

ANALYSIS OF SIGNALS IN SEMICONDUCTOR CRYOGENIC DARK
MATTER SEARCH DETECTORS AND NEW CONSTRAINTS ON
INTERACTIONS OF LIGHT DARK MATTER WITH ELECTRONS

by

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Abstract

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In this dissertation, an attempt was made to address the problem of the nature of dark matter using an experimental approach. An analysis of data from the Super Cryogenic Dark Matter Search (CDMS) HVeV silicon chip detector, equipped with transition-edge phonon sensors of high collection efficiency, is presented. Various particle theories of dark matter are tested against the data, yielding competitive exclusion limits for the interaction strengths of light (sub-GeV) dark matter candidates with electrons. To do this, a comprehensive understanding and modeling of the rich physics involved at every step from energy deposition to the observable readout signal was necessary.

The precise measurement of phonon energy deposition allowed by the TES provides possible advantages over simple ionization counters like Skipper-CCDs used in DAMIC-M and SENSEI experiments. After a brief review of the signal models considered and assumptions behind them, and an overview of the technical details of the experimental apparatus, an analysis of the data taken in March of 2022 is presented (HVeV Run 4). Of course, special attention to calibration and detector response modeling is required to take full advantage of such precise calorimetry. Further, a brief study of the potential reach of the SuperCDMS experiment at SNOLAB to the solar axion signal is presented. I end with a discussion on low-energy backgrounds and the future of the HVeV program as it approaches optical scale resolution.

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It must be acknowledged that this dissertation was only possible because of various actors and influences in my life. I firstly thank my parents for prioritizing education, valuing scientific knowledge, and fostering an environment at home where discussions about virtually any topic were tolerated. Without their constant support, I wouldn't be here. I also would like to express gratitude for the State of Kuwait, which established me and hosted my family for generations since they left Palestine. I would like to thank my ancestors, who remained faithful and persevered through all kinds of adversity, including wars and invasions. I thank my sisters for always being there for me, listening to all my rants. And I thank all my teachers at all levels of my education, who encouraged me to excel in the sciences. I am also grateful to all the professors at the University of Toronto for their excellent instruction, providing me with a strong background in physics since my undergraduate years and captivating my interest in the study of nature.

Throughout my graduate studies, I have crossed paths with some of the smartest people I have met so far, including my good friends Sreekar, David, Shaaban, Jared, Jonathan, Ahmed, Milica, Tae, and Tim, people without whom I would not be the same person I am today (for better or for worse). I have learned so much from our discussions and random talks, and the transition into graduate school would have been much rougher without them. And of course, my closest research partners, Enze Zhang and Amirata Sattari, who were there with me at every step in this journey as my trusted collaborators. And last but not least, I would like to thank my supervisors, starting with my MSc advisor professor David Curtin, who gave me the pleasure of working with him on the optimization of MATHUSLA detector currently under construction at CERN, teaching me valuable lessons in quantum field theory, collider simulations, and various beyond standard model theories. I especially thank my PhD advisor, professor Miriam Diamond, for always supporting me as her student, putting me through the trying but necessary lessons and exercises that honed my skills as an experimental physicist. Miriam always fostered an encouraging environment, where ideas and concerns were heard. Taking the lead in the implementation of SuperCDMS software infrastructure on Compute Canada systems, and together with countless staff and students, she tirelessly worked to ensure quality access to scientific education, including for ethnic minorities.

I would also like to extend my gratitude to my research family, namely the extended SuperCDMS group at the University of Toronto including Ziqing Hong, Pekka Sinervo, Brigit and Stefan Zatchler for their wise advice and immense help. Professor Hong always believed in me, even when I didn't believe in myself. The data presented in this thesis would not be here without the entire SuperCDMS collaboration, including Valentina Novati, Ben Schmidt, Enectali Figueroa-Feliciano, Tom Ren, Kyle Kennard, Matthew Wilson, who worked tirelessly on HVeV Run 4 data taking and previous analyses, providing their invaluable expertise to guide our analysis. Special thanks to committee members professors Peter Krieger and Yonatan Kahn for their invaluable feedback on this paper.

Foreword

In 1897, Sir Joseph John Thomson demonstrated the first concrete evidence for the existence of subatomic particles (“corpuscles”) that carry negative electric charge, using observations from various cathode ray tubes. Today, the electron is a well-known particle of mass $510.99895000 \pm 0.00000015$ keV, according to the Particle Data Group, and it sits theoretically as a fundamental Dirac fermion within the Standard Model (SM) of particle physics, a quantum field theory (QFT) often thought of as an effective theory in need of a high energy completion. In the time between, known as the twentieth century, a vast multitude of discoveries occurred in physics, including but not limited to: the quantum nature of light by 1905; the existence of nuclei in atoms by 1911; the relativistic nature of gravity by 1915; hadrons as constituents of nuclei, and the existence of antimatter by the 1930s; partons as constituents of hadrons, and electroweak unification in the 1960s; discovery of W and Z massive bosons of the weak force in the 1980s; culminating in the discovery of the heaviest fundamental fermion known to date (the top quark), as well as the miniscule masses of neutrinos (through their flavour oscillations) in the 1990s. These discoveries (except those pertaining to gravity) are all described today within the framework of the SM. The crown jewel of the SM was the discovery of the Higgs boson at the Large Hadron Collider (LHC) in 2012, solving the non-unitarity problem in weak interactions.

The theoretical inventions accompanying said discoveries, conveniently summarized in the SM of particle physics and General Relativity (GR), model all observed fundamental phenomena. On one hand GR provides a framework to describe the manifold of spacetime itself, while the SM concerns itself with the description of energy in its various manifestations (except gravitational and dark). The complete reconciliation of these two approaches remains a problem in physics, its most popular solution being a testable theory of quantum gravity. Considering gravity in the context of GR, combined with the accumulation of increasingly precise cosmological observations in the 20th century, it was possible to develop the *concordance model*, also known as the Λ CDM model of cosmology, explaining structure on the largest observable spacetime scales. The observation of the accelerating expansion of the universe points to “dark energy” described in Einstein’s cosmological constant (Λ), and observations of the cosmic microwave background (CMB) radiation, surveys of astronomical objects, as well as recent observations of weak gravitational lensing all point to the existence of cold dark matter (CDM). It is estimated that 70% of the energy in the universe consists of dark energy, 25% of dark matter, and a mere 5% exists as baryonic (“normal”) matter. Further, the anti-baryonic matter is nowhere to be found, which is a problem if one assumes that the baryonic matter was created via SM interactions (which do not allow this level of matter-antimatter asymmetry). In addition to these problems,

theoretical difficulties exist in the SM of particle physics, even when gravity and cosmology are neglected, notably the hierarchy problem and the strong CP problem.

In the same vein as Henry Cavendish's 1797 experiment, which determined Newton's gravitational constant by *direct detection* of the feeble forces between lead balls, we embark on an ambitious endeavour to measure the coupling constants of DM particles with SM particles in a semiconductor substrate. In the former case, Cavendish had Newton's law which explains celestial object movements, and he knew the nature of his (massive, uncharged) lead balls, so he was able to deduce the coupling constant of gravity. In our case, we have Einstein's GR to explain the cosmological observations, inferring the dark matter galactic halo, including some properties like its local mass density. We do not know the composition of this halo, but we can eliminate some possibilities. Combining our knowledge of the nature of our detectors with various plausible particle theoretical hypotheses on the nature of dark matter, one may measure coupling constants between DM and SM particles, provided that they are *strong enough* compared to confounding effects. These coupling constants are supposedly nonzero as forces can be mediated by quantum corrections in the QFT paradigm. This brings us to the other similarity we share with Cavendish: we must go to great lengths to reduce noise and backgrounds, in order to be able to measure such slight forces. Background effects cannot be fully eliminated, so they must also be modeled. Null results are common in direct detection experiments, which instead end up excluding additional parameter space of the tested theories.

Table of Contents

Abstract	ii
Acknowledgements	iii
Foreword	iv
List of Figures	ix
List of Tables	xii
Statement on Collaborative Work	xiii
1 The Problem of Dark Matter	1
1.1 Let There be Fields	1
1.2 Particle Dark Matter Candidates	9
1.2.1 Weakly Interacting Massive Particles	11
1.2.2 Wave Dark Matter	14
2 A Direct Detection Solution	18
2.1 Overview of Direct Detection Experiments	20
2.1.1 Neutral Particle Detection	21
2.1.2 Wave Detection	24

2.2	The Cryogenic Dark Matter Search	24
2.2.1	SuperCDMS SNOLAB Experiment	31
2.2.2	The SuperCDMS HVeV Program	34
2.2.3	HVeV Run 4 at NEXUS	36
2.2.4	Filtering Techniques	42
2.2.5	Standard Event Selections	47
2.2.6	Standard Live-time Selections	49
3	Investigating Detector Response Effects using HVeV Run 4 LED Data	51
3.1	Calibration Data Acquisition	54
3.1.1	Triggering on LED Events	55
3.1.2	Data Selections for LED Calibration Data	58
3.2	Detector Response Fits	60
3.2.1	Cross-Talk Subtraction	61
3.2.2	Comparing the Models	65
3.2.3	Summary of CT and II Results	66
3.2.4	Calibrating DM Search Data	67
3.2.5	Calibration Uncertainty	67
3.3	Resolution Modelling	70
4	HVeV Run 4 Analysis: Search for Dark Matter - Electron Interactions	72
4.1	Calculating Trigger Efficiencies	72
4.2	Data Selections	73
4.2.1	Live-time Cuts	73
4.2.2	Event Quality Cuts	79
4.3	Calibration Using LEDs	82
4.4	Exclusion Bounds	84
4.4.1	Limit Setting Methods	85
4.4.2	Including the Effect of DM Attenuation in Earth's Crust	88

5	Detection Prospects for Solar Axions in SuperCDMS SNOLAB	94
5.1	SuperCDMS SNOLAB as a Helioscope	94
5.2	Solar Axion Event Rate	95
5.2.1	A tension in the literature	97
5.2.2	A semi-classical treatment	98
5.3	Projected Sensitivity of SuperCDMS SNOLAB to Solar Axions	105
6	Conclusions	107
6.1	On the background rate of HVeV detectors	107
6.2	On the elevated rate periods in HVeV Run 4	110
6.3	Looking Forward	115
	References	118
	APPENDICES	138
A	Axio-electric calculations	139
A.1	Relation between field density and axion flux	139
A.2	Green's functions derivation	140
A.2.1	The vacuum Green function	142
A.2.2	Including absorption	143
A.3	Differential rate of photon production	144

List of Figures

1.1	Cosmic microwave background anisotropy	5
1.2	Schematic of the radiation-dominated universe	6
1.3	Large-scale structure simulation	8
1.4	Weak gravitational lensing survey	9
1.5	Galactic rotation curves and dark matter halo	12
1.6	Particle dark matter candidates	17
2.1	Various approaches for dark matter detection	20
2.2	Sensitivity of direct detection experiments	23
2.3	Quasiparticle trap-assisted Electrothermal feedback Transition edge sensor	25
2.4	Phonon production in a voltage biased substrate	26
2.5	Cryogenic Dark Matter Search phonon-only detectors	30
2.6	Cryogenic Dark Matter Search detector with phonon and charge readout .	31
2.7	Setup of the SuperCDMS SNOLAB experiment	33
2.8	Projected sensitivity of SuperCDMS SNOLAB detectors	34
2.9	Various QET mask designs for HVeV 1 cm ² Silicon detectors	35
2.10	Quasiparticle traps and transition edge sensor	36
2.11	Schematic of charge propagation in a semiconducting crystal	37
2.12	Rendering of detectors in their new detector housing	38
2.13	Images of the setup of HVeV Run 4	40
2.14	TES resistance vs. bias current	41

2.15	Comparison of final setups of detector towers for NEXUS Runs 13 and 14 .	42
2.16	Example of HVeV Run 4 readout Trace	44
2.17	Pulse template and noise PSD used in the optimal filter	45
2.18	Comparison of Threshold Triggering Algorithms	46
2.19	Pulse amplitude distributions in NFC1 detector	48
2.20	Mixing chamber temperature excursion	49
2.21	Baseline TES current of readout traces	50
3.1	Illustration of various detector processes	53
3.2	Both sides of an LED box used in HVeV Run 4.	54
3.3	LED response to an applied triangle test signal	55
3.4	Sample readout trace with LED source	56
3.5	LED calibration fit	57
3.6	Power spectral densities from various datasets in HVeV Run 4	58
3.7	LED event pulses	59
3.8	Average LED pulse shapes	62
3.9	Estimation of cross-talk amplitude	63
3.10	Cross-talk estimates in various LED datasets	64
3.11	Detector response modelling fit	65
3.12	Correlation matrices of detector response fits	66
3.13	Summary of charge trapping and impact ionization estimates	68
3.14	Energy calibration uncertainty	69
3.15	Energy resolution estimates	71
4.1	Estimates of the trigger efficiency	73
4.2	Event rate as a function of time	75
4.3	Distribution of minimal time differences between NFC1 events	77
4.4	Event passage fractions after applying live-time cuts	78

4.5	Pulse χ_{red}^2 distribution for Cs-137 x-ray data	80
4.6	An event failing the reduced- χ^2 event quality cut	81
4.7	Event quality cut signal efficiency	82
4.8	Pulse amplitude spectra from NR13 and NR14 datasets	83
4.9	Threshold test statistic	89
4.10	Dark matter exclusion bounds derived from HVeV Run 4	92
4.11	Effect of overburden on event rate	93
5.1	Solar axion event rate in SuperCDMS SNOLAB Germanium iZIP detectors	97
5.2	Benchmark sensitivity of SuperCDMS SNOLAB to solar axions	106
6.1	Event rate in background data sets from NR14	108
6.2	Empirical fit of NR13 dark matter search spectrum	109
6.3	An outer band of events	111
6.4	Position of background events in NR13 dark matter search spectrum	112
6.5	Low energy event rates from NR13 at different crystal voltage	113
6.6	Elevated event rate period in DM search data	114
6.7	LED event times relative to the closest non-LED event	116
A.1	Sun emitting axions towards Earth	147
A.2	Kinematics of solar axion conversion to x-rays	147

List of Tables

2.1	SuperCDMS initial payload summary	32
2.2	HVeV Run 4 payload summary	41
3.1	Parameters of the simultaneous photon hits model	61
4.1	Table of nuisance parameters	88

Statement on Collaborative Work

The precise measurement of fundamental particles and their interactions involves the use of sophisticated hardware (and software), the operation of which requires collaborative work. Additionally, a proper application of theoretical predictions is necessary, and consultation with theorists from various fields is often needed. In the following chapters, I present various analyses which could not have been done without the work and input of various collaborators. In the interest of academic integrity, I list exactly what my contributions were.

Chapters 1 and 2 are mostly composed of reviews of the literature on DM theory and detection, which were greatly aided by discussions with professors Miriam Diamond and Ziqing Hong. The subsequent two chapters focus on HVeV Run 4, the data-taking of which was done at Fermilab, with the great team there including Valentina Novati, Benjamin Schmidt, Bruce Hines, Nick Mast, Matthew Hollister and Kyle Kennard conducting the bulk of the data-taking, with some crucial real-time analysis being done by the team at the University of Toronto. Ziqing Hong, Yongqi Wang, and Mason Buchanan were also involved in the design and testing of the photon calibration apparatus described in Ch 3. The data analysis of the calibration data in Chapter 3 was mostly conducted by myself with help from Matthew Wilson and Alexander Zastyevev, while the dark matter search analysis of Chapter 4 was a joint effort between two main analyzers including Enze Zhang and myself, with help from Huanbo Sun. Of course, the general direction of the analyses was defined by the designers of the run and many Principal Investigators in the SuperCDMS collaboration, including the aforementioned contributors, in addition to Enectali Figueroa-Feliciano.

In Chapter 5, the potential of solar axion detection at SuperCDMS SNOLAB is investigated and the reach of the experiment calculated, with help and guidance from professors Miriam Diamond, Pekka Sinervo, and Belina Von Krosigk. The code developed by Danika Macdonald was used to numerically evaluate theoretically derived event rates, and the insights of Amirata Sattari are also recognized. A calculation from first principles of the theoretical event rate of solar axions in crystal detectors was conducted to resolve an apparent conflict in the literature. This was led by professor John Sipe, who's insights on the relevant length scales were crucial. While professor Sipe set up the problem in clear terms and provided the mathematical machinery to solve the problem analytically, the solution was cast in the language of particle physics by myself. Finally, I conclude in Chapter 6 with an empirical analysis of low-energy backgrounds, which was completely self-driven. I also contributed to the SuperCDMS infrastructure including data acquisition, event reconstruction and data analysis software, which will not be outlined in the following text.

Chapter 1

The Problem of Dark Matter

In the next few chapters, we will attempt to directly detect a galactic halo of dark matter particles. In this introductory chapter, I outline why we are inclined to do so. We start by reviewing the cosmological evidence for dark matter, followed by a brief review of where a dark matter particle may sit in the framework of the standard model (SM) of particle physics, invoking well-motivated beyond standard model (BSM) scenarios.

1.1 Let There be Fields

The framework of general relativity (GR) allows for modeling the universe as a whole according to Einstein's field equations [1]:

$$G_{\mu\nu} = \Lambda g_{\mu\nu} + \kappa T_{\mu\nu}, \quad (1.1)$$

with Λ and κ being the cosmological and Einstein gravitational constants respectively, $T_{\mu\nu}$ our stress-energy density tensor, $g_{\mu\nu}$ the metric tensor, and $G_{\mu\nu}$ the Einstein tensor which takes into account the geometry of spacetime. Parameter Λ is considered a physical constant (subject to increasing scrutiny [2]), commonly interpreted as an intrinsic energy of the vacuum of spacetime. The precise nature of this dark energy is unknown, and the smallness of its apparent value from redshift surveys [3] is called the cosmological constant problem [4]. The observed rate of expansion of the universe is encoded in the Hubble constant $H_0 \simeq 70 \text{ km/s/Mpc}$, with an apparent tension between different measurement strategies [5]. This universal mechanics is taken for granted for the rest of this thesis. Our interest as particle physicists is in the mass/energy density $T_{\mu\nu}$. Matter and (non-gravitational)

energy are described by the standard model (SM) of particle physics. Combining Eq. 1.1 with the SM and astronomical observations, one can deduce the constituents of $T_{\mu\nu}$. A history of matter and energy may, in principle, be reconstructed.

The SM of particle physics is a quantum field theory (QFT) that describes the known ‘fundamental’ particles (leptons, quarks, gauge bosons and the Higgs boson), and their quantum interactions. Fundamental particles are point-like, lacking the sub-structure seen in a composite particle. Free particles are said to be excitations of their respective quantum field, making up independent Fourier modes of the field defined over spacetime. The idealized excitation is an asymptotic state with well-defined momentum and energy, and interaction probabilities are often calculated using perturbation theory by considering the quantum amplitude of an initial state ending up as a final state, assuming a Lagrange density $\mathcal{L}$. The density $\mathcal{L}$ is made up of field operator terms that define the fields’ dispersion in free space (the kinetic and mass terms), as well as products of different flavored field operators (the interaction terms). Each term in $\mathcal{L}$ must obey Poincare spacetime symmetries, as well as other ‘internal’ symmetries. The defining symmetry of the SM is the local gauge symmetry $SU_C(3) \times SU_L(2) \times U_Y(1)$, which defines how the leptons and quarks interact with each other through the exchange of the gauge bosons. The Higgs boson is involved in the spontaneous breaking of the $SU_L(2) \times U_Y(1)$ symmetry into electromagnetism $U_{EM}(1)$ and the short-range weak force mediated by massive bosons. The breaking takes place when the Higgs field acquires a vacuum expectation value (VEV) with an observed value of $v \sim 250 \text{ GeV}$. The VEV is often referred to as the Higgs condensate which permeates spacetime, providing mass for fundamental fermions through Yukawa coupling [6].

All directly observed non-gravitational energy is made up of SM particles. It may not be straight forward to go from quarks to nuclei as there are some strong interactions and perturbation theory fails. Two examples of techniques used to treat the strong interactions at low energies are lattice field theory and effective field theories (EFTs). EFTs are low-energy approximations of the true theory. EFTs have ultra-violet (UV) completions, theories that remain valid and self-consistent at high energy, but reproduce the EFT at low energy. In fact, the SM itself is often considered an EFT in need of completion. Problems of fine-tuning and naturalness (which will be discussed in more detail in Section 1.2) motivate extensions to the SM, the most popular of which have been supersymmetric theories where each SM particle has a superpartner (see [7] for a review of supersymmetry). The minimal supersymmetric model (MSSM) includes only the SM and its superpartners, along with an extended Higgs sector (and its superpartners). A discrete symmetry called R-parity is additionally imposed to satisfy flavour physics constraints [8]. In the MSSM, supersymmetry is broken explicitly without reference to the mechanism involved, making it an EFT as well. Many varieties of supersymmetry (SUSY) have been proposed, including

B-L SUSY [9], grand unification SUSY [10], all the way up to supergravity [11] which can be related to string theory [12], a leading candidate for a quantum theory of gravity [13].

One may look to the oldest photons to deduce the abundance of various forms of energy in the early universe. The cosmic microwave background (CMB) radiation is an almost isotropic (after the subtraction of Earth’s rest frame’s effects) flux of photons of mm scale wavelengths, which has been surveyed to ever-increasing precision by modern space telescopes [14]. The energy spectrum of CMB photons is consistent with a Boltzmann (thermal) distribution. According to the best of understandings, these are the oldest photons that are visible to us today: blackbody photons emitted before the ‘recombination’ event in the evolution of the universe. As the universe expanded and cooled, the plasma of hadrons and leptons combined to form (light) neutral atoms. After recombination, the thermal photons that were in the plasma were free to reach us today, experiencing a redshift as spacetime expanded. There are observed ‘hot-spots’ in the CMB (see Figure 1.1); areas of solid angle that receive hotter radiation than others. These represent (after the subtraction of confounding effects that contribute to the cosmic infrared background [15, 16, 17], the effect of voids [18, 19], etc.) regions of space that contained a relatively higher particle number density in the primordial plasma. These fluctuations are said to have been the seeds of the large scale structure that we see today. The angular spectrum of these fluctuations in the CMB provides further limits on the constituents of the early universe.

To see how the abundances of different species affect the CMB, careful accounting of the various species is necessary. We denote the energy density of species i by ρ_i , and as a fraction of the critical density $\Omega_i \equiv \rho_i / \rho_{crit}$ with $\rho_{crit,0} = \frac{3H_0^2}{8\pi G}$ in the late universe (current epoch). The Friedmann equation for an isotropic universe relates the Hubble parameter H to the total energy density ρ_{tot} and scale factor a :

$$H^2 = \frac{8\pi G}{3} \rho_{tot} - \frac{k}{a^2} \quad (1.2)$$

where G is Newton’s gravitational constant, and k is the curvature parameter. If $\Omega_{tot} = \sum_i \Omega_i = 1$, then $k = 0$ and the universe is geometrically flat. There are three known components contributing to $\Omega_{tot} = \Omega_\Lambda + \Omega_r + \Omega_m$, where Ω_Λ is the dark energy density set by the cosmological constant, Ω_r is the radiation density and Ω_m is the mass density. Combining the Planck space telescope’s CMB data with results from Baryon acoustic oscillations (BAO) from galactic redshift surveys [20, 21], one finds that $1 - \Omega_{tot} = 0.001 \pm 0.002$, consistent with a flat universe. This presents a fine-tuning problem as Ω_{tot} is required to be very close to 1 in the early universe. This is solved by cosmic inflation models [22]. The late-universe energy density of dark energy is estimated at $\Omega_\Lambda = 0.6847 \pm 0.0073$, and that of matter is $\Omega_m = 0.315 \pm 0.007$ most of which is consistent with a pressure-less cold

(non-relativistic) dark matter (CDM) [14]. As dark energy and dark matter dominate the total energy of the universe, the standard model of cosmology is referred to as Λ CDM. Other properties such as the number of relativistic fermions $N_{\text{eff}} = 2.99 \pm 0.17$ may be deduced from the CMB when combined with BAO observations [14], and the result is consistent with three generations of neutrinos in the SM.

According to the concordance (Λ CDM) model of cosmology, the surface of last scattering that represents the ‘source’ of the CMB photons was around 300 thousand years after the singularity (see Figure 1.2). Earlier still, nucleosynthesis produced the relative abundances of light nuclei in the early universe. Thanks to the standard model of particle physics which has been extensively studied at energies exceeding 10 TeV, we can extrapolate further back to a time before hadrons, when a plasma of high-energy ($> \Lambda_{QCD} \sim 0.1$ GeV) quarks and gluons permeated space. At higher energies still, we reach the weak energy scale $\Lambda_{\text{weak}} \sim 250$ GeV. Before the electroweak SM cross-over [24] (or BSM phase transition [25]), fundamental particles were massless. Electroweak baryogenesis attempts to explain the baryon asymmetry [26, 27] using electroweak symmetry-breaking dynamics, but it requires BSM physics to explain the observed level of asymmetry [28]. Higher energy scales that have not been confirmed in laboratories include a scale $\Lambda_{GUT} \sim 10^{16}$ GeV where the three forces of the standard model unify under a grand unified theory (GUT), and the scale $\Lambda_{\text{Planck}} \sim 10^{19}$ GeV at which quantum gravity becomes important.

Thus the story of the early universe is a story of many interacting fields, all described by an *action* $S = \int \mathcal{L} d^4x$. The action may experience a discontinuity in its derivatives at some critical temperatures, such as the weak and GUT scales. At these points, the system is not necessarily in thermal equilibrium, and the dynamics may be non-trivial. Away from these hypothetical phase transitions, particle species maximize the total entropy through scattering, annihilation or absorption, achieving thermal equilibrium. Some particles may decouple or *freeze out* from the thermal bath, as is the case in neutrino decoupling which occurs when the expansion rate of the universe exceeds the rate of weak interactions ($\Lambda_\nu \sim 1$ MeV). After neutrino decoupling, the ratio of neutrons to protons freezes in at ~ 0.2 neutrons per proton [30]. The change in neutron numbers is then dominated by the weak decay of neutrons, until stable nuclei are formed. The formation of stable nuclei may not have proceeded until well after electron-positron annihilation and the cooling of the residual photons below ~ 0.1 MeV (dependent on the baryon-to-photon ratio η).

