a less aggressive detector upgrade and increased exposure (green, 0V 1cm³ Si detectors with **Det B** upgrade and times twenty exposure). Current constraints are shown in solid light grey and the expected sensitivity for SuperCDMS SNOLAB is shown in dark grey for Ge and Si. For ALP dark matter the parameter space indicated by a hint from stellar cooling [287] is shown in yellow. The pink shaded region for dark photons is excluded by stellar constraints [255]. For LDM, sharp theory targets from Ref. [49] are shown, assuming $M_{A'} = 3m_\chi$ where A' is the dark photon mediator and χ is the dark matter. There are several dark matter targets for the heavy mediator.

and therefore mass reach. For these detectors, the sharply rising WIMP spectrum is easily separable from the flatter ER backgrounds using the PLR method. Therefore, both the

ERDM and NRDM programs would benefit highly from exploring the 0V R&D options.

Any potential future SuperCDMS sensitivity projections should consider a more careful implementation of the II and CT model. The SuperCDMS Detector Monte Carlo (DMC) is a simulation tool under development to provide an understanding of detector response. Though the DMC is still some time from completion, this would eventually be the ideal replacement for part of the Matlab-based sensitivity code dealing with the detector response.

It is also interesting to consider the future beyond these upgrade scenarios. In Ref. [177] we describe a variety of new R&D directions including new detector materials with lower band gap like InSb (0.1 eV), or Dirac materials (e.g. ZrTe₅) with band gaps on order of a few meV [289]. Another avenue for dark photon-coupled LDM is optical phonon creation (single phonon excitations) in polar materials such as Al₂O₃, SiO₂, and CaWO₄. The maximum kinetic energy transfer is related to the energy of the lowest optical mode, making low optical mode materials of the most interest [290].

The next few years will be an exciting time for SuperCDMS. With the commissioning of the SNOLAB experiment beginning in 2024, we will soon be able to set constraints on the dark matter models for which we have provided projections, and potentially others. There is a long road ahead in R&D to reach the most aggressive of our detector upgrade scenarios, but guided by our forecasting efforts, we have plenty of avenues to explore and vast dark matter parameter space to uncover.

Chapter 6

Athermal PhotoDetector Characterization at the CUTE Facility

Understanding detector performance and maintaining and verifying consistent behaviour is a vital part of dark matter detection. The calibration of energy scales requires consistent behaviour between calibration data and low background data (assuming the data are not acquired concurrently). Any changes in the detector performance can impact the interpretation of the data. For an example relevant to SuperCDMS, when measuring the detector response to ERs using ^{133}Ba calibration data and to NRs with ^{252}Cf data, we want to be assured that the detector response is the same between the calibration data sets and in the low background data. We can make measurements of various parameters to characterize detector behaviour for this purpose.

This chapter concerns data taken at the CUTE facility between December 2019 and March 2021. I was part of a data taking team consisting of several people, and was personally involved in the collection of all described CUTE data within this chapter. I would like to emphasize that this is a team effort that would not have been possible without contributions from many individuals. In this chapter I describe my own analysis work, but also refer to analyses performed by R. Underwood (as described in his thesis [183]) and A. Pradeep (currently only available internally to the SuperCDMS collaboration). As with data taking, the analysis is largely a team effort and there are several other individuals who have made

significant contributions to the analysis of CUTE data. This work was heavily supported by the CUTE and SNOLAB onsite crews, particularly through the COVID-19 pandemic. I make comparisons of the results to work performed by C. Fink and S. Watkins at SLAC.

Quoted uncertainties on detector parameters are either automatically propagated through the QETpy and RQpy packages, or determined from non-linear least squares fits and standard error propagation when multiple data sets are combined. An uncertainty of 10% on the shunt resistance is assumed throughout.

6.1 Motivation

This chapter describes the characterization of a CPD-style detector named PD2. These single channel athermal phonon detectors are described in Section 3.4.3. The analysis of a second detector named CPD is described in Appendix C. This detector has the same QET design as PD2, but was intended to have lower T_c for improved energy resolution (unfortunately, the opposite turned out to be true, with CPD having a higher T_c), and had not been operated prior to operation at CUTE in Run 16.

PD2 has been well characterized at SLAC with results published by C. Fink, S. Watkins et al. in Ref [241] and described in further detail in theses by Fink [291] and Watkins [202]. Several relevant parameters from these works are shown in Table 6.1, including DC parameters determined from IV measurements, parameters determined from complex impedance measurements, and T_c and G_{ta} , which are determined from a fit of a power law to the measured bias power as a function of bath temperature. At SLAC an uncertainty of 10% is assumed on R_{sh} ; for the following analysis I assume the same uncertainty.

The various parameters are described in some detail in Chapter 3, but I shall review them briefly here. The four resistances are the shunt resistance R_{sh} responsible for voltage-biasing the TES, the parasitic resistance R_p in the TES circuit, the resistance of the TES R_n in the normal regime, and the resistance of the TES R_0 at a specific operating point. P_0 is the bias power of the TES at the operating point. G_{ta} is the thermal conductance of the

TES to the absorber, and T_c is the critical temperature of the TES. The natural thermal time constant is here referred to as τ_0 (this was referred to as τ in Chapter 3). The baseline energy resolution is represented by σ_0 , and the phonon-collection time is τ_{ph} . Finally, the phonon-collection efficiency is ϵ_{ph} .

Parameter	Value
R_{sh} (m Ω)	5 ± 0.5
R_p (m Ω)	8.7 ± 0.8
R_n (m Ω)	88 ± 10
R_0 (m Ω)	31 ± 3
P_0 (pW)	3.85 ± 0.45
G_{ta} (nW/K)	0.48 ± 0.04 (stat.) $^{+0.49}_{-0.00}$ (syst.)
T_c (mK)	41.5 ± 1.0 (stat.) $^{+10}_{-0}$ (syst.)
τ_0 (μ s)	1700 ± 200
τ_{ph} (μ s)	20
ϵ_{ph}	$13 \pm 1\%$, $17 \pm 1\%$
σ_0 (eV)	3.86 ± 0.04 (stat.) $^{+0.19}_{-0.00}$ (syst.)

Table 6.1: PD2 parameters determined at SLAC from IV curves, complex impedance data, and bias power measurements. Measurements from complex impedance data are presented at the 35%- R_n bias point. G_{ta} and T_c are determined by fitting a power law to the bias power as a function of bath temperature. Table from Ref. [241].

A search for SI-nucleon coupled low mass dark matter was also performed with PD2 at SLAC [242], setting constraints for masses above 93 MeV/ c^2 . PD2 was operated at CUTE in the hopes of improving upon this result and performing a dark matter search in the low background environment. Data from CUTE Run 15 were used to set dark matter constraints by R. Underwood as part of his PhD thesis [183].

The event rate at low energy was observed in PD2 over the course of CUTE Runs 11, 14, and 15. The event rate decreased over time during each of the runs, but also did not decrease between the end of Run 14 and the beginning of Run 15, indicating that this is likely not some kind of radioactive background decaying over time. Figure 6.1 shows the event rate

over time in three different energy bins, where the described behaviour is most prominent in the lowest energy bin. It was hypothesized that the high observed event rate was due to stress fractures in the crystal relaxing over the course of each run. Such micro-fractures can produce signals which mimic signals from particle interactions.

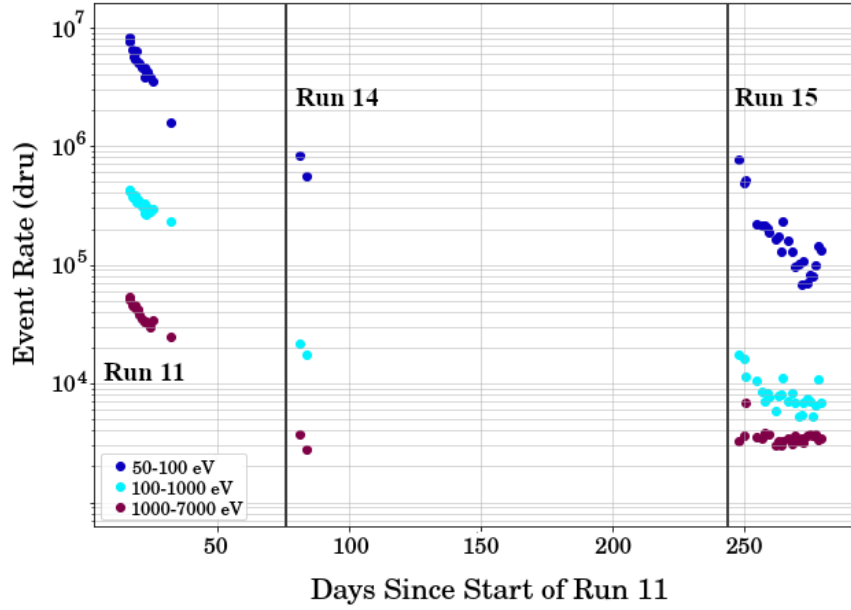


Figure 6.1: The event rate of PD2 over time, measured in different energy bins, adapted from Ref. [183]. The black vertical lines indicate the beginning of Run 14 and Run 15. Events are binned by energy, with colour representing a low (red), intermediate (cyan), and high (navy) energy range. The event rate can be seen to decrease over time during each run.

To test the hypothesis that the low energy background is related to stress fractures, we attempted to adjust the clamping scheme of the crystal in the detector housing. The CPD-style detectors are held in a hexagonal housing with clamps at each of the six corners holding the crystal in place. These clamps alternate facing upwards and downwards. Figure 6.2b shows an example of this nominal clamping scheme. Spacers were added to the clamps in an attempt to reduce the stress on the crystal. Furthermore, we received an additional CPD-style detector, referred to as CPD. In this run we added an ^{55}Fe source directed at

the noninstrumented face of PD2 for energy calibration. The source is collimated and attenuated by several layers of aluminium foil. There are three potential calibration peaks including the ^{55}Fe K- α and K- β lines at 5.889 keV and 6.49 keV and a peak at 1.486 keV from Al fluorescence. PD2 was calibrated with the Al fluorescence line at SLAC using a similar setup. However, the rate of the Al line was too low to be observed in the PD2 energy spectrum in CUTE Run 18.

At SLAC the presence of an additional $\mathcal{O}(100)\,\mu\text{s}$ fall-time was observed in the complex impedance response, and an additional long 3.2 ms thermal fall-time was observed in measured pulses. A similar response was found for PD2 at CUTE. However, this response became more pronounced with the new clamp configuration, with the third fall-time in the complex impedance response increasing to millisecond timescales and the long thermal fall-time increasing in prominence for measured pulses. Seemingly, the addition of the spacers decreased the thermal coupling of the absorber crystal to the bath.

In response to the degraded PD2 detector response following the clamp adjustment, we attempted to adjust the clamps again on both PD2 and CPD in Run 19. The clamps on PD2 were adjusted from a pattern alternating upwards and downwards clamps at each of the 6 corners to two clamps (one upwards and one downwards) on each of three corners in an attempt to limit the bending of the crystal, but still holding the detector firmly for good thermal contact. With CPD we used a different clamping scheme to attempt to reproduce the changes seen on PD2 between Run 15 and Run 18. Focus was placed on putting contact with the clamps only on the flat surfaces rather than on the sides of the crystal. Figure 6.2 shows the clamping scheme for both detectors. The ^{55}Fe source was also removed in this run to focus on the detector behaviour.

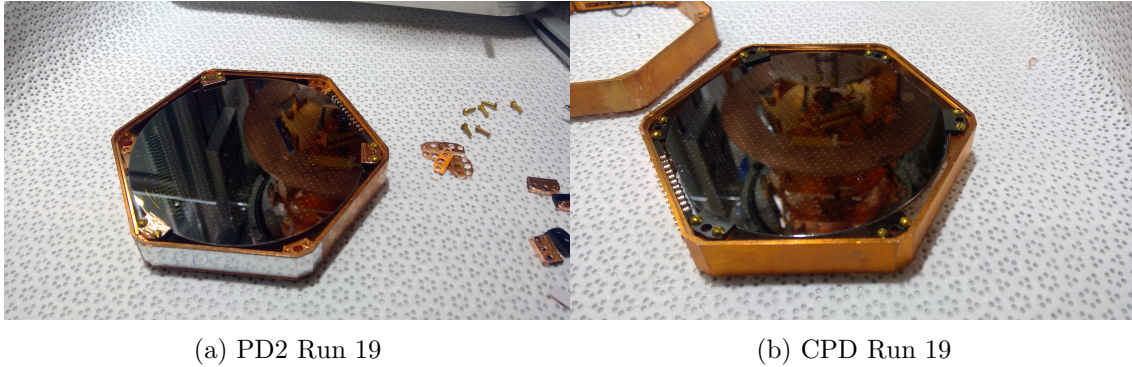


Figure 6.2: The clamping schemes for PD2 (left) and CPD (right) in CUTE Run 19. The cylindrical crystal detector sits in the hexagonal copper housing, held in place by clamps at the corners. Two clamps (one upwards, one downwards) are used on three corners for PD2, while CPD follows the nominal scheme (used for PD2 at SLAC and in Runs 11–15) of clamps alternating upwards and downwards on all six corners. A copper spacer is also partially shown next to CPD, which is used to stack and separate the detectors while keeping them in the same optical cavity.

The noise response of PD2 significantly worsened with the introduction of high levels of low frequency noise. The presence of a long thermal fall-time is still observed in both the complex impedance measurements and direct measurements of pulse shapes. CPD did not replicate the behaviour seen on PD2 in Run 18, with the fall-times measured from the complex impedance remaining consistent with Run 18. However, the noise behaviour changed, inducing a change in the high frequency complex impedance similar to PD2.

In Run 20 the clamps were adjusted for a final time. One of the clamps on PD2 was found to be slightly loose after Run 19, so the clamps were tightened (but the configuration was kept the same) in the hopes that this would address the low frequency noise. No adjustments were made to the clamps on CPD. For this run, the ^{55}Fe source was put back into the detector stack, this time directed at CPD. Four layers of Al foil were removed from the collimator to increase the event rate and improve the chances of observing the Al fluorescence peak, which was able to be partially resolved in the CPD Run 20 energy spectrum. PD2 continued to exhibit large amounts of low frequency noise and was mostly

neglected during this run.

In CUTE Run 16 we measured the critical temperature T_c of both PD2 and CPD by applying a low amplitude test signal to the TES and heating the mixing chamber stage slowly, ideally allowing the crystal time to reach thermal equilibrium. We note the point at which we observe a phase shift in the test signal response as the value of T_c . The PD2 T_c was determined to be (40 ± 1) mK, while the T_c of CPD is (66 ± 2) mK.

The collection efficiency for each detector was measured using the calibration peaks from the ^{55}Fe source, a result described further in Section 5.4.4. This was performed by R. Underwood [183] for PD2 in CUTE Run 18. We additionally measured G_{ta} for PD2. The results for PD2 are shown in Table 6.2 and compared to the measurements at SLAC from Ref. [241]. The measured values of G_{ta} and T_c are consistent, but at CUTE we observed a much lower phonon-collection efficiency. Note that the exact value of the phonon-collection efficiency is highly dependent on the method used to calculate it, a point which will be elaborated upon later.

Parameter	PD2-SLAC	PD2-CUTE	CPD-CUTE
G_{ta} (nW/K)	0.48 ± 0.04 (stat.) $^{+0.49}_{-0.00}$ (syst.)	0.48 ± 0.06 (R18)	-
T_c (mK)	41.5 ± 1.0 (stat.) $^{+10}_{-0}$ (syst.)	40 ± 1 (R16)	66 ± 2 (R16)
ϵ_{ph}	$13 \pm 1\%$	$6.83 \pm 0.1\%$ (R18)	$\sim 10\text{-}20\%$ (R20)

Table 6.2: PD2 and CPD parameters determined at CUTE compared to the PD2 values measured at SLAC from Ref. [241].

Characterizing detector performance is vital for understanding the expected sensitivity and for interpreting acquired data. Throughout this chapter I describe various measurements used to characterize PD2. In Section 6.2 I summarize the CUTE runs relevant to this analysis. DC characteristics of the detector are determined in Section 6.3; of particular importance is the bias power in transition. Differences in bias power at the same transition point can indicate the presence of parasitic power, and measurements of bias power over

the transition region can also be an indication whether or not we are in a regime of pure electrothermal feedback. Bias power measurements are a useful tool for checking whether or not a detector is behaving consistently, or whether there are noise/power sources affecting the performance. Section 6.4 describes complex impedance measurements. These provide a more complete picture of detector behaviour and response, which includes characteristic fall-times related to the electrical and thermal behaviour of the TES. In Section 6.5 the fall-times and rise times are measured using pulses and compared to the fall-times from the complex impedance measurements. At SLAC there was an additional $\mathcal{O}(\text{ms})$ fall-time not observed in the dIdV, the presence of which is verified at CUTE.

In Section 6.6 we model the noise in the different resistance regimes. Noise is another indicator of detector performance, as the baseline energy resolution is directly related to the noise. Noise modelling can tell us if there are additional noise sources beyond the expected noise from first principles based on measured detector parameters. In this section we also discuss energy calibration and collection efficiency. The latter is a useful tool for energy calibration when there is no direct calibration method. Assuming a detector has consistent performance, one can determine the collection efficiency in a run with a calibration source, and then calibrate the energy scale in other runs using the measured energy from the recorded signal together with sensor parameters and parameters of the readout circuitry and the known collection efficiency. This is highly reliant on consistent detector behaviour, and proved to be detrimental to the interpretation of the dark matter search results from Ref. [183]. Finally, Section 6.7 describes measurements of the thermal conductance between the absorber and the bath G_{ab} and the thermal conductance between the TES and the absorber, G_{ta} . These measurements were performed to validate the behaviour observed in the complex impedance measurements.

6.2 Summary of CUTE Runs

Table 6.3 provides an overview of the runs at CUTE relevant to this chapter. Runs 12 and 13 were failed cooldowns. Note that in Run 16 the T_c for CPD was determined, but the run was very short due to a planned power outage at SNOLAB and is not included in the table. The setup was identical to Run 18. Run 17 was a failed run due to a failure with a LN2 dewar; air was introduced into the fridge as the LN2 was used too rapidly and not refilled in time. The payload in each run did not only consist of the CPD detectors, but also up to three TES chips (small devices instrumented with TESs), T5Z2 (a Soudan iZIP detector previously used for CDMSlite Runs 1 and 2), and G124 (a SNOLAB-style HV pathfinder detector).

Run Number	Date (YYYY-MM-DD)	Relevant Payload	Purpose
Run 11	2019-12-11 – 2020-01-20	PD2	PD2 characterization
Run 14	2020-02-21 – 2020-03-05	PD2	PD2 characterization
Run 15	2020-08-06 – 2020-09-21	PD2	PD2 characterization and dark matter search
Run 18	2020-10-23 – 2020-12-04	PD2 & CPD	PD2 adjusted clamps, PD2 energy calibration with ^{55}Fe source, CPD characterization and comparison to PD2
Run 19	2020-12-11 – 2021-01-12	PD2 & CPD	Further adjustment of PD2 clamps, adjustment of CPD clamps
Run 20	2021-03-03 – 2021-03-31	PD2 & CPD	PD2 clamps tightened, CPD energy calibration with ^{55}Fe source

Table 6.3: A summary of CUTE runs relevant to this work, along with the relevant payload and goal.

6.3 DC Characteristics

Section 3.2.2 introduced how the DC characteristics of TESs can be determined from IV curves, measuring the TES current I as a function of bias voltage V . By varying the bias voltage from a high magnitude that keeps the TES in the normal state and decreasing down to zero voltage bias, we can determine the parasitic resistance R_p from the superconducting region and the normal resistance R_n from the normal regime. IV sweeps are important for maintaining a consistent bias point between different runs and even over time within an individual run. The other main use of IV analyses is the calculation of the TES bias power. Under ideal conditions the bias power should be consistent between runs and constant throughout the transition region. A lower bias power can indicate the presence of parasitic power.

In the following Section I describe the analysis of IV curves for PD2 using the QETpy and RQpy packages [205, 292] developed by the Pyle group. IV sweeps were taken alongside complex impedance measurements in Runs 11, 14, 15, 18, and 19. Figure 6.3a shows a summary of these IV curves. There are clear differences in the TES response to bias voltage between the Runs. Run 14 and Run 15 are consistent as expected for no changes occurring between the runs, but the clamp adjustments in subsequent runs change the TES response. Run 11 is an outlier in that there were no changes to the clamping scheme between Run 11 and Run 14, but there is a significant difference in detector response. The readout cables were changed after Run 11, which affects the parasitic resistance and the slope of the IV curve in the superconducting regime, but this would not affect the bias power. The reason for the difference in bias power is unclear, but may be related to the hypothesized stress fractures.

Figure 6.3b shows the calculated bias power for PD2 as a function of bias voltage. One can observe a clear difference in the bias power between the different runs, though Run 14 and Run 15 are consistent. The bias power was highest in these two runs and decreased in the subsequent runs when clamp adjustments had been made. The significant low frequency

(LF) noise introduced in Run 19 is likely responsible for the significant degradation in bias power between Run 18 and Run 19. Note that the bias power is expected to be constant in the transition region for a TES dominated by electrothermal feedback. This is not observed, except for in Run 15.

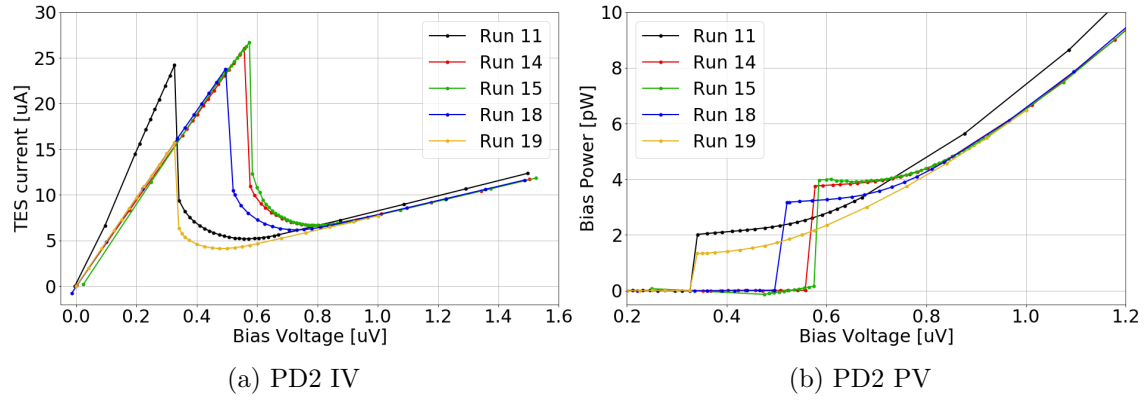


Figure 6.3: Summary of *IV* (left) and *PV* (right) curves for PD2 across different CUTE runs. The bias power is highest in Runs 14 and 15, and lowest in Run 19, where higher bias voltage is required to keep the TES in transition.