Since we are interested in dark matter, let us list the times in the universe’s history when it may have played an important role. We focus on times $t \gtrsim 10^{-24}$ s after the singularity, when the temperature had dropped well below the Planck scale Λ_{Planck} and Λ_{GUT} :

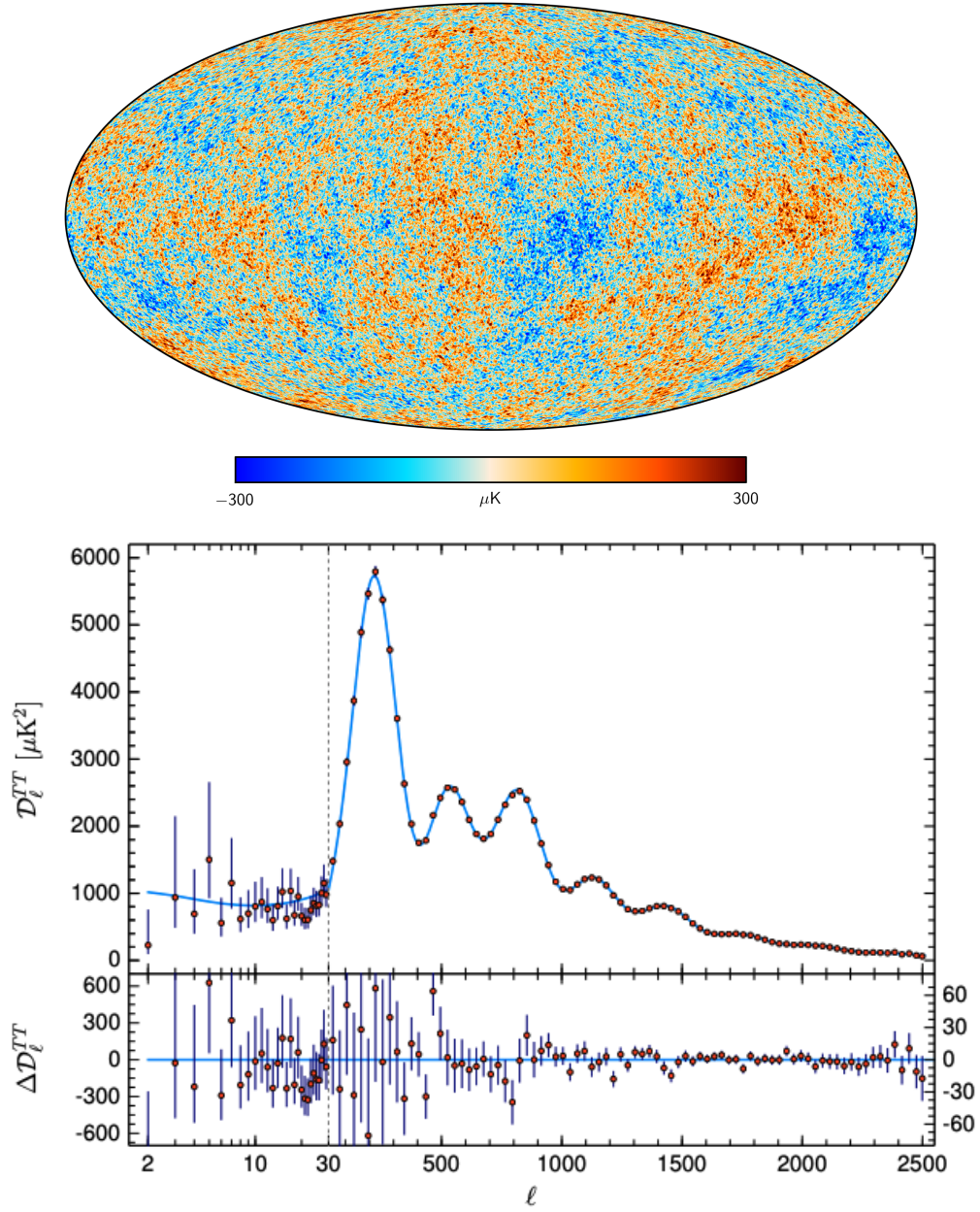


Figure 1.1: Top: Differences (anisotropies) in the measured cosmic microwave background's temperature relative to the mean as observed by the Planck space telescope [23]. Bottom: The angular power spectrum of the temperature anisotropies are used to extract cosmological parameters, including the relative energy densities $h^2 \Omega_c = 0.120 \pm 0.001$ and $h^2 \Omega_b = 0.0224 \pm 0.0001$ for cold dark matter and baryonic matter, respectively [14].

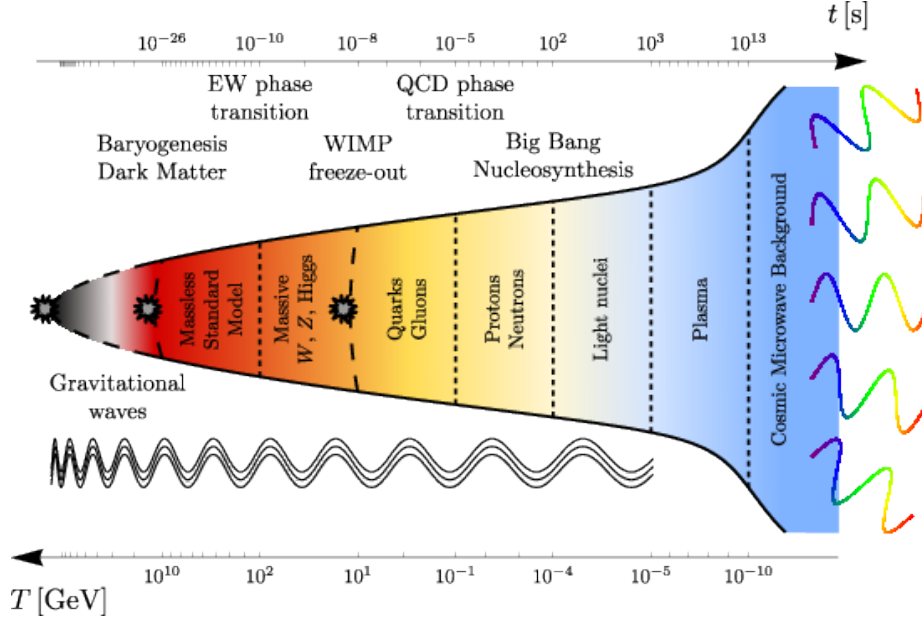


Figure 1.2: Schematic of the early radiation-dominated universe (up to recombination era), with important particle physics milestones highlighted. Retrieved from [29].

1. **Initial abundances:** The decay of the inflaton field [31, 32, 33], and the breaking of the GUT symmetry [34, 35], are hypothesized events in the very hot early universe, which produced an initially unknown relative abundance of SM particles and dark matter, because of dependence on the specifics of the UV completion of the SM. However, after the universe expands and cools down to temperatures $T \lesssim \Lambda_{GUT}$, the various fields would soon reach thermal equilibrium and standard model relative abundances will be set mostly by the electro-weak force. The particles are massless, so kinematic considerations are not yet relevant, and the electro-weak force was long-range. Part of the baryon asymmetry may have been generated by this time [36]. However, this is not easily testable, as the energy scales involved are inaccessible to Earth laboratories, and the opaqueness of the universe at those times prevents telescopes from looking further back than the CMB surface of last scattering. Gravitational waves may be used to test some of this [37], particularly if primordial blackholes are involved [38, 39], or if strong phase transitions occurred at those high scales [40]. Thermal dark matter particles could have already been produced by this stage as noted in Fig. 1.2. The DM number density would be dominated by the DM self-annihilation cross-section $\sigma_{DM+DM \leftrightarrow anything}$ under thermal production.

2. **Electroweak baryogenesis:** If a net baryon number is generated at temperatures $\Lambda_{GUT} > T \gtrsim \Lambda_{weak}$, then an extension of the weak sector is necessary to account for the level of baryon asymmetry observed [41]. The possibility of *asymmetric dark matter* (where the ratio of DM particles to anti-particles is not 1:1) is relevant here, as the baryon asymmetry may be the result of a common mechanism [42, 43]. Dark matter may carry lepton number (e.g. DM as a sterile right handed neutrino [44, 45]) and be involved in leptogenesis before baryogenesis [46, 47]. This can fit into the framework of supersymmetry [48, 49], where DM may be composed of a stable neutral supersymmetric particle. DM particles may also be produced ‘non-thermally’ via a dynamic process, such as the decay of a heavy parent (super)particle [50].
3. **Big bang nucleosynthesis:** When the temperature drops down to $T \lesssim \Lambda_{QCD}$, the strong nuclear force forms baryons out of quarks. The most stable baryons are the proton and neutron, which interact via pion exchange. After further cooling, the baryons fuse to form stable atomic nuclei heavier than ${}^1_1H^+$ (single protons), mostly deuterium, helium and lithium nuclei. These nuclei existed in a plasma along with electrons. Absorption lines from Lyman- α observations constrain the primordial relative abundance of light nuclei [51, 52], confirming this picture of big bang nucleosynthesis (BBN), but problems remain with the abundance of lithium [53]. This places severe constraints on the electromagnetic interactions of the dark matter, as its self-annihilation would cause a delay in the formation of stable nuclei, resulting in an over-abundance of hydrogen compared to helium.
4. **Recombination era:** When the universe cools down to $T \sim \Lambda_{ionization} \simeq 15\text{eV}$, recombination occurs, where the Baryonic matter forms neutral atoms that interact lightly with light. At this time, the universe is hundreds of thousands of years old. This is the time of the surface of last scattering for the thermal photons of the plasma of nuclei and electrons. The thermal spectrum of the CMB constrains the decay of heavy unstable particles into photons close to recombination. This puts limits on dynamical dark matter models with long-lived particles (LLPs) [54].
5. **Large-scale structure formation:** After baryonic matter forms neutral atoms, light becomes essentially decoupled from matter. Thus matter clumps and collapses under gravity to form nebulae, star systems, galaxies, and galaxy clusters that make up the large-scale structure of the universe. From observation, the universe appears to be homogeneous and isotropic over scales of $\sim$ billions of light-years (cosmological principle). Filament structures made out of galaxies and inter-galactic gases arise (Figure 1.3), yielding voids of size $\sim$ billions of light-years. These include the recently discovered Quipu structure [55] which spans over a billion light-years. Dark

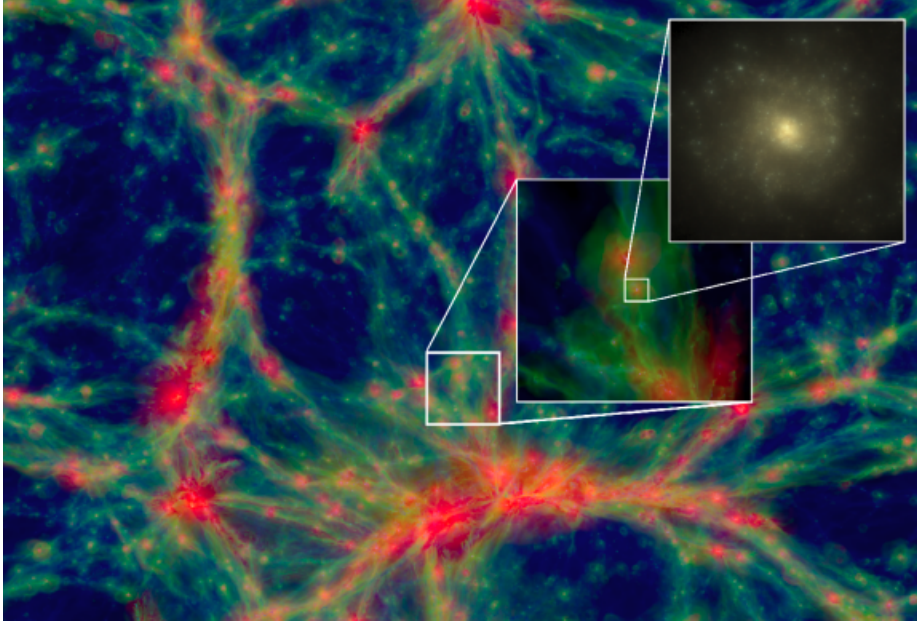


Figure 1.3: Result from a hydrodynamic simulation of large-scale structure formation (EAGLE project, see [56]). A high intensity indicates higher gas density, while the colour indicates temperatures as follows: blue indicates a temperature $T < 10^{4.5}$ K, green indicates $10^{4.5} \text{ K} < T < 10^{5.5} \text{ K}$, and red is $T > 10^{5.5} \text{ K}$. Inset axes show a series of zoom-ins onto a single galaxy. A strategy to observationally map these filaments is detecting faint extra-galactic UV light [58].

matter plays an essential role in the formation of large-scale structure. In fact, dark matter-only simulations of structure formation do a good enough job of predicting galactic formation. This includes hydrodynamic simulations [56] as well as N-body simulations which simulate the gravitational collapse of ‘super-particles’ of a thousand times solar-mass [57].

Thus it appears that dark matter has been affecting the universe since the dawn of time, as told by the CMB radiation coming from deep space. There are also vast regions of extra-galactic space in the late universe which exhibit behaviour consistent with there being localized energy densities that are collision-less. Weak gravitational lensing, or the statistical analysis of gravitational lensing effects caused by many galaxies and clusters, allows for the mapping of dark matter in the sky. See Figure 1.4 for the so-called ‘bullet cluster’. The infamous colorized image outlines regions of varying mass density based

gravitational lensing, overlaid with the distribution of light-emitting matter. The effects of dark matter are therefore omnipresent across spacetime. The cosmological evidence suggests that the universal matter density is mostly comprised of a cold, weakly interacting, almost collision-less, non-baryonic matter. Otherwise, modifications to general relativity (Eqn. 1.1) may be necessary. Many modified gravity hypotheses have also been proposed in the literature, which can explain some of the evidence for dark matter [59, 60]. In the next section, we continue to describe the observed effects of dark matter on smaller astronomical distance scales, followed by a brief overview of particle dark matter candidates.

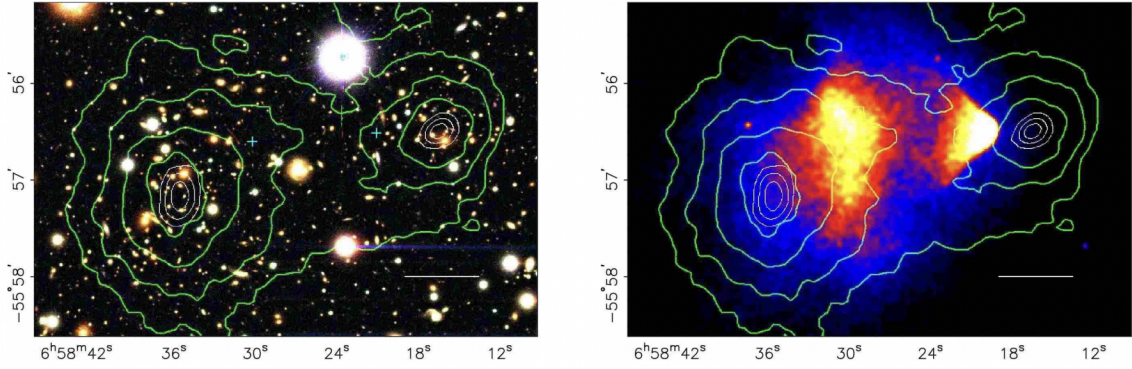


Figure 1.4: Green contours indicating a decreasing gradient of matter density in space surrounding two colliding galaxy clusters. On the left, the contours are displayed on the galactic view (the distortions in galaxy shapes due to gravitational lensing produce these gradient lines). The right sub-figure shows the same dark matter contours, but with a heat-map of intergalactic plasma density, obtained through observation of extra-galactic UV light. The two distinct maps in the right figure indicate that there are non-luminous components of light-bending matter that are ahead of the luminous matter in the trajectory before the collision (indicating weakly interacting or ‘collision-less’ matter). The results are taken from Ref. [61].

1.2 Particle Dark Matter Candidates

In listing key events in the history of the universe in the previous section, hypothetical dark matter particles were, perhaps prematurely, alluded to. From the cosmological evidence we have provided so far, a valid question to ask is whether it could be possible that dark matter is made up of astronomical objects that simply do not emit/strongly absorb light, such as

massive compact halo objects (MACHOs), which are not bound to a star system, including rogue planets, gas giants, and black holes. The number density of these objects would be so low that they are essentially collision-less. One argument against MACHOs making up a major component of dark matter would be that the CMB power spectrum implies the existence of a collision-less matter before recombination, when there were no atoms, let alone large gravitationally collapsed objects. Still, one may demand more direct, late-universe evidence that dark matter is not composed of MACHOs. The ‘dark’ gravitational lensing observed in Figure 1.4 may well have been generated by MACHOs. And the CMB fluctuations attributed to dark matter may have resulted from primordial black-holes (PBHs). A direct constraint on MACHOs is best obtained by looking at gravitational effects at sub-galactic (astronomic) scales.

Indeed, the first indication of the existence of dark matter came from anomalous galactic rotation curves, where the luminous matter distribution (with reasonable assumptions about the light-to-mass ratio for baryonic matter) could not explain the speeds of stars on the outer edges of galaxies [62] (see Fig. 1.5). From these curves one may obtain a ‘global’ estimate of the DM density as a function of radius from the galactic center by assuming a spherical halo. A spherical halo is a reasonable first-order assumption because the collision-less dark matter cannot dissipate energy through radiation and collapse into a disk as in the case of baryonic matter during galaxy formation. Tracer stars are used to deduce local densities outside the galactic disk ($\rho_{dm, ext}$ in Fig. 1.5) to deduce whether the DM halo is oblate (more disk-like) or prolate [57]. If MACHOs made up the majority of this DM halo density, then one would expect a higher rate of micro-lensing effects when a MACHO passes between earth and a star within our galaxy than is observed [63]. This leaves us with the possibility of PBHs that are massive enough ($\gtrsim 10^{15}$ g) to be stable on time-scales of the age of the universe. Microlensing observations indicate that black holes are unlikely to make up a large fraction of dark matter [64].

So, from cosmological and sub-galactic scales, it is apparent that there is a type of matter that remains unaccounted for by the known particles of the SM or by non-luminous astronomical bodies. It is therefore difficult to escape the possibility of a particle that is described by BSM physics. This particle must be weakly interacting to be collision-less and evade BBN limits. The SM has theoretical problems which inspire UV completions that naturally contain particles that can satisfy these requirements, as will be discussed in Sec. 1.2.1 and 1.2.2. Other possible explanations for anomalous rotation curves are available, such as modified Newtonian dynamics (MOND) [65], but these often struggle with explaining *all* of the evidence which includes the cosmological evidence in the previous section. However, it is worth noting that cosmology has some standing problems, including new observations of larger galaxies forming earlier in the universe than is expected under

Λ CDM, which happens to be a prediction of MOND [66].

If we accept the existence of the galactic dark matter halo, the quantities that characterize it would be of interest. To first order, the speed distribution of the constituent particles would be set by virialization, whereby the dark matter particles which acquire a speed larger than the galactic escape velocity v_{esc} leave the galaxy, while the gravitationally bound particles reach stable equilibrium, resulting in a local speed distribution that is close to a Maxwell-Boltzmann distribution that is truncated at v_{esc} . The Maxwell distribution is peaked at $v_0 \sim \mathcal{O}(v_t(R_0))$ where v_t is the tangential velocity at the solar system's radial position within the galaxy R_0 . Finally, the instantaneous velocity of Earth with respect to the galaxy $v_E(t)$ adds a fluctuating ‘wind’ of DM particles as the Earth revolves around the sun and the sun shoots within the galaxy. Ref. [57] summarizes the methods of estimating the local dark matter density and velocity profile, including simulations, rotation curve and tracer star/gas observations, deriving a velocity distribution $f(\vec{v}|v_{esc}, v_0, \vec{v}_E(t))$ with local mass density $\rho_{DM} \sim 0.3 \text{ GeV/cm}^3$ [67] which we shall assume for the rest of the thesis. Note that f is independent of DM particle mass. While this DM halo dominates the total mass of the Milky Way, the local energy density of DM is less than a trillion times the kinetic energy per unit volume of boiling water at standard atmospheric pressure!

Other properties of the DM halo can be derived from astronomical data. Dark matter self-interaction can affect the DM density profile within a galaxy potentially resulting in gravitational lensing signals [68]. And from the bullet cluster in Fig. 1.4, studies have attempted to draw conclusions about self-interaction of DM [69]. In what follows, we will focus on detecting the particles that make up the Milky Way's DM halo. But we must know what to look for: 1) the particles have to be well-motivated and fit into the framework of the SM; 2) the particles must plausibly be produced in the early universe leaving the late universe with the observed DM relic density $h^2 \Omega_c$ ($h = \frac{H_0}{100 \text{ km/s/Mpc}}$ is the reduced Hubble constant), with the DM's history being consistent with what is known of the cosmological history (listed in Sec. 1.1). We will discuss various particle DM candidates in the remainder of this chapter. The mass ranges of these particles are summarized in Fig. 1.6.

1.2.1 Weakly Interacting Massive Particles

Suppose that dark matter is produced before or around electroweak symmetry breaking through the decay of the inflaton field, breaking of the GUT symmetry, etc. The dark matter would soon thermalize with other fields, with the abundance of DM decreasing gradually as the universe cools down and the pair production of DM particles from the annihilation of others becomes increasingly kinematically inaccessible. If the DM remains in

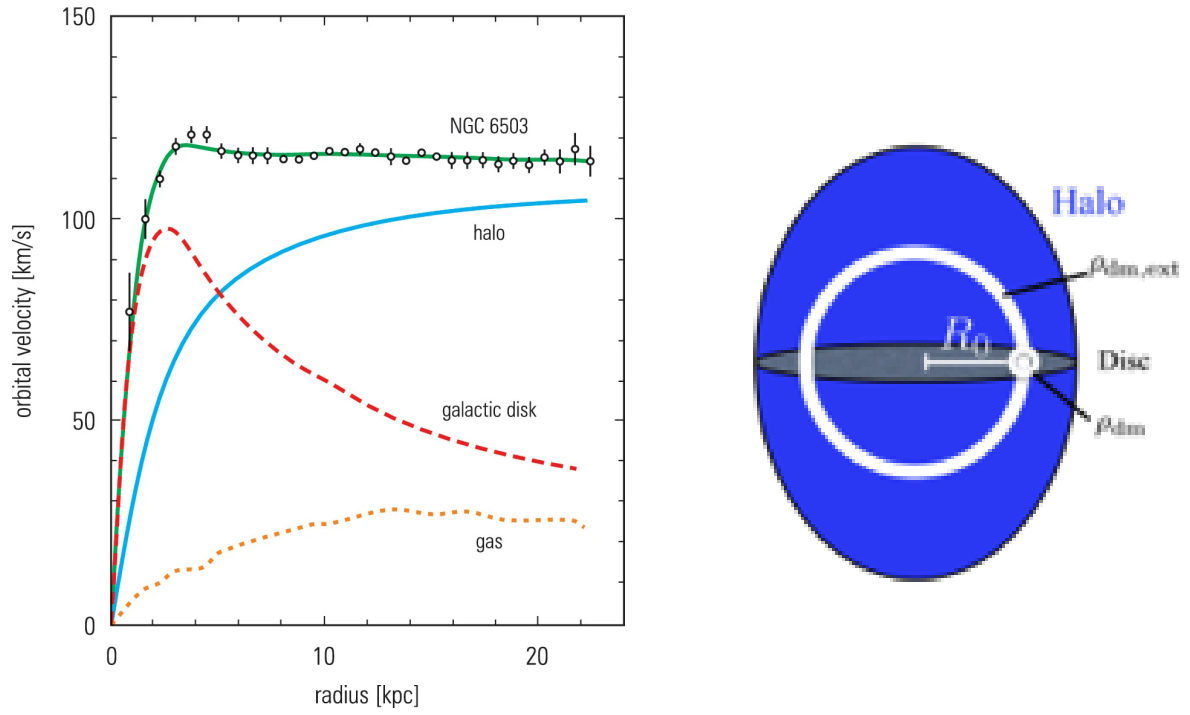


Figure 1.5: Left: Galactic rotation curves from NGC 6503 [70]. Right: Illustration of the galactic DM halo [57].

thermal equilibrium indefinitely with other fields, it would continue self-annihilating until well past photon decoupling (‘surface of last scattering’) from Sec. 1.1. However, when the expansion rate H (Hubble parameter) exceeds the rate of self-annihilation Γ , the DM starts to ‘freeze out’ at temperature $T_{f.o.}$ and thermally decouple from the bath, resulting in a relic density of CDM $h^2 \Omega_c \simeq 0.12$. The freeze-out temperature is the solution to $\Gamma = n_{eq} \langle \sigma v \rangle \sim H$, where $n_{eq} \sim (m_{DM} T)^{3/2} e^{-m_{DM}/T}$ is the number density of DM particles at equilibrium with the bath. If the DM is already cold by temperature $T_{f.o.}$, then the thermally averaged annihilation cross-section $\langle \sigma v \rangle \sim \sigma \sqrt{x} e^{-x}$ where $x = m_{DM}/T$. And in the radiation-dominated era $H \sim T^2/M_{Planck}$ where M_{Planck} is the Planck mass. The relic density of thermal cold dark matter is then given by [71]:

$$h^2 \Omega_c = \frac{1}{200} \left(\frac{m_{DM}}{T_{f.o.}} \right) \left(\frac{10^{-8} \text{ GeV}^{-2}}{\sigma_{DM+DM \leftrightarrow anything}} \right). \quad (1.3)$$

The fact that weak-scale DM cross-sections and masses plugged into Eq. 1.3 produce the cosmologically estimated relic density of DM is known as the WIMP (weakly interacting massive particle) miracle¹. This was taken as a hint that weak-scale particles make up DM, and the combination of DM masses and cross-sections which give $h^2 \Omega_c \sim 0.1$ are taken as benchmark parameters for experiments to probe. A well-motivated WIMP is realized in a popular solution to the Hierarchy Problem of the SM. According to the SM, the mass of the Higgs boson is highly sensitive to high-energy quantum corrections, making its small mass compared to the Planck scale a fine-tuning problem. A natural solution is to cancel out these quantum corrections by introducing superpartners for all SM particles: bosons have fermionic superpartners and vice versa (supersymmetry). The lightest supersymmetric particle (LSP) in supersymmetric completions of the SM is stable if R-parity (see Sec. 1.1) is conserved, making it a viable WIMP DM candidate. Particles that may conceivably constitute the LSP include the lightest neutralino (a combination of the superpartners of the electroweak and Higgs bosons), and the sneutrinos (the scalar superpartners of neutrinos), but bounds on the invisible decays of the Z-boson exclude the sneutrino as the LSP² in the MSSM [72]. The gravitino (fermionic superpartner of a hypothetical graviton) appears in supergravity theories [11], and may also constitute the LSP [73].