The measured values of R_p , R_n , and R_0 and P_0 at close to the 35%- R_n operating point from each run are compared to SLAC in Table 6.4. In general, at CUTE we operate the detectors at the 40%- R_n bias point, though the bias power should be similar between the two bias points. At CUTE we observe a significantly higher R_n than at SLAC; the reason for this is not clear as this is a detector dependent quantity that does not depend on the readout electronics. A possible reason for the discrepancy could be the assumed value of R_{sh} . We assume the same value as at SLAC; however, a value of $\sim 4 \text{ m}\Omega$ at CUTE or $\sim 6 \text{ m}\Omega$ at SLAC could explain the discrepancy. The assumed value of R_{sh} impacts the calculation of other parameters; for example, R_p , R_n , V_b , and P_0 .

The value of R_0 varies between the runs because of the finite step size of the *IV* sweep; the bias point closest to 35%- R_n is shown. We also observe a higher R_p in all runs other than Run 11, though this is not particularly surprising because this is entirely dependent on the readout electronics, and between Run 11 and Run 14 the wiring was extended with

additional (non-superconducting) wires and connections, which can affect the resistance.

The bias power is consistent with SLAC during Runs 14 and 15, slightly lower in Run 18 with the first clamp adjustment, and significantly lower in Run 19 with the next clamp adjustment. The bias power in Run 11 is also low, possibly due to the presence of stress microfractures creating signals which heat the TES.

Parameter	SLAC	Run 11	Run 14	Run 15	Run 18	Run 19
R_p (m Ω)	8.7 ± 0.8	8.5 ± 0.9	16.4 ± 1.6	16.3 ± 1.6	15.8 ± 1.6	15.7 ± 1.6
R_n (m Ω)	88 ± 10	108 ± 12	107 ± 13	108 ± 13	107 ± 13	109 ± 13
P_0 (pW)	3.85 ± 0.45	2.08 ± 0.26	3.77 ± 0.56	3.86 ± 0.48	3.19 ± 0.46	1.33 ± 0.19
R_0 (m Ω)	31 ± 3	36 ± 5	38 ± 6	37 ± 6	41 ± 6	40 ± 6

Table 6.4: PD2 DC parameters, R_0 and P_0 at $35\%R_n$. A value of R_{sh} of 5 ± 0.5 m Ω is assumed. Similar bias power to SLAC is observed in Run 14 and Run 15. The bias power is lower in the other runs, particularly Run 19 where there was significant LF noise.

6.4 Complex Impedance Measurements

Complex impedance measurements can be used to monitor the consistency of detector behaviour over time, whether between individual series or between runs. These measurements allow us to determine the time constants from the electrical and thermal differential equations governing the TES behaviour and can indicate if something has changed in the thermal or electrical behaviour of the detector or readout circuit. We can also verify the assumption of being in a regime where electrothermal feedback dominates the TES response, i.e. the high loop gain limit where power excursions are compensated entirely by a decrease in Joule heating.

The theory behind the complex impedance response is described in detail in Chapter 3. These measurements take a square-wave response at a given bias voltage and measure the TES response. The known square-wave response is used to deconvolve this measured response, and the resulting complex impedance is fit in the frequency domain. This requires

prior knowledge of R_p and R_0 , which can be determined from IV curves and/or from fitting the square-wave response in the normal and superconducting regions, and of R_{sh} . As described in Ch. 3, the analysis of IV and dIdV sweeps is streamlined in the RQpy and QETpy analysis packages maintained by the Pyle group. An initial fit to the complex impedance is performed, and then improved with a priors fit. This method is described in detail in C. Fink's thesis [291]. Two models are included, a 2-pole model and a 3-pole model, the latter representing the inclusion of an additional thermal block as described in Section 3.2.2. As we shall see in this section, the 3-pole model best describes the PD2 complex impedance response in all cases.

There is some degeneracy in the 3-pole model in that the response can describe either the intermediate-block model described in Section 3.2.2 (in which there is an additional thermal block between the TES and the bath) or the hanging-block model (in which there is an additional thermal block coupled to the TES but not the bath). In this work I assume that the intermediate-block model is more appropriate, as this could reasonably describe the setup of the TES and absorber detector, with the detector playing the role of the additional thermal block. Typically one assumes a very large conductance between the absorber and the bath, G_{ab} (or $g_{1,b}$ in the intermediate-block model) compared to the thermal conductance between the TES and the absorber, G_{ta} ($g_{tes,1}$ in the intermediate-block model), in which case the intermediate-block model reduces back to the two-block model.

Figure 6.4 shows the response in the time domain for PD2 for each run at the bias point closest to $40\%-R_n$ taken in each $IV/dIdV$ sweep. A close view of the response in Run 11 is also shown to highlight the difference between the 2-pole and 3-pole fits, where the 3-pole model captures an additional fall-time. The 2-pole and 3-pole model fits are both shown, and the 3-pole model can be seen to best describe the data in all runs; however, the additional fall-time is significantly longer in Runs 18 and 19. Note that the response is measured over a longer time to capture this long fall-time - the period of the square-wave was increased for this purpose. The noise in Run 19 is particularly poor and the quality of

the fits is compromised, but they are shown for completeness. These results imply that the thermal conductance between the absorber and the bath may have changed with the clamp adjustments between Run 15 and Run 18, and again between Run 18 and Run 19.

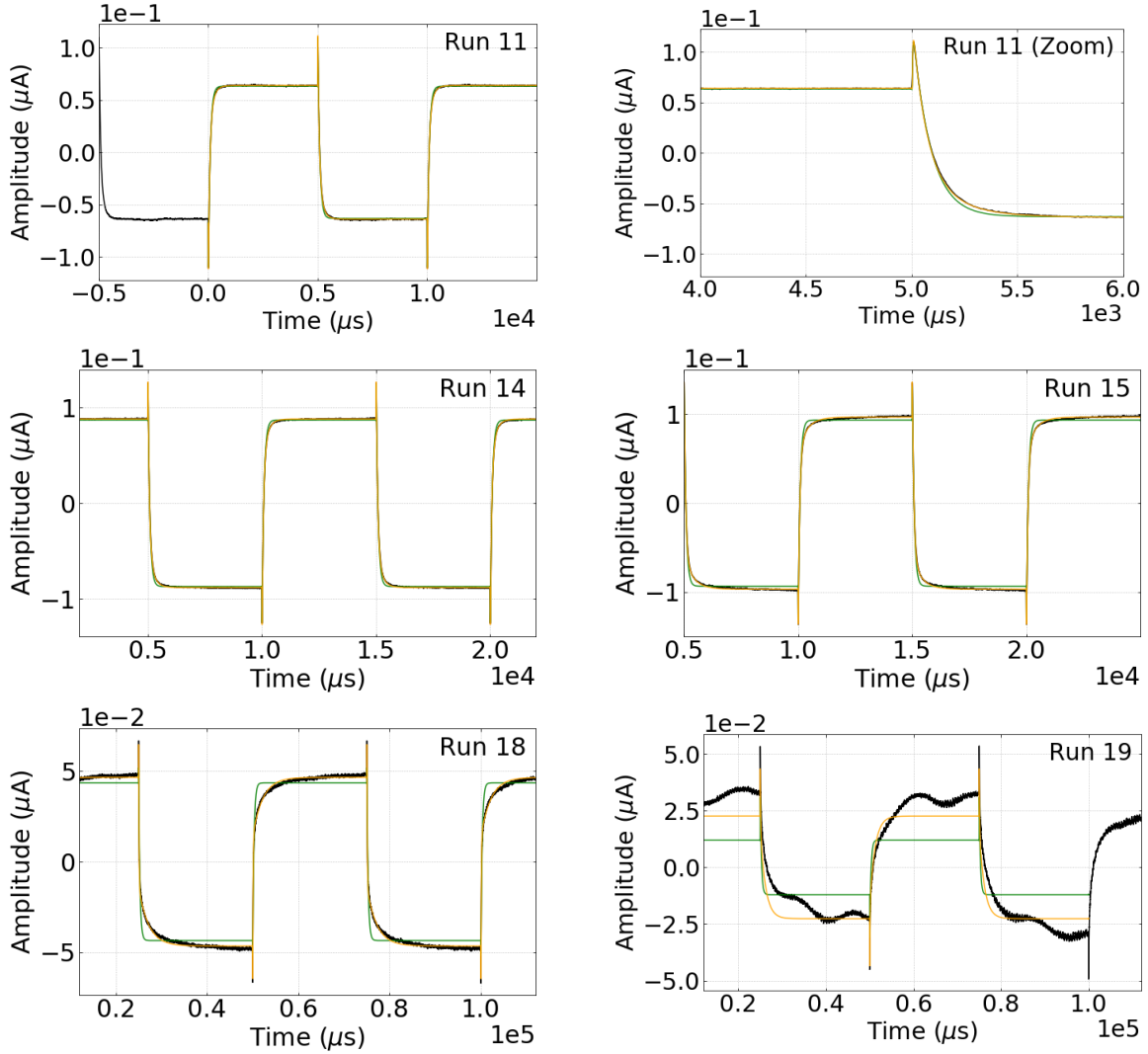


Figure 6.4: Summary of 2-pole (green) and 3-pole (yellow) model fits to the dIdV response (black) in the time domain for bias points close to 40%- R_n for PD2 in each run. The top right plot shows a close view the Run 11 fit, demonstrating how the 3-pole model best describes the data. The drastic degradation of the noise is apparent for Run 19. The test signal frequency is 100 Hz in Runs 11, 14, and 15, and 20 Hz in runs 18 and 19 to capture more of the slow signal response. The test signal amplitude was also decreased by a factor of ~ 2 for the latter two runs.

Figure 6.5 shows the fits to the complex admittance (reciprocal of the complex impedance) in the frequency domain for Runs 14, 18, and 19, with the magnitude and phase shown separately. The 3-pole model describes the response well in Run 14, but in Run 18 we can see that the response degrades at high frequency, an effect resulting from noise. This effect is exacerbated in Run 19.

The 3-pole fit parameters are given in Table 6.5, including the physical time constants τ_+ (electrical time constant) and τ_- (thermal time constant) as described in Section 3.2.2, as well as a third time constant τ_n . Overall the behaviour in Runs 14 and 15 is very similar to that observed at SLAC. The main fall-time of $\sim 65 \mu\text{s}$ is roughly consistent with the $58 \mu\text{s}$ value measured at SLAC [241]. At SLAC a secondary thermal fall-time of $370 \mu\text{s}$ was observed, which is most consistent with the $390 \mu\text{s}$ from Run 15. The electrical time constant τ_+ is also consistent with the $3 \mu\text{s}$ measured at SLAC [202]. After the clamp adjustments, we clearly see very long extra thermal fall-times in Run 18 and Run 19, and the first thermal fall-time also increases slightly.

Note the low measured loop gain, $\mathcal{L}_I$, particularly in Run 19. We typically expect to have high $\mathcal{L}_I$ and many of the assumptions of TES behaviour are dependent on this parameter being large. The parameter determined from the fit is $\mathcal{L}_I$, assuming an intermediate-block thermal model. Finally, we can see from the increasing ratio $G_{ratio} = G_{ta}/(G_{ta} + G_{ab})$ that the thermal conductance of the absorber to the bath is decreasing with each run.

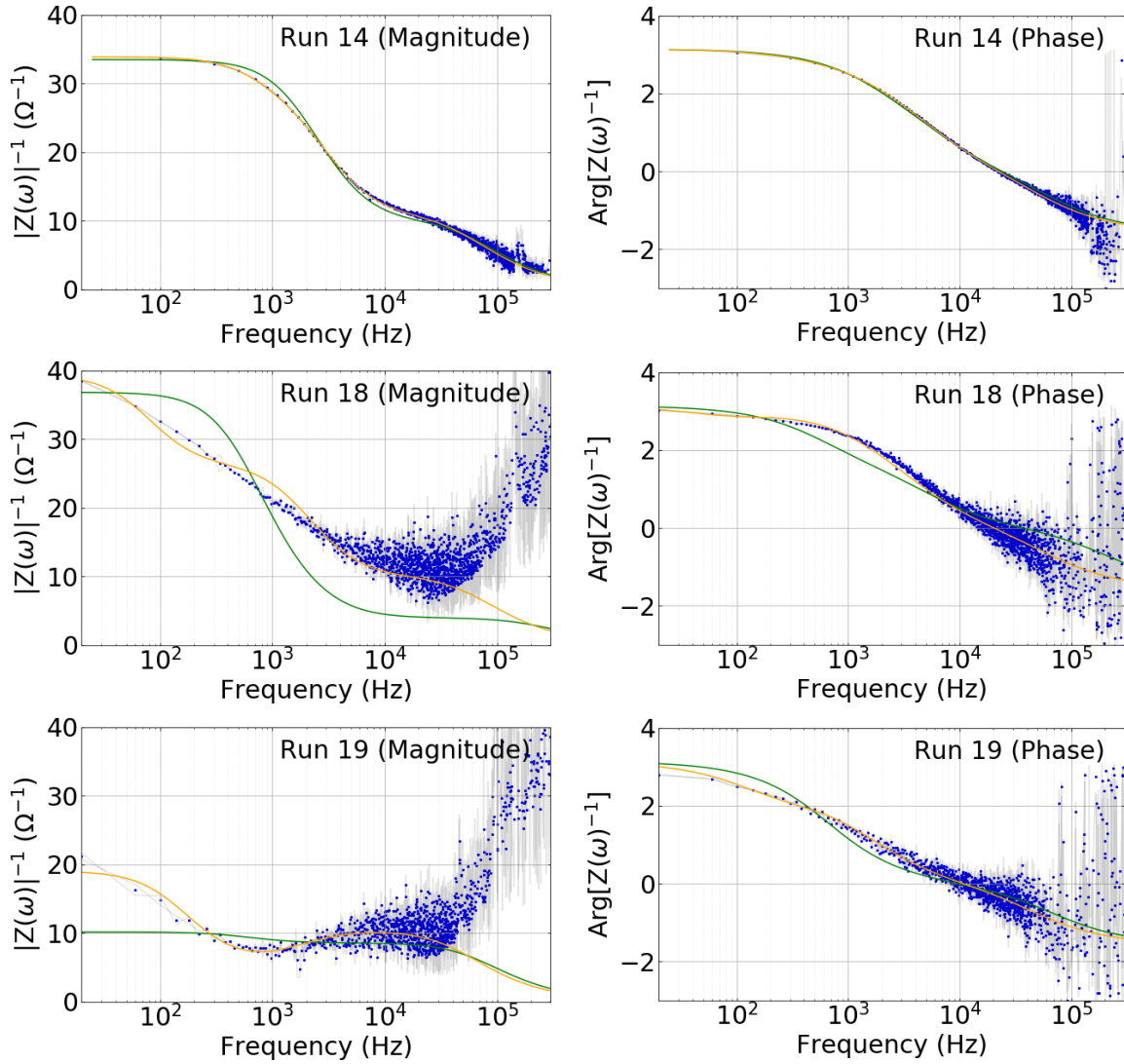


Figure 6.5: PD2 2-pole (green) and 3-pole (yellow) model fits to the complex impedance (mean: blue points, 1σ bounds: grey lines) in the frequency domain for a TES bias point close to $40\%-R_n$ in Run 14. The magnitude (left) and phase (right) are both shown, with the magnitude of the complex impedance showing that the 3-pole model best describes the data.

6.5 Pulse Shapes

The time constants from the complex impedance measurements can provide us with information about the electrical and thermal behaviour of the TES, but when fitting the PD2 pulse shapes in the time domain at SLAC, an additional long (3.2 ms) time constant was

Parameter	SLAC	Run 11	Run 14	Run 15	Run 18	Run 19
τ_+ (μs)	3	2.5	2.9	3.1	2.5	3.4
τ_- (μs)	58	64	65	66	91	85
τ_n (μs)	370	170	210	390	21801	1170
β	1.1 ± 0.1	0.89 ± 0.01	0.86 ± 0.01	0.88 ± 0.01	1.20 ± 0.01	0.76 ± 0.07
L (nH)	190 ± 10	208 ± 1	271 ± 1	276 ± 1	251 ± 3	331 ± 22
$\mathcal{L}_I$	80 ± 5	7.1 ± 0.03	15.94 ± 0.05	16.29 ± 0.07	8.29 ± 0.04	1.65 ± 0.03
G_{ratio}	-	0.320 ± 0.008	0.434 ± 0.003	0.472 ± 0.002	0.651 ± 0.004	0.71 ± 0.02
τ_0 (μs)	1700 ± 200	239.7 ± 1.2	368.5 ± 1.3	360.3 ± 1.7	251.6 ± 1.5	137.6 ± 5.0

Table 6.5: PD2 dIdV parameters from a 3-pole fit model close to 40% R_n , including the three time constants τ_+ , τ_- , and τ_n , the current sensitivity β , inductance L , the (intermediate-block model) loop gain $\mathcal{L}_I$, and the natural thermal time constant τ_0 (referred to as τ in Chapter 3). G_{ratio} is the ratio of the thermal conductances, $G_{ta}/(G_{ta} + G_{ab})$, assuming $G_{ta} = 0.48 \text{ nW/K}$ from SLAC [241]. The values from SLAC are also shown for comparison, though the non-fall-time parameters are from a 2-pole model fit.

observed [241]. They also observed the expected $58 \mu\text{s}$ and $370 \mu\text{s}$ fall-times from the dIdV measurements. Time constants should ideally be consistent with the thermal time constant from the dIdV response, and the presence of an extra long fall-time at SLAC was hypothesized to be related to the direct absorption of phonons with energy smaller than the Al superconducting bandgap into the TES.

At CUTE we can also measure the time constants from pulses using low background data. For PD2 in Run 15 I use the same series used in Ref. [183] for a low mass dark matter search, and for Run 18 I use pulses from the series used to calibrate the energy scale in the same reference. The energy scale was calibrated for these series in both runs. For PD2 pulses in Runs 11, 14, and 19 I selected several good quality low background series and roughly calibrated the energy scale using

$$E \approx \frac{1}{\epsilon_{ph}} \frac{dP}{dI}(0) \int \Delta I dt, \quad (6.1)$$

where ΔI is the baseline-subtracted measured pulse in current. This approximation is valid at low energies where the ΔI^2 component of the power is negligible. The rough calibration is acceptable as the main goal is to compare the pulse shapes of roughly similar energy.

The measured data format is a trace length of 52 ms with the pulse trigger centred near 26 ms. I truncate the fit from ~ 26 ms to 32 ms to capture the long fall-time while avoiding the end of the trace where noise dominates the response. I select events with (roughly) calibrated energy from 1-2 keV, avoiding noise and most saturation effects. I chose this region because the SLAC pulse shape study was performed on events in the Al fluorescence peak at 1.486 keV, so this is a good point of comparison. Pulses are selected using trigger cuts to ensure that the relevant detector issued a trigger, and a flat χ^2 cut is used to select good events.

For each run I calculated the mean pulse shape and fit three different pulse models. I also calculate the mean pulse shape for each series and fit the same models, using the standard deviation on the fit parameters as an uncertainty on the final fit parameters of the mean trace, which dominate over the statistical uncertainty from the fit method.

In each case I perform fits using 1-, 2-, and 3-fall-time models. The former two models do not describe the observed data well as they do not capture the presence of the long thermal tail, and are only shown here to demonstrate the difference compared to the 3-fall-time model. The 3-fall-time model is given by

$$f(t) = -(A_1 + A_2 + A_3)e^{-(t-t_0)/\tau_r} + A_1e^{-(t-t_0)/\tau_1} + A_2e^{-(t-t_0)/\tau_2} + A_3e^{-(t-t_0)/\tau_3}, \quad (6.2)$$

where t_0 is a time offset accounting for the start position of the pulse, the A_i are amplitudes (we actually only need to fit for two amplitudes, as we normalize the pulse height to 1), τ_r is the rise time (this should be close to $\tau_{ph} = 20 \mu\text{s}$ measured at SLAC), and the τ_i are the fall-times. The 1- and 2-fall-time models can be determined from this model by setting A_3 and/or A_2 to zero.

We can also estimate the energy associated with each fall-time component τ_i by comparing the integral of the pulse with just that component ($-A_ie^{-(t-t_0)/\tau_r} + A_ie^{-(t-t_0)/\tau_i}$) to the integral of the total pulse. The integral is truncated at 26 ms from the start of the

pulse.

In Table 6.6 I show the fitted parameters of the 3-fall-time model to the mean pulse from each run for PD2. The rise time is slightly higher than expected, particularly in Runs 14 and 15, compared to the measured $20\,\mu\text{s}$ at SLAC [241]. The first fall-time in each run other than Run 11 is also shorter than the fall-time determined from the dIdV, which was determined to be $\sim 65\,\mu\text{s}$ for Runs 11 to 15 and $\sim 90\,\mu\text{s}$ for Runs 19 and 20. We see a secondary fall-time of $\mathcal{O}(100)\,\mu\text{s}$ in each run. This is similar to the observed additional fall-time in the dIdV from Runs 11-15, though in Runs 18 and 19 the additional fall-time was $\mathcal{O}(\text{ms})$. We do observe the presence of a very long 2-4 ms thermal fall-time in all runs, but this is a dominating component in Runs 18 and 19.

Parameter	SLAC	Run 11	Run 14	Run 15	Run 18	Run 19
$\tau_r\,(\mu\text{s})$	20	23.3 ± 0.7	35.8 ± 1.7	35.3 ± 3.1	25.9 ± 4.4	23.0 ± 5.7
$\tau_1\,(\mu\text{s})$	58	148 ± 1	36.0 ± 1.9	35.7 ± 3.1	51 ± 9	57 ± 15
$\tau_2\,(\mu\text{s})$	370	481 ± 4	169 ± 21	180 ± 20	265 ± 44	311 ± 105
$\tau_3\,(\mu\text{s})$	3000	3860 ± 70	1640 ± 430	2780 ± 450	2980 ± 200	2840 ± 780
$\%E_1$	70	26.6%	23.4%	22.8%	22.9%	9.6%
$\%E_2$	22	41.1%	51.5%	53.0%	32.0%	18.1%
$\%E_3$	8	32.3%	25.2%	24.2%	45.0%	72.3%

Table 6.6: Fall times of PD2 pulses measured using a 3-fall-time model in each CUTE run. The approximate fractional energy contained within each fall-time is also shown. There is a long thermal time constant in each run, but in Runs 18 and 19 the fraction of energy contained within this long tail is significantly larger than in the other runs. All three fall-times measured in Run 11 are longer than those measured in the other runs. The time constants and approximate energy contributions determined at SLAC are shown for comparison [241].

At SLAC a dominant fall-time of $58\,\mu\text{s}$ with 70% energy contribution was found, while the energy contribution was about 8% and 22% from the $370\,\mu\text{s}$ and 3 ms components, respectively. If we compare the first two components measured together in CUTE Runs 11 to 15 we see roughly 70-75% of the energy contribution, with the remaining 25-30% in the long tail. This is a slightly lower contribution than at SLAC, but roughly consistent. In Runs 18 and 19 the contribution from the long tail increases dramatically. This slower energy

collection could potentially explain the reduced collection efficiency of 6.83% measured at CUTE compared to the 13% determined at SLAC.

In Figure 6.6a I show an example of the 1-, 2-, and 3-fall-time models for Run 15. The 1-fall-time and 2-fall-time fits do not capture the long thermal fall-time, which is clearly seen in the logarithmic scale. The mean PD2 traces for each run are shown in Figure 6.6b with the fitted 3-fall-time models. Run 14 is almost indistinguishable from Run 15 and is not shown. There is a clear increase in the energy contribution between Run 15 and Run 18, and then in Run 19 with the changes in the clamp configuration. Run 11 is somewhat anomalous with a much longer initial fall-time.

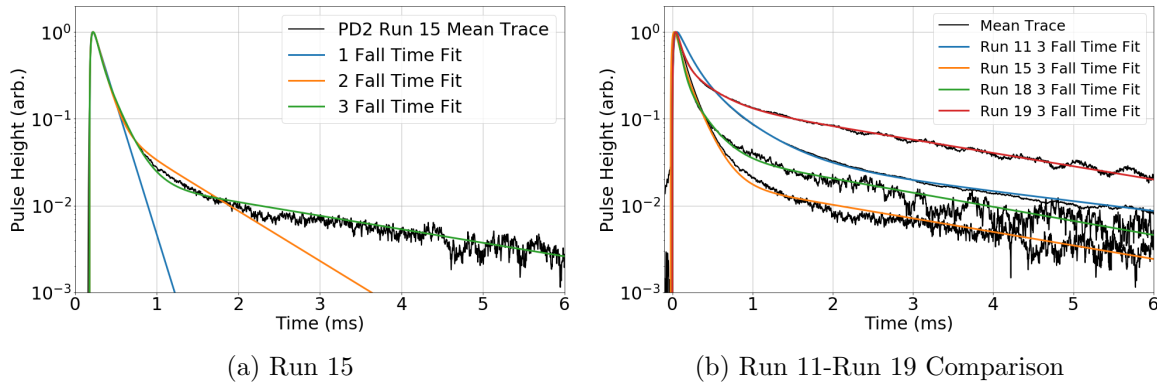


Figure 6.6: (Left) 1-, 2-, and 3-fall-time fits to the mean PD2 pulse shape from CUTE Run 15. (Right) Comparison of the mean pulse shape and 3-fall-time fit for PD2 in CUTE Runs 11, 15, 18, and 19. The pulse shape in Run 14 is almost identical to that in Run 15, and is thus omitted.

Figure 6.7 compares the average pulse shape as a function of optimal filter amplitude for PD2 in Run 18. The longer fall-time is more prominent at higher energies. This behaviour was also observed at SLAC, and corresponds to a local saturation effect manifesting as the dominant fall-time increasing with energy. High energy events push nearby QETs into the normal resistance regime, and slows down the response of the whole detector because of the single channel readout.

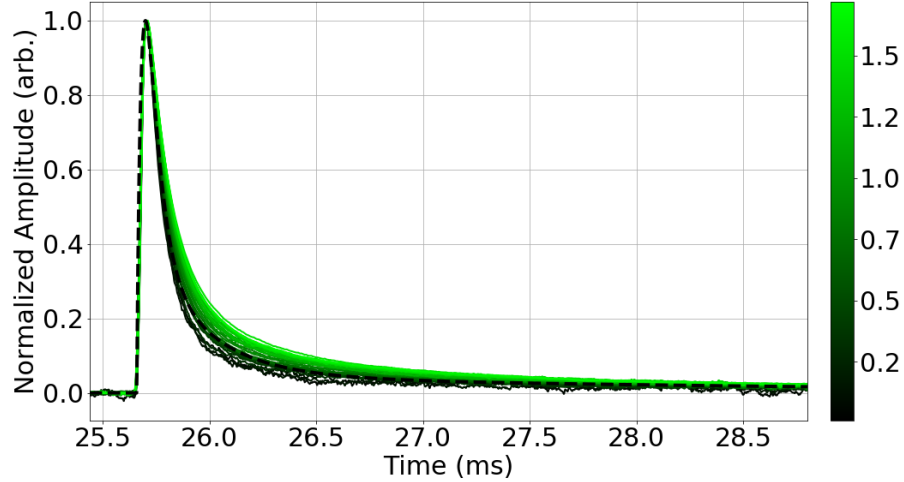


Figure 6.7: Averaged and normalized pulse shapes for PD2 in Run 18. The colour indicates the reconstructed optimal filter amplitude, with bright green indicating the highest amplitude and black indicating the lowest amplitude. The fall-time lengthens with increasing amplitude. The pulse shape fitted to 1-2 keV events is shown for comparison (black dashed). An amplitude of $0.2 \mu\text{A}$ corresponds to $\sim 0.5 \text{ keV}$, and an amplitude of $1 \mu\text{A}$ corresponds to $\sim 3 \text{ keV}$.

6.6 Noise and Energy Resolution

Noise modelling is a useful tool to evaluate the presence of excess noise beyond what is expected from first principles based on the measured detector parameters. This is fundamental noise which cannot be reduced without changes to the detector or readout circuit, and other noise sources are typically environmental. The lowest noise possible is desirable as this limits our baseline energy resolution, which in turns affects the low energy sensitivity.

Using the measured complex impedance parameters¹ β , $\mathcal{L}$, and τ_0 , we can model the noise in transition and compare it to the measured noise. The various noise components and how they are modelled in the QETpy package are described in Section 3.2.3, and include

¹QETpy uses the fit parameters from the 2-pole model rather than the 3-pole model. The impact is negligible, affecting the TFN noise and TES Johnson noise modestly in regimes where they are subdominant to the load noise and SQUID noise. Modelling the TFN and TES noise for the 3-pole model is complicated by requiring knowledge of the absorber temperature as well as the thermal conductances at the temperatures of the TES, the bath, and the absorber.

SQUID noise, TES Johnson noise, TFN noise (which is dependent on the thermal coupling to the bath), and load noise from the load electronics. The noise is typically presented as the square root of the power spectral density.

The normal noise is modelled first to determine the SQUID noise parameters. Here negligible load noise is assumed, and the TES noise is modelled with the known R_n . The superconducting noise can then be fit to model the load noise and determined the effective load temperature, T_{load} . With all needed parameters to model the transition noise now known, we can compare the modelled noise to the measured noise.

Furthermore, using the measured noise and the complex impedance response the expected baseline energy resolution can be determined from optimal filter theory:

$$\sigma_0 = \left[\int_0^\infty \frac{d\omega}{2\pi} \frac{4 |p(\omega)|^2}{S_P(\omega)} \right]^{-1/2}, \quad (6.3)$$

where $S_P(\omega) = \frac{dP}{dI}(\omega) S_I(\omega)$. The current sensitivity $\frac{dP}{dI}(\omega)$ is determined from the complex impedance measurement, and $S_I(\omega)$ is the measured current noise. Recall

$$p(\omega) = \epsilon_{ph} / (1 + i\omega\tau_{ph}) \quad (6.4)$$

is the pulse shape with collection efficiency ϵ_{ph} . The uncertainty on the baseline phonon energy resolution is estimated by varying the parameters used to calculate dP/dI from the complex impedance fitting. The parameters are drawn from a normal distribution using the fit uncertainties. This gives a distribution of the baseline energy resolution. The central value and uncertainty band are taken from this distribution.

6.6.1 Noise Modelling

Figure 6.8 shows examples of the normal and superconducting noise models for PD2 in CUTE Run 14. There are unmodelled electronic noise peaks, particularly at ~ 330 Hz (with harmonics). This noise has been confirmed as originating from an environmental source:

the controller for the Peltier cooler [180, 181]². The normal and superconducting noise are similar in the other runs.

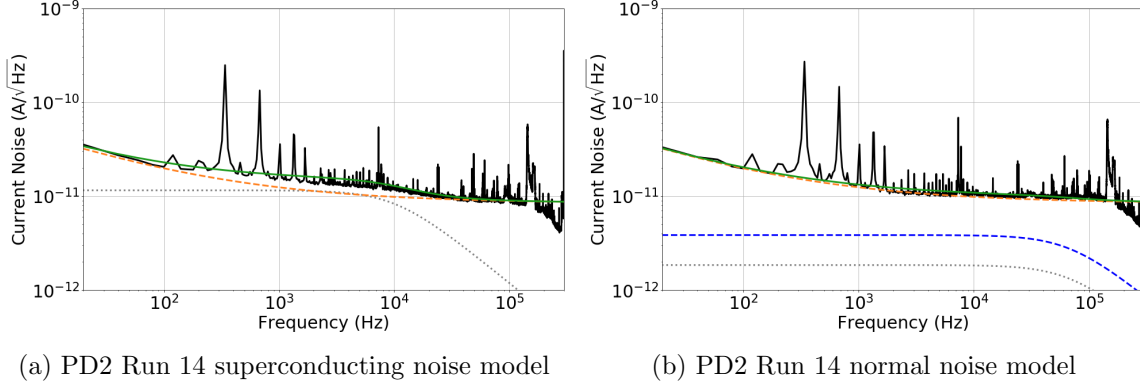


Figure 6.8: Superconducting (left) and normal (right) noise measurement and model for PD2 from CUTE Run 14. Modelled components include TES Johnson noise (blue dashed), load noise (grey dotted), and SQUID noise (orange dashed). The total noise is also shown (green solid).

Figure 6.9 shows the current and power noise with the modelled noise components for PD2 in CUTE Runs 14 and 19. The power noise is determined by converting the measured current noise using the current sensitivity dP/dI . In both runs the noise is undermodelled, a phenomenon that was also observed at SLAC [241] and noted by other experiments [293]. At SLAC the excess noise was modelled as additional TES Johnson noise and an environmental noise component. They claim that the excess noise could potentially be modelled by a more complicated thermal model. The additional TES noise hypothesis does not well describe the excess observed at CUTE, but in Figure 6.9 I also show the effect of increased TFN noise (scaled by factor of 3), which would occur if the value of the thermal conductance G (assumed to be G_{ta}) is higher than expected. This additional noise describes the noise well in Runs 14, 15, and 18, but the noise in Runs 11, and 19 is elevated even further, particularly in Run 19 as seen in Figures 6.9c and 6.9d.

The elevated noise in Runs 11, and 19 is present only when the detectors are in transition,

²The Peltier cooler is responsible for cooling the circulation turbo pump

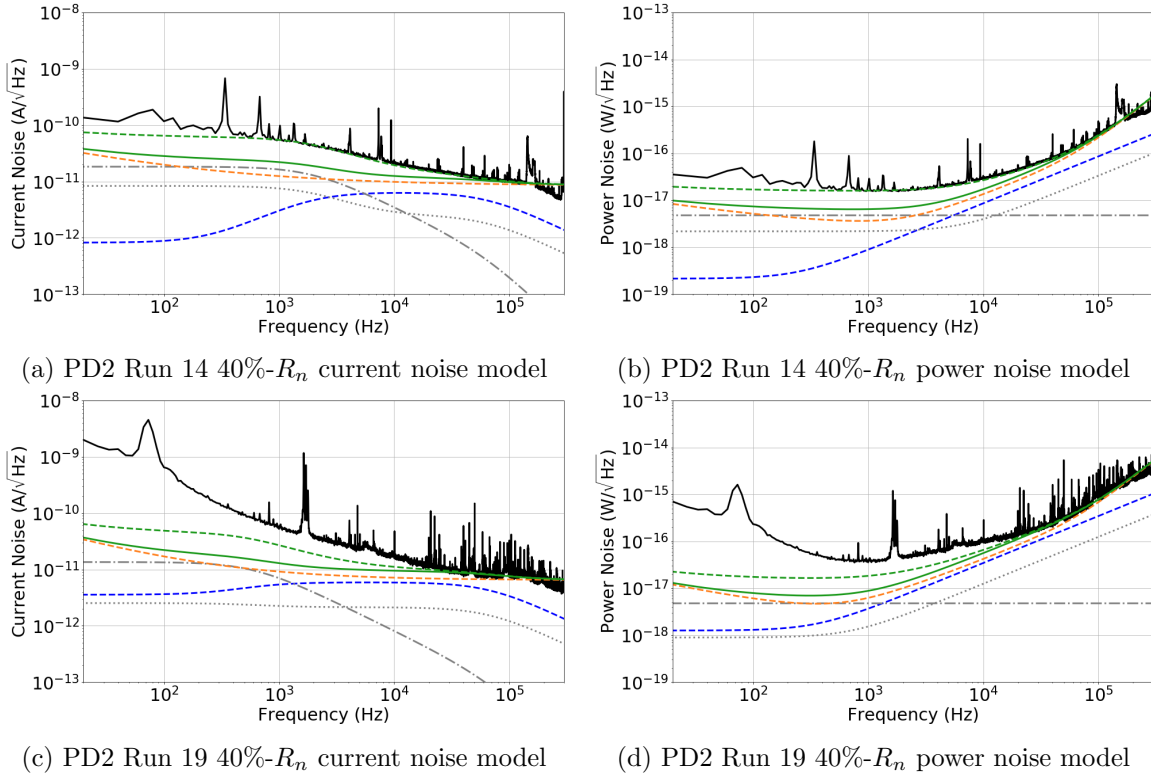


Figure 6.9: (left) Current and (right) power noise models for PD2 in CUTE Run 14 (top) and Run 19 (bottom) data. The overall noise level is slightly elevated compared to SLAC. Modelled components include TES Johnson noise (blue dashed), load noise (grey dotted), TFN noise (grey dash dot), and SQUID noise (orange dashed). The total noise (green solid) is also shown. An example of the total noise including an increased TFN noise component is also shown (green dashed) which describes the noise well in Runs 14 and 15, but still under-models the noise in the other three runs.

indicating that the noise is power coupled (related to the heating of the TES while in transition). Power coupled noise sources include radio frequency (RF) and IR radiation, or even vibration. PD2 shared an optical cavity with the other CPD-style detector (see Appendix C, which did not show the same elevated noise, so radiation is not a likely culprit. Though we have previously seen that smaller detectors are not as sensitive to vibration as larger detectors [180], the adjusted clamping scheme may have made the detector susceptible to vibration, increasing the LF noise. The elevated noise in Run 11 may be related to the hypothesized stress fractures.

6.6.2 Baseline Energy Resolution

The expected baseline energy resolution determined from the measured current noise, the current sensitivity dP/dI , and a single phonon-collection time ($\tau_{ph} = 20 \mu s$) pulse shape is shown as a function of transition point (represented by $\%R_n$) for each run in Figure 6.10. The 13% collection efficiency determined at SLAC is assumed, though the collection efficiency was also determined at CUTE in Run 18, as described below. The expected resolutions for Runs 14 and 15 are consistent and reach a minimum of ~ 5 eV around 30–40%- R_n . The resolution is higher in the other three runs. Run 19 has a large uncertainty band because of the high frequency behaviour of the complex impedance, imparting a larger uncertainty on the dIdV fit parameters.

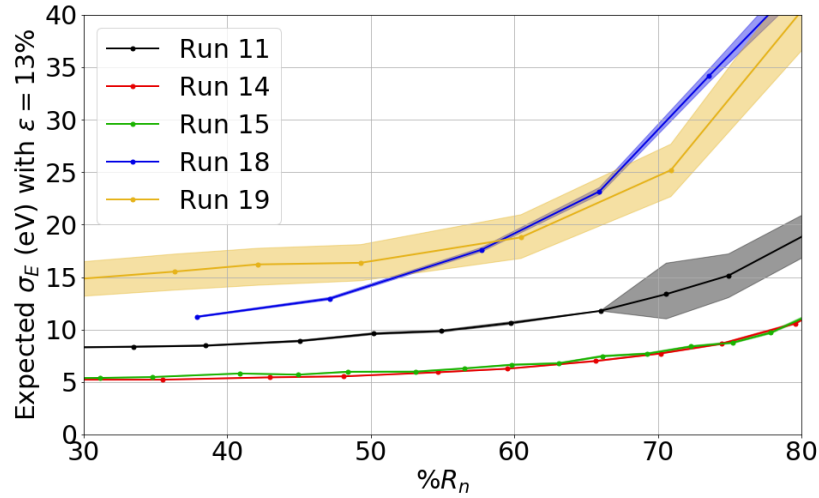


Figure 6.10: Summary of the PD2 expected baseline energy resolution for each CUTE run vs. transition point (determined by $\%R_n$). These estimates assume a collection efficiency of 13% as measured at SLAC, which may be appropriate for Runs 14 and 15 where the detector response is similar to SLAC. However, the collection efficiency was measured in Run 18 as 6.83%. The expected energy resolution can be determined by rescaling the resolution by a factor of 13/6.83.

An energy calibration for PD2 was performed in Run 18 by R. Underwood as part of his PhD thesis in Ref. [183]. During this run, the ^{55}Fe source was directed at PD2, providing

distinct peaks at 5.889 keV and 6.49 keV. At energies this high, the main energy estimator from the pulse amplitude, the optimal filter amplitude (or OFamps, see Appendix A on optimal filter theory) is impacted by saturation effects. The energy scale was therefore calibrated using the absorbed energy. This may be calculated using the equation for absorbed energy from Chapter 3,

$$E_{abs} = \int_0^T [(V_b - 2I_0 R_L) \Delta I - R_L \Delta I^2] dt, \quad (6.5)$$

where I_0 is the baseline TES current, ΔI is the measured current signal, and T is the truncation time of the integral. At SLAC the integral was truncated to $7\tau_-$ to minimize the effect of noise on the integral. In doing so, the long thermal time constant of the pulse is cut out and the measured absorbed energy is lower than if one were to integrate until the pulse had returned to the equilibrium baseline. This effect is seen in the quoted collection efficiency: the 13% value is determined from the truncated integral, while the total collection efficiency is estimated to be 17%, with 70% of the phonons being collected by the fast $20 \mu s$ collection time. In CUTE Run 18 we integrated out to $750 \mu s$, roughly $8\tau_-$ (where $\tau_- = 90 \mu s$ in Run 18).

The absorbed energy, being calculated from the integral of the pulse, is less prone to saturation effects. The calibrated energy scale using the optimal filter amplitude, E_{OF} is determined by fitting the absorbed energy E_{abs} as a function of optimal filter amplitude with the following model

$$E_{abs}(E_{OF}) = -b \cdot \log(1 - E_{OF}/a). \quad (6.6)$$

This model was used to correct for saturation effects at SLAC. The saturation-corrected optimal filter amplitude is calibrated to the ^{55}Fe peaks, and the collection efficiency can be extracted based on the expected energies of the peaks compared to the measured energies. A collection efficiency of 6.83% was determined, significantly lower than the 13% determined

at SLAC. Figure 6.11 shows the model fit to the absorbed energy and the optimal filter amplitude parameter. The ^{55}Fe peaks are clearly visible and distinguishable structures at high optimal filter amplitude. The position of the peaks relative to the other events in the measured energy distribution indicates a different pulse shape. Events near the centre of the detector have faster phonon collection and therefore a sharper rise-time than events closer to the edges of the detector. The 6.83% collection efficiency was assumed for Run 15 in Ref. [183], where R. Underwood calibrated the energy scale using the same saturation model.

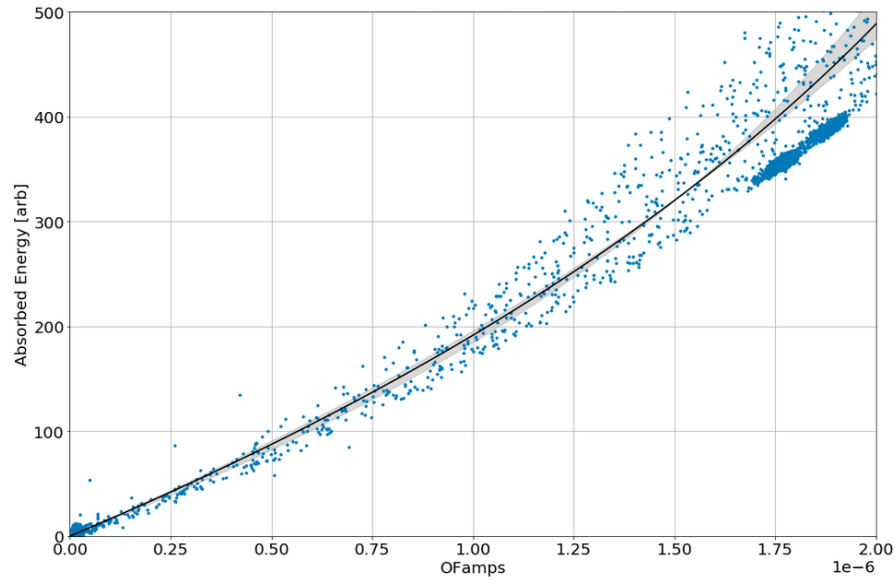


Figure 6.11: Absorbed energy vs. optimal filter amplitude for PD2 in CUTE Run 15 from Ref. [183]. The best fit line (black) to the data (blue points) includes an uncertainty band (grey). The two ^{55}Fe peaks are clearly visible around $1.7 - 1.9 \mu\text{A}$. The position of these structures compared to the rest of the distribution indicates that they are surface events; this is a demonstration of the position dependence of the pulse shape.

With this energy calibration, we can directly measure the baseline energy resolution for Runs 15 and 18 and compare to the expected energy resolution, as shown in Table 6.7. The ‘calibrated’ energy resolution is determined by measuring the resolution of the OFamps0

distribution of random trigger events (see Appendix A for a description of OFamps0). Because the behaviour of PD2 in CUTE Run 15 is more consistent with the behaviour at SLAC than in CUTE Run 18, I provide energy resolutions in Run 15 using both measurements of the collection efficiency.

Energy Resolution	Run 15		Run 18
	$\epsilon = 6.83\%$	$\epsilon = 13\%$	$\epsilon = 6.83\%$
Calibrated (eV)	11.8 ± 0.4	6.1 ± 0.2	11.6 ± 0.4
Expected (eV)	11.1 ± 0.06	5.82 ± 0.04	21.3 ± 1.4

Table 6.7: Expected energy resolution from complex impedance and noise modelling compared to the measured energy resolution with calibrated energy scale. The resolutions for Run 15 are shown for two different values of the collection efficiency, ϵ : 13% as measured at SLAC and 6.83% as measured at CUTE in Run 18.

The expected energy resolution in Run 15 is roughly consistent with the calibrated energy resolution as expected from optimal filter theory using a single $20 \mu\text{s}$ phonon-collection time. The assumed collection efficiency is irrelevant to the comparison as both quantities are scaled in the same way.

The expected energy resolution from optimal filter theory in Run 18 is significantly larger than the measured energy resolution assuming the measured 6.83% collection efficiency. Note that there is not a significant difference in the noise measurement used to determine the expected energy resolution and the noise in the series used for calibrating the energy scale and measuring the energy resolution. At SLAC the total collection efficiency when integrating out to effectively infinity was found to be 17%, and the expected and measured energy resolutions were found to be inconsistent. It was determined at SLAC that there was an additional 3.2 ms long phonon-collection time. When integrating to $7\tau_-$ ($406 \mu\text{s}$) the 13% collection efficiency was determined, indicating that roughly 30% of the collected energy is associated with the longer fall-times (8% for the $370 \mu\text{s}$ fall-time and 22% for the 3.2 ms fall-time).