¹The mass range for WIMPs is $10 \text{ TeV} \gtrsim m_{DM} \gtrsim 1 \text{ GeV}$. The upper bound is derived from the Lee and Weinberg unitarity bound while the lower bound comes from the fact that $\sigma v \sim \frac{m_{DM}^2}{m_Z^4}$ (with m_Z being the Z-boson mass) becomes exceedingly small to produce the relic density thermally, leading to ‘overclosure’ of the universe ($h^2 \Omega_c > 0.12$).

²In direct detection experiments which are the main topic of this thesis, we are insensitive to the specific flavour of WIMPs. I present this information here only for reference.

Evading the WIMP Miracle

The WIMP benchmark discussed has been thoroughly probed by experiments as will be shown in Ch. 2. Light ($\lesssim 10$ GeV) dark matter particles can arise in dark sector completions of the SM which involve the addition of new force carriers. While this may be done within the context of SUSY [74], minimal completions with an additional ‘dark photon’ are useful in the context of experimental searches [75]. If the dark mediator is of similar mass or heavier than the DM particle, then the freeze-out scenario may still produce the relic abundance if asymmetric DM is assumed [43], or if the DM is a scalar or a Majorana fermion [76, 77]. But for the case of a light dark mediator the DM is never in thermal equilibrium with the SM, but may be produced via gradual freeze-in from annihilations of SM particles into DM [76]. Which SM particles couple to this mediator depends on the specific theory, as DM can couple mainly to leptons (leptophilic DM [78]), or in other cases to electromagnetically charged SM particles in general (through the mixing of the dark photon with the SM photon [79]). If the dark mediator is stable on cosmological time scales, it may also make up part of the DM density, as will be described below in Sec. 1.2.2.

1.2.2 Wave Dark Matter

If dark matter is composed of a bosonic field, its particles will behave like energy quanta. Instead of scattering with matter, they would mediate interactions between fermions, and may be directly absorbed by fermions thus acting as energy packets. Dark matter may well be composed (at least partially) of such sub-GeV bosons, being produced in the early universe during a phase transition associated with the breaking of the generating symmetry of the bosons. In the low mass range $\lesssim 1$ eV, the DM is conveniently described by the wave picture [80].

Massive Gauge Bosons

The dark photon introduced above to mediate forces between sub-GeV dark matter is a gauge boson corresponding to a broken $U_D(1)$ local symmetry [79]. The dark photon would mix kinetically via the operator $\mathcal{L}_{\text{mix}} = \frac{\varepsilon}{2} B_{\mu\nu} F'^{\mu\nu}$ with either the hypercharge gauge field or the electromagnetic field [81], where B and F' are the SM and dark gauge field strength tensors. The mixing results in weak interactions between DM and SM fermions, or in the case of massless dark photons, DM would behave as a ‘millicharged particle’ with fractional charge. Dark photons are stabilized against decay into two SM photons by charge conjugation and parity (CP) symmetry [82], and may make up part of

the DM relic abundance. Collider searches constrain the interaction strengths of heavy ($100 \text{ GeV}/c^2 \gtrsim m_D \gtrsim \text{MeV}/c^2$) BSM gauge bosons, as those would show up as resonances in pair production cross-sections or leave missing energy signatures if the bosons are long-lived [83, 84, 85]. Anomalous electromagnetic decays of Beryllium have been interpreted as a sign of the existence of an $\mathcal{O}(10) \text{ MeV}$ dark mediator [86]. Lighter dark mediators (down to keV masses) can alter charge distributions of nuclei, producing constraints on their coupling strengths [87].

Axions and Axion-like Particles

Another well-motivated boson that could make up dark matter is the QCD axion. The axion is a pseudo-scalar pseudo-Nambu-Goldstone boson which results from the breaking of an additional global $U(1)_{PQ}$ symmetry which is broken at high energy [88]. The strong nuclear force's Lagrangian appears invariant under chiral unitary transformations of the light quark fields (up, down and strange, neglecting the electroweak force). This does not appear to be the case observationally, as an effective CP symmetry-breaking term appears under chiral transformation due to quantum effects [88]. This CP-violating term would generate a neutron electric dipole moment (EDM) which is not observed [89, 90]. The smallness of the strength of this term is a fine-tuning problem. The spontaneous breaking of the $U(1)_{PQ}$ symmetry generates a vacuum expectation value for the axion field which precisely cancels the CP-violating term. Thus, the presence of the axion field can solve the strong CP problem. The effective axion interactions with photons and electrons are:

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}m_a^2\varphi^2 + J^\mu A_\mu + \mathcal{L}_{a\gamma\gamma} + \mathcal{L}_{ae}, \quad (1.4)$$

where φ is the axion field, $A_{\mu\nu}$ is the electromagnetic field and $F_{\mu\nu}$ is its strength tensor, while $\mathcal{L}_{a\gamma\gamma}$ and $\mathcal{L}_{ae}$ specify the interaction terms for axions with photons and axions with electrons, respectively [88, 91]:

$$\begin{aligned} \mathcal{L}_{a\gamma\gamma} &= -\frac{g_{a\gamma\gamma}}{4}\varepsilon^{\mu\nu\alpha\beta}\varphi F_{\mu\nu}F_{\alpha\beta}, \\ \mathcal{L}_{ae} &= g_{ae}\frac{\partial_\mu\varphi}{2m_e}J_5^\mu, \end{aligned} \quad (1.5)$$

where J^μ and J_5^μ are the vector and axial electromagnetic current densities. Electrons would radiate (through processes analogous to Bremsstrahlung or Compton scattering) axions if $g_{ae} > 0$. Photons can convert to axions in the presence of an external electromagnetic field through the 3-point interaction with coupling $g_{a\gamma\gamma}$. These interactions would

therefore contribute to stellar cooling [92], as any radiated axion may freely escape stellar cores providing some of the strongest constraints on a wide range of light axion masses ($0 < m_a \lesssim \mathcal{O}(10 \text{ keV})$) [93].

The QCD axion's interaction strengths are inversely proportional to its mass [88] given the relation to $m_a f_a \simeq m_\pi f_\pi$ where m_a , m_π are the masses of the axion and pion respectively while f are their decay constants. Axion-like particles (ALPs) are generalizations of the QCD axion that can have a wider range of masses and can be motivated from string theory [94]. Observations of blackhole spins are used to probe axions with masses as low as $\sim 10^{-14} \text{ eV}$ [95]. Cosmic ALPs with a mass $\gtrsim 20 \text{ eV}$ may decay via the interaction $g_{a\gamma\gamma}$ to photons which can be probed using telescopes that look for excess radiation (extragalactic background light, galactic x-rays [96]). Constraints may also be derived from the fact that some parameter combinations may result in the overabundance of axions as DM [97, 98]. ALPs may also be produced in Earth labs, providing a probe that is independent of cosmological and astronomical models [99]. For example, heavy ($\gtrsim \text{MeV}$ -scale and up to GeV-scale) ALPs may leave missing energy signatures or excess decay products in high-energy particle colliders [100, 101].

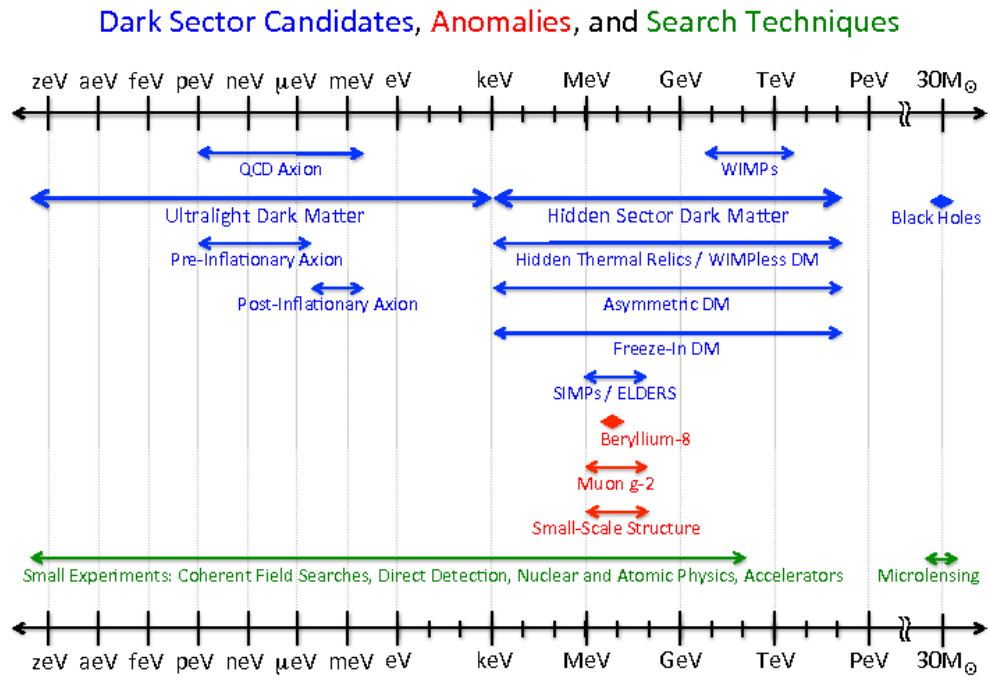


Figure 1.6: Particle dark matter candidates in various mass range [77].

Chapter 2

A Direct Detection Solution

In Chapter 1, I presented the cosmological evidence for dark matter (DM) that is well documented in the literature. We saw how dark matter, which is estimated to make up the vast majority of all matter in the universe, can collapse and form halos permeating galaxies, which alleviates their rotation curve problem. The WIMP miracle and other hints motivated many experiments that look for non-gravitational evidence of dark matter. Complementary approaches have been taken (Figure 2.1), including collider searches [75, 102], where the channel of interest is dark matter production rather than scattering with standard model (SM) baryonic matter. There also exist indirect methods of dark matter detection, such as looking for particle flux excesses or anomalous heating due to the accumulation of dark matter particles around and within a large celestial body [103, 104, 105, 106, 107], or even the production of a dark matter candidate by a celestial body [108, 109, 110, 111]. [112, 113]. In this chapter (and for the rest of this thesis) we will focus on *direct detection* techniques [114, 115, 116, 117, 118, 119, 120, 121], specifically the effect of the galactic dark matter halo on SuperCDMS semiconducting substrates [122, 123, 124, 125].

Direct detection experiments assume a halo of DM particles, measuring the rare recoil events between the particles and the interaction medium. If a cold (non-relativistic) DM particle of mass m_{DM} recoils off of an initially stationary component of the interaction medium, say a nucleus of mass m_N , then the energy transfer ΔE_N as a fraction of DM mass particle is given by:

$$\frac{\Delta E_N(p_N)}{m_{DM}} \equiv f(\eta, \beta, \theta) = 2\beta^2 \cos^2 \theta \frac{\eta}{(1 + \eta)^2} \leq \frac{2\beta^2 \eta}{(1 + \eta)^2}, \quad (2.1)$$

where $\eta \equiv \frac{m_{DM}}{m_N}$. Thus the ‘kinematic matching’ of the target with the DM mass under consideration is of the essence. For a target much heavier than the DM ($\eta \rightarrow 0$), the energy deposition as a fraction $f(\eta, \beta, \theta)$ of DM mass falls as linearly with η . The rate of this interaction is related to the the DM-nucleus cross-section $\sigma_{\chi N}$ as:

$$R = \int \frac{d\sigma_{\chi N}}{dE_R} \cdot n_{DM} \cdot v \cdot f(v, v_{esc}, v_0, v_E) \cdot d^3v \quad (2.2)$$

where v is the speed of the DM particle, f is the probability density of local DM having velocity v (dependent on the dispersion v_0 , speed of Earth in the galactic frame v_E and truncated at the galactic escape velocity v_{esc}), and the number density $n_{DM} = \rho_{DM}/m_{DM}$ in terms of the local DM density estimated in Ch. 1 at $\rho_{DM} \sim 0.3 \text{ GeV/cm}^3$ [57]. [126] In turn, the nuclear recoil differential cross-section is [75]:

$$\frac{d\sigma_{\chi N}}{dE_R} = \frac{m_N A^2 \sigma_{\chi n}^0}{2\mu_{\chi n}^2 v^2} F_N(E_R), \quad (2.3)$$

with F_N being the nuclear form factor.

Eqn. 2.3 is only one out of many possibilities for the form of the interaction of DM with nuclei; the relation is strictly valid for a spin-independent (SI) point-like interaction with nucleons through an effective 4-point operator. Many other nuclear interactions can be considered under the paradigm of nonrelativistic effective field theory (NREFT) [127], some being spin-dependent (SD) interactions represented by chiral operators. In this thesis, we focus on SI interactions of DM with baryonic matter. In the case of light-DM experiments that search for electron recoils, the phase space is more complicated than in the nuclear case, as the bound state electrons are in a superposition of different momentum states, and modeling the shell structure becomes necessary [76]. Absorption rates of dark bosons are calculated in terms of the photo-electric cross-section [128]. One may utilize the DM ‘wind’ experienced in Earth’s reference frame relative to the galactic halo which causes ‘annual modulation’ in the event rate of Eq. 2.2. Alternatively, one may use directional detectors [129] which provide angular information depending on the DM wind direction. In the following sections, we focus on direct detection by measurement of recoil event rates and energy distributions. In this approach, the modeling of background sources in the ‘region of interest’ (the expected range of DM energy depositions) is required for dark matter discovery.

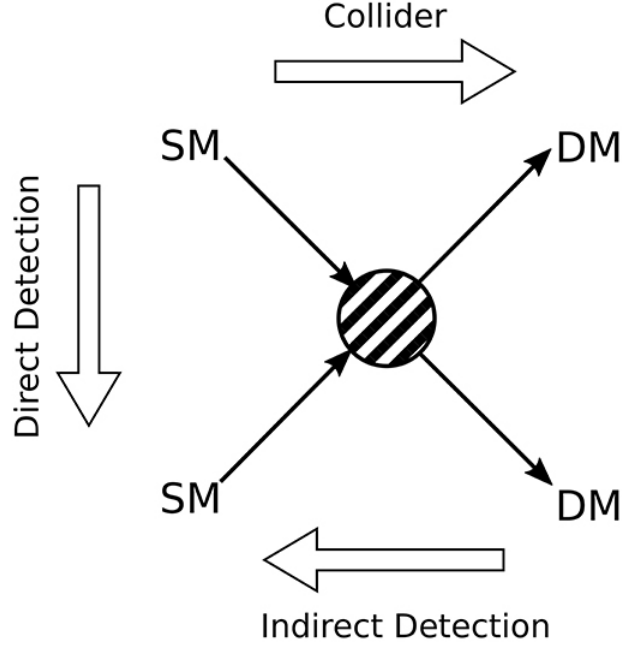


Figure 2.1: Representation of different approaches to dark matter detection [130].

2.1 Overview of Direct Detection Experiments

Technologies used in the detection of particles have evolved significantly since the invention of the cloud and bubble chambers in the early and mid 20th century. Charged particles passing through and depositing energy in fluids held at critical phases leave a trail of nucleation sites which result in condensation or bubble formation, allowing for particle path reconstruction. Paired with a background magnetic field, the charged particles follow curved paths of radii related to their mass to charge ratio. Bubble chambers were crucial in the discovery of countless hadronic particles produced in early accelerator experiments. Modern particle detectors rely on a plethora of electronic devices, including trackers made up of many pixel modules (such as charge coupled devices, resistive plate chambers and photomultiplier tubes), and calorimeters that measure the energy deposited by charged particles passing through them. The combination of these devices allowed the construction of giant machines that extract detailed information about particle collisions at a high readout frequency, culminating in the success of the Large Hadron Collider (LHC) in the discovery of the Higgs boson beyond an *unreasonable* doubt [131].

2.1.1 Neutral Particle Detection

In the framework of trackers and calorimeters, weakly interacting neutral particles are rarely directly observed. In collider experiments, unstable neutral particles manifest as resonances in the production cross-sections of final state charged particles, while stable, weakly interacting neutral particles appear as missing energy. Direct detection of weakly interacting neutral particles is mostly the domain of fixed-target, rare-event search experiments. To detect an incident weakly interacting neutral particle, one most commonly relies on secondary ionization produced by the initial interaction. This is usually followed by the detection of Cerenkov or scintillation light produced by the charged secondaries, as was done in the Sudbury Neutrino Observatory (SNO) and the Main Injector Neutrino Oscillation Search (MINOS) experiments [132, 133], among others. Most early dark matter searches also relied on scintillation signals, including DAMA which used NaI crystals [134], which was later followed by the use of liquid scintillators. The noble elements are well suited for scintillation as they are chemically inert and readily form excimers upon energy deposition, which then undergo radiative de-excitation. The largest such detectors now use many tonnes of liquid noble gasses as interaction media, and have excluded large regions of WIMP parameter space, such as the LUX-ZEPLIN (LZ) and the DEAP-3600 experiments which use liquid Xe and Ar, respectively [135, 136].

Nuclear recoils provide an enriched sample of neutral particle interactions thanks to the electromagnetic shielding provided by electrons, eliminating backgrounds from charged particles and photons if the recoil type can be discriminated. Liquid Xe experiments achieve nuclear recoil discrimination by comparing prompt scintillation and latent surface electroluminescence, while liquid Ar experiments exhibit pulse shape differences due to different electronic excitations by nuclear versus electron recoils. Time projection chambers (TPCs) detect charge and scintillation signals, with the advantage of sensitivity to direction [137]. Solid state detectors can be instrumented to directly measure the produced ionization. For example, semiconducting crystals can be instrumented with electrodes which collect the produced ionization, provided that a bias voltage is applied. Cryogenic bolometers instrumented with the most sensitive thermometers can detect phonons produced by particle recoils. For such solid detectors, complementary readout is often required for nuclear recoil discrimination. For example, CDMS and EDELWEISS detectors implement both phonon and charge readout [123, 114], while the CRESST experiment uses scintillating bolometers (CaWO_4 interaction medium) [138].

Over the past couple of decades, WIMPs (Sec. 1.2) with weak-scale masses have been thoroughly probed at typical Z-boson exchange cross-sections, yielding numerous null results [76]. Experiments of ever-increasing exposure are pushing towards sensitivity to

cross-sections $\lesssim 10^{-48} \text{ cm}^2$, beyond which the ‘neutrino fog’ would significantly limit the effect of increased exposure in many experiments (see Figure 2.2). Sub-GeV (non-WIMP) dark matter may be a part of a rich dark sector (see Sec. 1.2), motivating experiments to minimize energy thresholds (the minimum energy that can be measured efficiently and reliably by the detector). Solid state detectors cannot compete with the exposure of liquid noble gas experiments, but tend to have lower energy thresholds. The threshold is dependent on energy resolution (the precision of energy estimate, which is fundamentally limited by random fluctuations in the energy input over the time scale of the measurement, discussed below). Recall that the velocity distribution of dark matter particles depends on a dispersion v_0 related to local galactic orbital velocity (see Sec. 1.2), which in turn depends on the overall matter distribution in the galaxy but is independent of DM particle mass m_{DM} . Thus the (in principle) detectable energy of dark matter particles making up the DM halo is $\propto m_{DM}$. Unless the particles are bosonic (directly absorbed [139] in the case of keV-scale dark photons and ALPs introduced in 1.2.2), only a fraction of the detectable (kinetic) energy is deposited (Eqn. 2.1), constrained by kinematics.

Research and development of new dark matter detectors is ongoing, with gram-scale charge-coupled devices (CCDs) used by DAMIC-M and SENSEI as well as CDMS HVeV silicon chip bolometers pushing semiconducting substrates to their energy threshold limit by resolving single electron-hole pairs [141, 119, 142] and searching for MeV-scale light dark matter (LDM). Superconducting substrates are being investigated as interaction media for even lighter candidates in the keV mass scale [143]. Figure 2.2 summarizes the sensitivity of past and future direct detection experiments to the DM-nucleon cross-section. The case can be made that for sub-GeV fermionic dark matter, looking at electron recoils may extend sensitivity without lowering energy thresholds due to the kinematic matching observed in Eqn. 2.1. On the exposure frontier, high purity Ge experiments such as LEGEND [144, 145] that primarily search for neutrino-less double beta decay ($0\nu\beta\beta$), while lacking nuclear recoil discrimination, can be used to search for electron-recoil DM (ERDM) candidates such as milli-charged particles [146]. (The rate of electron recoils dominates that of nuclear recoils under those models. Dark bosons discussed in Sec. 2.1.2 below are another example. Nuclear recoil discrimination would not be of much use for these models).

Bubble chamber technology also continues to prove its worth in the PICO-60 experiment (optimized for SD interactions), where superheated C_3F_8 fluid is used as an interaction medium. Bubble nucleation by gamma and beta electron recoils is highly disfavoured, while the rejection of alpha events is done using acoustic sensors [120]. The Scintillating Bubble Chamber (SBC) experiment [147] uses scintillation to improve the energy resolution. Finally, spherical proportional counters (SPCs) used in experiments such as the NEWS-G [148] employ a gaseous interaction medium to search for sub-GeV dark matter.

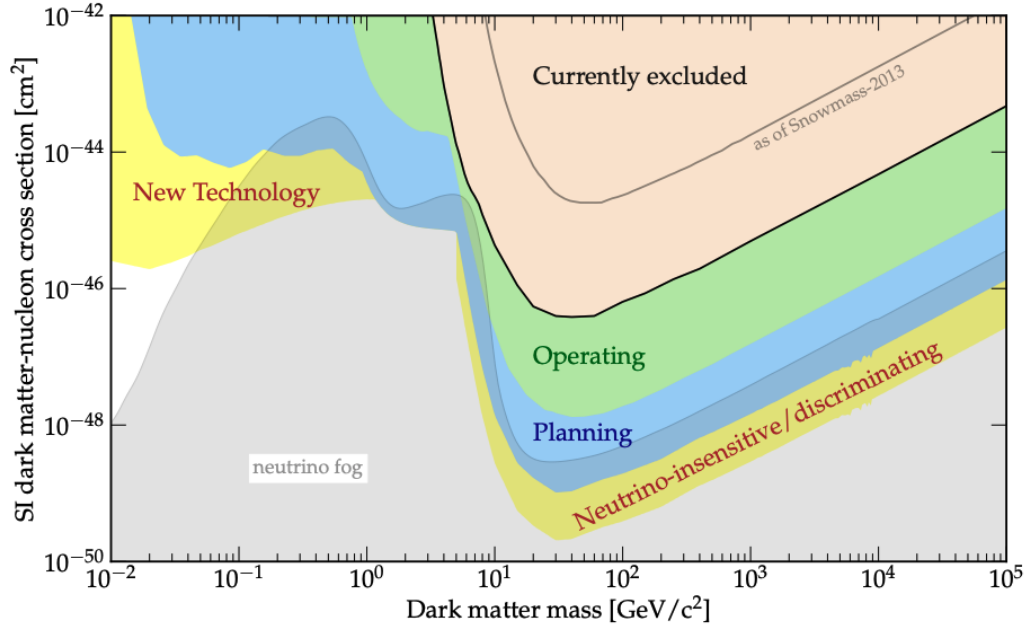


Figure 2.2: Summary of current and projected experimental upper bounds on DM-nucleon cross-section [140]. The direct detection searches are limited at high mass by a decreasing number density (to keep local DM mass density constant), and at low mass by the non-zero energy threshold. The neutrino fog is an irreducible background due to neutrinos from various sources which is discussed in Sec. 2.2.1.

These experiments complement each other in covering various ranges of DM parameter space.

2.1.2 Wave Detection

As discussed in Sec. 1.2.2, the properties of bosonic DM of $\lesssim$ eV scale mass are conveniently captured in the wave picture. If the galactic DM halo is made up of such a field, the approaches described above for neutral particle detection may not be optimal. Ultra-light cold DM axion is probed by haloscopes such as ADMX [149], which uses microwave resonance cavities that capture radiation from axion conversion in a magnetic field. Broad-band searches such as ABRACADABRA [150] look for an induced oscillation in a strong magnetic field. Quantum sensors that employ smaller resonance cavities and detect single photons using qubits are investigated in the literature [151]. Gram-scale CCDs and bolometers may also detect $\gtrsim 1$ eV/ c^2 halo axions via direct absorption under the assumption that the axions couple directly to electrons [141, 119, 142]. Boosted light axions may be produced in the sun, motivating solar axion searches such as the CAST helioscope [152] which uses strong magnetic fields and photomultipliers to detect x-rays from axion conversion. While this is an indirect approach to DM search, direct detection experiments such as CDMS are sensitive to much the same solar axion parameter space (through the axion-photon coupling $g_{a\gamma\gamma}$, see Sec. 1.2.2) as helioscopes [153]. High exposure Ge $0\nu\beta\beta$ detectors such as the proposed LEGEND-1000 [145] also show promise for use as helioscopes [154].