If we consider a ‘total’ collection efficiency of 17%, use a model with two phonon collection times, and assume 78% of phonons are collected by the $20\ \mu\text{s}$ phonon-collection time we obtain an expected resolution of $10.87 \pm 0.06\ \text{eV}$, which is more compatible with the measured value of $11.6 \pm 0.4\ \text{eV}$. The slow phonon-collection can be observed in Figure 6.12, which shows the absorbed energy for events with calibrated optimal filter amplitude between 5.25 and 7 keV. The two ^{55}Fe peaks are visible as two distinct bands. The absorbed energy rapidly rises over the first millisecond, but continues to rise at a slower rate over the entire pulse length of 26 ms instead of flattening out as expected from a single phonon-collection time.

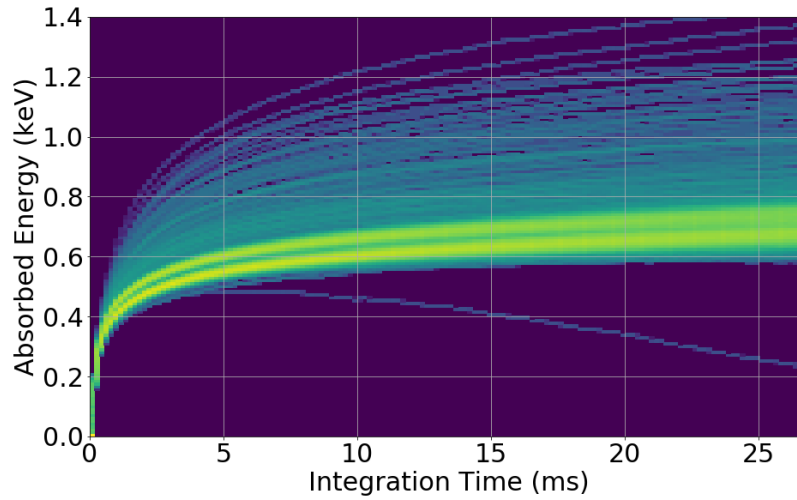


Figure 6.12: Absorbed energy for events in the ^{55}Fe calibration peaks for PD2 in CUTE Run 18 as a function of integration range. The colour indicates event density, with purple indicating no events and light green indicating the highest event density. The two peaks are easily distinguishable as two similarly shaped distributions but separated in absorbed energy. At low integration times the peaks are not readily separable, and at high integration times the resolution begins to degrade and once again it becomes challenging to separate the peaks. The distribution of events with higher absorbed energy than the calibration peaks are events with different pulse shape than the ^{55}Fe events, indicating different positions in the detector. The ^{55}Fe events are directed at the centre of the detector, and have a faster pulse shape than events nearer the edge of the detector.

One final effect to consider is that the calculation of absorbed energy is contingent on the assumption of high loop gain. In Run 18 we measured $\mathcal{L}_I = 8.28 \pm 0.04$. In the high loop gain limit the change in TES temperature approaches zero. Below this limit we would also have to consider the change in power flowing from the TES to the bath as the TES temperature changes. It is possible that we are close to the low loop gain regime and the absorbed energy calculation is inaccurate. Irwin also notes that this calculation is only appropriate for small signals, and in general more direct methods of energy calibration should be used [200].

6.6.3 A Note on Shunt Resistance

As previously mentioned, we have assumed the design value of the shunt resistance $R_{sh} = 5\text{ m}\Omega$, but differences in the calculated normal resistance between SLAC and CUTE may indicate that a value of $4\text{ m}\Omega$ at CUTE might be more appropriate. The assumed shunt resistance does not impact parameters such as the time constants, but dP/dI and R_L are affected; the former means that the expected baseline energy resolution would be lower than the values provided by a factor of $4/5$, and latter affects the absorbed energy calculated in Equation 6.5, also by a factor of $4/5$. While this would improve our apparent baseline energy resolution, it could not explain the low collection efficiency as the absorbed energy would also be reduced.

6.7 PD2 Thermal Conductance Measurements

In an attempt to understand the presence of the additional long fall-times, we performed measurements of the thermal conductance between the absorber and the bath, G_{ab} , and the thermal conductance between the TES and the absorber, G_{ta} . A weak thermal link between the absorber and the bath could account for the observed behaviour. We are able to compare the results with measurements from SLAC; in particular, G_{ta} should be consistent at CUTE as this is intrinsic to the detector. In the following discussion we refer to several temperatures: T_b is the bath temperature, T_a is the temperature of the absorber,

and T_0 is the TES temperature.

6.7.1 Thermal conductance between bath and absorber G_{ab}

Measurements of G_{ab} typically require two TESs on the same absorber, where one acts as a heater for the absorber and the other as a thermometer. By keeping T_a constant and measuring the power P_{ab} flowing from the absorber to the bath, G_{ab} can be measured. However, PD2 is a single channel device and this method cannot be used to measure G_{ab} . Instead, we note that G_{ab} is related to the natural time constant τ and the heat capacity C_v of the absorber by

$$G_{ab} = \frac{C_v}{\tau}. \quad (6.7)$$

We can attempt to measure τ using the thermal tails of high energy pulses. A similar analysis was performed at SLAC using the tails of muon events, measuring a 16.9 ms thermal fall-time. This corresponds to G_{ab} of 11.4 nW/K. However, the depth of SNOLAB vital to the low background environment at CUTE becomes our weakness - we have no (or very few) muons events to study. There is a small population of high energy events which begin to saturate the detector, and we can also use the LEDs in the detector housing to create high energy pulses. Both methods have caveats: the former has very limited statistics, while in the latter case the LED events are not yet well understood. I show both methods to demonstrate that the results are comparable.

Figure 6.13 shows the resulting fits to the long tails of LED events and a small population of five high energy events which saturate the detector. In both cases I fit the mean trace of each distribution, though for the latter I also fit each trace individually and take the standard deviation of the measured fall-time as the uncertainty. I use a 2-fall-time model, which describes the response better than a 1-fall-time model, taking the longer fall-time as the thermal time constant.

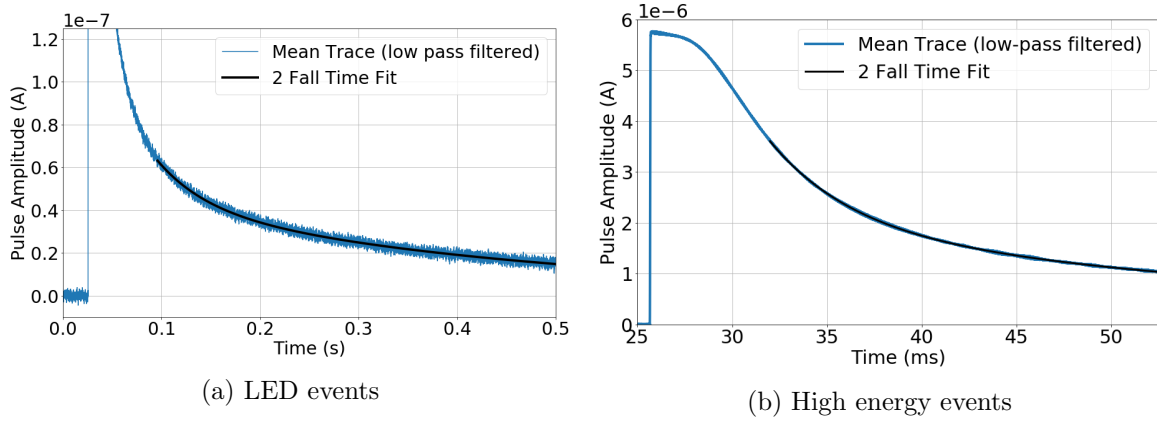


Figure 6.13: Fits of the long thermal tail in (left) LED pulses and (right) high energy pulses in low background data for PD2 in CUTE Run 18. The mean pulse (blue) is shown with the fit (black). Note the difference in time scales for both; the LED pulse is shown in seconds while the high energy events is shown in milliseconds. A 2-fall-time model is used for both. The distorted pulse shape from saturation is clearly visible for the high energy events.

The heat capacity is calculated using Debye theory as

$$C_v = 12\pi^4 \frac{k_B}{5A} \rho V N_A \left(\frac{T}{\Theta_D} \right)^3, \quad (6.8)$$

where $k_b = 1.38 \times 10^{-23}$ J/K is the Boltzmann constant, $A = 0.028$ kg/mol and $\rho = 2329$ kg/m³ are the atomic mass and density of Si, respectively, $V = 4.56 \times 10^{-6}$ m³ is the volume of PD2, $N_A = 6.022 \times 10^{23}$ mol⁻¹ is Avogadro's number, $T = 40 \pm 10$ mK is an upper limit on the estimated absorber temperature (note this will give an upper limit on the heat capacity and therefore on G_{ab}), and $\Theta_D = 625$ K is the Debye temperature of Si. The heat capacity of PD2 is calculated to be $C_v = 193$ pJ/K.

Table 6.8 shows the results of the tail fits and the calculated G_{ab} for each method. The two measurements are roughly consistent, and significantly lower than the value of G_{ab} determined at SLAC. This is strong evidence of a weak thermal link between the absorber and the bath, which could be a result of the loosened detector clamps.

Method	τ (s)	G_{ab} (nW/K)
LED	0.42 ± 0.02	0.46 ± 0.02
High energy events	0.5 ± 0.05	0.39 ± 0.04
SLAC (muons)	0.0169	11.4

Table 6.8: The measured thermal fall-times from LED and high energy events and the corresponding G_{ab} for PD2 using $C_v = 193$ pJ/K. The value determined at SLAC was calculated by measuring the fall-time of muon tails.

6.7.2 Thermal conductance between absorber and TES G_{ta}

In contrast to the measurement for G_{ab} , measurement of G_{ta} can be performed with a single TES, but is dependent on having already measured G_{ab} . We use the following relationships between the power flowing from the TES to the absorber P_{ta} , and the power flowing from the absorber to the bath P_{ab} and the temperatures:

$$P_{ta} = K_{ta} (T_0^n - T_a^n), \quad n = 5, \quad (6.9)$$

$$P_{ab} = K_{ab} (T_a^n - T_b^n), \quad n = 4, \quad (6.10)$$

where the given n are the expected values ($n = 5$ for electrothermal coupling and $n = 4$ for phonon coupling) and T_0 should be close to the critical temperature of the TES. We cannot measure the absorber temperature directly, but G_{ab} is related to T_a by

$$G_{ab} = \frac{dP_{ab}}{dT_a} = nK_{ab}T_a^{n-1}. \quad (6.11)$$

By measuring the temperature of the bath we can calculate T_a using Equations 6.10 and 6.11 and the already measured value of G_{ab} .

In equilibrium, the bias power of the TES P_0 satisfies

$$P_0 = P_{ta} = P_{ab}. \quad (6.12)$$

We can measure the TES power directly at various values of T_b using IV sweeps, and calculate G_{ta} using Equation 6.9. Here I assume roughly 10% uncertainty on P_0 from R_{sh} and a very conservative 10% uncertainty on T_b , which is measured using two thermometers. We performed IV sweeps at 8 different bath temperatures, including the fridge base temperature, allowing some time to wait for the fridge to stabilize before performing the IV sweep. I include a conservative uncertainty to account for the possibility that the detector tower is not yet in thermal equilibrium.

Figure 6.14 shows the measured bias power (equal to the power flowing from the TES to the absorber) as a function of the calculated T_a , assuming $G_{ab} = 0.46 \pm 0.02$ from the LED measurement. I perform two fits, one constraining $n = 5$ as expected, and the other allowing n as a free parameter. The resulting fit parameters are shown in Table 6.9 with comparison to the measurement from SLAC.

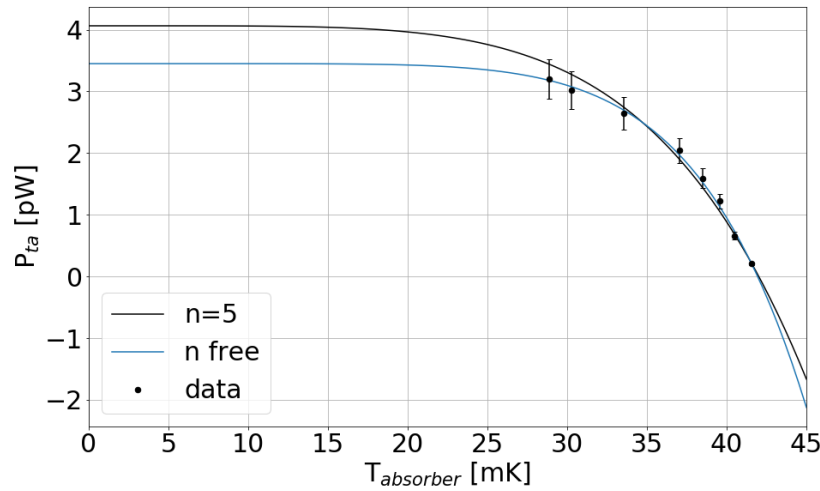


Figure 6.14: Fits to PD2 G_{ta} data (black points). Models constraining $n = 5$ (black solid) and allowing n as a free parameter (blue solid) are compared. A 10% uncertainty on P_{ta} is assumed.

The fit allowing n as a free parameter has significant uncertainty, though the model appears to have a better overall fit as seen in Figure 6.14. The constrained $n = 5$ fit is

Fit	K_{ta} (W/K ^{<i>n</i>})	T_0 (mK)	n	G_{ta} (nW/K)
n fixed ($n = 5$)	$(3.1 \pm 0.1) \times 10^{-5}$	42.0 ± 0.5	5 (fixed)	0.48 ± 0.06
n free	$(8.3 \pm 31) \times 10^{-3}$	41.9 ± 0.06	6.48 ± 1.1	0.56 ± 29
SLAC	0.48 ± 0.04 (stat.) ^{+0.49} _{-0.00} (syst.)			

Table 6.9: Fit results for a PD2 G_{ta} measurement with two different models, one constraining $n = 5$ and the other allowing n as a free fit parameter. A 10% uncertainty on P_{ta} and T_b is assumed.

highly consistent with the measurement from SLAC, and for both fits T_0 is consistent with the measured T_c as expected. The consistency of this measurement with SLAC helps to bolster confidence in the estimate of G_{ab} , which is used to calculate T_a .

In Section 3.2.2 we discussed the idea of a TES model which introduces an additional thermal block, which can introduce an additional fall-time to the TES response. The ‘intermediate’ model consists of an additional thermal body between the TES and the bath. If we consider the absorber as the additional thermal body, we have an additional fall-time with contribution weighted by the ratio of the thermal conductances

$$G_{ratio} = \frac{G_{ta}}{G_{ta} + G_{ab}}. \quad (6.13)$$

At SLAC the measured values of $G_{ab} = 11.4$ nW/K and $G_{ta} = 0.48$ nW/K give $G_{ratio} = 0.04$; the strong link between the bath and the absorber means that there is negligible contribution from an additional fall-time. However, in our case G_{ta} and G_{ab} are very similar, and the weak connection between the bath and the absorber introduces an additional fall-time. The measured G_{ratio} at CUTE assuming $G_{ta} = 0.48 \pm 0.06$ nW/K and $G_{ab} = 0.46 \pm 0.02$ nW/K is 0.51 ± 0.03 . This is lower than the value determined from the complex impedance measurement in Run 18, 0.651 ± 0.004 , which implies a lower G_{ab} or higher G_{ta} than measured here. Given the consistency of the measurement of G_{ta} between SLAC and CUTE and the expectation that this parameter should not change, it is more likely that the measured G_{ab} is an underestimate, or that the intermediate-block model is not a full representation of the detector behaviour.

6.8 Discussion

The performance of PD2 at CUTE in Runs 14 and 15 closely matches that at SLAC, with the observation of a $\mathcal{O}(\text{ms})$ long thermal fall-time comprising roughly 30% of the pulse energy. Like at SLAC, we observed the presence of a low energy excess event rate, though the event rate was found to change over time in Runs 11, 14, and 15. One hypothesis for this behaviour is stress fracture events. The clamping scheme was adjusted in an attempt to improve the performance and reduce the background; however, the first attempt in Run 18 introduced a longer fall-time in the dIdV and more energy contribution to the long tail, and a second attempt in Run 19 dramatically increased the LF noise. The energy contribution to the longer fall-time also increased in this run.

The phonon-collection efficiency of PD2 was measured as 6.83% during Run 18 by R. Underwood, a significantly lower value than the 13% measured previously at SLAC. This was the only run for which we have a calibration source for PD2 and can directly calibrate the energy scale for PD2 at CUTE. The importance of the collection efficiency is shown in Figure 6.15, where the change in assumed energy scale has a dramatic impact on the sensitivity of PD2 to low mass SI nucleon-coupled dark matter. Because of the similarity in behaviour of PD2 between SLAC and CUTE Run 15 versus the dramatically different behaviour in CUTE Run 18, the larger collection efficiency seems more appropriate for the Run 15 limit. This highlights the importance of being able to consistently calibrate the energy scale of a detector by maintaining identical detector behaviour between calibration and dark matter-search data periods, or by having a direct method of calibration during the dark matter-search data periods. PD2 was later operated at Berkeley with the clamps returned to the nominal configuration and the dIdV response returned to what was observed at SLAC.

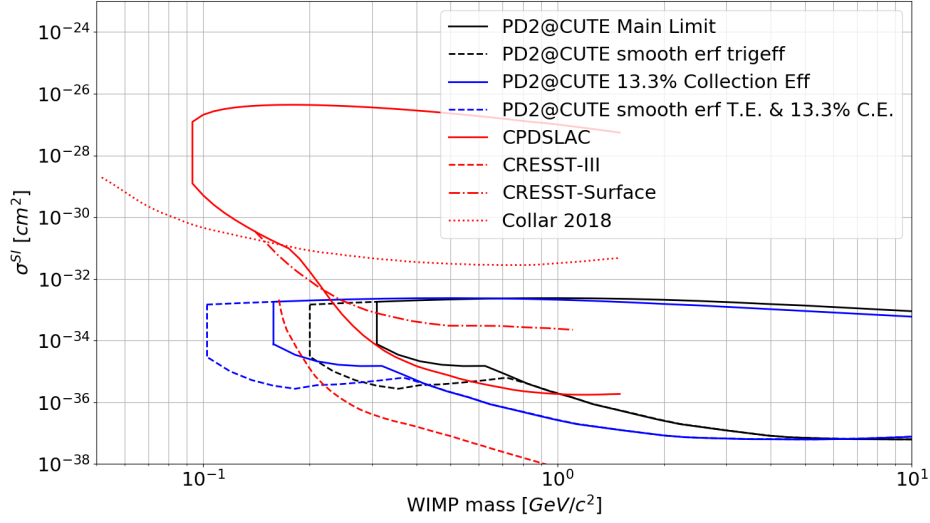


Figure 6.15: PD2 CUTE Run 15 upper 90% CL limit on SI nucleon-coupled dark matter assuming a 6.83% collection efficiency (black solid). This limit imposes a sharp threshold of 21 eV, compared to the limit calculated with a smoothed error function for the trigger efficiency (black dashed). The importance of the assumed collection efficiency is shown with the same limit calculated assuming a 13.3% collection efficiency with fixed threshold (blue solid) and smoothed error function trigger efficiency (blue dashed). The upper sections of the limit represent constraints from the overburden, calculated via a Poisson limit. Shown for comparison are the limit set with the same detector at SLAC [242] (red solid), and limits from CRESST-III [85] (red dashed), CRESST Surface [244] (red dash-dot), and Collar et al. [294] (red dotted). Figure from Underwood’s thesis [183].

Unfortunately, due to the elevated noise observed in Run 18 and the uncertainty of the collection efficiency in Run 15, it is difficult to evaluate whether or not the adjustments made to the clamping scheme had the intended effect of reducing the low energy excess background. Figure 6.16 demonstrates this effect by comparing the calibrated energy spectrum from CUTE Run 15 under the assumption of the two different collection efficiencies to the Run 18 spectrum. Though the Run 18 spectrum has a lower rate (despite the ^{55}Fe source) above 60 eV, the low energy excess can be interpreted as either reduced (assuming a 6.83% collection efficiency in Run 15), or worsened or hidden by the noise (assuming a 13% collection efficiency in Run 15).

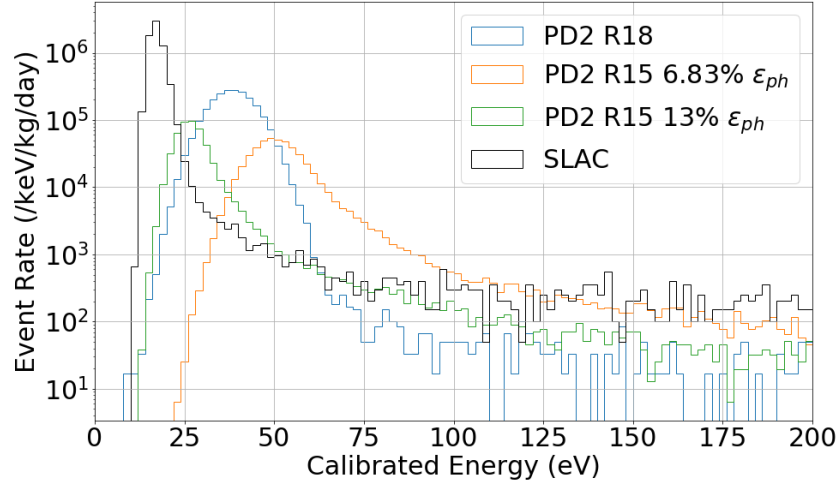


Figure 6.16: The PD2 energy spectrum measured at CUTE in Run 18 assuming a collection efficiency of 6.83% (blue) and in Run 15 assuming a collection efficiency of 6.85% (orange) and 13% (green). The spectra are compared to the spectrum measured at SLAC (black) [242]. The uncertain collection efficiency in Run 15 makes understanding the effect of the clamp adjustments in Run 18 non-trivial. The background rate above ~ 60 eV is lower in Run 18 than the other measurements, but the interpretation of the low energy excess in Run 15 changes with the assumed collection efficiency. The lower energy threshold at SLAC can be seen here.

We attempted to use another CPD-style detector with the same sensor layout as PD2 but higher T_c , called CPD, as a comparison to replicate the effects on PD2. First we measured the properties and response in Run 16, and then adjusted the clamping scheme in an attempt to replicate the effects observed on PD2 while still reducing stress fractures. Like with PD2, we observed the presence of a third pole in the dIdV response, but to a much lesser extent than in PD2, and we were unable to replicate the introduction of the long fall-time in the dIdV, possibly due to the higher T_c making this a less sensitive device. This detector also has a $\mathcal{O}(\text{ms})$ thermal time constant in the measured pulses. Throughout the runs with CPD, this detector was affected by a time-varying 10 kHz noise. An analysis of this detector is given in Appendix C.

Chapter 7

Conclusion

An abundance of astrophysical and cosmological evidence suggests that the majority of the matter in the Universe is non-baryonic and ‘dark’ [2], yet no discovery of this dark matter has been made and its nature remains elusive. With the lack of a discovery of the WIMP, a favoured dark matter candidate, more emphasis is being placed on other dark matter candidates and interaction channels, including couplings to electrons. The focus of the work of this thesis is electron-interacting dark matter.