2.2 The Cryogenic Dark Matter Search

The Cryogenic Dark Matter Search (CDMS) is a decades-long international program that aims to directly detect dark matter using cryogenic semiconducting crystal bolometers. The interaction media include Si and Ge crystals, which produce ionization at a low threshold of the medium’s band-gap $E_{gap} \sim \mathcal{O}(1 \text{ eV})$. The crystals are instrumented with **transition-edge sensors** (TESs) of eV-scale resolution, making CDMS detectors optimal for low-mass ($m_{DM} \lesssim 10 \text{ GeV}$) searches. In addition to interaction medium complementarity, some detector modules include charge readout, offering robust nuclear recoil discrimination using the ionization yield (charge produced relative to total energy deposited) as a function of recoil energy $Y(E_R)$. CDMS I (1998-2002) was the first experiment in the CDMS series [155]. It took place in a shallow underground facility at Stanford University (SUF). It was followed by CDMS II (2003-2009) at a mine in Soudan, Minnesota [156]. Then there

was SuperCDMS Soudan (2009-2014) with larger detectors and exposure [157]. Finally we have SuperCDMS SNOLAB, which started construction in 2018 with larger detectors, more room for detector towers, and better background controls [158]. The SuperCDMS SNOLAB setup is described in Section 2.2.1.

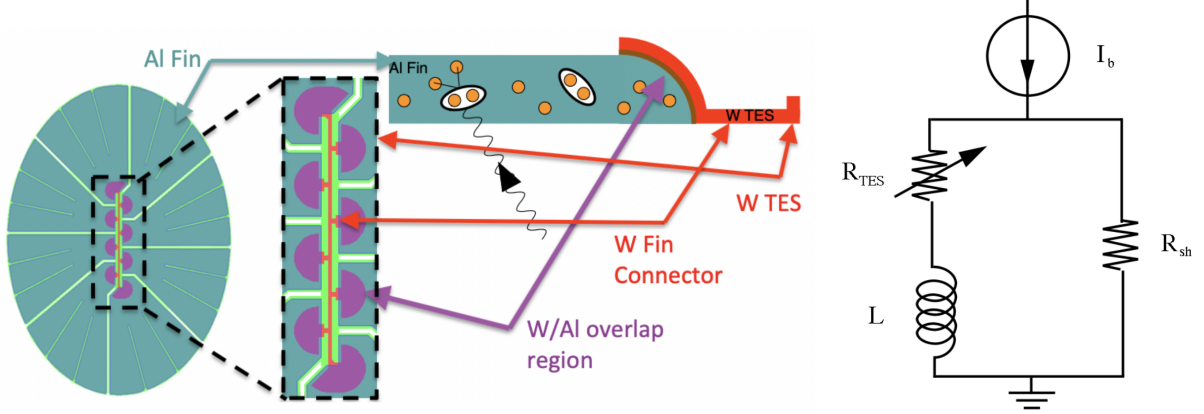


Figure 2.3: Left: Schematic of a Quasiparticle trap assisted Electrothermal feedback Transition edge sensor (QET) used in CDMS detectors for phonon detection. Right: the overall TES circuit with bias current I_b , shunt resistor R_{sh} , the variable transition edge sensor (TES) R_{TES} , and the inductance L used for readout by a DC SQUID.

A CDMS detector is composed of an interaction medium (a high purity *Si* or *Ge* crystal) that is equipped with Quasiparticle-trap-assisted Electrothermal-feedback Transition edge sensors (QETs), operated at millikelvin temperatures (Fig. 2.3). When a particle deposits energy to a component (an atom, a nucleus, or an electron) of the crystal, relaxation of the initial perturbation site releases recoil phonons and charge carriers (Fig. 2.4). In semiconductors, energy at least equal to the bandgap (between the top of the valence band and the bottom of the conduction band) is necessary to release electron-hole (e^-h^+) pairs ($E_{gap} \sim 1.31$ eV for silicon at millikelvin temperatures [159]). If an electric field is applied across the substrate, the charge carriers will be drifted across the detector, reaching a terminal velocity $\sim v_{sound}$, beyond which the charge carrier would lose energy via Neganov-Trofimov-Luke (NTL) phonon production faster than it can receive energy from the electric field (interactions of the fast charge carrier with the lattice are resonantly enhanced) [160]. Thus the NTL effect can be used to amplify the phonon signals of ionization events [161]. Finally, the e^-h^+ pairs would recombine at the surfaces, releasing their kinetic and band-gap energies as phonons (Si and Ge, being indirect semiconductors, are inefficient at producing luminescence). Thus by energy conservation the total phonon

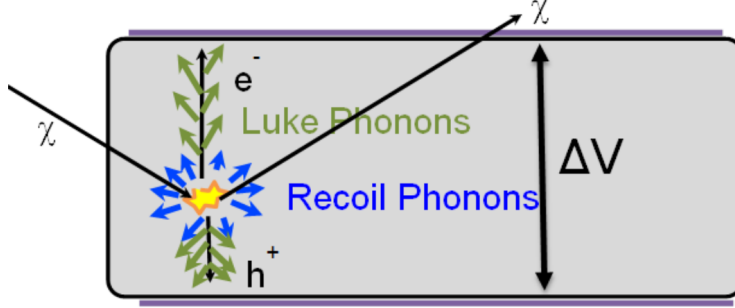


Figure 2.4: Illustration of phonon and ionization production due to particle χ 's recoil against a nucleus in a semiconducting crystal biased with a voltage ΔV . If electron-hole (e^-h^+) pairs are produced, they are drifted across the substrate while releasing Neganov-Trofimov-Luke (NTL) Phonons in addition to prompt recoil phonons.

energy E_t of an event is given by:

$$E_t = E_r \left(1 + Y(E_r) \frac{e \Delta V}{\varepsilon_{e-h^+}} \right), \quad (2.4)$$

where E_r is the primary energy deposition or recoil energy, ε_{e-h^+} is the mean energy required per e^-h^+ pair produced (in the limit of many pairs produced), and $Y(E_r)$ is the ionization yield of nuclear recoils relative to electron recoils. Of course, the total phonon energy is not always the same as there are stochastic processes involved in the production and propagation of e^-h^+ pairs. Equation 2.4 gives the first moment (expectation value) of the total energy in the limit of many e^-h^+ pairs ($N_{e-h^+} \gg 1$). In that limit the second moment (or variance) is conveniently encoded in the Fano factor $F = \sigma^2/\mu$ of charge carrier production [162], which would be $F = 1$ if each charged pair was produced randomly and independently. In Si, the Fano factor measured using x-rays is $F \sim 0.155$ [163]. The mean energy per e^-h^+ pair for silicon is $\varepsilon_{e-h^+} \sim 3.8 \text{ eV}$ [164], which is notably larger than the band-gap because of various inefficiencies in the conversion of the primary recoil particle's energy into ionization. The quoted values for F and ε_{e-h^+} are asymptotic (high energy) values used in the statistical parametrization of charge carrier production. Precise ionization models based on simulation and empirical data are required when $N_{e-h^+} \lesssim 10$, where temperature-dependent effects may also be taken into account [165].

If the interaction deposits energy $E_r > E_{gap}$ to an electron ($Y = 1$), then a primary charge carrier pair is produced, with excess energy (above the band-gap) partitioned be-

tween the electron and hole according to the electronic shell structure of the material and available phase space. The initial electron and hole then scatter with other bound electrons, releasing a cascade of e^-h^+ pairs through impact ionization [166]. But each secondary scattering also produces phonons, so $\varepsilon_{e^-h^+}$ is mostly set by the relative rates of phonon production and impact ionization [165]. In the case of nuclear recoils, one must model and/or measure the ionization yield function $Y(E_r) \leq 1$ which gives the efficiency of ionization relative to electron recoils. The Fano factor of charge-carrier production also varies for nuclear recoils compared to electron recoils [167]. An estimate for $Y(E_r)$ may be derived using the Lindhard model [168, 169], which is valid at high recoil energies ($E_r \gtrsim 10$ keV). The model uses the formalism of stopping powers to calculate the average energy lost to secondary ion-ion recoils, treating ion and electron scattering separately, assuming ionized electrons do not produce nuclear recoils, and that energy transfers to ions are much larger than those to electrons [170]. It is also assumed that E_r is much larger than the subsequent energy transfers to ions, which in turn is much larger than the electron binding energy [171]. A solution for the ionization yield with optimizable parameters may then be derived. Significant deviations from the Lindhard parametrization have been observed at low recoil energies, including in CDMS neutron calibration data [172, 173].

The athermal phonons from the initial recoil, as well as NTL phonons from charged secondaries, propagate through the crystal. If the crystal is harmonic, lattice sites can be treated as independent oscillators, and the phonons propagate through the crystal bulk or reflect from boundaries without losing energy until they are absorbed. Higher order phonon-phonon interactions found in real crystals allow for *anharmonic down-conversion*, whereby a phonon may split into two or more phonons of lower energy as it propagates across the crystal [174]. Once a phonon is as likely to down-convert into two phonons of lower energy as it is to fuse with another into a higher energy phonon, it is said to have thermalized. Thermal phonons are abundant within any solid at non-zero temperature, following a thermal Bose-Einstein distribution peaked at $k_B T \sim \mathcal{O}(\mu\text{eV})$ at millikelvin temperatures. Some of the athermal phonons are absorbed by the superconducting aluminium fins shown in Figure 2.3, breaking Cooper pairs (states of 2 electrons bound by phonon mediated force, see [175] for review of BCS theory) and creating superconducting quasi-particles. Phonons must have energy larger than the aluminium superconducting energy gap ($\sim 180 \mu\text{eV}$) to be absorbed by the fins (at 10 mK operating temperature, thermal phonons are not absorbed by the fins). Of course, the athermal phonons must also be incident on the aluminium fins to be absorbed by them. Thus, some phonon energy may escape the crystal as heat, particularly through un-instrumented crystal surfaces and into the detector holders.

The aluminium fins are coupled to tungsten transition edge sensors (TESs) with critical temperature $T_c \sim \mathcal{O}(100)$ mK, held at transition between superconducting and normal state by a bias current I_b (Fig. 2.3). The interface between the aluminium and tungsten has a smaller energy gap than the aluminium bulk, which helps trap the superconducting quasiparticles near the tungsten TES [176], and once they recombine they heat the TES, raising its resistance thus decreasing the current I_s passing through it, resulting in a deficit in ohmic heating which is our signal. This results in a negative electro-thermal feedback that sees the TES cool down and return to *baseline* within a time τ_{BW} as the power balance is restored in the TES [177]:

$$\begin{aligned} P_{bath} + P_{Joule} + P_{parasitic} &= 0, \text{ quiescent baseline} \\ P_{bath} + P_{Joule} + P_{parasitic} + P_{phonon} &= C \frac{dT}{dt}, \end{aligned} \tag{2.5}$$

where P_{bath} is the heat lost by the TES to the thermal bath within which it resides, and P_{Joule} is the Joule heating by the current I_s passing through the TES, and $P_{parasitic}$ is an environmental power source (electromagnetic fields, including from communication devices). When the bath temperature stabilizes, and the bias current I_b is pumped, the quiescent (equilibrium) state is achieved if no other power source is present. In the quiescent state, the TES current I_s is stable around its baseline value I_0 (within noise and parasitic power fluctuations). During a particle hit the phonons generated (and absorbed) will be an additional power source P_{phonon} , and the TES temperature as a function of time is described in the derivative in Eqn. 2.5, where C is the heat capacity of the TESs. The result is a fast-rising pulse (as the relevant processes delivering energy to the TESs propagate at the speed of sound v_{sound} in the substrate, the rise-time is $\tau_{rise} \sim L_{thick}/v_{sound} \sim \mathcal{O}(\mu s)$ for detector of thickness $L_{thick} \sim \mathcal{O}(\text{cm})$). The pulse has a longer fall-time ($\tau_{BW} > \tau_{rise}$) depending mostly on the strength of the electrothermal feedback, which is energy-dependent. In different runs, the parasitic power may vary, so a different I_b may be needed to achieve the same “working point” on the Resistance-Temperature (RT) curve of the TES.

To characterize the TES response, one varies I_b and measures the baseline $I_s = I_0$ at each bias. This can be converted to other TES quantities by finding the total resistance

and potential difference:

$$\begin{aligned}
\frac{1}{R_{total}} &= \frac{1}{R_{sh}} + \frac{1}{R_{TES} + R_p}, \\
\frac{1}{V_{total}} &= \frac{1}{I_b \cdot R_{total}} = \frac{1}{I_b \cdot (R_{sh})} + \frac{1}{I_b \cdot (R_{TES} + R_p)} = \frac{1}{I_s \cdot (R_{TES} + R_p)}, \\
\Rightarrow \frac{I_b}{I_s} &= 1 + \frac{R_{TES} + R_p}{R_{sh}},
\end{aligned} \tag{2.6}$$

where R_p is the parasitic resistances of the TES. The shunt resistor shown in Fig. 2.3 has a known resistance of R_{sh} . At superconducting phase, $R_{TES} = 0$ ($R_{p,TES}$ can be deduced), while in the normal phase $R_{TES} = R_n$ where $R_n > 0$ is a constant.

The CDMS bolometers are therefore classified as sampling calorimeters which measure a fraction ϵ_p of the produced phonons. The parameter ϵ_p is called the *phonon collection efficiency*, and it can be optimized by varying aluminium fin and TES lengths and overall coverage (Fig. 2.10), in addition to minimizing the surface area of the (un-instrumented) side-walls. The efficiency ϵ_p can be measured by first converting event pulse amplitudes into an absorbed energy estimate E_{abs} using circuit properties [125]:

$$E_{abs} = \left(1 - 2\frac{R_l}{R_l + R_0}\right) I_b R_{sh} \int (I_0 - I_s(t)) dt + R_l \int (I_0 - I_s(t))^2 dt, \tag{2.7}$$

where $R_l = R_{sh} + R_p$ and R_0 is the baseline TES resistance, R_p is the parasitic resistance, R_{sh} is the shunt resistance, I_b is the bias current, I_0 is the baseline TES current and $I_s(t)$ is the instantaneous TES current at time t . The collection efficiency is found by evaluating the fraction $\epsilon_p = E_{abs}/E_t$ where E_t is the total expected phonon energy from Eqn. 2.4. The constant ϵ_p bakes all the processes (collection efficiency by the aluminium fins, quasiparticle diffusion efficiency and conversion to heat energy in the tungsten) into an overall constant.

In a nuclear recoil search, one may use the measured value of $Y(E_R)$ to remove the vast majority of electron recoil backgrounds (right side of Fig. 2.6). Since Equation 2.4 has 2 unknowns (E_R and $Y(E_R)$), an independent measurement of ionization is required to find $Y(E_R)$ for a single event, given E_t from the phonon sensors. The measured yield is defined as $Y(E_R) = E_Q/E_r$, where $E_Q = N_{e^{-}h^{+}} \epsilon_{e^{-}h^{+}}$ is the measured ionization signal. The SuperCDMS detectors that read charge and phonons simultaneously are of the interleaved Z-Sensitive Ionization and Phonon-mediated (iZIP) type. The charge carriers are collected by electrodes on each face of an iZIP detector (left side of Fig. 2.6, showing charge collection electrodes interleaved with linear QET arrays). These electrodes provide a potential difference ΔV across the crystal, with the detector faces being held at equal

and opposite potentials. Optimized electrode distributions ensure a constant electric field across the fiducial volume of the detectors. The electrodes feed into high-electron-mobility transistors (HEMTs) for signal amplification in SuperCDMS SNOLAB detectors, while silicon junction field-effect transistors (JFETs) were used for that purpose in previous CDMS iterations. Another type of SuperCDMS detector is the high voltage (HV) detector which only measures phonons. The QET arrays are more densely packed on an HV detector (no charge collection electrodes), while the NTL gain is maximized by applying a high potential difference across the crystal (see Eq. 2.4 and Fig. 2.4 and surrounding text). The recoil energy threshold of the HV detector is thereby lowered significantly, albeit at the expense of nuclear recoil discrimination. The first CDMS run to utilize the HV mode (using iZIP detectors with increased bias voltage) was called CDMS low ionization threshold experiment (CDMSlite) [124].

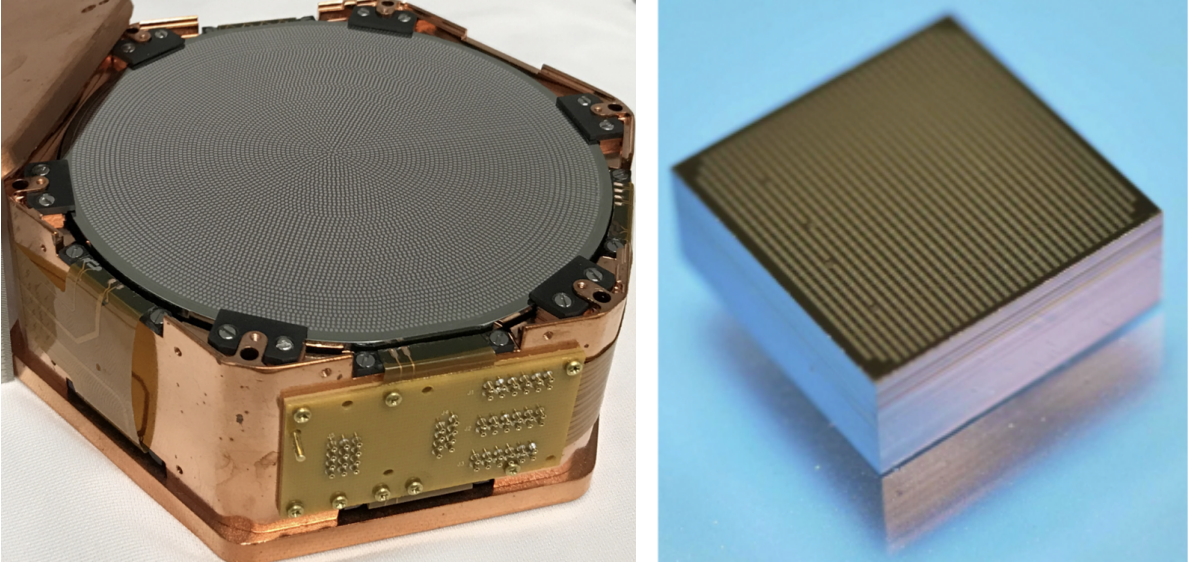


Figure 2.5: Left: SuperCDMS SNOLAB Ge high voltage (HV) detector instrumented with QET arrays, shown in its detector holder. SNOLAB detectors have a diameter of 10 cm. Right: SuperCDMS Si HVeV (1 cm side length) prototype showing the QET instrumented face. Lacking charge readout, these detectors cannot discriminate nuclear recoils but have high resolution and low thresholds.

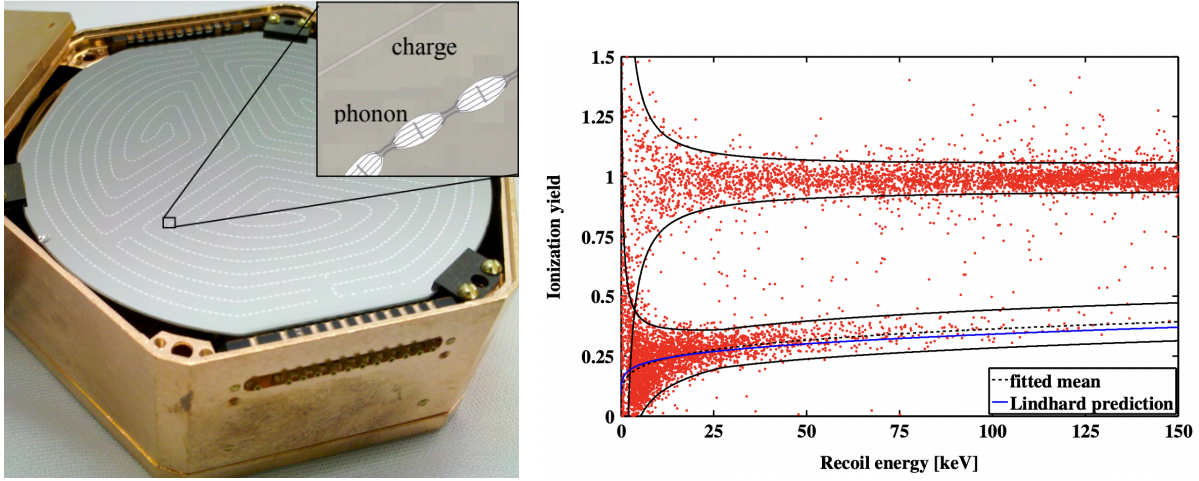


Figure 2.6: Left: Image of a SuperCDMS Soudan Ge interleaved z-sensitive ionization and phonon-mediated (iZIP) detector of diameter 10 cm in its holder[178]. Right: Distinct bands in ionization yield for nuclear recoil discrimination possible due to combination of charge and phonon readout. [179].

2.2.1 SuperCDMS SNOLAB Experiment

Situated 2 km underground in the Creighton Mine in Sudbury ON, Canada, with an overburden providing ~ 6000 meters of water equivalent (m.w.e.) shielding, SuperCDMS SNOLAB's main goal is to probe WIMPs in the $\lesssim 10$ GeV mass range through nuclear recoils [123]. This low-mass (compared to the weak scale) sensitivity is made possible through the reduced energy thresholds of the ~ 10 cm wide detectors compared to tonne-scale experiments which focus on weak-scale masses. Sub-GeV DM can also be probed with SuperCDMS SNOLAB through electron recoils. The experimental setup of SuperCDMS SNOLAB is shown in Figure 2.7. The experiment is built on a seismic platform to reduce vibrational backgrounds from the surrounding rock. An initial payload of 24 3.3 cm thick detectors of 10 cm diameter sit in four copper detector towers, which in turn sit in concentric copper cans which make up the 'SNOBOX'. A dilution refrigerator is thermally linked to the inner copper cans. The SNOBOX is shielded from magnetic fields by a mu-metal shield. It is further surrounded by a high-density polyethylene (HDPE) neutron shield, which is surrounded by lead bricks to shield against gamma radiation. The outermost layer of shielding is composed of water tanks and polyethylene for additional neutron shielding [158].

The initial payload for SuperCDMS SNOLAB is composed of germanium and silicon

Tower Number	Detector Type	Ge	Si
1	iZIP	6	0
2	HV	4	2
3	HV	4	2
4	iZIP	4	2

Table 2.1: SuperCDMS initial payload summary.

substrates. The different substrates provide complementarity as the silicon detectors have lower energy thresholds, while using germanium allows higher exposure per unit volume due to its larger atomic mass. The detector types are either iZIP (left side of Fig. 2.6) or HV (left side of Fig. 2.5). Table 2.1 summarizes the contents of each detector tower in the initial payload. SuperCDMS detectors are event position sensitive as the QET arrays are separated into channels (6 channels per face). SuperCDMS iZIPs are fitted with interleaved phonon sensors (QET arrays) and charge collection electrodes. The QETs are grounded while the charge collection electrodes are biased, such that the electric field points across the detector bulk but is parallel to the faces when close to said faces, potentially allowing for surface event rejection [180]. The HV detectors on the other hand have densely packed QETs on both faces, maximizing phonon collection efficiency and energy resolution. The high crystal bias voltage means most of the phonons are NTL phonons, so the detector is indirectly measuring ionization through phonon emission. The high voltage stretches the ER and NR spectra differently according to the respective ionization yields, resulting in partial removal of ER backgrounds in NR searches. Also, if single e^-h^+ peaks are resolved, NR peaks would be shifted to higher energy compared to ER peaks.

Background Sources

While creative limit-setting methods exist for the case of an unknown background [181], a decisive dark matter discovery requires a comprehensive understanding of background events. The most significant source of background events is due to the presence of impurities within the semiconducting interaction media. Traces of tritium (3H) are present in all substrates, which decays via β emission with a lifetime of a couple of years. In silicon detectors, ^{32}Si traces also decay through β emission over a period of years. Cosmogenically activated Germanium including ^{71}Ge decays via electron capture, producing event spectra that are sharply peaked around binding energies of K, L and M shell electrons. Other experiment materials such as the copper used in detector towers can be contaminated with radio-isotopes or cosmogenically activated, adding alpha, beta and gamma backgrounds.

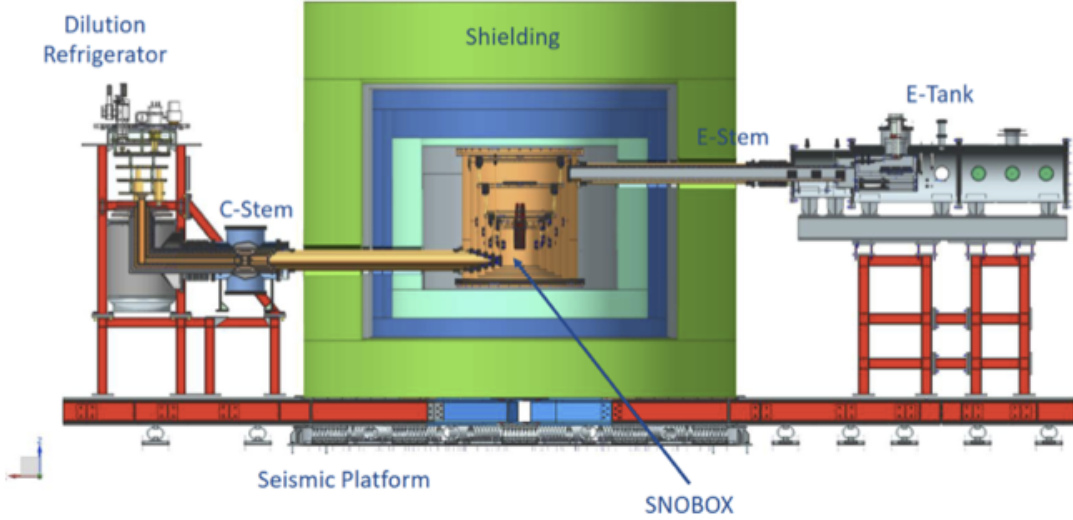


Figure 2.7: Setup of the SuperCDMS SNOLAB experiment [158]. Outermost shield (green) is composed of water tanks and polyethylene, followed by a thin (gray) layer of aluminium acting as a radon diffusion barrier, followed by the lead shield (blue), followed by the inner polyethylene layers (turquoise).

Radon gas can decay and implant daughter particles such as ^{210}Pb on detector surfaces. Radon can be purged from line-of-sight regions surrounding the detectors by displacement using nitrogen gas. Radio-isotopes in the experimental environment are a source of gammas and neutrons, as will be elaborated upon shortly.

The energy spectra of the background sources are estimated using GEANT4 [182] based simulations, and the magnitude of the background event rates is estimated by assaying the materials [123]. This information is then used to generate the projected sensitivity shown in Figure 2.8, which also shows the neutrino fog [183] beyond which sensitivity scales weakly with increasing exposure. Solar, cosmic and atmospheric neutrinos coherently scatter with nuclei [184, 185, 186], presenting an irreducible background to nuclear recoil DM searches. Target complementarity can help disentangle neutrino and DM signals [187]. Other cosmogenic backgrounds are easier to shield against. The atmospheric muon flux is reduced millions of times thanks to the 2km of overburden above the SNOLAB facility. The cavern in which the experiment sits is, however, itself a background source, as trace radio-isotopes (namely ^{238}U and ^{232}Th) in the rock produce gammas and neutrons along their decay chains. Gamma rays are also produced in the chain of decay of ^{222}Rn gas in the cavern. Neutrons may also be produced via spallation, either by alpha particles or

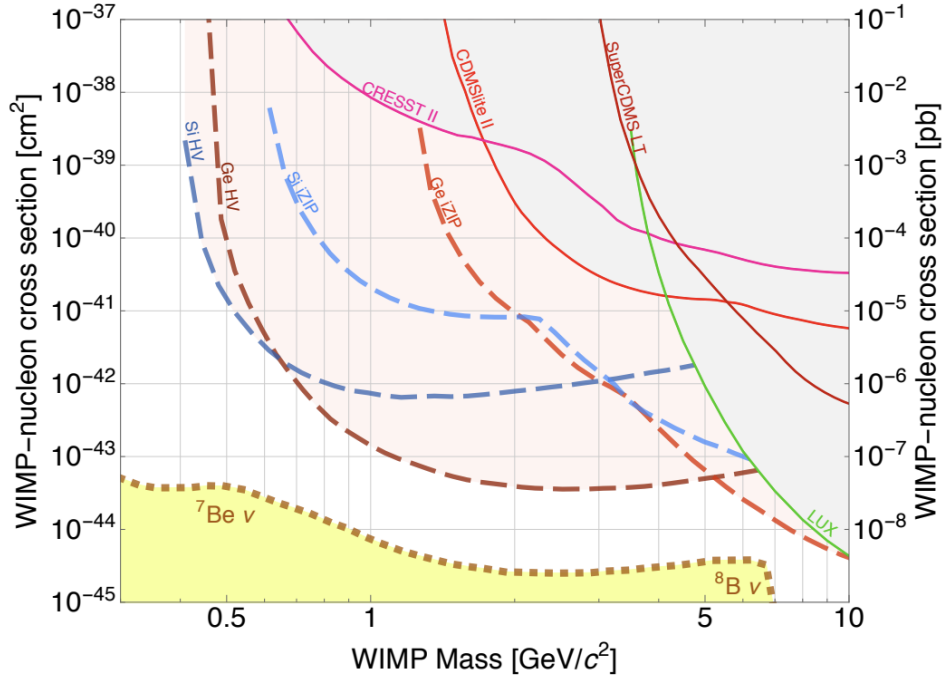


Figure 2.8: Projected sensitivity of different SuperCDMS SNOLAB detectors to the DM-nucleon scattering cross-section, assuming a 4 year net exposure of 4 detector towers [123].

muons [188, 189]. These cavern neutrons and gammas are mitigated by the shields shown in Fig 2.7. And in the case of iZIPs, non-neutron backgrounds can be discriminated against using the ionization yield, or by surface event discrimination [180].

2.2.2 The SuperCDMS HVeV Program

The high voltage with eV-scale resolution (HVeV) detectors were developed by the CDMS collaboration to study various detector effects in a simple and controlled environment. This includes the study of the non-ionizing low energy excess (0QLEE) observed in many low-threshold experiments [190], measuring the ionization yield at low energies [173], and exploring the potential of single- e^-h^+ pair production as a dark matter signal [191]. The HVeV detectors are optimized for phonon collection efficiency and resolution. The small thickness of the wafers used in HVeV detectors (4mm) reduces the surface area of side walls, limiting phonon energy loss through them [174]. By varying the TES and Al fin

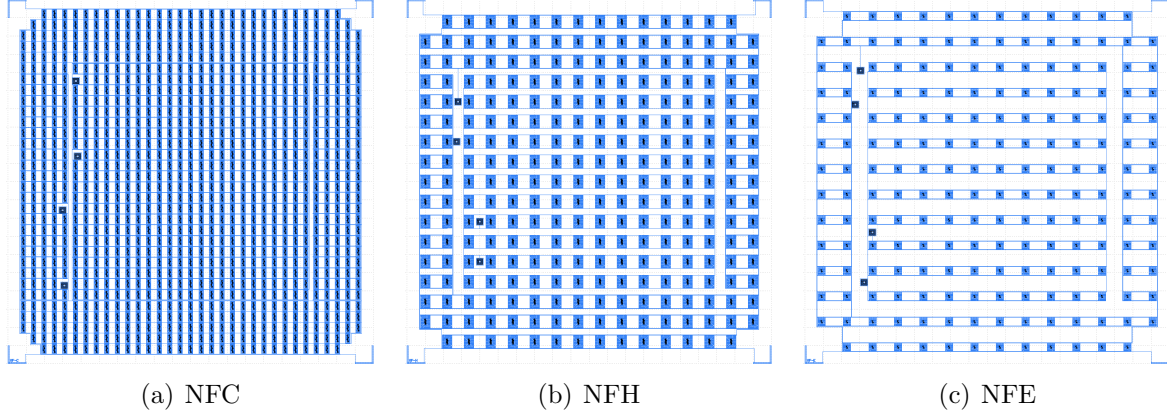


Figure 2.9: Various QET mask designs for HVeV 1 cm² Silicon detectors. Blue rectangles each indicates a single QET with aluminium fins and a tungsten TES at the center (black), while the light blue lines indicate the aluminium scaffolding which connects QETs within a single channel in parallel. The central square covering half the area of each mask is the inner QET channel, while the remaining edge areas fall under the outer QET channel.

lengths that make up a QET (Fig. 2.10), phonon collection can be optimized, with trade-offs between resolution and dynamic range [125]. Figure 2.9 shows various QET mask designs that are instrumented on an HVeV silicon wafer. The QET arrays are divided into two channels (inner and outer), each covering half the surface area of one face. The other face is covered by an aluminium grid which is used to provide crystal bias voltage. The right side of Fig. 2.5 shows an image of an HVeV detector.

The precision of our energy estimate (energy resolution) is limited by noise, as well as the proportion of phonons we sample relative to the total number of phonons produced in an event. The threshold of a detector is either the total phonon energy of one e^-h^+ pair or a multiple of the *baseline resolution*, whichever is larger. The phonon energy resolution σ_p can be estimated by:

$$(\epsilon_p \sigma_p)^2 = 2 k_B G T_0^2 \tau_{BW} \quad (2.8)$$

where G is the thermal conductance of the TES, k_B is the Boltzmann constant, T_0 is the operating temperature, and τ_{BW} , ϵ_p are as defined earlier in Sec. 2.2 [177, 125]. After achieving eV-scale resolution, and upon applying a high voltage $\mathcal{O}(100)$ V, single- e^-h^+ peaks are resolved (Figs. 2.19(a), calibration datasets of Ch. 3). In the calibration datasets presented in Ch. 3, inter-peak events are present. These events are most generally classified as *high/low NTL amplification* events. The currently accepted hypothesis is that, due to

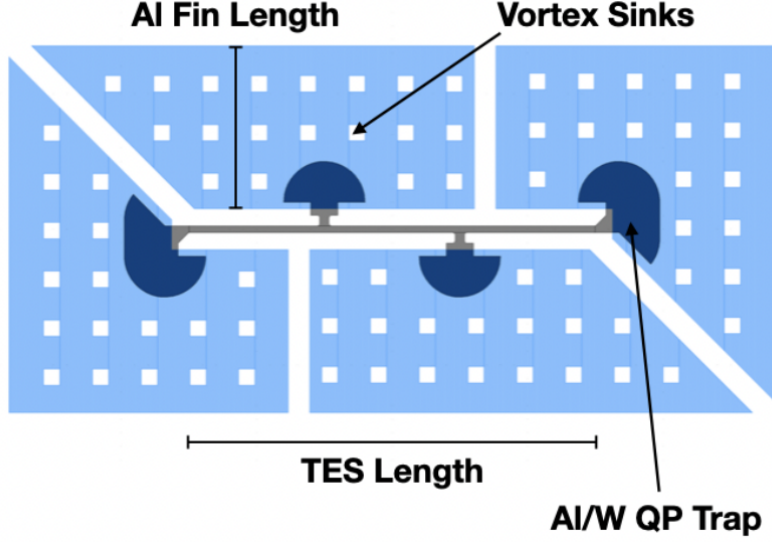


Figure 2.10: Diagram of a tungsten TES (grey) connected with aluminium fins (blue), forming a QET. The lengths of fins and TES are varied to optimize collection efficiency, resolution and dynamic range [125]. Vortex sinks on the superconducting aluminium fins disrupt persistent currents which may be induced by background magnetic fields.

impurities (which result in charge-carrier sources and sinks) [192], some charge carriers get trapped in the sinks (becoming loosely bound to the impurities), causing incomplete (lower) amplification of the event. These loosely bound charges themselves can also be released with energies less than the band-gap, causing higher amplification than expected (see Fig. 2.11). These effects are called *charge trapping and impact ionization* (CTII), and are modeled most simply using a probabilistic model with two parameters: the charge trapping fraction f_{CT} and impact ionization fraction f_{II} [193, 194]. The modelling of this phenomenon is discussed at length in Ch. 3.

2.2.3 HVeV Run 4 at NEXUS

HVeV Run 1 produced world leading exclusion limits for low-mass electron-recoil dark matter [196]. Subsequently, HVeV Run 2 used new detector designs which improved the energy resolution down to ~ 3 eV [125]. However, exclusion curves did not improve dramatically, the high background event rate being the most important limiting factor [197]. Burst events – a type of event where pulses in the TES current ($I_s(t)$ in Eqn. 2.7) happen

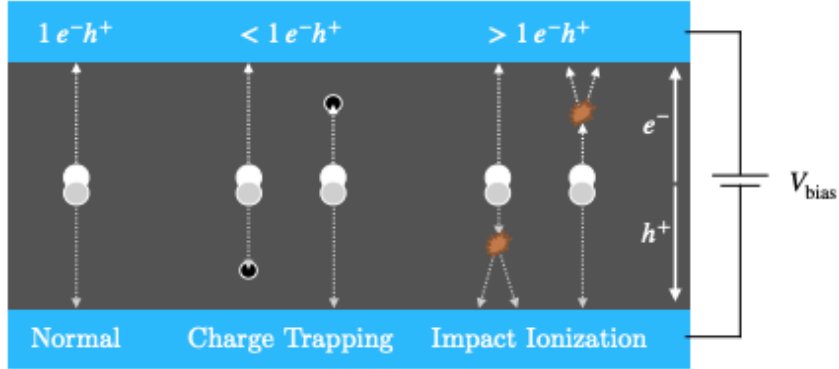


Figure 2.11: Schematic of charge carrier traversal of the semiconducting substrate. Full NTL amplification does not always occur. Figure retrieved from [195]

in quick succession – constituted a major fraction of background events. HVeV Run 3 employed a setup of two detector pairs in order to take advantage of the correlation between events in different detectors to reduce this background and improve dark matter limits [142]. HVeV Run 3 firmly established that these background events were *external* to the detectors. Indeed, HVeV Run 4 came to confirm this, as this background was eliminated by the deployment of a light-tight detector holder. The evidence seems to point to a luminescent source with a line of sight of the detectors in previous runs, likely a PCB material (possibly SiO_2 , see [198, 199, 200, 201]).

Experimental Setup

In January 2022, in the Northwestern EXperimental Underground Site (NEXUS) at Fermilab (Batavia, IL), data taking began for HVeV run 4. The site is 107m underground (in the MINOS cavern), and the overburden provides 225m of water equivalent shielding, reducing the atmospheric muon rate by a factor of ~ 200 [202]. In this low-recoil-energy search ($E_r \lesssim 20 \text{ eV}_{\text{ee}}$), this depth suffices as cosmogenic background depositions lie outside the region of interest, so the main purpose of the overburden is to minimize disruption of the detectors by high energy cosmogenic secondaries (which may saturate the detectors for extended periods of time), thereby maximizing live-time. Combining all that was learnt from previous runs, the run’s aim was to vastly improve dark matter sensitivity by reducing the background event rate. A new detector holder hosts four 0.93 Si ($1 \text{ cm}^2 \times 1 \text{ mm}$) wafers in close proximity to increase chances of coincident events (Fig. 2.12). Two of the

detectors have the same QET mask type C [125], while the other two are of type E and H. The detector holder is made up of two parts, each holding a pair of detectors side by side. Fig. 2.13(a) shows the bottom part of the holder and the back (HV aluminium grid) faces of NFC1 and NFE detectors, while Fig. 2.13(b) shows the upper part of the holder and the front (QET instrumented) faces of NFH and NFC2. Assembling the two parts together, we have the QET instrumented faces opposing each other, and the back faces of all detectors point outwards.

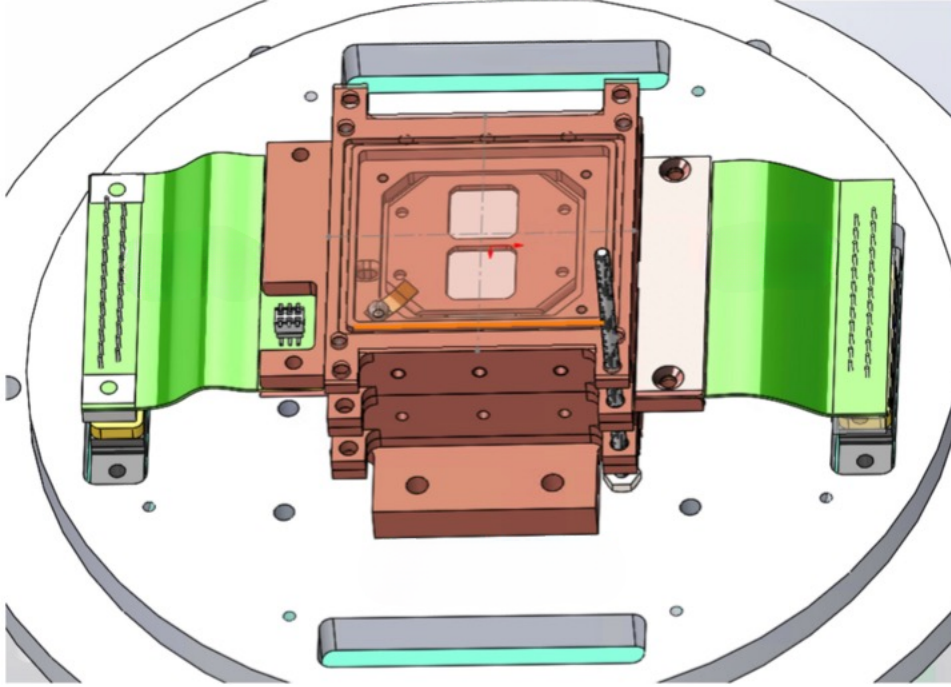


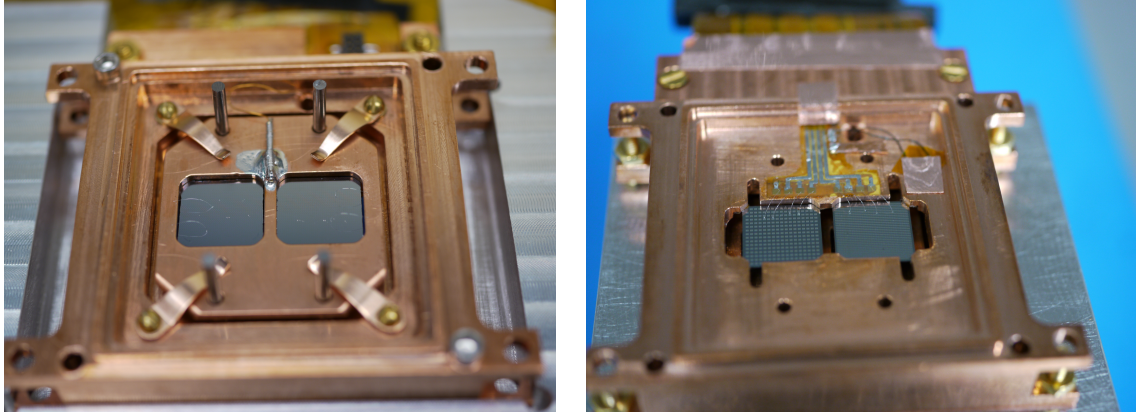
Figure 2.12: Rendering of detectors in their new light-tight detector housing (before attaching the top copper lid). The back (aluminium-grid) faces of detectors NFH (top) and NFC2 (bottom) can be seen (two small squares), with readout connectors on both sides (green). Unseen are NFC1 (under NFC2), while NFE (under NFH).

The detector holder is fixed inside a copper cavity (Fig. 2.13(c)), which in turn is mounted to a CryoConcepts dry (cryogen-free) dilution refrigerator (Fig. 2.13(d)). A dilution refrigerator achieves milli-kelvin temperatures in its mixing chamber (MC), where ^3He and ^4He are pre-cooled and mixed, establishing a phase boundary. Instead of relying on cryogenics like liquid helium and nitrogen for pre-cooling stages, the dry fridge uses a pulse tube cryocooler holds the first two stages at 60 K and 4 K. To achieve a low-vibration

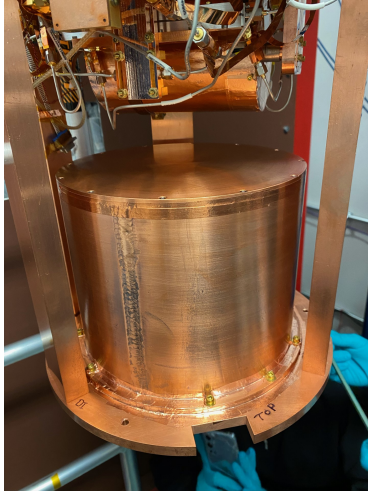
environment, the pulse tube is mechanically decoupled from the fridge using Ultra-Quiet TechnologyTM [203]. In the still stage of the fridge, the ^3He is further cooled to $\sim 870\text{ mK}$, followed by the cold plate at $\sim 80\text{ mK}$, and then finally fed into the MC. At the 1 Kelvin stage of the fridge, an A4K shield from Amuneal [204] shields the sensitive QETs from magnetic fields. The refrigerator resides in a class-10000 (FED-STD-209E [205]) clean room, covered with a METGLAS blanket for additional magnetic shielding. In addition to upgrading the detector holder, a new lead shield (Fig. 2.13(e)) with a minimum thickness of 4 inches was installed to reduce background events due to gammas from decays within the surrounding rock of the cavern. Each phonon channel is read out by a single-stage DC Superconducting Quantum Interference Device (SQUID), which is a pair of Josephson junctions that acquires a potential difference when biased by a current $I > I_c > 0$ where I_c is the critical current. The potential difference across a SQUID is very sensitive to external magnetic fields [206]. The SQUIDs are shielded from magnetic interference by a mu-metal and niobium box.

HVeV Run 4 was made up of two separate periods of data taking named NEXUS Run 13 (NR13) and NEXUS Run 14 (NR14). Running from January to April of 2022, NR13 included the dark matter search data acquired at 100 V bias. In addition, NR13 took high energy calibration data using a $3.14\text{ MBq }^{137}\text{Cs}$ gamma source at 0 V and 100 V crystal bias, as well as various other testing data sets. The low energy calibration strategy of HVeV Run 4 involved the use of LEDs shining on the detector backsides. Between NR13 and NR14, installation of the newly designed LED boxes (Fig. 3.2) occurred, and low-energy calibration data was taken in July. This separation was done to mitigate light leakage and achieve the lowest possible background rate in the DM search data. Details of the LED apparatus and data acquisition are described in Section 3.1. As a NEXUS Run begins, the fridge mixing chamber would be cooling down by a couple mK to its final temperature of 11 mK, the SQUIDs of the readout circuit are ‘locked’ (magnetic flux changes are offset by a feedback inductance, the feedback current is the signal which is digitized) and testing datasets are initially taken. One of the most important are “IV curve” datasets, where the TES bias current is varied and the signal current (I_s in Eqn. 2.7) is measured.

Through simple circuit relations described in Eqn. 2.6, one may derive the resistance vs. bias current curves of the superconductor sensors. Fig. 2.14 shows the TES resistance curves at the end of NR13 as well as for NR14 at various stages of the run. For NR13, the bias current for each channel was set so that the resistance of the TES circuit is 40% of the value of its resistance at its normal (non-superconducting) state R_n . Background datasets were also taken with varying high voltage (HV) bias. The bias of 100 V was chosen as a sweet spot since it sufficiently amplifies the ionization events away from background events of the non-ionizing low-energy excess and the sub- e^-h^+ peaks (both discussed in



(a) Back-sides of detectors NFC1 (left) and NFE (right), instrumented with aluminium grids which provide the high voltage bias across the wafers. Fine wires are bonded to the aluminium grid. (b) Front-sides of detectors NFH (left) and NFC2 (right), instrumented with 2 channels of QET arrays (each). Each QET channel has a pair of wires which provide the TES bias currents.



(c) The copper housing.



(d) The dilution refrigerator.



(e) The lead shield.

Figure 2.13: Images of the setup of HVeV Run 4.

Ch. 6), while not being unnecessarily high to mitigate leakage of charge carriers across the semiconductor medium (charges can tunnel from electrodes into the crystal [192, 207]). With these tests completed, the DM search data was taken over a 12 day period, yielding a raw exposure of 10.8 g days per detector. Fig. 2.15(a) shows the final setup for NR13, which also included Cs-137 x-ray source data sets taken at 0V and 100V bias.

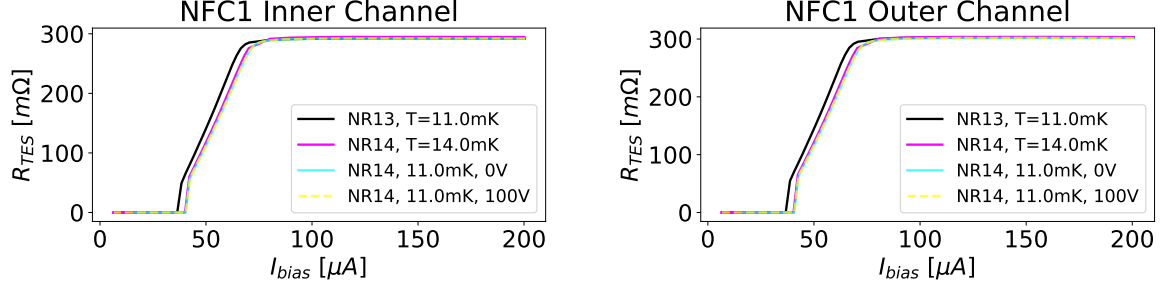
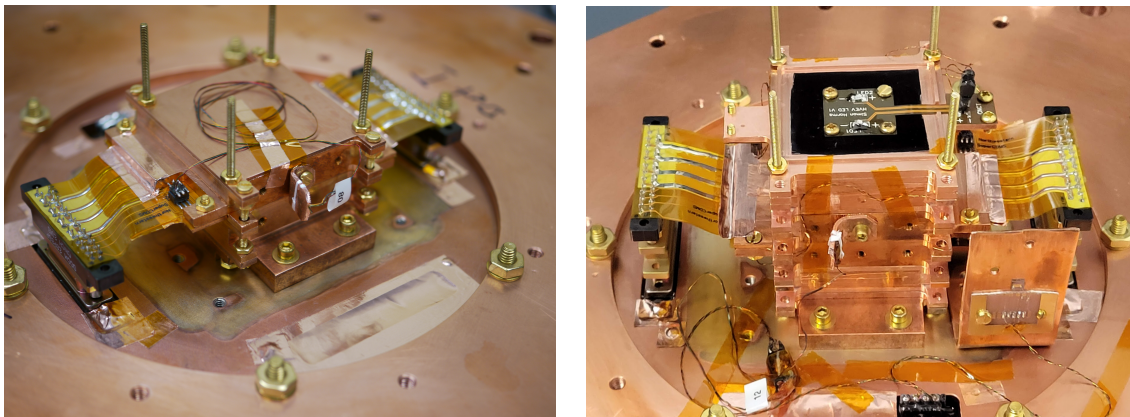


Figure 2.14: The TES resistance of the inner (left) and outer (right) channels of detector NFC1 of HVeV Run 4 (normal resistance $\sim 300\text{ m}\Omega$) as a function of bias current. The curves for NR14 are shifted to the right, meaning a higher bias current is needed in NR14 to achieve the same percentage of transition towards normal resistance as NR13. This could indicate an additional parasitic power source in NR13 compared to NR14 ($P_{\text{parasitic}}$ from Eqn. 2.5).

Detector	L_{TES} (μm)	L_{fin} (μm)	R_n ($\text{m}\Omega$)	σ_p (eV)	E_{sat} (keV)
NFC1	150	60	300	2.79(3)	18(2)
NFC2	150	60	300	2.26(2)	17(2)
NFE	25	100	300	4.94(5)	2.6(3)
NFH	100	125	900	2.65(3)	6.0(6)

Table 2.2: HVeV Run 4 payload summary. Each detector is 0.93 g of high-purity silicon, instrumented with various QET masks shown in Fig. 2.9.