The SuperCDMS experiment uses cryogenic semiconductor (Ge and Si) detectors to search for dark matter interactions with SM matter. The previous generation operated at SUL and set world leading and competitive constraints on nucleon-coupled SI WIMP dark matter [139, 145, 152]. Constraints on bosonic dark matter including dark photons and ALPs were set using iZIP and CDMSlite data acquired at SUL from 2012 to 2015 [162], the first analysis of its type on SuperCDMS SUL data. This work is described in detail in Chapter 4 of this thesis. This analysis required the adaptation of the iZIP data selection criteria for ER interactions, as the data had previously been analyzed with a focus on NR interactions. The main sensitivity comes from the detectors operated in CDMSlite mode, which have lower energy threshold and no yield discrimination - all events are assumed to be NRs for WIMP dark matter searches and were interpreted as ERs for this work. We were unable to set competitive limits using the iZIP detectors, but with sensitivity to higher

energies these detectors are also sensitive to dark Compton interactions. With the CDMSlite detectors we were able to constrain new parameter space at the time of publication.

The next generation of SuperCDMS is currently under construction at SNOLAB, with commissioning planned for 2024. Chapter 5 describes forecasting efforts for SuperCDMS SNOLAB to determine the expected sensitivity to different electron-interacting dark matter models, including dark absorption of dark photons or ALPs, and LDM. This work is also described in Ref. [177], which includes forecasts for NRDM. The HV detectors will be able to achieve lower mass reach and push several decades down in cross-section/coupling parameter space for each channel. In this work we also explored a variety of different upgrade scenarios for the future beyond SuperCDMS SNOLAB, making use of the same facility. We considered three different background upgrade scenarios of increasing cost and difficulty to implement, including a highly aggressive fully underground detector lifecycle to mitigate cosmogenic backgrounds. Background improvements beyond the most basic upgrades were found to provide marginal improvements to sensitivity compared to what can be achieved with detector-based upgrades. We also considered three different detector upgrade scenarios with increasing difficulty, starting with in-hand technology that is too late for SuperCDMS SNOLAB, as well as two more aggressive scenarios, the latter of which requiring significant R&D. Different detector sizes were considered in the forecasting process, including SNOLAB-sized detectors, smaller HVeV-sized detectors, and detectors of an intermediate size. The smaller detectors provide improved energy resolution, and therefore lower energy threshold. Finally, two different detector types are considered: 0V and HV, which provide similar overall sensitivity. Both detector types can be pursued as a form of risk management, as achieving the most aggressive detector upgrade scenarios will require mitigating the leakage background in the HV detectors and significantly decreasing the baseline energy resolution of the 0V detectors. Beyond these upgrade scenarios, the push to even lower thresholds will likely require a new approach for SuperCDMS using alternative materials with lower bandgaps.

Before SuperCDMS SNOLAB begins commissioning (and long before any upgrade scenarios will be implemented), the detectors will be tested at the CUTE facility, a low background test facility located at SNOLAB. Operation at HV and calibration of the Si detectors will be important testing milestones. Chapter 6 describes the testing and analysis of a CPD-style detector named PD2, a single channel athermal phonon sensor previously operated at SLAC [241]. The presence of a low energy background in these types of devices has been observed at SLAC [241], CUTE, and at UMASS [180], and is thought to be due to stress fractures. At CUTE we adjusted the clamping scheme of PD2 in an attempt to reduce this background, but inadvertently decreased the thermal contact, introducing a long fall time, as well as low frequency noise. A low phonon collection efficiency of 6.83% was measured at CUTE in Run 18 compared to the 13% measured at SLAC. This discrepancy, as well as the dramatic difference in the detector performance between Runs discussed in detail in this thesis, complicated a dark matter search performed with this detector at CUTE [183]. The assumed collection efficiency impacts the energy scale calibration, which in turn affects the dark matter sensitivity. This thesis demonstrates the importance of understanding detector performance and calibration for dark matter analysis.

This work places emphasis on dark matter models beyond the canonical WIMP, with a search for dark absorption in SuperCDMS Soudan data and projections for SuperCDMS SNOLAB and upgrade scenarios for several ERDM models. The importance of understanding detector performance is also demonstrated. The next few years will be an exciting time in dark matter research with the upcoming testing of the SuperCDMS detectors at CUTE and the full SuperCDMS SNOLAB experiment coming online soon after. Looking even further forward, the SuperCDMS SNOLAB facility will extend the search for dark matter with vast science reach for low-mass dark matter.

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Appendix A

Optimal Filtering

SuperCDMS raw data are digitized pulse traces from the phonon and ionization sensor readout, from which we want to extract the energy and other information about an event. The measurement has irreducible noise which in particular limits the measurement of low energy events. Optimal filter theory attempts to estimate the amplitude of a signal with a known profile and reconstruct the true pulse from noisy data by minimizing a χ^2 parameter in the frequency domain. This technique is superior to a time domain χ^2 minimization because there are correlations in the time domain which are not present in the frequency domain.

optimal filter theory has been extensively derived and described in many CDMS theses including but not limited to Refs. [143, 202, 215, 295, 296]. Here I provide a brief overview of the simplest case and generalized extensions used for SuperCDMS analyses. The following will treat Fourier transforms in the continuous limit, but the same formulae can be derived in the discrete case and is what we use in practice.

A.1 The OF and OF0

We assume that the measured signal $S(t)$ can be represented by the sum of the desired signal and Gaussian noise $n(t)$ as

$$S(t) = aA(t) + n(t), \quad (\text{A.1})$$

where $A(t)$ is the assumed shape, or template, of the signal normalized to an amplitude of 1 and which is scaled by some amplitude, a . The noise is assumed to be stationary, as in it does not change in the presence of the signal. The signal shape $S(t)$ can be calculated theoretically from the TES parameters, but in practice it is generally constructed from the average of many good quality pulses. The pulse shape is assumed to be constant.

We calculate the chi-square between the measured and expected signal in frequency space weighted by the power spectral density of the noise, $J(f)$

$$\chi^2 = \int_{-\infty}^{\infty} \frac{|\tilde{S}(f) - a\tilde{A}(f)|^2}{J(f)} df, \quad (\text{A.2})$$

where $\tilde{S}(f)$ and $\tilde{A}(f)$ are the Fourier transforms of the measured signal and expected signal shape, respectively, and a is constant to be determined. The optimal estimator of the amplitude a is determined by minimizing the χ^2 and solving for a , giving

$$\hat{a} = \frac{\int_{-\infty}^{\infty} \tilde{A}^*(f) \tilde{S}(f) / J(f) df}{\int_{-\infty}^{\infty} |\tilde{A}(f)|^2 / J(f) df}. \quad (\text{A.3})$$

The variance of the best fit value of a can also be determined through $\sigma_a^2 = \left[\frac{1}{2} \frac{\partial^2 \chi^2}{\partial a^2} \right]^{-1}$, which is

$$\sigma_a^2 = \left[\int_{-\infty}^{\infty} \frac{|\tilde{A}(f)|^2}{J(f)} df \right]^{-1}. \quad (\text{A.4})$$

We have so far assumed that the pulse template and the measured signal are aligned in time, but this is often not the case; there may be a slight shift in the actual start time

of a measured pulse. This result can be generalized for an arbitrary time offset, t_0 , i.e. if template pulse starts at $t = 0$, the shifted template pulse $e^{-i\omega t_0} \tilde{A}(f)$ starts at $t = t_0$, where $\omega = 2\pi f$. In this generalized case, the χ^2 can be written

$$\chi^2 = \int_{-\infty}^{\infty} \frac{|\tilde{S}(f) - ae^{-i\omega t_0} \tilde{A}(f)|^2}{J(f)} df. \quad (\text{A.5})$$

The optimal estimator of the amplitude is found in the same manner as before and can be shown to be

$$\hat{a}(t_0) = \frac{\int_{-\infty}^{\infty} e^{i\omega t_0} \tilde{A}^*(f) \tilde{S}(f) / J(f) df}{\int_{-\infty}^{\infty} |\tilde{A}(f)|^2 / J(f) df}. \quad (\text{A.6})$$

We define the optimal filter function $\phi(f)$ as

$$\phi(f) = \frac{\tilde{A}^*(f)}{J(f)}, \quad (\text{A.7})$$

which is dependent only on the noise and the expected pulse shape. Eq. A.6 becomes

$$\hat{a}(t_0) = \frac{\int_{-\infty}^{\infty} e^{i\omega t_0} \phi(f) \tilde{S}(f) df}{\int_{-\infty}^{\infty} |\tilde{A}(f)|^2 / J(f) df}, \quad (\text{A.8})$$

where the numerator is just the inverse Fourier transform of $\phi(f)S(f)$ and can be numerically calculated for all time shifts of the template by a Fast Fourier transform [202]. It is helpful to rewrite Eq A.5 as

$$\chi^2 = \int_{-\infty}^{\infty} \frac{|\tilde{S}(f)|^2}{J(f)} df - a(t_0)^2 \int_{-\infty}^{\infty} \frac{|\tilde{A}(f)|^2}{J(f)} df, \quad (\text{A.9})$$

where we see that minimizing the chi-square corresponds to a time shift that maximizes the largest pulse amplitude. Only $A(t_0)$ needs to be maximized to also find the time delay. We can also determine the variance on the amplitude in this case

$$\sigma_{t_0}^2 = \left[a(t_0)^2 \int_{-\infty}^{\infty} \omega^2 \frac{|\tilde{A}(f)|^2}{J(f)} df \right]^{-1}. \quad (\text{A.10})$$

In SuperCDMS data processing, we refer to this method as the optimal filter algorithm and attempt to find the maximum amplitude in a specific time window. However, we also use the reconstructed energy of random triggers to estimate our baseline energy resolution, which should naively be expected to be a distribution centred on 0 energy. However, the optimal filter algorithm will preferentially select towards non-zero amplitudes and result in a biased measurement. Thus SuperCDMS processing also includes an algorithm using the no time delay optimal filter method, which we refer to as OF0.

A.2 Variants on the Optimal Filter

A.2.1 Non-Stationary OF

In the above discussion we had assumed that the noise is stationary, i.e. the Fourier modes are independent. However, this may be an oversimplification of what we observe in measured data as we often have time dependent noise (note that the noise in general does have time dependence, but that we previously assumed that the shape was constant. Here we are concerned with the shape of the noise varying in time). The NSOF is a generalization optimal filter which takes this behaviour into account.

This method can also be used to account for variation in the pulse shapes. The measured pulses at Soudan are highly uniform in shape at the tail of the pulse, but have position dependence at the beginning of the pulse because of the sensor layout. The goal of the NSOF is to de-weight this contribution in this chi-square. This is done by taking many good quality pulses with the same start time, creating an average template, and subtracting this template from the traces to obtain residuals. The residual is treated as time dependent

noise. The chi-square can be written as

$$\chi^2 = \sum_{ff'} \left[\tilde{S}^*(f) e^{i2\pi f t_0} - a \tilde{A}^*(f) \right] \{ \tilde{\mathbf{V}}(f, f')^{-1} \} \left[\tilde{S}(f') e^{-i2\pi f' t_0} - a \tilde{A}(f') \right], \quad (\text{A.11})$$

where $\tilde{\mathbf{V}}(f, f')^{-1}$ is the autocovariance matrix, which is nondiagonal. Frequencies which have high correlation end up being de-weighted as $\tilde{\mathbf{V}}(f, f')^{-1}$ is small for high correlations between frequencies f and f' . The optimal amplitude can be shown to be

$$\hat{a}(t_0) = \frac{\sum_{ff'} \tilde{S}^*(f) e^{i2\pi f t_0} \{ \tilde{\mathbf{V}}(f, f')^{-1} \} \tilde{A}(f')}{\sum_{ff'} \tilde{A}^*(f) \{ \tilde{\mathbf{V}}(f, f')^{-1} \} \tilde{S}(f')}. \quad (\text{A.12})$$

Compared to the optimal filter algorithm, the NSOF algorithm is a better energy estimator with improved energy resolution.

A.2.2 NxM Generalization and The Two Template OF

Thus far this discussion of optimal filter theory has only considered 1 template being fit to a single trace. However, optimal filter theory can be generalized to M templates being fit to N traces, termed $N \times M$. The measured signal S , amplitude a , template A , and noise n are generalized to vectors and a matrix,

$$\begin{bmatrix} S_1 \\ \vdots \\ S_N \end{bmatrix} = \begin{bmatrix} a_1 & \dots & a_M \end{bmatrix} \begin{bmatrix} A_{11} & \dots & A_{1N} \\ \vdots & \ddots & \vdots \\ A_{M1} & \dots & A_{MN} \end{bmatrix} + \begin{bmatrix} n_1 \\ \vdots \\ n_N \end{bmatrix}. \quad (\text{A.13})$$

Using Einstein summation convention where Greek indices are over the N traces and Latin indices are over the M templates, the chi-square can be generalized as

$$\chi^2 = \sum_f \left[\tilde{S}^{*\alpha} - e^{i2\pi f t_0} a^j \tilde{A}_j^{*\alpha} \right] \{ \tilde{\mathbf{V}}_{\alpha\beta}^{-1} \} \left[\tilde{S}^\beta - e^{-i2\pi f t_0} a^k \tilde{A}_k^{*\beta} \right], \quad (\text{A.14})$$

where $\tilde{\mathbf{V}}_{\alpha\beta}$ is the covariance matrix between the noise of the different traces. We minimize the χ^2 by solving the N equations

$$\hat{\mathbf{a}} = \mathbf{P}^{-1} \cdot \mathbf{q}(t), \quad (\text{A.15})$$

where we have defined the matrices

$$P_{jk} = \sum_f \tilde{A}_j^{*\alpha} \{\tilde{\mathbf{V}}_{\alpha\beta}^{-1}\} \tilde{A}_k^\beta \quad (\text{A.16})$$

$$q_j = \sum_f \tilde{A}_j^{*\alpha} \{\tilde{\mathbf{V}}_{\alpha\beta}^{-1}\} \tilde{S}^\beta e^{i2\pi f t_0}. \quad (\text{A.17})$$

An important manifestation of the $N \times M$ generalization is the two template optimal filter, or 2T Optimal Filter (OF), which corresponds to the 1×2 case. This approach was developed in an attempt to model the fast initial rise of a pulse and the slow ballistic tail with separate templates. This method characterizes both the energy and position information of a pulse, and was used by the CDMSlite analyses to develop a radial parameter for position information. The slow template is the standard template determined by averaging many good quality pulses, and the fast template is the average of the residual pulses (negative residual peaks are inverted before averaging). The 2T algorithm was run separately on the individual channel traces as well as the total summed trace. The amplitude of the slow template on the total summed trace gives energy information and the amplitude of the fast template for the different channels encodes position information. Figure A.1 demonstrates the difference in pulse shapes between channels and how the 2T fit encodes the position information.

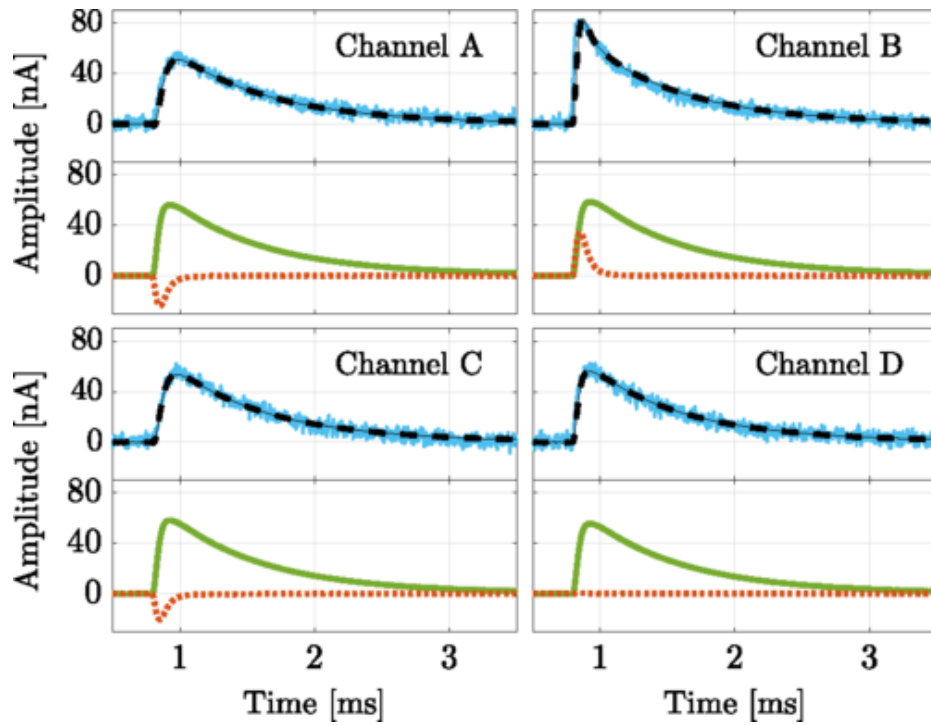


Figure A.1: The 2T OF fit on an example event from CDMSlite Run 2 [160]. The raw traces (blue) and fits are shown for the four phonon channels. The total fit (black dashed) is a linear combination of the fast template (orange dotted) and slow template (green solid). This particular event occurred under channel B as indicated by the largest fast amplitude and sharpest pulse shape.

Appendix B

Supplemental Plots for Sensitivity Forecasting

Many of the sensitivity forecasts calculated during the work of Chapter 5 were not shown for brevity, but are included in this Appendix for completeness. In the main Chapter only the **Bkg 1** scenario forecasts are shown as the improvements in sensitivity from the **Bkg 2** and **Bkg 3** scenarios are minimal compared to the improvements from the detector upgrade scenarios. Background improvements are even less important for LDM than for dark absorption, as the falling exponential spectral shape is more separable from the flat ER backgrounds.

The HV 1cm^3 detectors are generally not as sensitive as the larger detectors, with lower coupling strength sensitivity and similar mass reach to the SNOLAB-sized detectors in the **Det A** and **Det B** scenarios, and similar mass reach to the 10cm^3 detectors in the **Det C** scenario. Based on these results, the HV 1cm^3 detectors are likely not a good option over the larger HV detectors or the 0V 10cm^3 and 1cm^3 detectors. The forecasts for these detectors were also not shown in Chapter 5, but are included here.

B.1 Dark Absorption

Figures B.1 and B.2 show summaries of the 0V detector projections for dark photons and ALPs, respectively. Similarly, Figures B.3 and B.4 show the projections for the HV detectors.

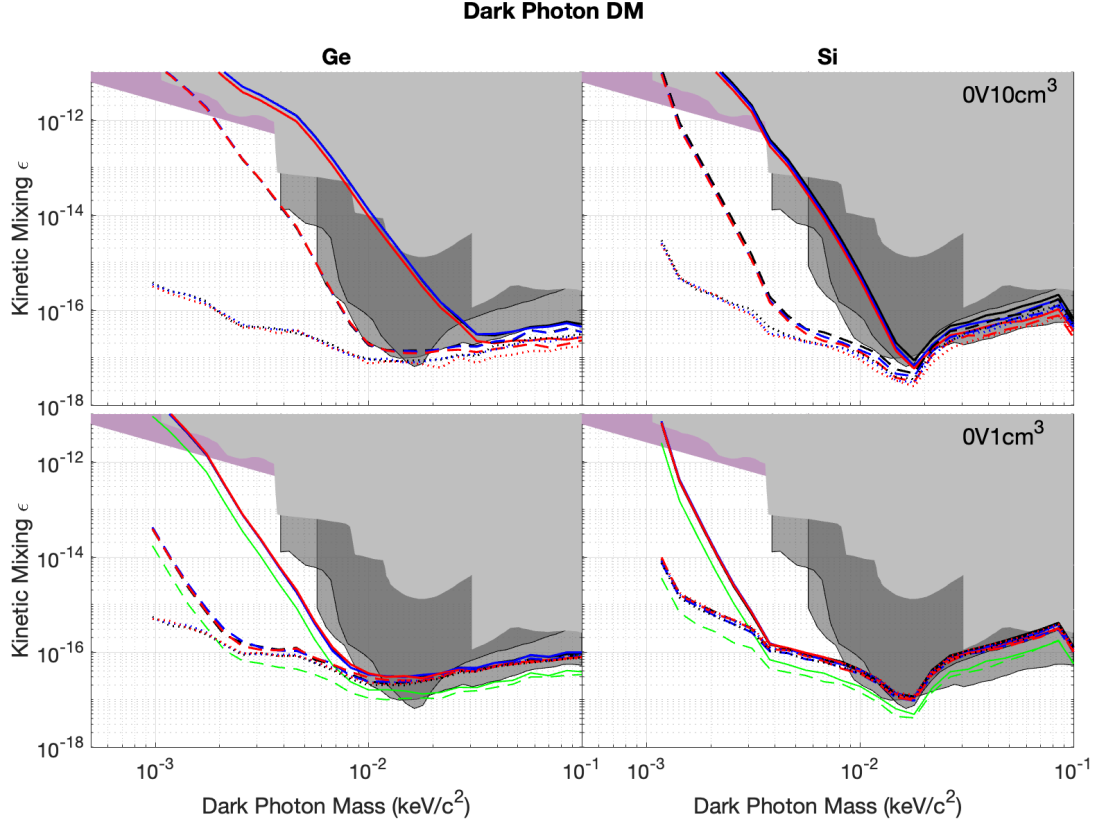


Figure B.1: Summary of forecasts for dark photon dark matter using 0V detectors for Ge (left) and Si (right). The 0V 10cm³ (top) and 1cm³ (bottom) detectors are shown separately. The line colour indicates the background scenario, Bkg 1, Bkg 2, Bkg 3 (black, blue, red), and the line style indicates the detector scenario Det A, Det B, and Det C (solid, dashed, dotted). For the 1cm³ detectors, an additional twenty times exposure is shown (green) for Det A and Det B, which would require use of the entire SuperCDMS cryostat. Current constraints are shown in solid light grey and the expected sensitivity for SuperCDMS SNOLAB is shown in translucent dark grey for Ge and Si. Stellar constraints are shown in pink.

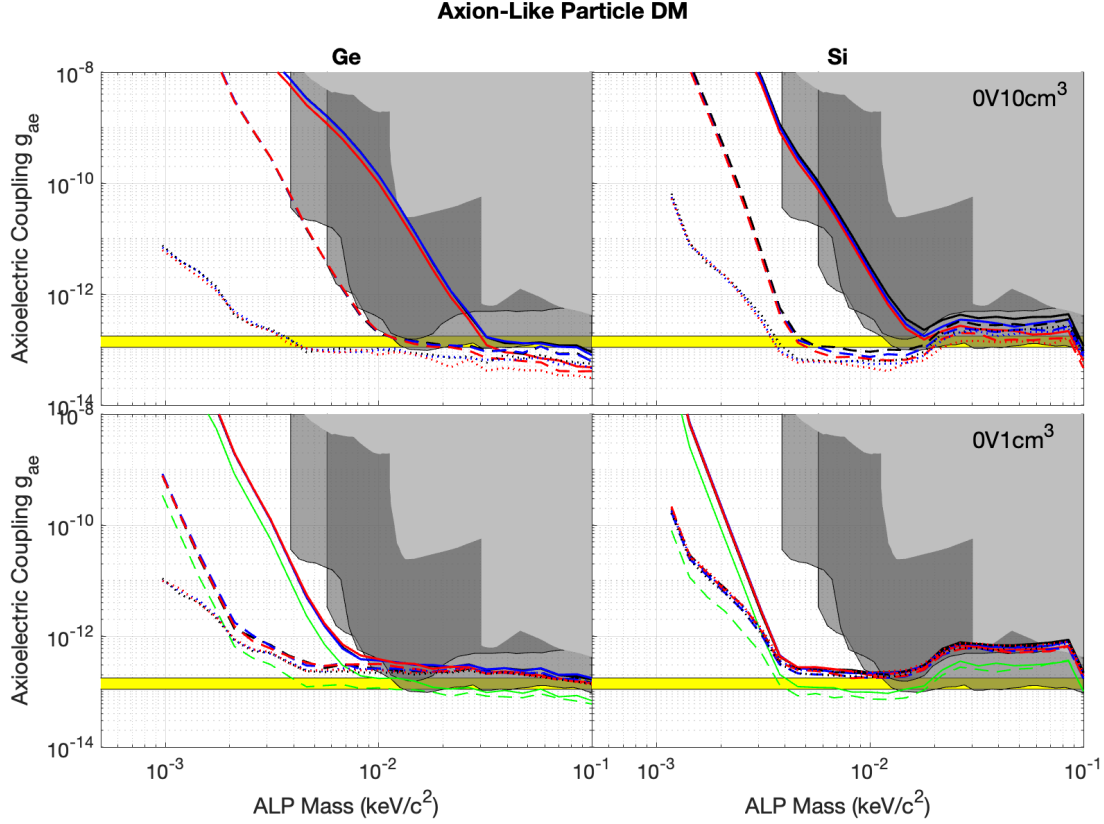


Figure B.2: Summary of forecasts for ALP dark matter using 0V detectors for Ge (left) and Si (right). The 0V 10cm³ (top) and 1cm³ (bottom) detectors are shown separately. The line colour indicates the background scenario, Bkg 1, Bkg 2, Bkg 3 (black, blue, red), and the line style indicates the detector scenario Det A, Det B, and Det C (solid, dashed, dotted). For the 1cm³ detectors, an additional twenty times exposure is shown (green) for Det A and Det B, which would require use of the entire SuperCDMS cryostat. Current constraints are shown in solid light grey and the expected sensitivity for SuperCDMS SNOLAB is shown in translucent dark grey for Ge and Si. The hinted parameter space from white dwarfs is shown in yellow.

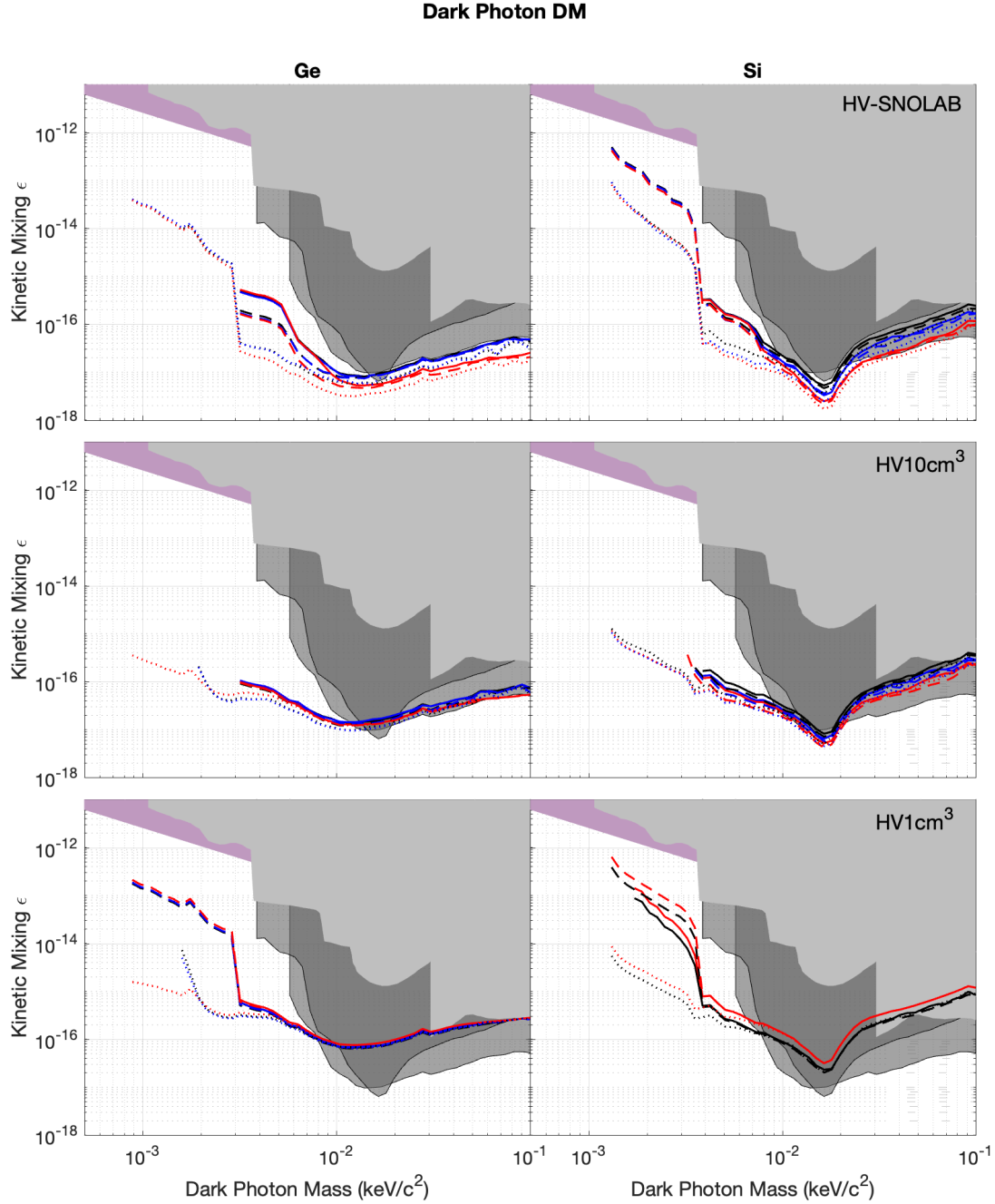


Figure B.3: Summary of forecasts for dark photon dark matter using HV detectors for Ge (left) and Si (right). The HV SNOLAB-sized (top), 10cm^3 (middle), and 1cm^3 (bottom) detectors are shown separately. The line colour indicates the background scenario, Bkg 1, Bkg 2, Bkg 3 (black, blue, red), and the line style indicates the detector scenario Det A, Det B, and Det C (solid, dashed, dotted). Current constraints are shown in solid light grey and the expected sensitivity for SuperCDMS SNOLAB is shown in translucent dark grey for Ge and Si. Stellar constraints are shown in pink.

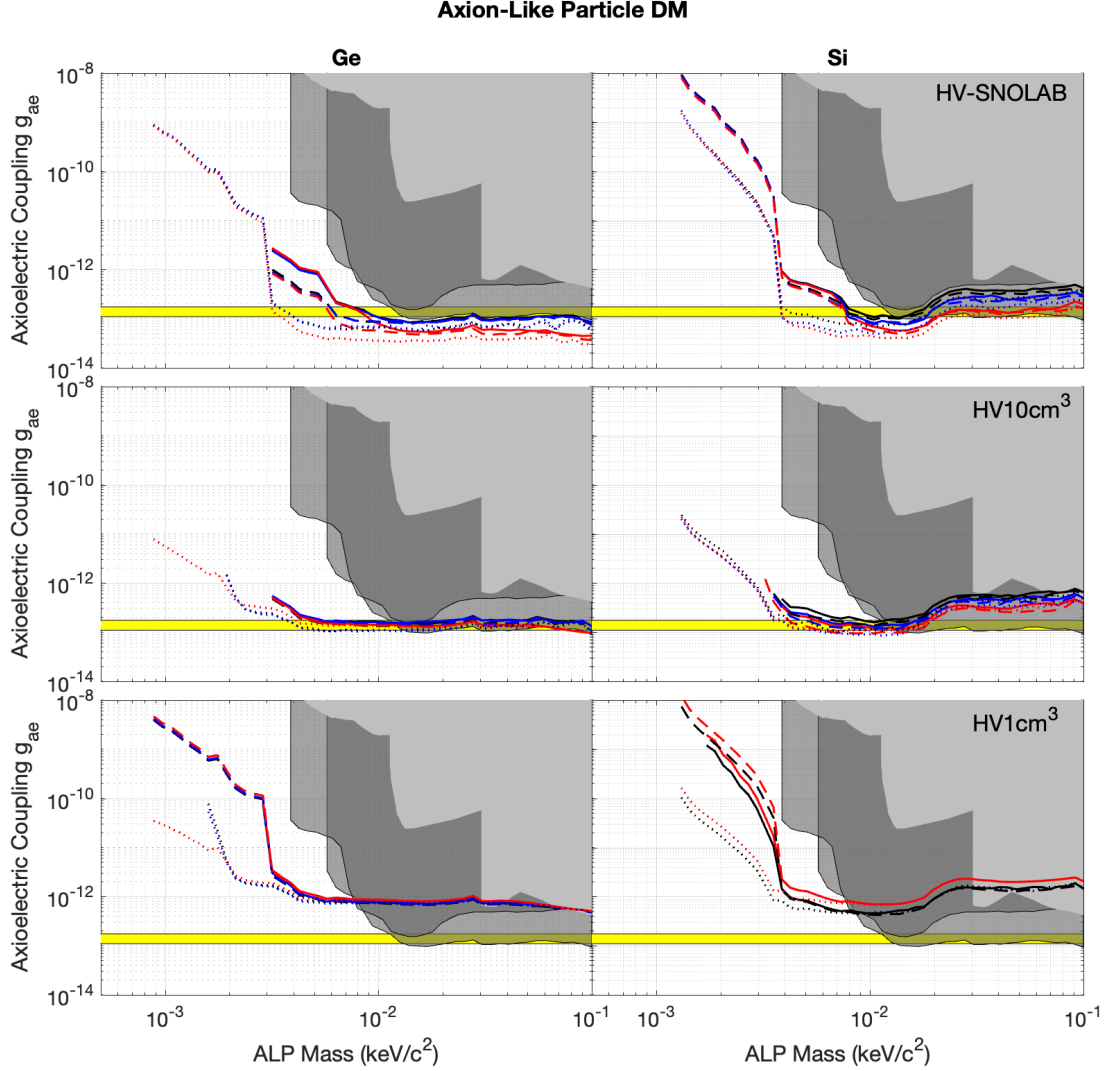


Figure B.4: Summary of forecasts for ALP dark matter using HV detectors for Ge (left) and Si (right). The HV SNOLAB-sized (top), 10cm^3 (middle), and 1cm^3 (bottom) detectors are shown separately. The line colour indicates the background scenario, Bkg 1, Bkg 2, Bkg 3 (black, blue, red), and the line style indicates the detector scenario Det A, Det B, and Det C (solid, dashed, dotted). Current constraints are shown in solid light grey and the expected sensitivity for SuperCDMS SNOLAB is shown in translucent dark grey for Ge and Si. The hinted parameter space from white dwarfs is shown in yellow.

B.2 Light Dark Matter

Figures B.5 and B.6 show summaries of the 0V detector projections for LDM coupled via a heavy and light dark photon mediator, respectively. Similarly, Figures B.7 and B.8 show the projections for the HV detectors.

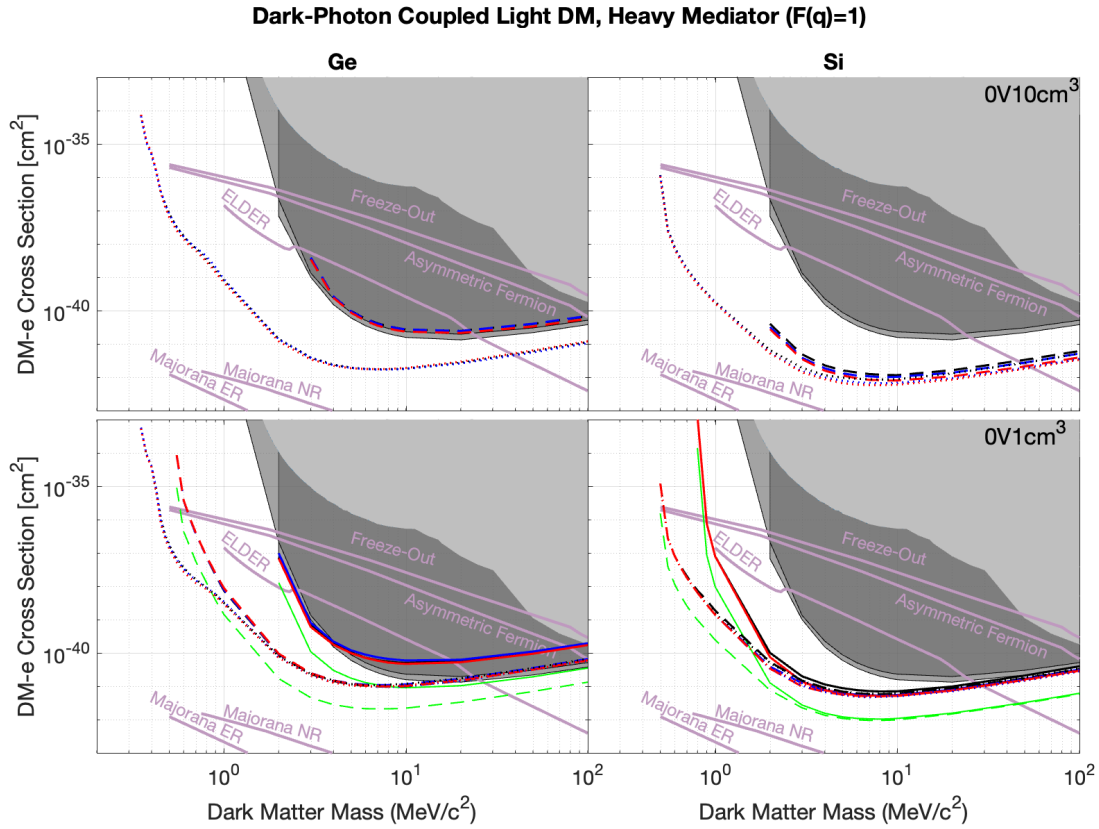


Figure B.5: Summary of forecasts for heavy mediator coupled LDM using 0V detectors for Ge (left) and Si (right). The 0V 10cm³ (top) and 1cm³ (bottom) detectors are shown separately. The line colour indicates the background scenario, Bkg 1, Bkg 2, Bkg 3 (black, blue, red), and the line style indicates the detector scenario Det A, Det B, and Det C (solid, dashed, dotted). For the 1cm³ detectors, an additional twenty times exposure is shown (green) for Det A and Det B, which would require use of the entire SuperCDMS cryostat. Current constraints are shown in solid light grey and the expected sensitivity for SuperCDMS SNOLAB is shown in translucent dark grey for Ge and Si. Sharp targets from Ref. [49] are shown in pink.

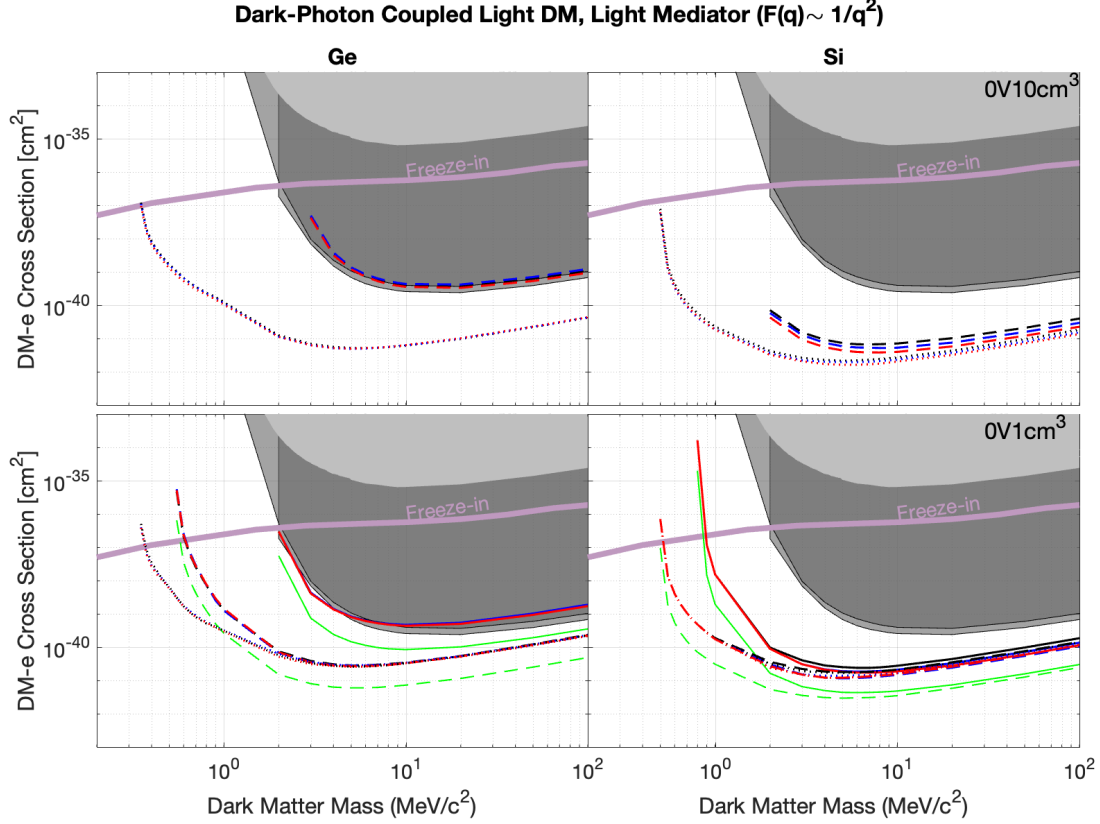


Figure B.6: Summary of forecasts for light mediator coupled LDM using 0V detectors for Ge (left) and Si (right). The 0V 10cm³ (top) and 1cm³ (bottom) detectors are shown separately. The line colour indicates the background scenario, Bkg 1, Bkg 2, Bkg 3 (black, blue, red), and the line style indicates the detector scenario Det A, Det B, and Det C (solid, dashed, dotted). For the 1cm³ detectors, an additional twenty times exposure is shown (green) for Det A and Det B, which would require use of the entire SuperCDMS cryostat. Current constraints are shown in solid light grey and the expected sensitivity for SuperCDMS SNOLAB is shown in translucent dark grey for Ge and Si. Sharp targets from Ref. [49] are shown in pink.

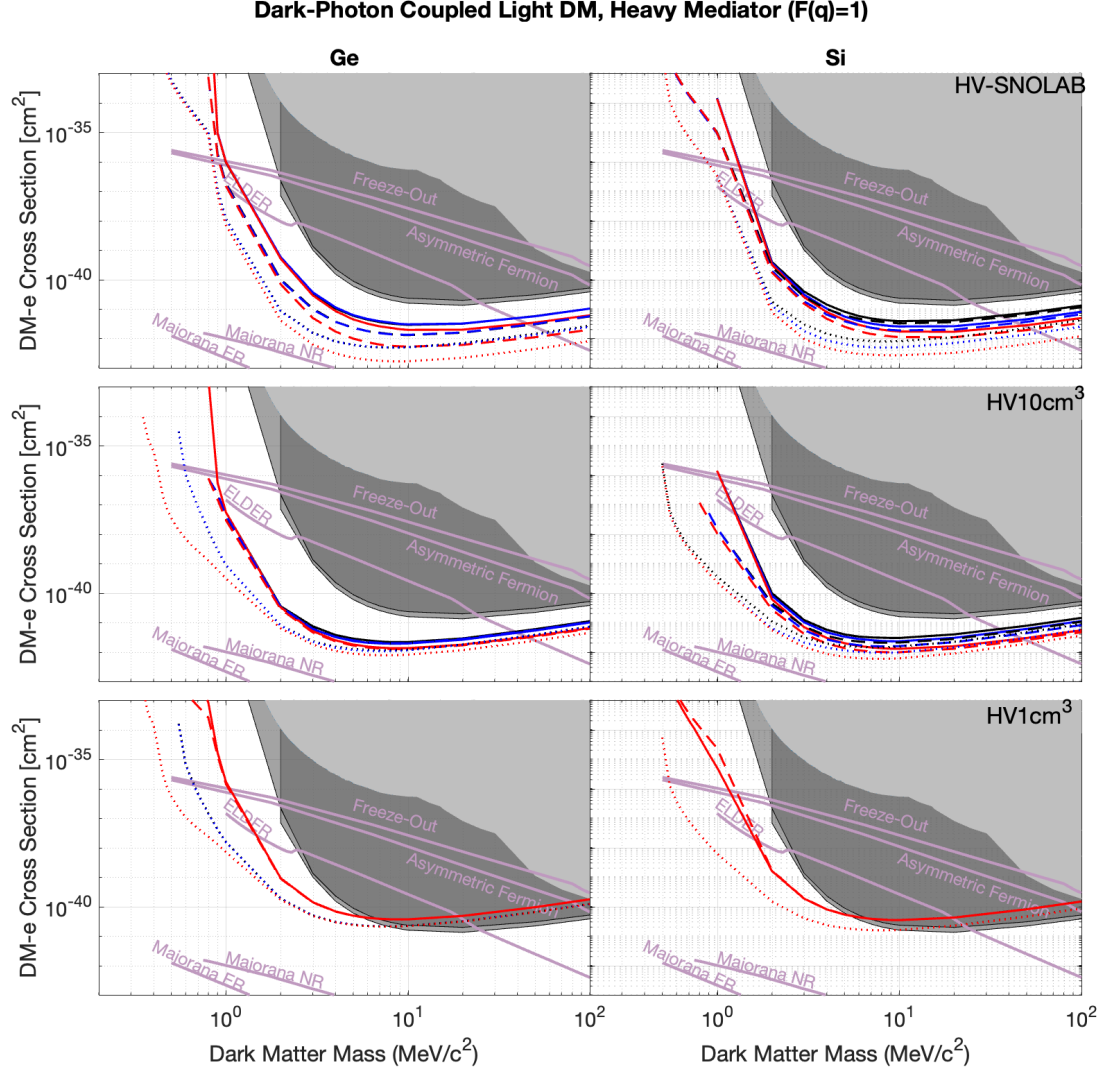


Figure B.7: Summary of forecasts for heavy mediator coupled LDM using 0V detectors for Ge (left) and Si (right). The HV SNOLAB-sized (top), 10cm^3 (middle), and 1cm^3 (bottom) detectors are shown separately. The line colour indicates the background scenario, Bkg 1, Bkg 2, Bkg 3 (black, blue, red), and the line style indicates the detector scenario Det A, Det B, and Det C (solid, dashed, dotted). Current constraints are shown in solid light grey and the expected sensitivity for SuperCDMS SNOLAB is shown in translucent dark grey for Ge and Si. Sharp targets from Ref. [49] are shown in pink.