In NR14 (final setup shown in Fig. 2.15(b)), a higher bias current was generally required to achieve 40% R_n (Fig. 2.14). Further, even when that benchmark resistance is achieved, the gain observed in background spectra (when comparing the mean $1e^-h^+$ pulse amplitude) did not match that of NR13. Therefore, datasets were taken at various bias currents to tune the observed $1e^-h^+$ peak position in background spectra of NR14 to those seen in NR13. Even after the tuning during data taking, the $1e^-h^+$ gains between NR13 and NR14 did not precisely match (see Fig. 2.19(a)), so a linear gain correction was needed to apply a calibration derived from NR14 on NR13. To quantify the systematic uncertainty due to such a correction, LED calibration data was not only acquired for one TES bias current, but also acquired datasets with the bias current slightly varied around the central value. Data sets varying the LED intensity were also acquired.



(a) Setup of NEXUS Run 13, with detectors enclosed in light-tight detector holder. The upper square copper lid covers the back faces of two detectors. (b) Setup of NEXUS Run 14, with detector holder replaced with the two LED square copper lids.

Figure 2.15: Comparison of final setups of detector towers for NEXUS Runs 13 and 14 before closing the copper housing.

After the TES currents are amplified by the SQUIDS, they are further amplified and digitized by a Detector Control and Readout Card (DCRC [208]) with a sampling frequency of 156.25 kHz. This is essentially the same data acquisition (DAQ) strategy as SuperCDMS SNOLAB. In HVeV Run 4, the raw data is continuously readout using the Maximum Integrated Data Acquisition System (MIDAS) software [209], yielding 0.5 second raw readout traces (Fig. 2.16) that cover the entire live-time. These digitized traces contained the TES current every $6.4 \mu\text{s}$. Triggering (the identification of events of interest) happened offline after data taking was completed. The first step in event reconstruction is to filter the raw traces to suitably maximize the signal-to-noise ratio. Then, one must identify events or hits (appearing as a pulse in the TES current), for which we use a triggering algorithm. Finally, we must calculate ‘reduced quantities’ for each pulse, upon which we apply a statistical analysis. The pipeline from raw digitized 0.5 s traces to reduced quantities for each pulse is outlined in detail in the following subsections.

2.2.4 Filtering Techniques

In order to remove noise components from our signal, we may apply various filtering techniques. In analyzing a time series, we usually want to suppress certain frequency com-

ponents in Fourier space, assuming that each mode is independent from the others. If a filter $\phi(f)$ is defined in the frequency domain, then it is simply multiplied with the noisy Fourier-transformed signal $v(f)$,

$$\phi(f) \cdot v(f). \quad (2.9)$$

The Optimal Filter (OF) [210], known as it is because it theoretically provides the most complete filtering, provided that one has a signal template and a noise template. To model the noise, we average power spectral densities (PSDs) of various *randomly triggered events*. These are events sampled randomly from each readout trace, with pulse rejection criteria (see 2.2.5) implemented to ensure the lack of a pulse in such a random event. For the OF we have $\phi(f) = s^*(f)/J(f)$, where $s^*(f)$ is the complex conjugate of the frequency domain signal template, and $J(f)$ is the noise PSD. One may instead use a Gaussian filter $\phi(f) = G(f) = \text{gauss}(f, \mu = 0, \sigma)$. To obtain a time-domain filtered trace, we calculate:

$$\frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-ift} [\phi(f) \cdot v(f)] df = \int_{-\infty}^{+\infty} [\phi(t - \tau) \cdot v(\tau)] d\tau \equiv (\phi * v)(t), \quad (2.10)$$

where the equality indicates that one may also find the time domain filter

$$\phi(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-ift} \phi(f) df \quad (2.11)$$

and then convolve it with the time domain noisy signal $v(t)$. Fig. 2.16 shows a 0.5 second readout trace from NR13, along with the OF-filtered counterparts.

Threshold Triggering

While the OF theoretically provides the maximum filtering, it has disadvantages, particularly for use in triggering. Because our PSDs have many high-frequency peaks (Fig. 2.17(b)), the time-domain filtered trace may suffer from ‘trigger echoes’ (fake pulses in the filtered trace). For this reason, we use a Gaussian-derivative filter in the triggering algorithm instead of the OF triggering shown in Fig. 2.16. The OF triggering takes as input the templates and a trigger threshold, while the Gaussian derivative triggering algorithm has a kernel $\phi(t)$ that is the derivative of a Gaussian. This produces the filtered derivative of the signal upon convolution with the noisy signal. Using the first derivative instead of the time series itself makes for a better rising-edge detection algorithm for ‘pile-up’ events where pulses occur in quick succession (see Fig. 2.18). The Gaussian-derivative filter takes as input the width of the Gaussian (we use a width of 6 digital time bins dt , $dt = 6.4 \mu s$),

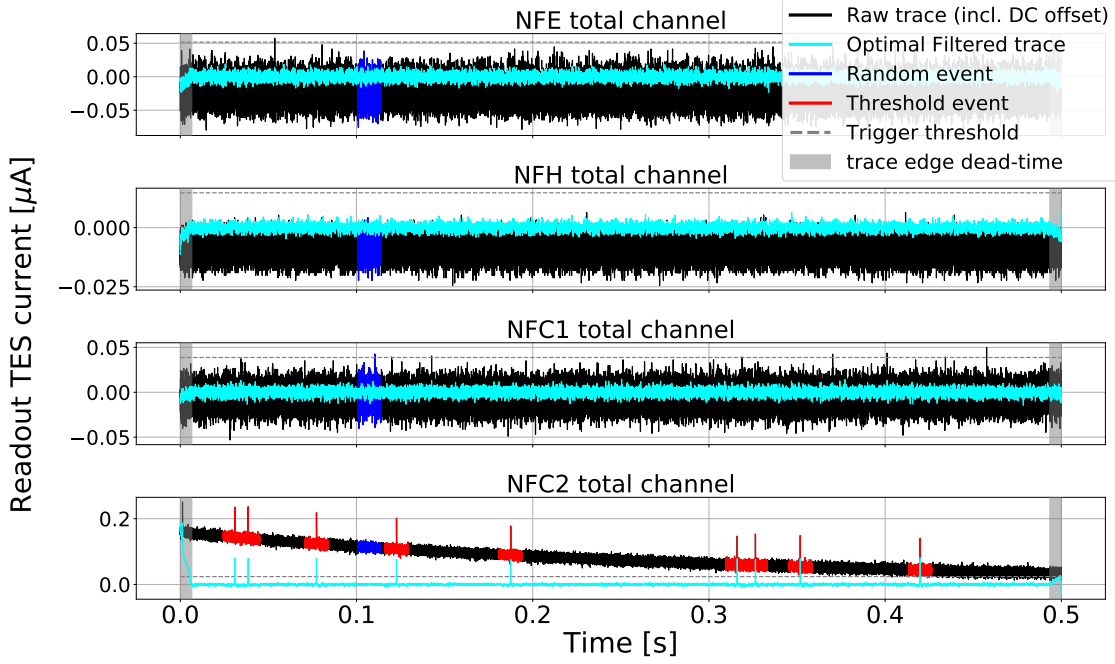
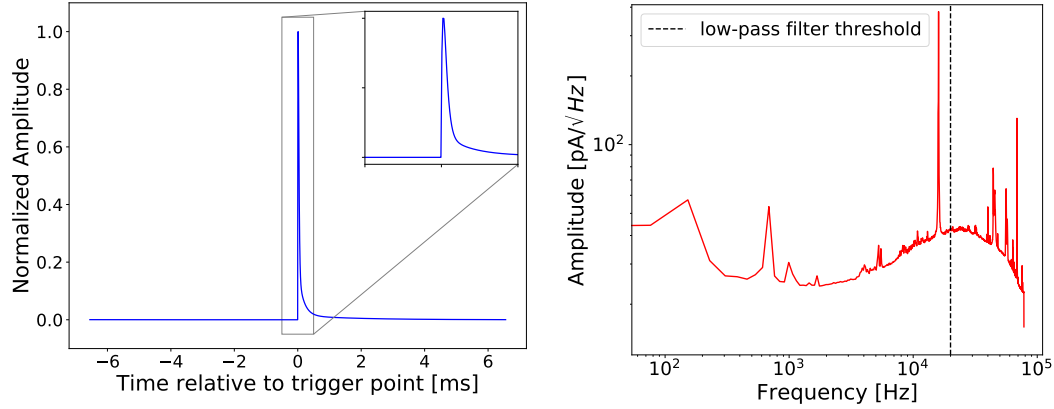


Figure 2.16: Example readout trace from the DM search data. In this example the filtered trace is found by applying the **optimal filter** (Eq. 2.12 and Fig. 2.17), and additionally **suppressing frequencies higher than 20 kHz**, as well as the **lowest frequency component** (for baseline current subtraction). In NFC2, we see the slowly decaying tail of a high energy event with smaller $1 e^- h^+$ events that are identified by the threshold trigger (red). The random event (blue) in NFC2 is skewed, so it is not used in PSD calculation. In this half-second trace, NFC2 is experiencing a period of dead time due to the TES baseline cut (Sec. 2.2.6). Also shown are the trace edge dead-time regions ($1024 dt \simeq 6.55$ ms on each end) that apply to every raw trace, as the filtering is incomplete at the edges of the time series. Note that the TES current is shifted by a DC offset at readout, so the values do not represent the actual current passing through the TESs at a certain time. This presents no issue since we are only interested in the change of the current from baseline I_0 in Eq. 2.7 (Fig. 2.21 shows that the mean baseline for NFC1 was slightly negative).

as well as a threshold that does not correspond to a certain amplitude in the original time series. While we run the processing on raw data with both triggering algorithms, Gaussian-derivative filter triggering is used for DM search. Finally, we implement a trigger hold-off so that two triggers do not occur within $6 dt$ of each other.



(a) Pulse template used in the optimal filter (OF) to reconstruct events in detector NFC1. It is the averaged $1 e^- h^+$ pulse shape from the dark matter search data from the same detector. The domain of the main graph is the pulse analysis window. It is normalized so the maximum is 1. (b) Mean power densities for NFC1 found from random events passing pulse-rejection cuts. Shown are the amplitude spectral densities ($\sqrt{\langle PSD \rangle}$) from dark matter search data. A low-pass hard-cutoff filter is also applied when applying the optimal filter.

Figure 2.17: Pulse template and noise PSD used in the optimal filter for NFC1 (blue). Both were acquired from NR13 dark matter search data.

Reduced Quantities

As can be seen in Fig. 2.18, the trigger point I_{trig} is identified for every triggered event within each readout trace. This is one of the reduced quantities (RQs) that we are left with after reconstruction. The trigger point sits at the center of a $2048 \cdot dt$ ($dt = 6.4 \mu s$) sample called the *pulse analysis window* (Fig. 2.17(a)), which will be analyzed to extract further RQs. Another important RQ is the OF amplitude of the event, which shall serve as our main energy estimator. The optimal filter can be seen as a frequency-domain fit, minimizing the reduced chi-squared:

$$\chi_{red}^2 = \sum_i \int_{-\infty}^{\infty} \frac{|v_i(f) - \sum_{\gamma} A_{\gamma} e^{-i\omega t_0} s_{i\gamma}(f)|^2}{J_i(f)} df, \quad (2.12)$$

where A_{γ} is the amplitude for template set γ and i is a channel index. (In practice we additionally suppress high-frequency > 20 kHz components when applying the OF, as

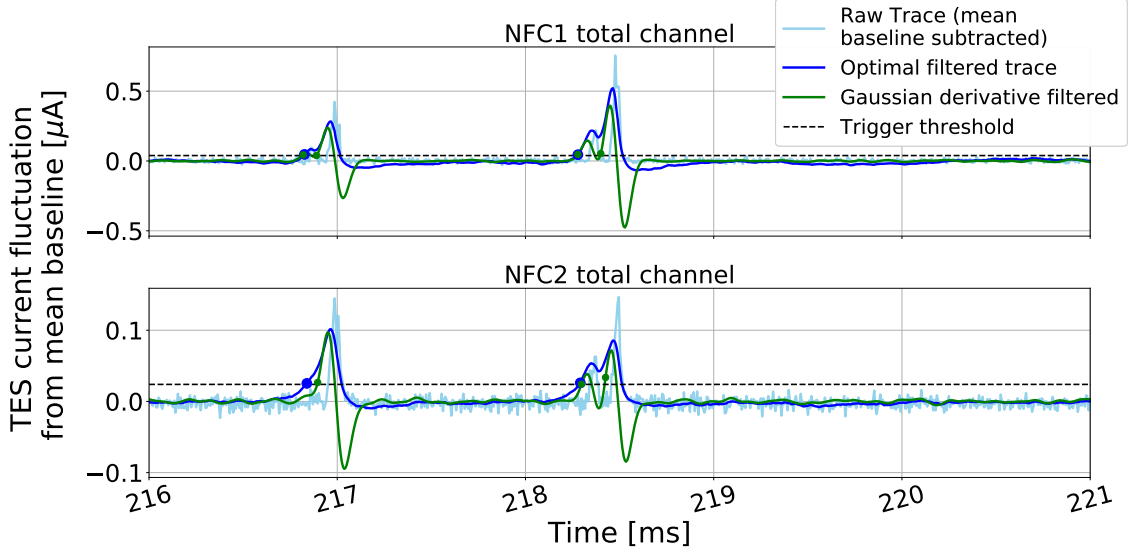


Figure 2.18: Comparison of different threshold triggers (this is an example of a **coincident burst** event which will be studied in Ch. 4. It is otherwise **not typical** for pulses to occur simultaneously in different detectors). The filtered time series may obscure the time difference between events that happen in close succession. Since we use a rising edge trigger, using the optimal filtered trace we will identify only two events in each detector (blue points) within the shown section of a readout trace, while the Gaussian derivative filtered trace crosses the trigger threshold four times (green points). The trigger point reduced quantity $\mathbf{I}_{\text{trig}}$ is taken as the index within a readout trace where the filtered trace is at a **maximum**. The mean baseline for NFC1 was taken from Fig. 2.21.

shown in Fig. 2.17(b)). In fact, the derivation of the OF kernel ϕ comes from this chi-squared fit, as shown in [210]. Eq. 2.12 shows the case of a simultaneous fit of different channels of uncorrelated noise with a set of templates $s_{i\gamma}$. In our case, we do not conduct a simultaneous fit of the amplitudes for the inner and outer channels. Instead, we sum the inner and outer channel traces and reconstruct the combined pulse $v_{\text{tot}}(t) = v_{\text{in}}(t) + \alpha v_{\text{out}}(t)$, where $\alpha = 1$ (same for triggering). Fig. 2.19(a) shows the total pulse amplitude (A_{OFL}) spectra from NR13 and NR14. Fig. 2.19(b) shows an optimization of the parameter α , justifying our choice as the channels have almost equal energy weights. The PSD J in this case is also from random events summed from both channels. We also use only one template $v_{\text{tot}}(t)$, which is the normalized mean pulse shape of $1 e^- h^+$ events. To reconstruct pulses from each channel separately (to assess relative channel weights, Fig. 2.19(b)), channel-

specific templates were used.

Another fit parameter is t_0 in Eq. 2.12, which allows for a shift around the trigger point, allowing for a better fit. We will make a distinction between two OF amplitude RQs: **OF0 amplitude** (A_{OF0}) is from the pulse fit with $t_0 = 0$, and **OFL amplitude** (A_{OFL}) is from a pulse fit with a limited allowed window of 6 samples $-6 dt < t_0 < 6 dt$, $dt = 6.4\mu s$. In that case, t_0 will correspond to the highest amplitude possible [210], making OFL estimation biased (prone to fitting noise fluctuations) in the case of low amplitude pulses. The PSDs and templates from NR13 DM search data were used to process NR14 data. This was done so that efficiency calculations from NR14 are controlled for reconstruction method. We will use all of the RQs discussed (I_{trig} , χ_{red}^2 , A_{OF0} , A_{OFL} , t_0) and more in the following statistical analyses. In order to ensure that the DM result is unbiased by the analyzers, only 30% of the live-time was visible at first, while 70% was blinded until the analysis method was fully developed. The blinding scheme was such that there were 15 minute periods of unblinded data in between 35 minute periods of blinded data.

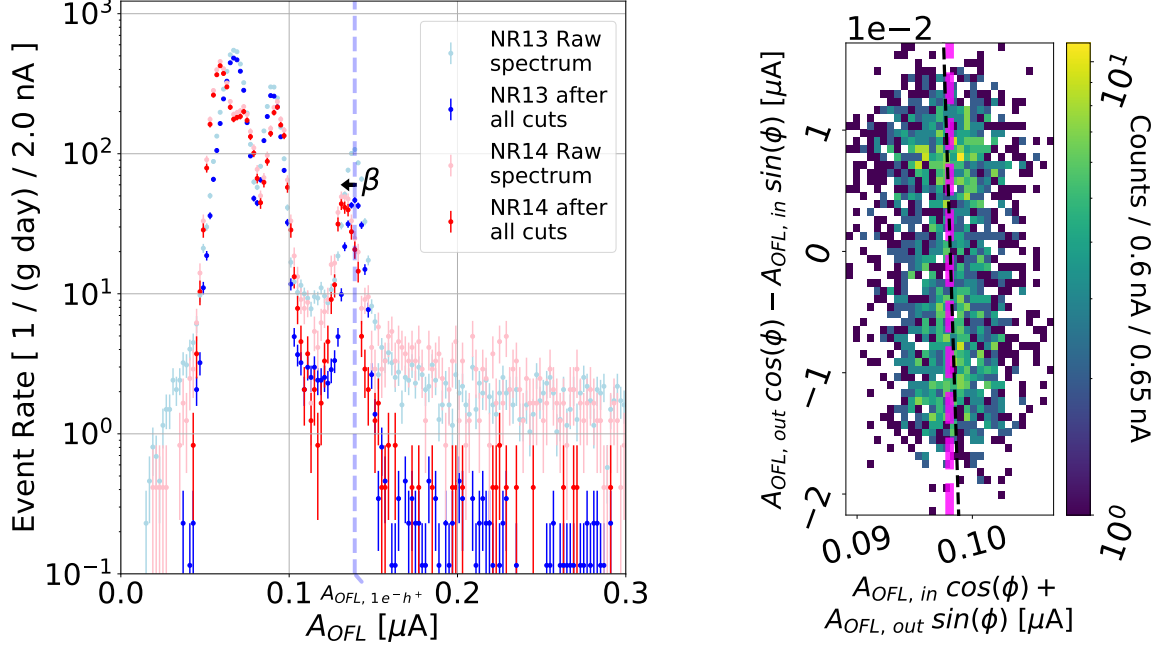
2.2.5 Standard Event Selections

Pulse Rejection Cuts (randoms only)

Randomly triggered events are used to assess noise conditions and produce a noise template for the OF, namely the noise PSD. Four cuts are applied to random events for pulse rejection. The cuts assess the range (max-min), mean baseline, slope and skewness of the randoms and removes outliers. These cuts are defined and applied automatically at the reconstruction stage (Section 2.2.3). The automated procedure is iterative: the skewness of the distributions of the four quantities is calculated after applying a symmetric cut around the median values, iterating until the skewness is below a certain threshold. Afterwards, the standard deviation and mean are calculated for each quantity, a 3σ cut is applied, and then the estimators are recalculated, repeating until the estimators stabilize.

Reduced- χ^2 Cut

The Reduced- χ^2 Cut uses the χ_{red}^2 RQ calculated by minimizing Eq. 2.12 to remove poorly reconstructed events from the data. The Reduced- χ^2 Cut is necessarily amplitude/energy dependent because of our choice of template as the mean $1 e^- h^+$ pulse shape: pulse fits will be worse (have a higher χ_{red}^2) for higher amplitude pulses. This reflects the energy dependent nature of the pulse shape due to 1) the non-linearity of the R-T curve and 2)



(a) Comparison of NR13 (full DM search dataset) and NR14 (b) The amplitudes of the inner and outer background pulse amplitude spectra in NFC1 (total channel channels are shown for $1e^-h^+$ events $v_{tot}(t) = v_{in}(t) + v_{out}(t)$). The $1e^-h^+$ amplitude in NR14 was from DM search data of NFC1 (rotated a factor of $\beta \simeq 0.96$ smaller than that of NR13, despite best by angle $\phi = \pi/4$). The vertical thick efforts to tune the QET bias to compensate for the parasitic magenta line is $A_{OFL, in} + A_{OFL, out} = A_{OFL, 1e^-h^+}$. This $1e^-h^+$ energy line is used in Ch. 3 to apply the LED calibration from NR14 to has a trend (black dotted line) with a slight tilt $\Delta \sim \frac{\pm\pi}{100}$ from the magenta vertical. This would suggest that we use $v_{tot}(t) = v_{in}(t) + \alpha v_{out}(t)$ to combine the channels, where $\alpha = \tan(\phi + \Delta) \sim 1.07$. The difference in resolution is minimal (< 0.02 eV), so this correction is forgone.

Figure 2.19: Pulse amplitude distributions in NFC1 detector

the variation of pulse fall-times which depend on the electro-thermal balancing processes. A series of amplitude-dependent 3σ upper bounds are found for the χ_{red}^2 distribution in Ch. 3 for LED data and in Ch. 4 for DM search data. To ensure a smooth cut boundary, the upper boundary values are fit as a function of OF amplitude using $\chi_{red}^2 < f_{cut}(A_{OF}) =$

$p_2(A_{OF})^{p_1} + p_0$. The functional form f_{cut} is an ansatz that fits the (A_{OF}, χ_{red}^2) trend for energies up to $\sim keV$.

2.2.6 Standard Live-time Selections

In this subsection we discuss two standard (analysis independent) live-time selection criteria used in HVeV Run 4. These are relevant for any analysis because they select periods of stable fridge temperature and TES baseline current (I_0 in Eqn. 2.7). Instabilities in either may affect the precision of phonon energy measurements.

Fridge temperature stability

During the runs, the temperature inside the mixing chamber of the fridge was maintained at 11.0 – 11.1 mK. This was measured by a variable-resistance thermometer and monitored throughout the run every 40 s. For a $\lesssim 10$ minute period (Fig. 2.20) during the DM search data taking, the thermometer registered a temperature of ~ 40 mK. This was the only recorded temperature excursion > 11.1 mK throughout the DM search data. To exclude data taken at this time, a 2 hour live-time cut is applied between 16 : 00 – 18 : 00 CDT. We cut about an hour before and after the excursion to ensure that the elevated temperature period is removed, as the clocks of the fridge monitoring and readout systems are not synced. The dead-time introduced by this cut is simultaneous across all detectors.

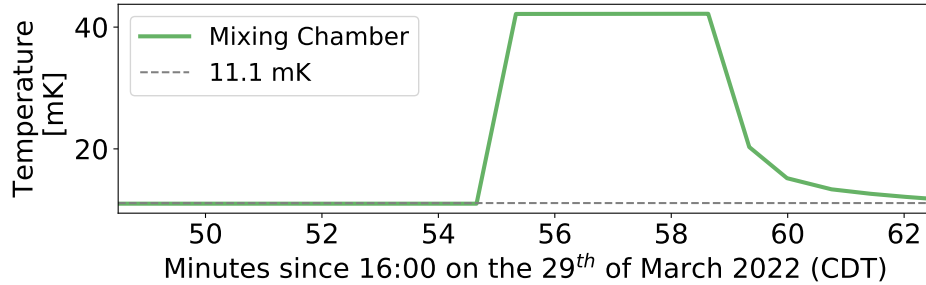


Figure 2.20: Mixing chamber temperature sampled every 40 seconds, zoomed around the brief excursion. The temperature exceeded 11.1 mK for a few minutes during the DM search data taking. The fridge temperature cut removes this period from the live-time.

TES baseline current stability

When the TESs are cooled and biased (Fig. 2.14), they are usually in a stable state with TES current being the quiescent baseline current, which is constant within noise. Any variation in the baseline current over long time scales indicates that the power balance has been upset, either by a temperature change, or because of a high energy event with a slow-falling tail, such as the one shown in Fig. 2.16. In other cases, electromagnetic radiation may input power in the TES directly, and we call that *parasitic power*. In any case, an elevated baseline affects reconstruction because it means the ‘working point’ on the R-T curve has moved away from its quiescent state, so the TES gain will differ. Thus, we define an algorithm that evaluates the mean baseline for each 0.5s raw trace, by first removing triggered pulses including their pulse analysis windows. Then the distribution of baseline values is fit with a Gaussian, and a 3σ cut is applied to remove violating traces from the live-time (specific for each detector). The values for the 0.5 second trace baselines in NFC1 for the entire DM search dataset are summarized in Figure 2.21.

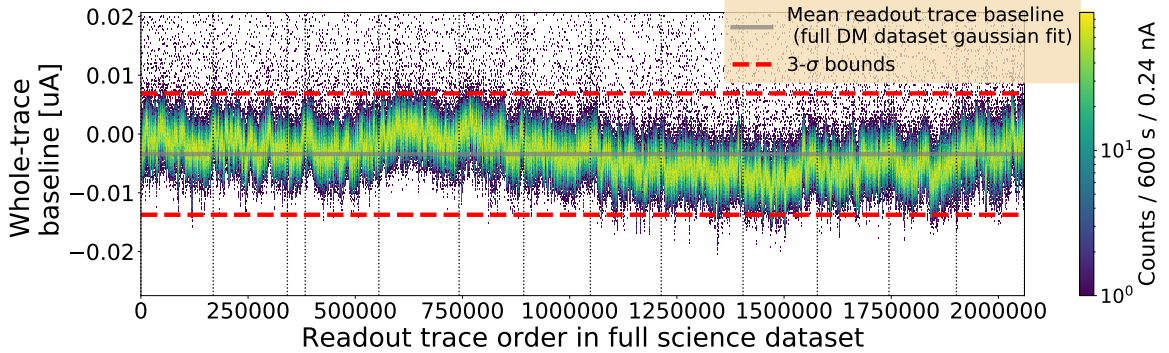


Figure 2.21: For each readout trace, triggered events and their pulse analysis windows are removed, and the rest of the TES current sample points are averaged, yielding the whole-trace baseline quantity for each 0.5 second in the DM search dataset. Shown are the acquired values for trace baseline in the NFC1 detector from the entire DM search dataset. The mean and 3σ boundaries (from a Gaussian fit of all trace baseline values) are also shown. Traces that lie outside of the red horizontal dotted lines are dead-time. Vertical dotted lines indicate when the run was stopped and re-started.