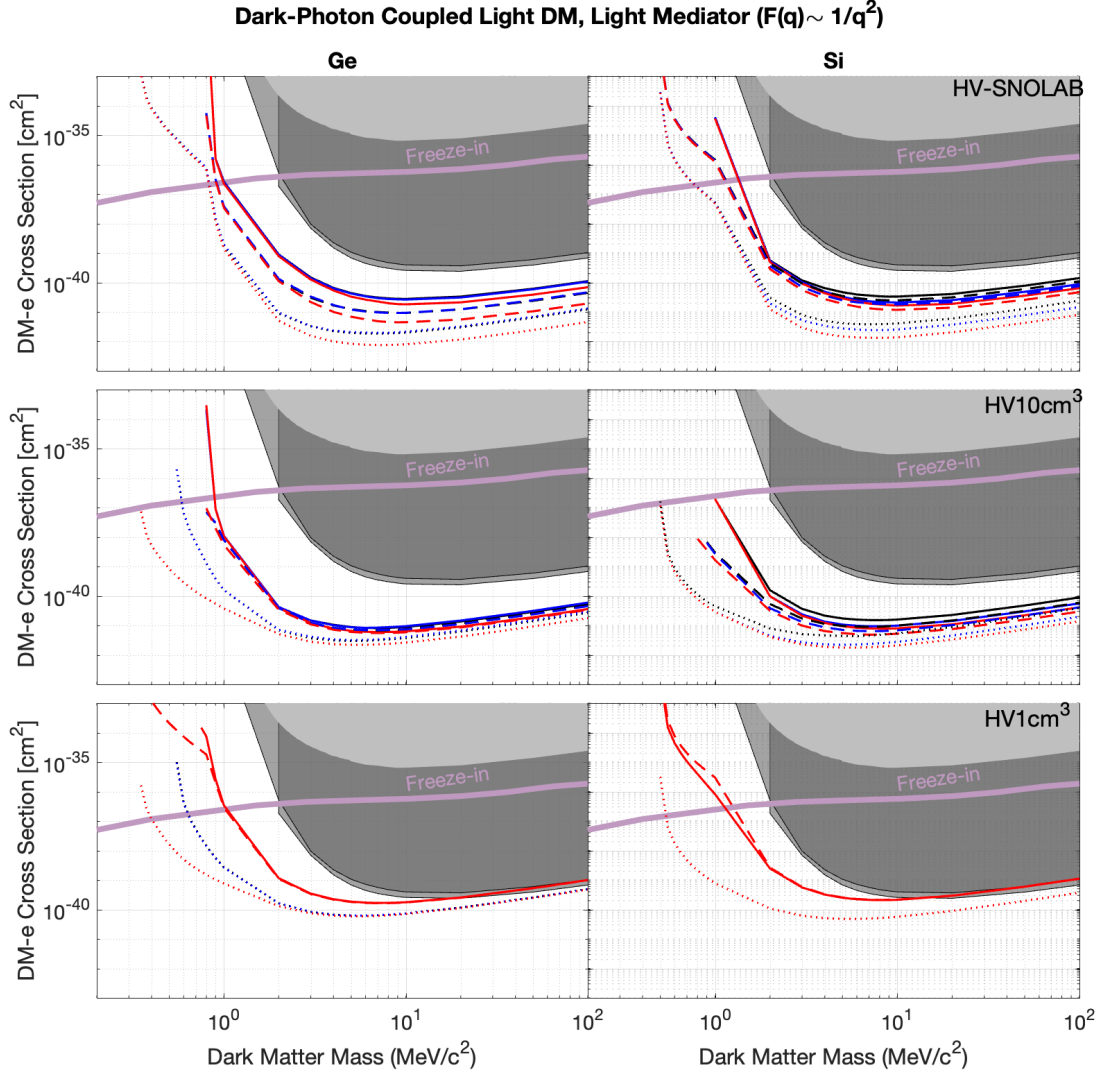


Figure B.8: Summary of forecasts for light mediator coupled LDM using HV detectors for Ge (left) and Si (right). The HV SNOLAB-sized (top), 10cm³ (middle), and 1cm³ (bottom) detectors are shown separately. The line colour indicates the background scenario, Bkg 1, Bkg 2, Bkg 3 (black, blue, red), and the line style indicates the detector scenario Det A, Det B, and Det C (solid, dashed, dotted). Current constraints are shown in solid light grey and the expected sensitivity for SuperCDMS SNOLAB is shown in translucent dark grey for Ge and Si. Sharp targets from Ref. [49] are shown in pink.

Appendix C

Testing of a Second Athermal Cryogenic PhotoDetector

The testing and characterization of the CPD-style detector PD2 at CUTE is described in detail in Chapter 6. There I briefly reference a second detector with the same design but higher T_c which was also operated during Runs 18, 19, and 20. This Appendix details the characterization of this detector in a similar manner to PD2. CPD was intended to have a lower T_c than PD2 and therefore improved energy resolution and threshold. However, the higher T_c means that this detector has less potential as a dark matter search detector, and the device was instead used to attempt to reproduce the effects of the clamp adjustments observed on PD2. The changes made to this detector between runs are outlined in Chapter 6.

C.1 CPD DC Characteristics

In Table C.1 the measured DC characteristics are shown for CPD with R_0 and P_0 close to the 40%- R_n bias point. This is chosen over 35% bias point for PD2 because this is the typical operating point at CUTE and we have no prior measurements to compare to. The readout electronics changed between Run 18 and Run 19, explaining the change in R_p . However, it is unclear why there is a change from Run 19 to Run 20. Other detector connectors had broken and were fixed between the runs, and it is possible that this channel was also adjusted. The bias power is consistent between the three runs.

Parameter	Run 18	Run 19	Run 20
R_p (m Ω)	22.9 ± 1.1	14.6 ± 0.7	10.7 ± 1.1
R_n (m Ω)	210 ± 12	213 ± 12	216 ± 23
P_0 (pW)	15.46 ± 0.99	14.76 ± 0.87	14.7 ± 1.7
R_0 (m Ω)	88.71 ± 5.7	85.15 ± 5.0	86.39 ± 9.8

Table C.1: CPD DC parameters, P_0 and R_0 at $40\%R_n$. A value of $R_{sh} = 5 \pm 0.5$ m Ω is assumed.

Figure C.1 shows a summary of the IV sweeps for CPD during the three runs in which it was operated, while Figure C.2 shows the RV and PV curves calculated from the IV measurement. Note that CPD requires a larger bias voltage than PD2 to reach the same point in transition, and CPD has significantly higher bias power. The bias power is consistent between the runs despite adjustments to the clamps between Run 18 and Run 19. The same behaviour should be expected between Runs 19 and 20, as the only change was the inclusion of the ^{55}Fe source facing CPD in Run 20.

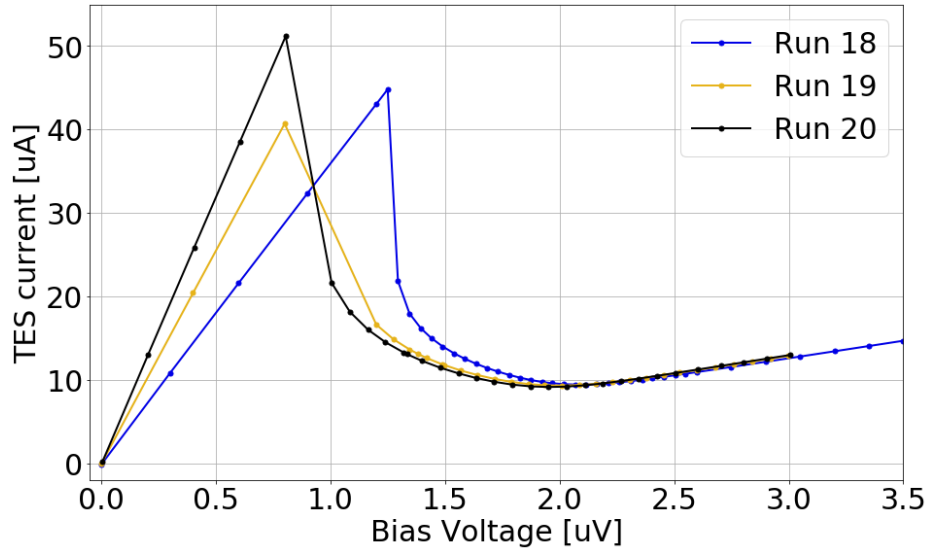


Figure C.1: Summary of IV curves for CPD across CUTE Runs 18, 19, and 20. The difference in R_p between the three runs is readily apparent from the difference in slopes in the superconducting regime.

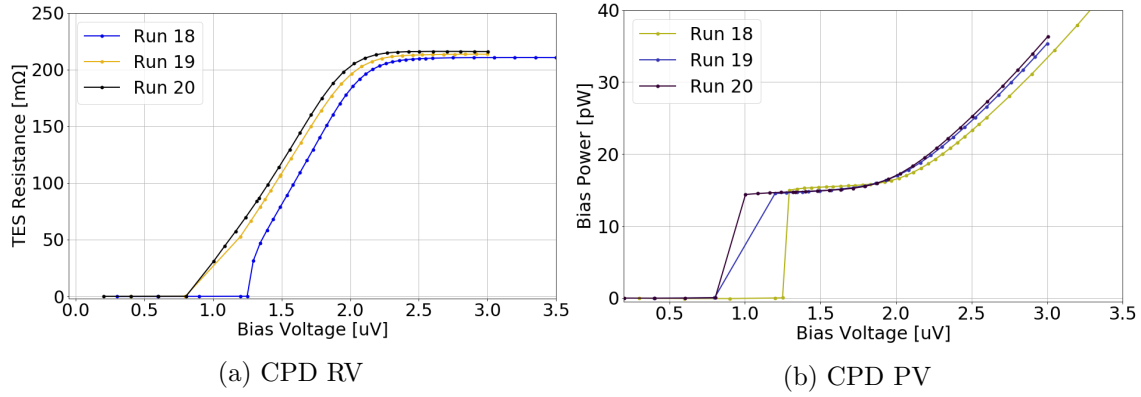


Figure C.2: Summary of (left) RV and (right) PV curves for CPD across different CUTE runs. Despite a difference in bias voltage required to achieve the same point in transition between runs, the bias power in transition is roughly constant in each run compared to the variation seen with PD2.

The overall DC response is more consistent between runs for CPD, with the main changes being due to difference in cabling between Runs 18 and 19 changing R_p and a shift in the bias voltage required to reach the same point in transition between the two runs.

C.2 CPD Complex Impedance

Figure C.3 shows the 2-pole and 3-pole fits to the average test signal response in the time domain for CPD in Run 18 only, as the time domain response in Runs 19 and 20 is almost identical. In Run 18, PD2 was at the 40%- R_n transition point as we were attempting to determine the response of the detectors while both were running. In Run 19 and 20, PD2 was initialized (no SQUID bias voltage or QET bias voltage) in order to prevent cross talk from the noisy detector. As with PD2, the 3-pole model best captures the detector response, indicating the presence of an additional thermal time constant, but the effect is much less pronounced than with PD2. Even in the runs with clamp adjustments the effect of the third time constant is consistent with Run 18, though the change in behaviour is visible at high frequency in the frequency domain. The main difference observable in the time domain is that the noise appears slightly elevated in Runs 19 and 20 compared to Run 18.

A close up view of the response is shown in Figure C.3, demonstrating that while the 2-pole fit seems reasonable on longer time scales, the 3-pole fit definitely captures a longer thermal time constant. Note that when comparing the results between runs that the test signal frequency differs.

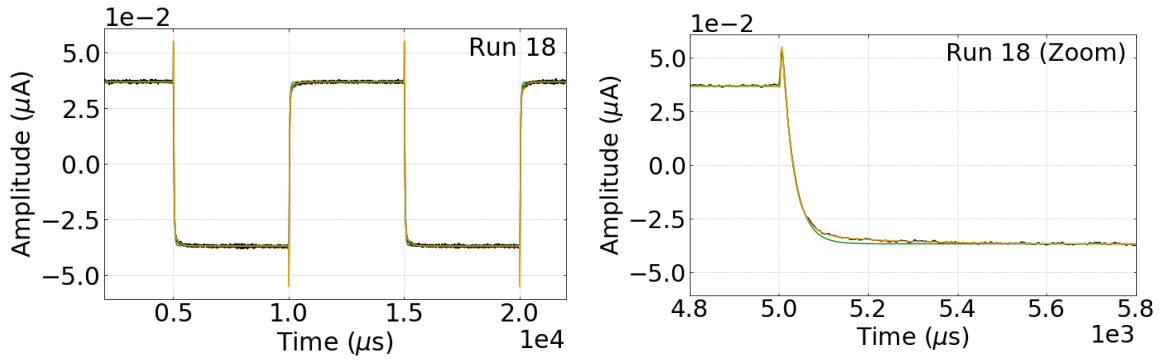


Figure C.3: 2-pole (green) and 3-pole (yellow) model fits to the CPD dIdV response (black) in the time domain for bias points close to $40\%-R_n$ for Run 18. The top right plot shows a close view the Run 18 fit, demonstrating how the 3-pole model seems to best describe the data. The response in Runs 19 and 20 is very similar.

Figure C.4 shows the complex impedance in the frequency domain with the two fit models for Runs 18 and 19. The magnitude and phase are shown separately. We can observe that in Run 19 a significant high frequency component was introduced to the complex impedance magnitude with the clamp adjustment, similar to PD2 in Run 18. During this run we had a large time varying 10 kHz noise, The response improved slightly in Run 20, but we also had the presence of this variable noise and the response was never quite as good as PD2 in Run 14 and Run 15.

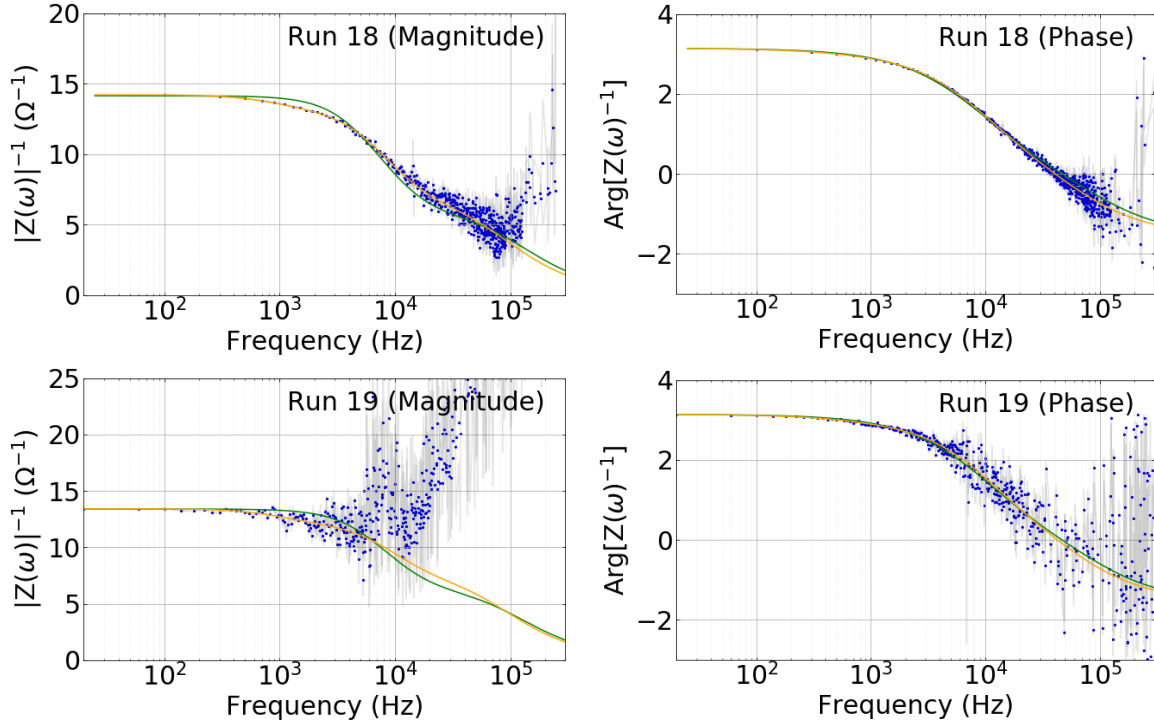


Figure C.4: 2-pole (green) and 3-pole (yellow) model fits to the CPD complex impedance (mean: blue points, 1σ bounds: grey lines) in the frequency domain for a TES bias point close to $40\%-R_n$ in (top) Run 18 and (bottom) Run 19. The (left) magnitude and (phase) are both shown, with the magnitude of the complex impedance demonstrating that the 3-pole model best describes the data. Between Run 18 and Run 19, the high frequency response changed somewhat dramatically.

Fit parameters from the 3-pole model are shown in Table C.2. The fall-times are faster than the PD2 dIdV fall-times, and are consistent between the three runs despite adjustments to the clamps between Runs 18 and 19 and the change in the high frequency response. The fit uncertainty on the parameters is larger than for PD2 because of the degraded high frequency response. With the adjustment of the clamps between Runs 18 and 19, the ratio of the thermal conductances also increases, indicating that the thermal conductance between the absorbed and the bath may have decreased as with PD2.

Parameter	Run 18	Run 19	Run 20
τ_+ (μs)	2.24	2.24	2.07
τ_- (μs)	22.6	18.3	17.4
τ_n (μs)	184.8	160.0	178.2
β	1.04 ± 0.04	1.01 ± 0.05	0.94 ± 0.04
L (nH)	357 ± 13	321 ± 19	287 ± 18
$\mathcal{L}_I$	19.8 ± 1.2	16.5 ± 1.0	23.9 ± 1.4
G_{ratio}	0.32 ± 0.01	0.44 ± 0.02	0.53 ± 0.04
τ_0 (μs)	202 ± 12	164 ± 11	236 ± 15

Table C.2: CPD dIdV parameters from a 3-pole fit model close to 40% R_n , including the three time constants τ_+ , τ_- , and τ_n , the current sensitivity β , the inductance L , the (intermediate-block model) loop gain $\mathcal{L}_I$, and the natural thermal time constant τ_0 (referred to as τ in Chapter 3). Also determined from the fit is G_{ratio} , the ratio of the thermal conductances, $G_{ta}/(G_{ta} + G_{ab})$.

C.3 CPD Pulse Fits

Table C.3 shows the fit parameters for the 3-fall-time fits for CPD. The first fall-time is similar to the thermal time constant from the dIdV measurements, while the second fall-time is slightly longer than the corresponding secondary thermal fall-time from the dIdV. The pulse shapes measured between the three runs are compatible unlike PD2, despite the clamp adjustments. Like PD2, there is a long thermal fall-time present that is not present in the dIdV. The energy contribution from this long tail slightly higher than observed for PD2 at SLAC (22%).

Figure C.5a shows the mean CPD pulse in Run 18 with the 1-, 2-, and 3-fall-time fits. The 3-fall-time fit results from each of the three runs are compared in Figure C.5b, indicating the similarity of the pulse shapes between the runs. We can also observe that CPD was noisier than PD2.

Parameter	Run 18	Run 19	Run 20
τ_r (μs)	30.3 ± 0.9	29.4 ± 1.6	25.5 ± 0.2
τ_1 (μs)	30.5 ± 0.9	29.6 ± 1.7	25.7 ± 0.4
τ_2 (μs)	244 ± 82	289 ± 91	258 ± 21
τ_3 (μs)	2100 ± 540	2600 ± 850	2520 ± 640
$\%E_1$	40.8%	38.5%	43.2%
$\%E_2$	22.5%	24.1%	22.8%
$\%E_3$	36.7%	37.5%	34.0%

Table C.3: Fall times of CPD pulses measured using a 3-fall-time model in each CUTE run. The approximate fractional energy contained within each fall-time is also shown. Like PD2, there is a long thermal time constant in each run, though for CPD the fractional energy attributed to each fall-time remains roughly constant between the runs. The overall pulse shape is also similar between the three runs.

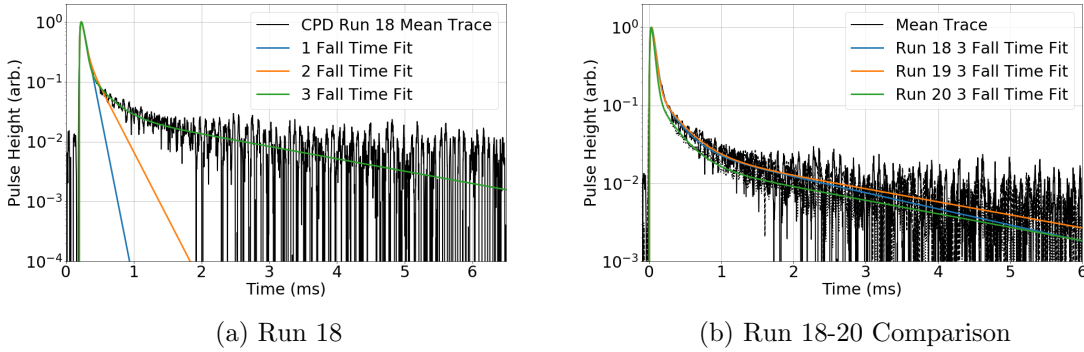


Figure C.5: (left) 1-, 2-, and 3-fall-time fits to the mean CPD pulse shape from CUTE Run 18. The 3-fall-time model best captures the long thermal tail. (right) Comparison of the mean pulse shape and 3-fall-time fit for CPD in CUTE Runs 18, 19, and 20. The mean pulse for each run is shown in black, with the line style indicating Run 18 (solid), Run 19 (dashed), and Run 20 (dotted).

Figure C.6 compares the average pulse shape as a function of optimal filter amplitude for PD2 and CPD. The longer fall-time is more prominent at higher energies for PD2 than for CPD. This behaviour on PD2 was also observed at SLAC, and corresponds to a local saturation effect manifesting as the dominant fall-time increasing with energy. High energy

events push nearby QETs into the normal resistance regime, and slows down the response of the whole detector because of the single channel readout. CPD is evidently less prone to saturation effects.

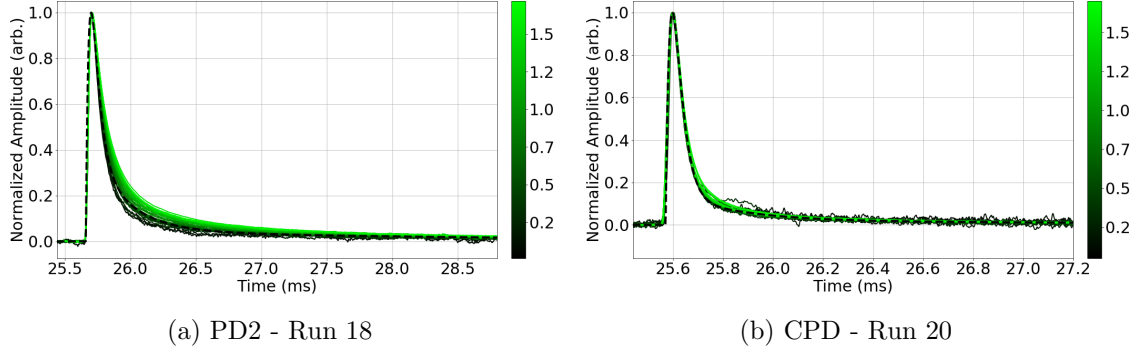


Figure C.6: Averaged and normalized pulse shapes for (left) PD2 in Run 18 and (right) CPD in Run 20. The colour indicates the optimal filter amplitude, with bright green indicating the highest amplitude and black indicating the lowest amplitude. For PD2 the fall-time lengthens with increasing amplitude. The pulse shape fitted to 1-2 keV events is shown for comparison (black dashed). An amplitude of $0.2 \mu\text{A}$ corresponds to $\sim 0.5 \text{ keV}$ for PD2 and 0.7 keV for CPD, and an amplitude of $1 \mu\text{A}$ corresponds to $\sim 3 \text{ keV}$ for PD2 and 4.2 keV for CPD.

C.4 CPD Noise

The noise is modelled for CPD in the same manner as for PD2, though we do not have a measurement of G_{ta} , which some of the noise components are dependent on. Instead, we can use scaling laws based on the measured PD2 G_{ta} and the known T_c of each detector.

We have

$$G_{ta}^{CPD} = G_{ta}^{PD2} \left(\frac{T_c^{CPD}}{T_c^{PD2}} \right)^{n-1} = 5.87 \text{ nW/K}, \quad (\text{C.1})$$

where $n = 5$.

Figure C.7 shows the measured normal and superconducting noise for CPD in Run 19 along with the modelled noise components. Recall that we determine the SQUID noise parameters from the normal noise and the load temperature T_l as shown in Table C.4. The

SQUID noise is not particularly well modelled in Run 18 as the noise level was elevated.

The normal and superconducting noise of CPD are slightly elevated compared to PD2, particularly in the 1-100 kHz range. We also observed the presence of a strange structure around 10 kHz which changed over time. This noise was present in all three runs.

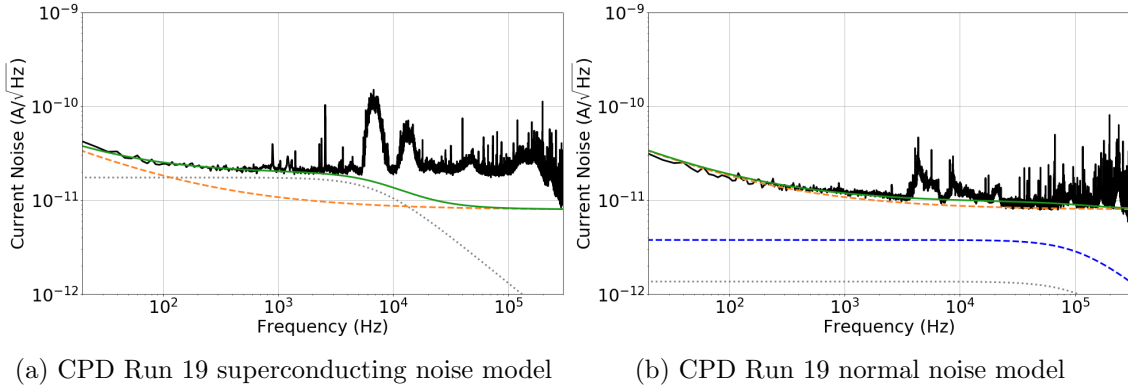


Figure C.7: (left) Superconducting and (right) normal noise measurement and model for CPD from CUTE Run 19. Modelled components include TES Johnson noise (blue dashed), load noise (grey dotted), SQUID noise (orange dashed), and the total noise is shown (green solid). There is significant high frequency noise in both regimes that is not modelled, particularly the superconducting regime. Large noise peaks around 10 kHz are visible.

Parameter	Run 18	Run 19	Run 20
DC_{SQUID} (pA/ $\sqrt{Hz}$)	0.0039	7.92	7.23
ω_{SQUID} (s ⁻¹)	5.5×10^{35}	163.6	98.7
n_{SQUID}	0.115	0.558	0.682
T_l (mK)	224.4	107.3	107.9

Table C.4: SQUID noise parameters and load temperature for CPD determined from normal and superconducting noise modelling. The Run 18 SQUID noise is not well modelled.

With the SQUID and load noise parameters extracted from the superconducting and normal noise fits, Figure C.8 shows the measured transition noise and modelled noise components for the 40%- R_n bias point. The current noise is directly measured and the power

noise is converted from the current noise using the dP/dI from the complex impedance measurement. The noise is fairly well described by the model in Run 18 other than the 10 kHz structure and other high frequency noise structures. In Runs 19 and 20, the noise is slightly undermodelled, but is overall better modelled than the PD2 noise in Run 19, and we do not see the same high levels of low frequency noise.

The expected energy resolution determined using the measured noise and current sensitivity and an assumed $20 \mu\text{s}$ phonon-collection time is shown in Figure C.9 for each of the three runs. The uncertainty bands are larger than determined for PD2 in most runs because of the noise impacting the high frequency response. Here a collection efficiency of 13% is assumed, which we shall see is a reasonable assumption. The energy resolution is higher than for PD2 in Runs 14 and 15 (also assuming a collection efficiency of 13%), which is expected given that this is a higher T_c device.

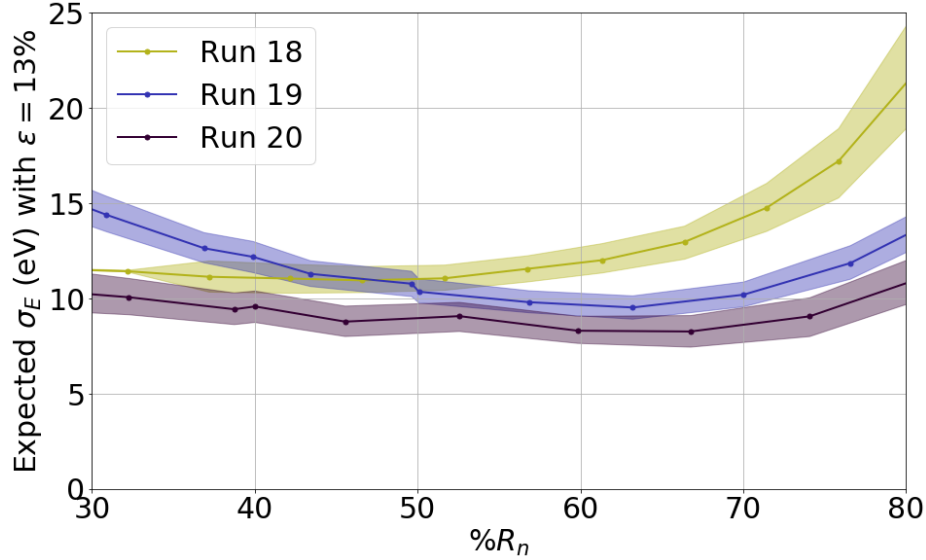


Figure C.9: Summary of the CPD expected baseline energy resolution for each CUTE run assuming a collection efficiency of 13%. Based on a measurement of the collection efficiency, $\sim 10\%$ collection efficiency may be more appropriate, which would increase the expected energy resolution.

An energy calibration model was determined for CPD in Run 20 by A. Pradeep using

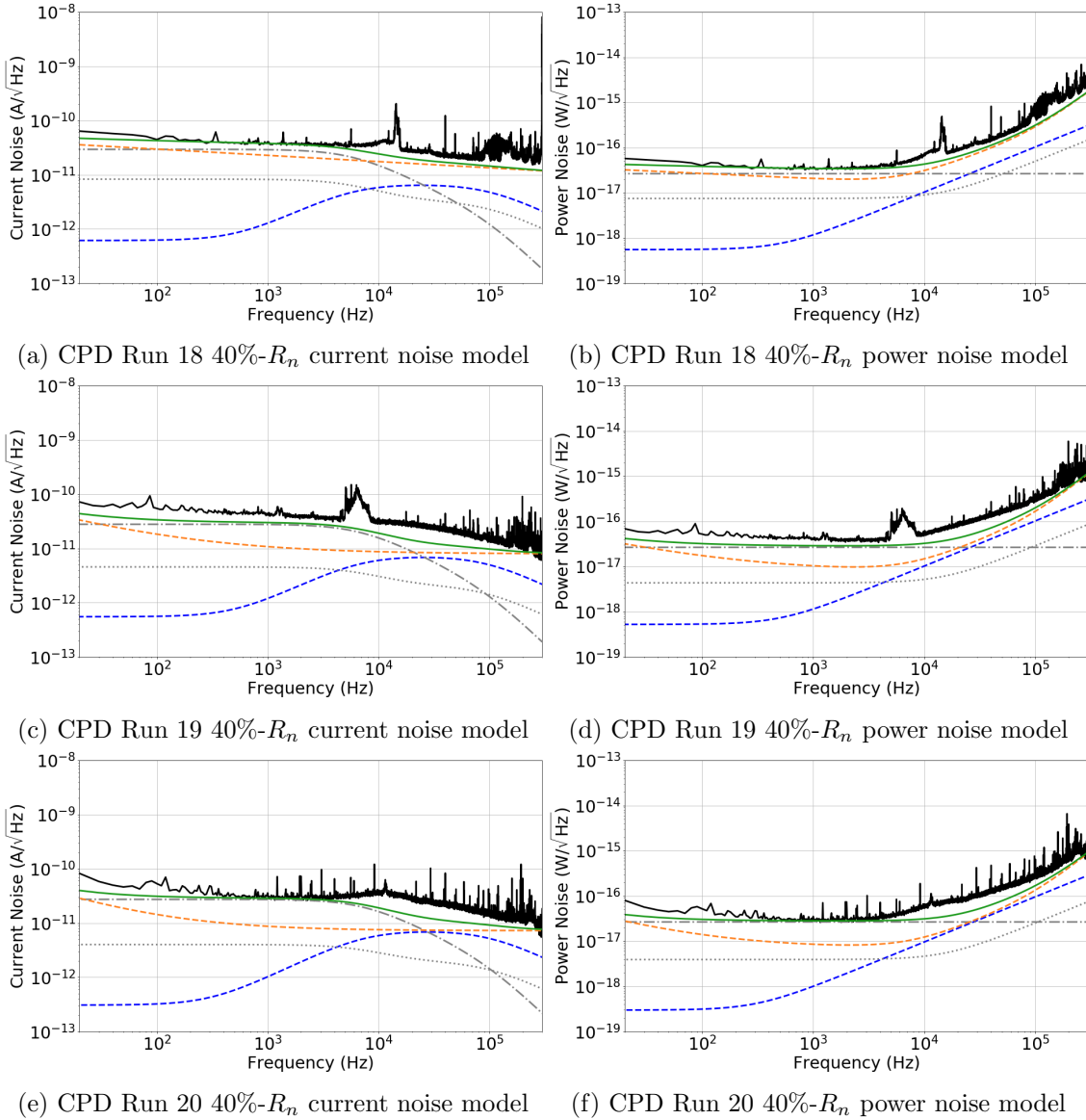


Figure C.8: (left) Current and (right) power noise models for CPD in CUTE Run 18 (top), Run 19 (middle), and Run 20 (bottom) data. The overall noise level is slightly elevated compared to SLAC. Modelled noise components include TES Johnson noise (blue dashed), load noise (grey dotted), TFN noise (grey dash dot), and SQUID noise (orange dashed). The total noise (green solid) is also shown.

the same saturation correction method as PD2 in Run 18. The ^{55}Fe source in this run was directed at CPD, meaning that the calibration peaks could be used to calibrate the energy scale.

The measured energy resolution using this calibration is compared in Table C.5 to the expected energy resolution from optimal filter theory. Assuming a 13% collection efficiency like PD2 at SLAC, the measured energy resolution is slightly higher than the expected energy resolution given a single phonon-collection time of $20 \mu\text{s}$. A collection efficiency of $\sim 10\%$ would give an expected energy resolution that is consistent with the measured energy resolution.

Parameter	Run 20
Calibrated (eV)	12.6 ± 0.4
Expected (eV, $\epsilon = 13\%$)	9.6 ± 0.8
Expected (eV, $\epsilon = 10\%$)	12.5 ± 1

Table C.5: Expected energy resolution of CPD from complex impedance and noise modelling compared to the measured energy resolution with calibrated energy scale. Expected values are shown considering two different values of the collection efficiency, ϵ . The expected energy resolution is consistent with the measured energy resolution for a collection efficiency of $\sim 10\%$.

Because of the additional long fall-time, it was found for PD2 at SLAC that the measured collection efficiency is highly dependent on the range over which the integral for the absorbed energy is performed. With the presence of the long fall-time observed in the CPD pulses, we should expect a similar behaviour for the collection efficiency. I measure the collection efficiency for CPD in Run 20 as a function of integration time using the ^{55}Fe calibration peaks, calculating the absorbed energy for 20406 events from 9 different series with optimal filter amplitude calibrated energy between 5 to 7.5 keV. To select events I apply a χ^2 cut, calculating the mean μ and width σ of the main χ^2 distribution and selecting events with χ^2 less than $\mu + \sigma$. I also apply a detector trigger cut, requiring that CPD issued the event trigger.

I designate events from 5 to 6.2 keV as from the 5.889 keV peak and events from 6.2 to 7.5 keV as the 6.49 keV peak events because the energy resolution of the absorbed energy quickly degrades and the peaks cannot be separated. Figure C.10 demonstrates this effect

of how the optimal filter amplitude can distinguish the two peaks relatively well, and how a low integration range can separate the peaks while with a longer integration range the peaks start to merge.

The absorbed energy as a function of integration time is shown in Figure C.11. The two peaks are distinguishable below an integration time of around 5 ms, but eventually the noise starts to dominate and the distribution widens. The absorbed energy rapidly increases over the first millisecond as the majority of phonons are collected by the fast rise time, then slowly continues increasing over the length of the trace.

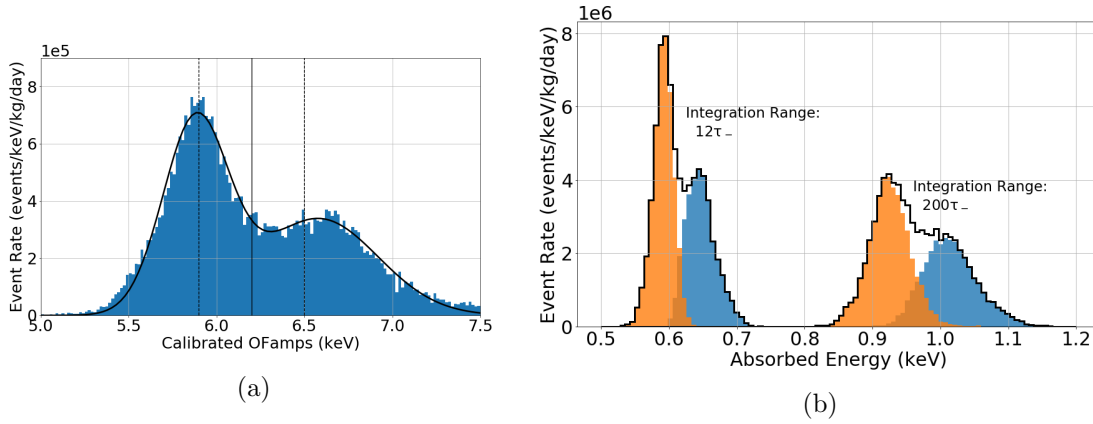


Figure C.10: (left) The calibrated optimal filter amplitude energy spectrum of the ^{55}Fe peaks (blue histogram) and fitted double gaussian model (black fit) is shown with the expected peak energy positions at 5.889 keV and 6.49 keV (black dashed vertical). A cut is made centred between the peaks at 6.2 keV to roughly separate the events. (right) The absorbed energy for two different integration ranges, 12 fall-times and 200 fall-times ($\tau_- = 17.4 \mu\text{s}$), demonstrating the degradation of the energy resolution with longer integration times. The total histogram (black line) is compared to events assigned to the 5.889 keV peak (orange, optimal filter amplitude < 6.2 keV) and events assigned to the 6.49 keV peak (blue, optimal filter amplitude > 6.2 keV).

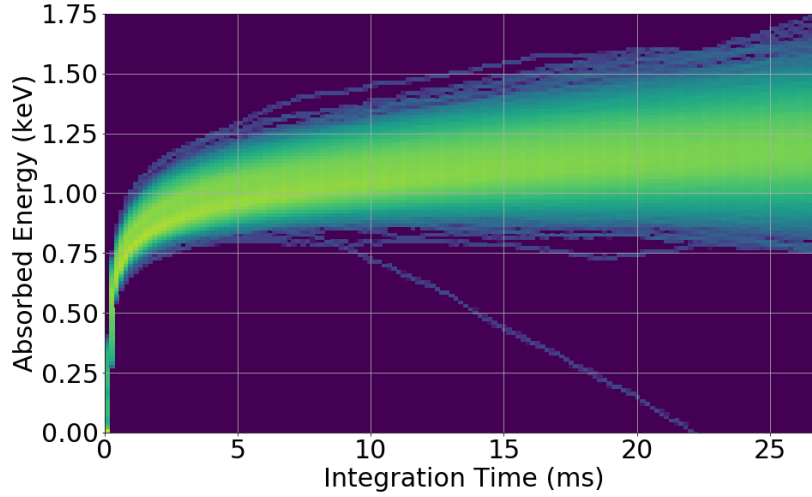


Figure C.11: Absorbed energy for events in the ^{55}Fe calibration peaks for CPD in CUTE Run 20 as a function of integration range. The colour indicates event density, with purple indicating no events and light green indicating the highest event density. The two peaks are distinguishable as two similarly shaped distributions but separated in absorbed energy over a range of $\sim 1\text{-}5$ ms, but the energy resolution degrades at higher integration ranges and the two distributions merge into one.

The collection efficiency is calculated for each peak as the mean absorbed energy of the events in the peak divided by the true energy of the peak. I choose this method rather than fitting for the peak energies with a double gaussian to avoid degeneracies in the regions where the resolution is too poor to distinguish the peaks. An uncertainty band is determined using the standard deviation of the absorbed energy. The final collection efficiency is given as the mean of the collection efficiencies determined from the two peaks, with the 1σ uncertainty band given through error propagation. Figure C.12 shows the collection efficiency for CPD as a function of integration time. It can be seen to increase throughout the length of the trace, resulting from the long thermal time constant.

Though less prone to saturation effects than the optimal filter amplitude, the measured absorbed energies of the peaks indicate slight nonlinearities as shown in Figure C.13. For each integration time, I also estimate the effect of saturation by fitting an exponential model

$f(E_{true}) = a(1 - e^{-b/E_{true}})$ to the mean absorbed energy of each peak, where the two peaks are separated by their optimal filter amplitude as before. One such fit is shown in Figure C.13 and compared to the linear Taylor expansion, demonstrating the nonlinear behaviour. The effect of this saturation on the collection efficiency can be seen in Figure C.12. I indicate the integration range at which a $\sim 10\%$ collection efficiency is determined, the range at which the measured and expected energy resolutions would be consistent. This corresponds to an integration range of 9.4 fall-times.

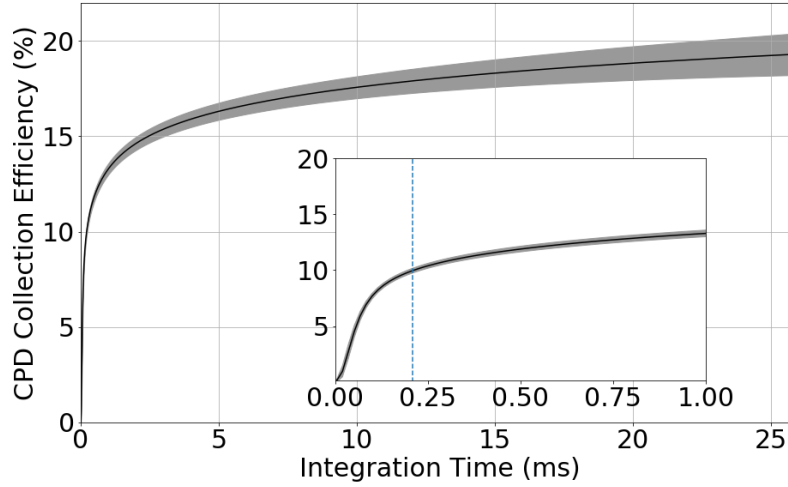


Figure C.12: The CPD collection efficiency measured in CUTE Run 20 as a function of pulse integration time. The collection efficiency and 1σ uncertainty band are determined for each ^{55}Fe calibration peak, and the mean collection efficiency (black solid line) and uncertainty band (grey shaded) are shown here. The collection efficiencies determined from the individual peaks are not shown because they are almost identical. The collection efficiency accounting for saturation effects is also shown (black dashed line). The inset plot shows the collection efficiency on a shorter integration time scale. The blue dashed line indicates an integration time of 9.4 fall-times ($\tau_- = 17.4 \mu\text{s}$), corresponding to a saturation-corrected collection efficiency of $10.1 \pm 0.2\%$.

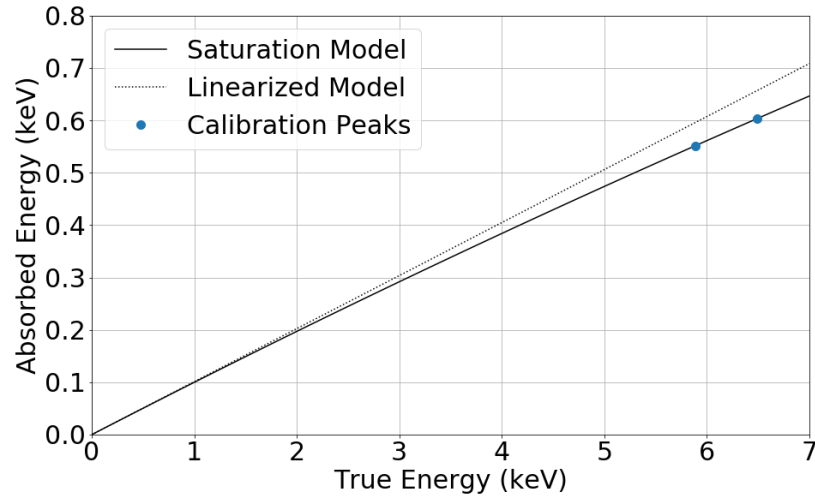


Figure C.13: Model correcting for saturation effects in absorbed energy for CPD in Run 20. The model (black solid) is fit to the absorbed energy of the ^{55}Fe calibration peaks (blue points) determined by integrating to $9.4\tau_-$. The linear Taylor expansion is shown for comparison (black dotted) to demonstrate the saturation.