Chapter 3

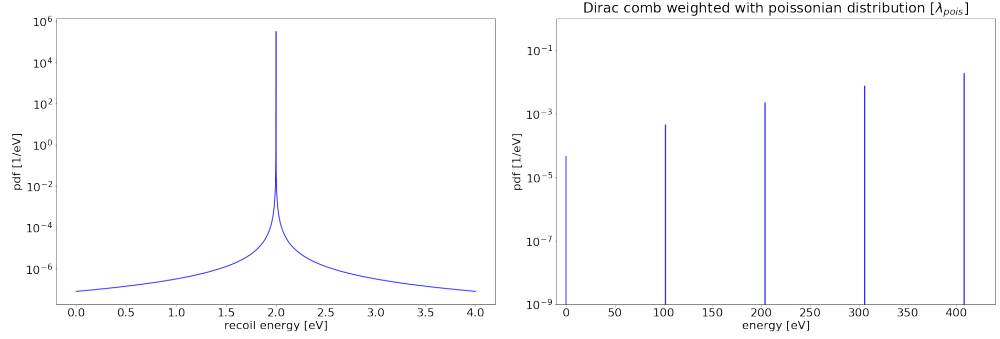
Investigating Detector Response Effects using HVeV Run 4 LED Data

A dark matter particle recoiling with a subatomic constituent of the crystal results in a chain of events which may lead to a TES current pulse that one observes. The mapping between pulse size and observed total energy is obtained through a calibration. On the other hand, the mapping between the basic initial interaction's recoil energy and the measured energy can be divided into distinct stages. At each stage of the transformation, a model is used. The mapping is not faithful in an event by event basis due to the stochastic nature of the processes involved; thus, the mapping is done on spectra in the recoil-energy domain, transforming them through a series of detector response modeling convolutions to the total phonon-energy scale while including various detector effects. The models adopted for the calibration of HVeV Run 4 go as follows (listed in chronological order of what is thought to take place in the crystal):

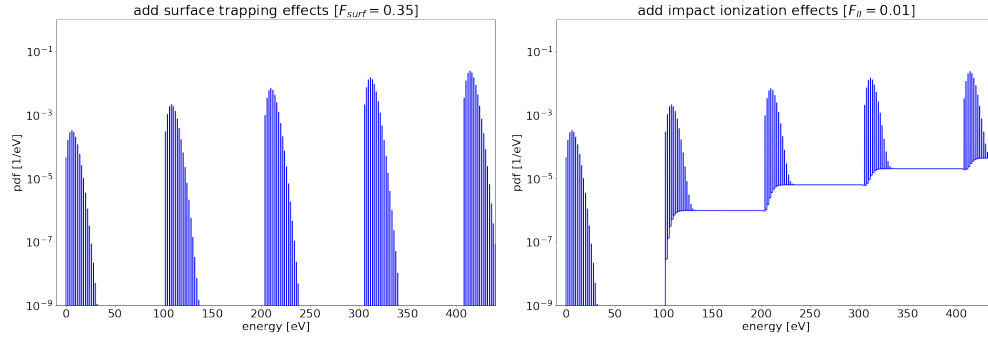
1. Energy is deposited to a component of the crystal (e.g. an electron) with a given probability distribution in recoil-energy $f(E_R)$, depending on the source ([76] for electron recoil DM). For 2 eV photons, see Fig. 3.1(a).
2. If the recoil energy E_R is higher than the band gap, electron-hole pairs are produced in the crystal. Must assume a charge production model giving the probability of producing any integer number of pairs given E_R , e.g. the Ramanathan-Kurinsky (RK) model [165]. For multiple simultaneous photons hits with number of photons per LED flash following Poissonian statistics with mean λ_{pois} , see Fig. 3.1(b).

3. Charges may promptly recombine near the surface of the crystal instead of traversing the crystal, releasing the photon energy as phonons without NTL gain phonons. This is referred to as a surface trapping effect [191, 194]. Could model surface trapping probabilistically using a fraction F_{surf} , indicating the probability of a surface e^-h^+ pair undergoing prompt recombination. Alternatively, one may use a constant energy shift to account for surface trapping events, as was done in the HVeV Run 4 LED calibration. (Surface trapping effect only relevant for HVeV R4 in the LED calibration, as the ~ 2 eV photons are very likely to be surface events). See Fig. 3.1(c).
4. Charges may get trapped in the bulk of the crystal (charge trapping), or they may free other bound charges (impact ionization). This is modeled probabilistically using the exponential CTII model [194]. See Figures 3.1(d) and 3.1(e) .
5. The detector is sensitive to the phonons produced by the initial recoil and the NTL phonons due to the charged pairs. The TES has a calculable baseline phonon resolution based on shape and material properties. This resolution is incorporated in the spectrum using convolution with a gaussian of width σ_{res} estimated in here. See Fig. 3.1(f).

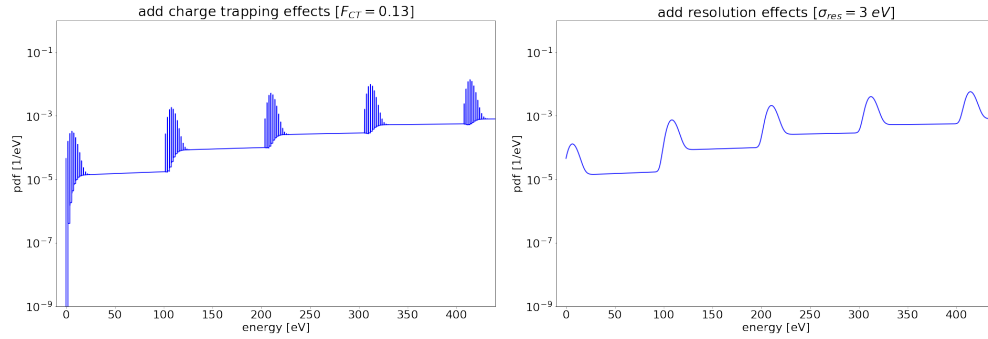
This chapter covers an analysis of NR14 LED data taken as part of HVeV Run 4. The setup of HVeV Run 4 was outlined in Section 2.2.3. With LEDs that provide $E_\gamma \sim 2$ eV photons aimed at the centers of the backsides (aluminium HV grid) of three different HVeV wafers, an extensive collection of datasets was acquired. This includes LED data taken for three different HVeV detectors (detector NFE experienced a malfunction during the LED run, no calibration data was acquired for it). For each of the detectors (NFC1, NFC2 and NFH), the LED intensity was varied along with the crystal bias voltage and TES bias currents. All of that data is analyzed below, to test the detector response models themselves and to estimate detector response parameters and map their dependence on run variables. The details of the LED apparatus, data-taking and event reconstruction are described below in Section 3.1. This is followed by investigations of steps 3-5 above, using the assumption that $f(E_R)$ from step 1 is peaked at $E_\gamma = 2.0$ eV. Finally, a discussion about the choice of calibration function and an evaluation of calibration uncertainties are presented.



(a) Recoil energy spectrum of single 2 eV photon (b) Total phonon energy from multiple photons



(c) Adding surface trapping (unamplified photons) (d) Adding impact ionization effect



(e) Adding charge trapping effect (f) Gaussian smearing (phonon resolution)

Figure 3.1: Illustration of detector processes listed at the beginning of Ch. 3.

3.1 Calibration Data Acquisition

Figure 2.15(b) showed the final setup of the LED data-taking run (NR14). During NR14, each detector was illuminated by an LED shining through a shortpass IR (SCHOTT KG3 [211]) filter covering a pinhole (Fig. 3.2(a)) pointed at the center of the detectors' back sides. Each LED was pulsed independently via BNC breakouts (Fig. 3.2(b)). To characterize the LED source, the wavelength of an LED identical to those used in NR14 was measured with a spectrometer at various temperatures, from room temperature (296 K), then liquid nitrogen boiling point (77 K) and liquid Helium (4 K). The greatest change in wavelength occurred between room temperature and 77 K, where the peak wavelength dropped from 630.3(2) nm down to 607.3(1) nm. At 4 K the wavelength settled to 605.7(1) nm. Since our LEDs are cold, this gives us an estimate for LED photon energy $E_{\text{photon}} = 2.05 \pm 0.02$ eV. Note that $E_g < E_{\text{photon}} < 2E_g$, where $E_g = 1.13$ eV is silicon's band-gap, ensuring that a single e^-h^+ pair is produced by each photon hitting the detector. To pulse an LED, it is driven by a 300 kHz sinusoidal signal and a single-cycle burst frequency of 10 Hz. The mean number of photons per LED flash (λ_{poisson}) was varied by changing either the driving signal's amplitude (peak-to-peak) or its DC offset. Fig. 3.3 shows the LED circuit's response to a triangle test signal.



(a) Bottom side of an LED box showing two pinholes each covered with an IR filter. (b) The top of an LED box, showing the wiring used to pulse each LED independently.

Figure 3.2: Both sides of an LED box used in NEXUS Run 14.

HVeV Run 4 employed a different calibration strategy compared to previous runs (by

using LEDs instead of lasers). The key advantage of the new strategy is that the Run 4 setup ensures photons hit the face of the detector to avoid incompletely amplified/collected sidewall events, yielding a purer dataset that is easier to model. However, having the light source so close to the detectors means readout wires are susceptible to capacitive cross-talk with LED wires, which must be modeled. In addition, taking DM search and LED calibration data in separate runs complicated the precise alignment of conditions between runs (see Fig. 2.14), which required the application of corrections (Fig. 2.19(a)) that introduced systematic uncertainties. The most comprehensive collection yet of photon calibration data was acquired during NR14, from which various detector response effects were measured, as will shortly be discussed in the following sections. This includes data sets of varying the LED intensity, taken in order to tease out the cross-talk amplitude from the degenerate effect of surface trapping. To vary the LED intensity while holding the cross-talk constant, the DC offset of the LED driving signal was varied, while the driving signal's peak-to-peak amplitude was held constant.

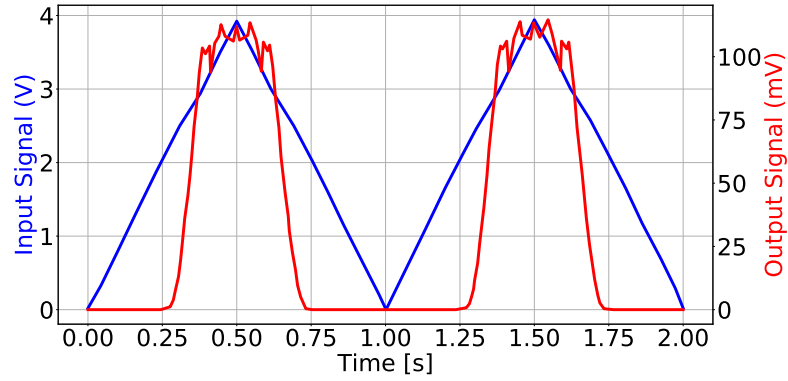


Figure 3.3: LED (in series with $50\ \Omega$ resistor) response to an applied triangle test signal of 1 Hz frequency, with LED submerged in liquid nitrogen.

3.1.1 Triggering on LED Events

The burst frequency (10 Hz) of the LED driving signal was known, so an LED interpolation trigger was developed to infer LED event times. This was necessary because the LED apparatus did not include a TTL (Transistor-Transistor Logic) channel for triggering on the LED driving signal. For each continuous period of LED data taking (with LED driving signal parameters held fixed), the algorithm starts by finding the times between each

threshold triggered event and the next 15 that follow it. For each time difference, 20 possible frequency values are calculated (each corresponding to an assumption of 1-20 test-signal periods having passed). The histogram of such values (300 per triggered event) has a peak at ~ 10 Hz, which we fit with a Gaussian. Next, events that have at least 2 events out of the previous 10 (or the next 10 if the former is absent) that differ by no more than 0.5 % from the expected times found from the Gaussian fit of event time differences are selected as the LED events that also pass the trigger threshold. To complete the LED interpolation algorithm, for each gap between those events, the proper number of triggers are forced (consistent with 10 Hz), with the trigger points for these sub-threshold LED events being equally divided in the LED event gap region.

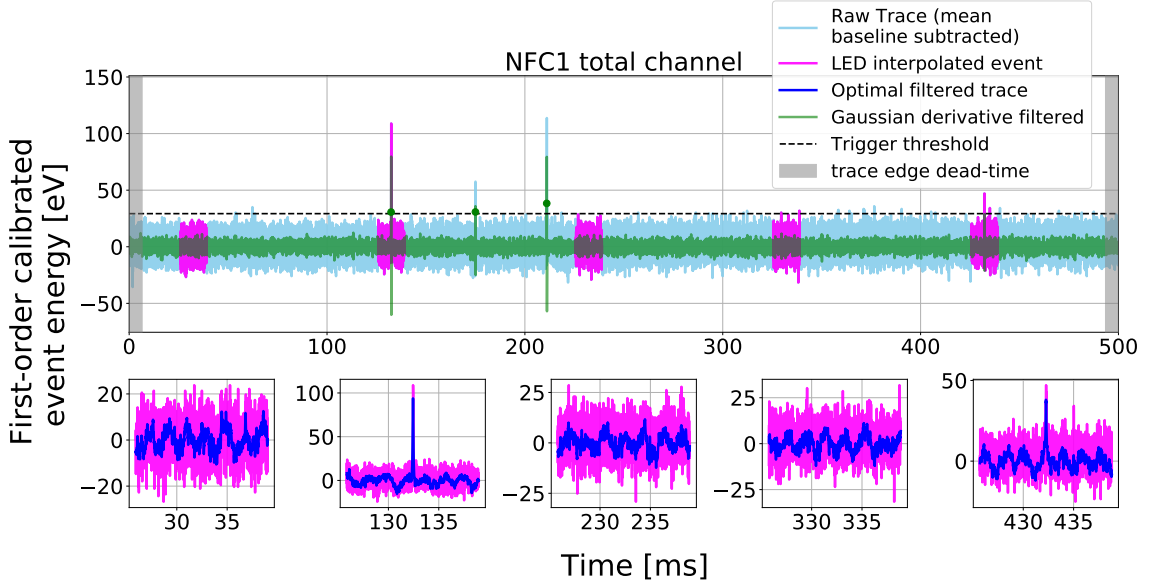


Figure 3.4: Sample readout trace from NFC1 calibration data set taken at 100 V crystal bias. Five LED pulse regions are identified by the LED interpolation trigger within this trace, which are highlighted in magenta in the upper figure with 5 zoom-in plots in the lower figures. The first-order (linear) calibration is discerned from the quantized amplitude spectrum of all LED pulses from the same dataset (Fig. 3.5 below). The second LED interpolated event from the left falls in the $1 e^-h^+$ range in Fig. 3.5. The event shown on the far right has an estimated total phonon energy of 50 eV (it is barely missed by the threshold trigger). The low frequency oscillations seen correspond to the ~ 500 Hz noise peak seen in Fig. 3.6(b), which is missing from the PSD from NR13 used in the optimal filter (Fig. 3.6(a)). The green dots indicate events that cross the trigger threshold.

This trigger allowed for the reconstruction of all LED events that occurred at the burst frequency of the LED’s bias signal, including sub-threshold and $0e^-h^+$ events. Fig. 3.4 shows a sample trace from an LED dataset, with an $E_{tot} \sim 50$ eV event eluding the threshold trigger but is identified by the LED interpolation trigger. Fig. 3.5 shows the spectrum of event amplitudes of LED interpolation triggered events from the same dataset which includes the trace shown in Fig. 3.4. In the fit of Fig. 3.5 the $E_{tot} \sim 50$ eV event from Fig. 3.4 is interpreted as a photon event which experiences charge trapping thus producing incomplete NTL amplification. Fig. 3.4 also indicates that events may frequently occur outside of the expected LED event times. In fact, there are many instances of an LED event failing event quality selections (Sec. 3.1.2) because of another (usually $1e^-h^+$) event within its pulse analysis window. Being careful with our treatment of these ‘LED coincidences’, one can use the LED events to additionally estimate the trigger efficiency, and to investigate the efficiency of event selection cuts (Ch. 4).

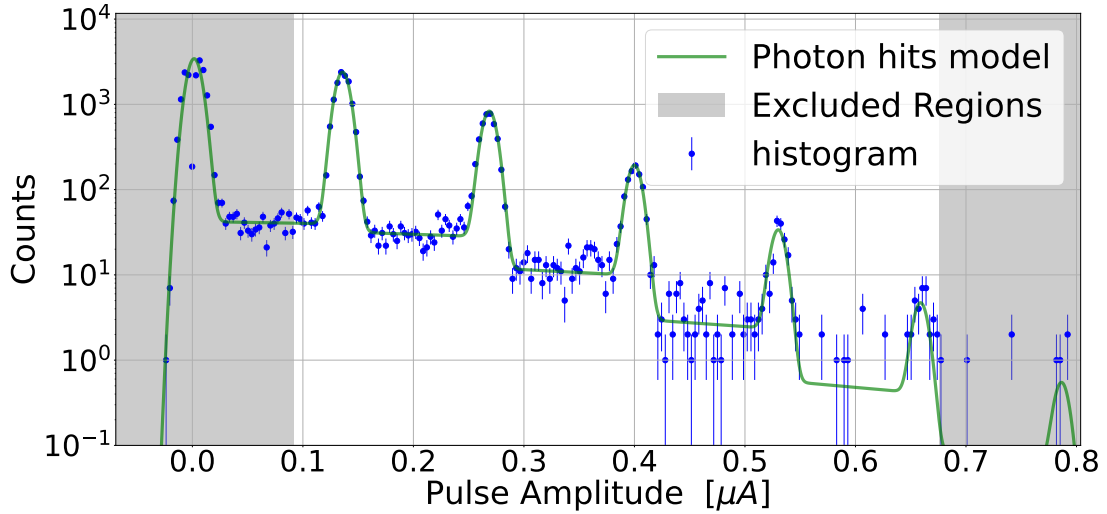


Figure 3.5: Calibration fit of the pulse amplitude (A_{OFL}) spectrum in NFC1 with crystal bias 100 V using the simultaneous photon hits model which was sketched in Fig. 3.1 and will be mathematically defined in Sec. 3.2 below. The result of this fit is used to calibrate the DM search data of Ch. 4. The $0e^-h^+$ peak is excluded as it is distorted in the A_{OFL} spectrum due to random noise fluctuations (see Sec. 3.2.1). Regions of low event count are also excluded from this binned fit.

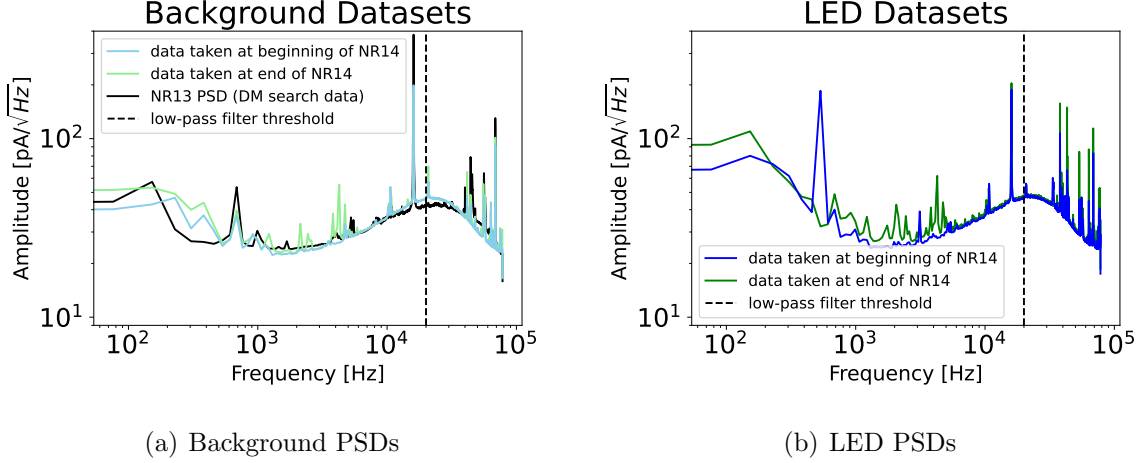


Figure 3.6: Power spectral densities from various datasets in HVeV Run 4. The NR13 PSD (black line) is used in the optimal filter in *all* datasets, including LED datasets.

3.1.2 Data Selections for LED Calibration Data

The mixing chamber temperature was stable throughout NR14, so no fridge temperature selection was required. We apply a TES baseline live-time selection for all LED data sets to ensure stable run conditions, similar to what is applied to DM search data in Sec. 2.2.6. Coincident events (which happen within each other's pulse analysis windows) were observed in LED data, which cause distinct bands in the χ_{red}^2 distribution (see Fig. 3.7). These events are characterized by having a main LED pulse followed by 1 e^-h^+ events. An energy-dependent χ_{red}^2 cut (introduced in Sec. 2.2.5) defined on the LED data removes events that have other events within their pulse analysis window, yielding a flat passage fraction. After the TES baseline and χ_{red}^2 cuts are applied to LED datasets, they may be used to calibrate the detectors. One may apply a coincidence event quality cut before the χ_{red}^2 if one is interested in calculating a cut efficiency (as will be shown in the DM search of Ch. 4), but for the purposes of calibration the χ_{red}^2 may do all the heavy lifting. These LED coincident events are discussed further in Ch. 6.

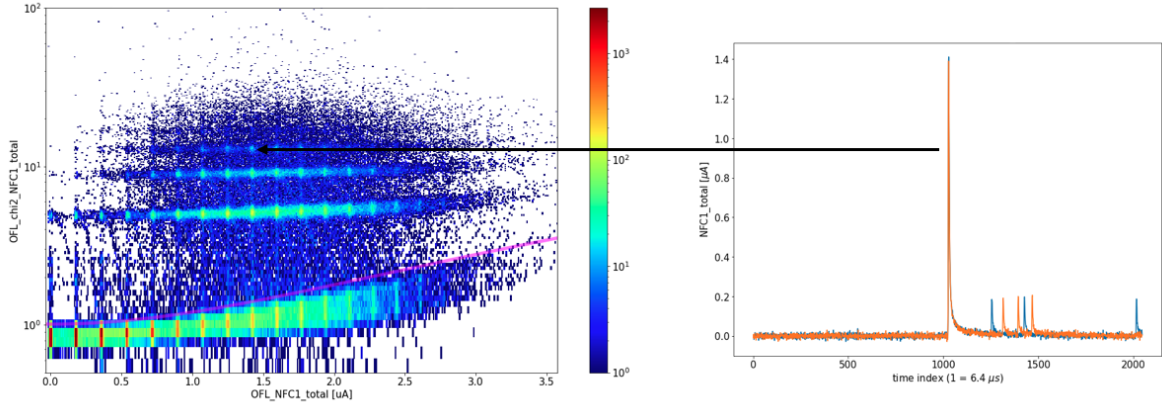


Figure 3.7: Left: The distribution of event counts in (A_{OFL}, χ_{red}^2) RQ (2.2.4) space. The lowest χ_{red}^2 band is bounded by the magenta line which defines the maximum χ_{red}^2 to pass the cut given A_{OFL} . The cut boundary is the parametrization $\chi_{red}^2 < f_{cut}(A_{OF}) = p_2(A_{OF})^{p_1} + p_0$ discussed in Sec. 2.2.5, with parameters p_i optimized to obtain a smooth estimate of the 3σ upper boundary of the lowest- χ_{red}^2 band. Right: Two event pulses (orange and blue) from the $8 e^- h^+$ peak, followed by three $1 e^- h^+$ events each. The black arrow shows where the events on the right fall in the 2D histogram on the left.

3.2 Detector Response Fits

To extract charge trapping and impact ionization (CTII) fractions, we must first construct a model for *simultaneous photon hits* and convolve with the appropriate detector response functions. While Fig. 3.1 shows energy spectra with partial application of different detector effects to illustrate the different physical processes involved, it does not show the order of steps in constructing the signal model itself. The multiple photon hits model is specified by one kernel, the one photon probability density function h_1 , which is convolved with itself N times to give the total energy (E_{tot}) spectrum due to $n \leq N$ photons simultaneously hitting the detector, $h_n(E_{tot}) = h_{n-1}(E_{tot}) * h_1(E_{tot})$. For a mono-energetic source of energy E_γ with $E_{gap} < E_\gamma < 2E_{gap}$, $h_1(E_{tot}) = \delta(E_{tot} - [E_\gamma + e\Delta V])$ is a Dirac delta function with ΔV being the crystal bias voltage (assuming a perfect detector with infinitesimal resolution and no trapping or impact ionization). This delta function gives $h_N(E_{tot})$ shown in Fig. 3.1(b), where the relative heights of the peaks are determined by the Poisson distribution of the number of photons emitted per LED flash λ_{Pois} . Fig. 3.1(c) shows $h_N(E_{tot})$ when $h_1(E_{tot}) = (1 - f_{surf})\delta(E_{tot} - [E_\gamma + e\Delta V]) + f_{surf}\delta(E_{tot} - E_\gamma)$, where f_{surf} is the surface trapping fraction which indicates the mean number of photon hits that will not result in any NTL amplification.

To include CTII effects, we utilize the same iterative approach to obtain composite probabilities. We use $h_1(E_{tot}) = (1 - f_{surf})p_{1e^-h^+CTII}(E_{tot}) * f(E_R = E_{tot}) + f_{surf}f(E_R = E_{tot})$ where $f(E_R) = \delta(E_R - E_\gamma)$ in the case of a mono-energetic source. The p.d.f. $p_{1e^-h^+CTII}(E_{tot})$ is derived by finding solutions to an exponentially decaying probability of survival in the total energy domain, for various combinations of CTII processes due to an initial surface event. A charge carrier must always traverse the entire crystal thickness for full NTL amplification, in contrast to the CTII model for bulk events used in the DM limit setting of Ch. 4 (where an e^-h^+ pair share the same drift length between them), see [194] for more details. The fraction f_{CT} (f_{II}) is the probability that a charge will undergo a charge trapping (impact ionization) process if it traverses the full crystal thickness or $f_k = 1 - e^{-T_k \cdot D_c}$, where T_k is the mean probability per unit length for process k and D_c is the crystal thickness. Finally, in order to fit the A_{OF} spectra we need to account for the nonlinearity of the TES response, so we use a calibration function $E_{OF} = F_{calib}(A_{OF}) = c_1 A_{OF} + c_2 A_{OF}^2$. Finally, a convolution with a Gaussian of width σ_{res} is effected to include phonon energy resolution effects. Now we have constructed what is referred to as the *extended* CTII model, which models unamplified events as depositing only the energy of the photon with no NTL amplification.

When surface trapping takes place, it can be shown that all e^-h^+ peak maxima are shifted by the same constant energy term which depends on the LED intensity λ_{LED} ,

Fit Parameter Description	Constant Shift Model	Extended Model
LED Intensity	$\lambda_{LED, app}$	λ_{LED}
Phonon Resolution [eV]	$\sigma_{res, app}$	σ_{res}
Charge Trapping Fraction	f_{CT}	
Impact Ionization Fraction	f_{II}	
Surface Trapping Fraction	$f_{surf} = 0$	f_{surf}
Constant Energy Shift [eV]	c_0	$c_0 = 0$
Linear Calibration Term [eV/ μA]	c_1	
Quadratic Calibration Term [eV/ μA^2]	c_2	

Table 3.1: Table of parameters in the simultaneous photon hits calibration and detector response model. Two options are shown for treating the ‘surface trapping’ which results in a shift in the energy spectrum of LED datasets, which are described in the text.

surface trapping fraction f_{surf} and the energy of the photons E_γ . This is given by $E_{shift} = \lambda_{LED} f_{surf} E_\gamma$ [212]. This allows us to use the simpler *constant shift* model if the surface trapping sub-peaks are obscured by a large resolution ($\sigma_{res} \gtrsim E_\gamma$, compare Fig. 3.1(e) and 3.1(f)). The energy shift can be taken into account using a fit parameter c_0 that is added to the energy function $E_{OF} = F_{calib}(A_{OF}) + c_0$ and setting $f_{surf} = 0$. The event energies would then be given by $E_{OF} - c_0 = F_{calib}$ with $F_{calib}(A_{OF} = 0) = 0$. Note that when using the constant shift model, the apparent lambda (from relative heights of e^-h^+ peaks) $\lambda_{LED, app} \sim (1 - f_{surf})\lambda_{LED}$ which is valid for $\lambda_{LED} \lesssim 10$ (λ_{LED} specifies relative heights of maxima in Fig. 3.1(b) while $\lambda_{LED, app}$ specifies those of Fig. 3.1(c)). The fitted resolution will be larger in the constant shift model than if we had used the extended model which accounts for the increased width due to unamplified events $\sigma_{res, app} > \sigma_{res}$ (see Fig. 3.1(c)). The main disadvantage of using the constant shift model is that it can lead to unphysical values of the fit parameters, for example an energy shift $-c_0$ larger than the expected E_{shift} . This should not affect the calibration, even though it neglects the correlations between various detector response parameters which are expected in the full underlying theory. The main disadvantage of the extended CTII model is the strong correlations between the parameters which may make it more difficult for fits to converge. Table 3.1 summarizes the fit parameters for both detector response models.

3.2.1 Cross-Talk Subtraction

Signs of cross-talk between readout and LED pulsing wires can be seen in the average pulse shape for LED events in Fig. 3.8. The high frequency oscillatory behaviour in the pulses is

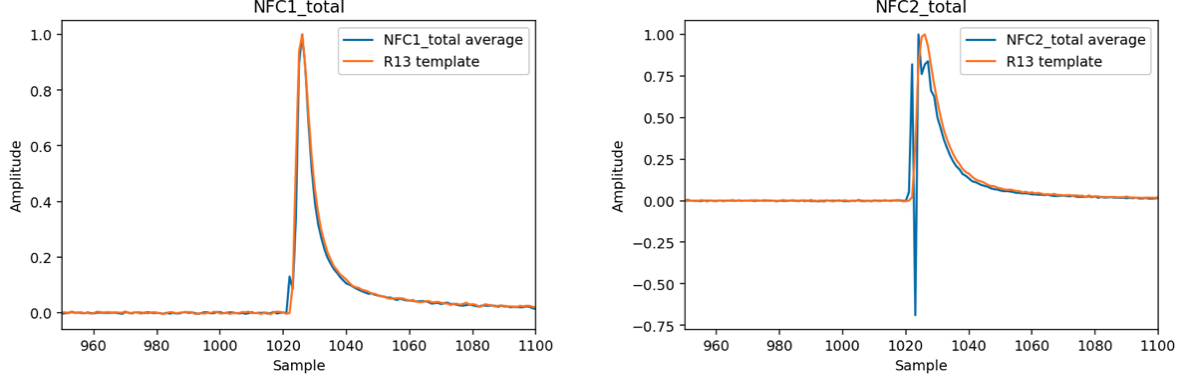


Figure 3.8: Average $1e^-h^+$ pulse shapes in NR14 LED data compared with the average pulses from NR13 DM search data. Left: for detector NFC1 and Right: for NFC2. Pre-pulse fluctuations are seen in the LED pulses, indicative of capacitive cross-talk between readout and LED wires. The cross-talk is the least prominent in NFC1.

removed by the low-pass filter (LPF) used in the event reconstruction stage. Even after the LPF is applied, the effect of the cross-talk is not fully removed, as can be seen when finding the mean amplitude of a $0e^-h^+$ event in the limit of small LED intensity. As discussed above, another effect which can cause a shift in the spectrum is the aforementioned surface trapping effect, which adds a constant energy $E_{shift} = \lambda_{LED} f_{surf} E_\gamma \simeq \frac{f_{surf}}{1-f_{surf}} E_\gamma \lambda_{LED,app}$ for $\lambda_{LED} \lesssim 10$. Multiple datasets with the same LED driving signal amplitude (peak-to-peak) were taken, while the LED intensity was varied only by varying the DC offset of the driving signal. Then the cross-talk amplitude (CTA) was estimated via a linear fit of the $0e^-h^+$ peak amplitude (from a Gaussian fit of the peak) as a function of $\lambda_{LED,app}$ (found from a constant shift model fit, see Table 3.1), where the CTA is the y-intercept. The CTA value is subtracted from each event amplitude. From these linear fits (see Fig. 3.9 for example) we can also estimate the surface trapping fraction using:

$$\frac{c_1 b}{E_\gamma} \simeq \frac{f_{surf}}{1 - f_{surf}}, \quad (3.1)$$

where b is the slope of the line and c_1 is the calibration constant in Table 3.1.

Fig. 3.9 shows the observed $0e^-h^+$ peak positions as a function of $\lambda_{LED,app}$, along with the linear cross-talk estimation fit the result of which will be used in the calibration (Sec. 3.2.4) relevant for the DM search data. The main calibration dataset (represented by point ‘1’ in Fig. 3.9, the pulse amplitude spectrum of which was shown in Fig. 3.5) used in

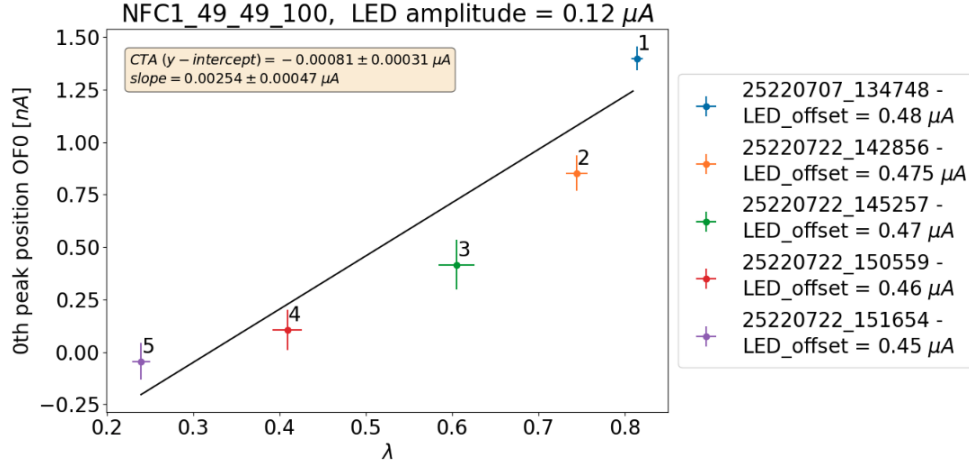


Figure 3.9: The $0e^-h^+$ peak amplitudes in datasets of varying intensity $\lambda_{LED,app}$ (NFC1 LED datasets at 100 V crystal bias). The intensity was varied only by varying the DC offset of the LED driving signal, while holding the signal’s peak-to-peak amplitude constant to hold capacitive cross-talk constant. A linear fit gives an estimate of the $0e^-h^+$ peak amplitude at zero LED intensity, which is taken as an estimate for the cross-talk pulse amplitude. The point labelled ‘1’ corresponds to the main calibration dataset (shown in Fig. 3.5) relevant for the DM search data, and was taken early on in NR14 (the legend labels the datasets as 25YYMMDD_hhmmss where 25 is the NEXUS site code, followed by the time of taking). The apparent curve is mostly due to dataset ‘1’ having an additional noise component acting as a parasitic power source, causing an unusually large fractional uncertainty on the estimated cross-talk ($\sim 100\%$ for this group of datasets) as discussed in the main text.

the DM search analysis was taken early on in NR14. After around two weeks, smaller LED datasets were taken as part of the LED intensity scan in order to calculate the cross-talk amplitude. By that time, however, noise conditions appear to have changed as seen in the disappearance of the 500 Hz noise peak seen in Fig. 3.6(b). The e^-h^+ peak widths also decreased for late-run LED intensity scan datasets, as will be discussed in Section 3.3. This change in noise conditions could also explain the additional shift in the main calibration dataset’s e^-h^+ peaks compared to the shifts observed in the datasets that lacked the 500 Hz noise peak (see point ‘1’ in Fig. 3.9 in comparison to other data points), which cannot be due to surface trapping. This is the largest source of uncertainty due to cross-talk amplitude subtraction for NFC1 datasets at 100 V crystal bias (relevant for DM search data). Only the calibration data for NFC1 taken at 100 V is affected by this; other datasets confirm

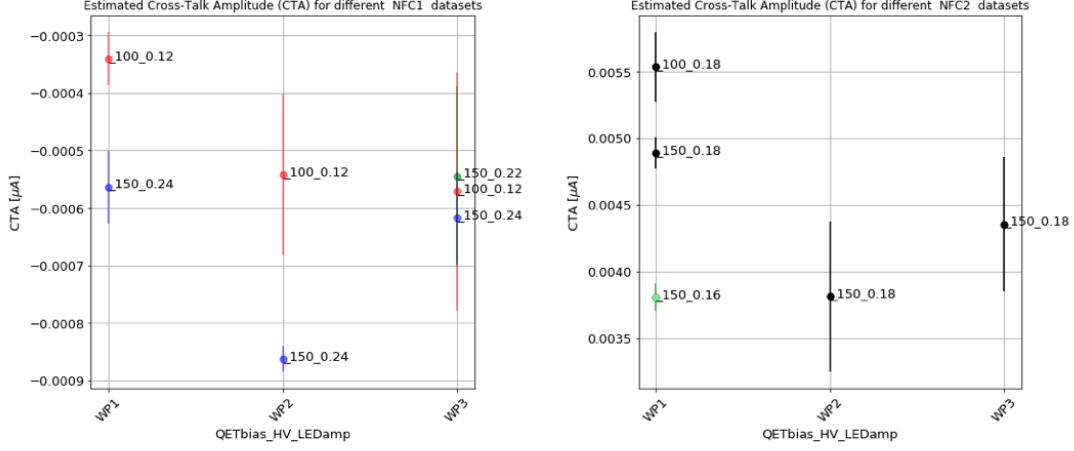


Figure 3.10: Cross-talk estimates for detectors NFC1 (left) and NFC2 (right) in NR14 LED data. The horizontal axes label different QET biases or ‘working points’ (WPs), while the points in the figures are labeled with the crystal voltage in volts and the driving LED signal’s amplitude (peak-to-peak). Each of these points is an estimate of the cross-talk amplitude from fits such as Fig. 3.9 where only the LED drive signal’s DC offset is varied, in an attempt to hold capacitive cross-talk constant. The apparent variation in NFC1 (left) between the CTA estimates as a function of crystal bias and varying QET biases indicates a component to the cross-talk that is not simply capacitive, so the maximum variation is taken as a systematic uncertainty in the cross-talk.

the linear relation between LED intensity and peak amplitude shift. Another source of cross-talk systematic uncertainty is the variation of estimated cross-talk with QET bias currents (Fig. 3.10), which challenges the assumption of simple *capacitive* cross-talk, and the maximum difference is taken as an estimate of that uncertainty. Instead of pure capacitive cross-talk, the cross-talk could also have a power coupling component which we do not model. Finally, a third cross-talk uncertainty is statistical uncertainty from the linear fits, which takes into account uncertainties in estimated $0e^-h^+$ peak amplitudes as well as $\lambda_{LED,app}$ from the constant shift fits to the LED spectra. The total uncertainty on the cross-talk amplitude will be combined in Section 3.2.5 with other sources of uncertainty on the energy scale.

All spectra used in the extraction of the CTAs are A_{OF0} , as we are interested in the $0e^-h^+$ peaks which are sub-threshold, so in the A_{OFL} spectra the $0e^-h^+$ peaks are de-

formed due to random noise fluctuations being of comparable magnitude to the pulses of interest (an OF fit with a time offset is biased towards higher amplitude values, see Appendix E of [210]). The $0e^-h^+$ peak also contains information about the LED intensity and surface trapping, while the region between the $0e^-h^+$ and $1e^-h^+$ strongly constrains the charge trapping fraction f_{CT} since it contains almost no impact ionization events. Thus, A_{OF0} spectra are also ideal for low-energy detector response modeling. However, when the LED intensity is very high ($\lambda_{LED,app} \gtrsim 5$), the higher energy peaks suffer significant loss of resolution in the A_{OF0} (no time-delay) spectrum, and the $0e^-h^+$ peak is not visible anyway, so the A_{OFL} spectrum is all that is needed (Fig. 3.11 shows a fit of a high LED intensity dataset).

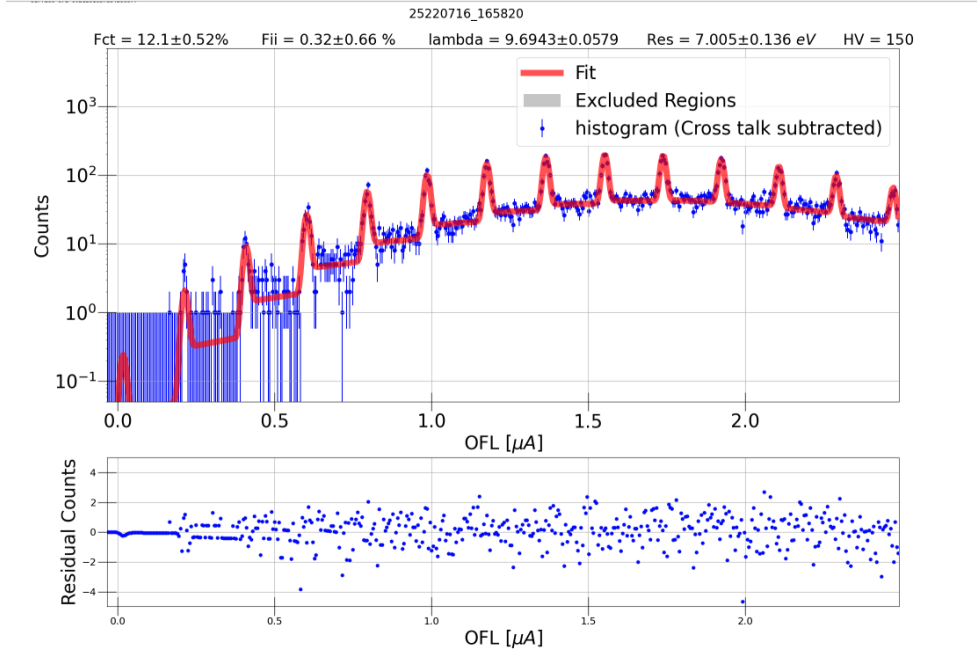


Figure 3.11: Top: Result of binned maximum likelihood fit of an LED dataset acquired in NR14 at 150 V crystal bias after applying the data selections in 3.1.2, using the constant shift model (parameters in Table 3.1). Bottom: residuals of the fit above (in units of σ).

3.2.2 Comparing the Models

After applying the χ^2_{red} and TES baseline cuts described in 3.1.2 on LED data, and subtracting the cross-talk amplitude found in the previous subsection, we proceed with fit-

ting all LED spectra using both the constant shift and the extended models. No statistically significant differences for the parameters of interest (calibration parameters and CTII parameters) were found between the two models. Since our energy resolution is large ($\sim 3 \text{ eV} > E_\gamma \simeq 2 \text{ eV}$), the hypothesized surface trapping sub-peaks in the extended model (recall Fig. 3.1(c)) are not visible, so the lack of direct information on the true value of f_{surf} (Table. 3.1) combined with the strong correlations in the extended model (Fig. 3.12) preclude us from using it reliably for calibration. A detector with improved resolution may resolve the sub-peaks and break these degeneracies.

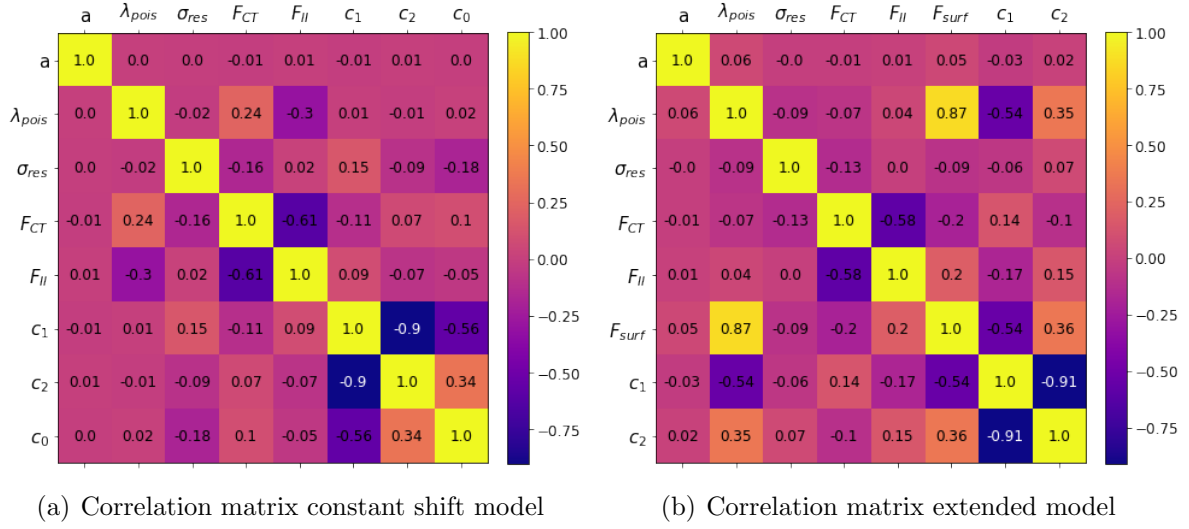


Figure 3.12: The correlation matrices from two models treating surface trapping differently (parameters summarized in Table 3.1). The correlations between the calibration constants c_1 and c_2 and surface trapping parameters c_0 or F_{surf} (lower right corners of the matrices) are unchanged between the models. However, the extended model results in strong correlations between λ_{LED} and calibration constants c_1 , c_2 and the surface trapping fraction F_{surf} .

3.2.3 Summary of CT and II Results

Charge trapping and impact ionization values were measured from various datasets taken at crystal bias of 100 and 150 V, with varying LED intensities. The results are summarized in Fig. 3.13, which shows that all values are statistically consistent with no apparent

dependence on voltage or LED intensity. The impact ionization values of best fit are almost all consistent with zero.

3.2.4 Calibrating DM Search Data

We use A_{OFL} as our energy estimator in the DM search analysis of Ch. 4, as all events of interest are well above the noise level ($E_{tot} > 50 \text{ eV} > 12 \sigma_{res}$), and relying on the trigger point without fitting for time-delay of the filtered trace does not take full advantage of the detector's timing resolution. Indeed, using A_{OFL} instead of A_{OF0} yields a lower value for the energy resolution. To calculate calibration constants for A_{OFL} , we fit the relevant A_{OFL} LED spectrum that was shown in Fig. 3.5 with the multiple photon hits model (excluding the $0 e^-h^+$ region), with all parameters in Table 3.1 fixed to the values acquired from an initial A_{OF0} fit (ideal for low-energy detector response modeling, see Sec. 3.2.1) of the same events which includes the $0 e^-h^+$ peak, except for the calibration constants c_1 , c_2 and the resolution parameter $\sigma_{res,app}$, which are floated. We use a calibration function $E_{OFL}(A_{OFL}) = \beta(c_1 A_{OFL} + c_2 A_{OFL}^2)$ for NFC1 data from NR13, where the central values for the parameters (from the fit of Fig. 3.5) are $(c_1, c_2) = (755.53 \text{ eV}/\mu\text{A}, 30.45 \text{ eV}/\mu\text{A}^2)$ and $\beta = 0.96$ is the linear gain correction needed to apply the NR14 calibration to NR13 (Section 2.2.3). This is the calibration used in Ch. 4 DM search, where the TES gain correction factor was calculated as $\beta = \frac{A_{1e^-h^+, NR13}}{A_{1e^-h^+, NR14}}$ (the ratio of $1 e^-h^+$ amplitudes from background datasets from the respective runs). The uncertainty on the energies is also needed for the DM analysis, and it is estimated in Section 3.2.5.

3.2.5 Calibration Uncertainty

The uncertainty on the energy scale after applying the calibration is estimated in this section. Three types of systematic uncertainties were identified: 1) uncertainty due to cross-talk subtraction; 2) uncertainty due to the assumption of a quadratic calibration function; and 3) uncertainty due to the linear TES gain correction applied to calibrate NR13 DM search data using NR14 LED data. The first two are taken into account as constant uncertainties: for 1) the estimated cross-talk amplitude's uncertainty converted to energy units, and for 2) the maximum residual for e^-h^+ peak energies compared to those predicted by the quadratic calibration function. Systematic 3) was calculated separately for each peak, and was quantified by applying a linear gain correction on data sets taken at varying QET biases within $\pm 4\%$ of the central value, taking the maximum deviation as the uncertainty estimate. Finally, the statistical uncertainty on the calibration constants

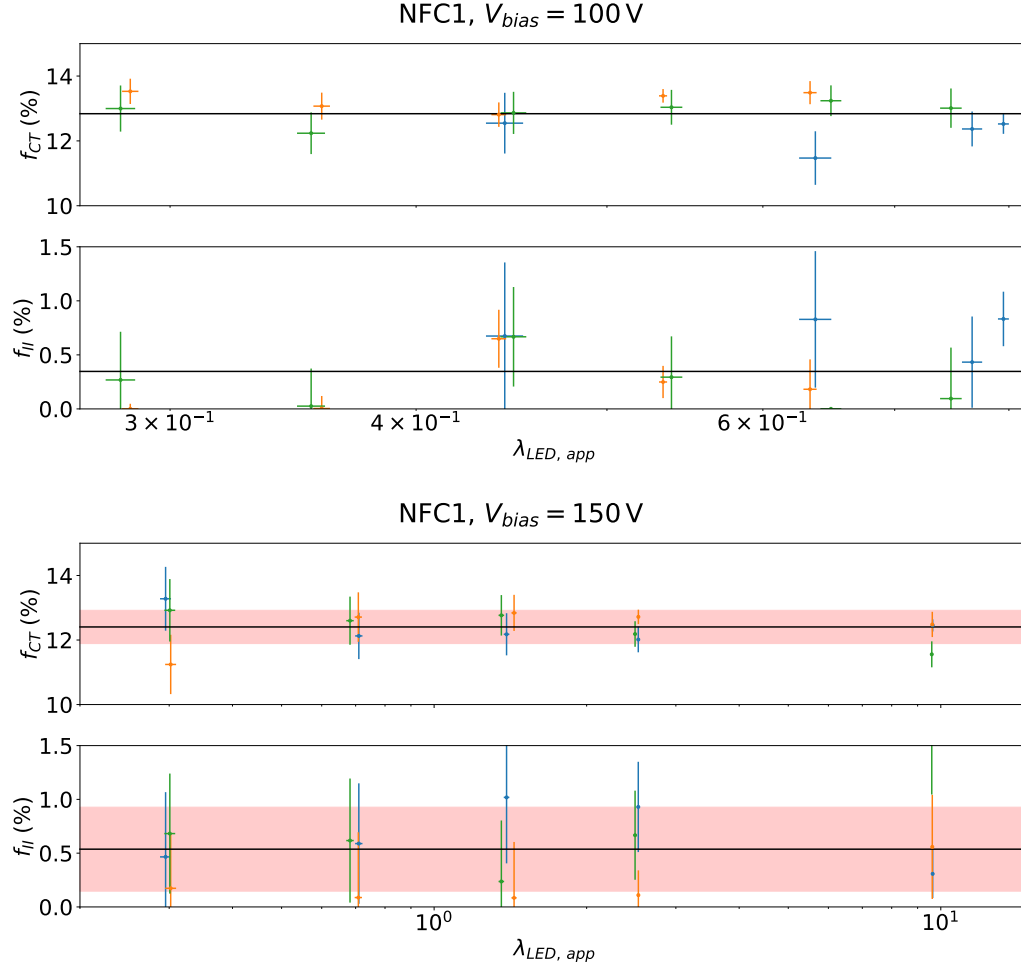


Figure 3.13: CT and II fractions from fits of various LED datasets of detector NFC1 taken in NR14. Upper two plots: CT and II fractions from datasets with crystal bias of 100 V. Lower two plots: same as upper two but crystal bias is 150 V. The horizontal black line indicates the mean of the point, while the shaded regions are cover the 1σ range where σ is the standard deviation of the points. No statistically significant variation is found in the CT/II fractions for varying LED intensity or crystal bias. Different coloured data points within each graph indicate data sets taken at different QET biases.

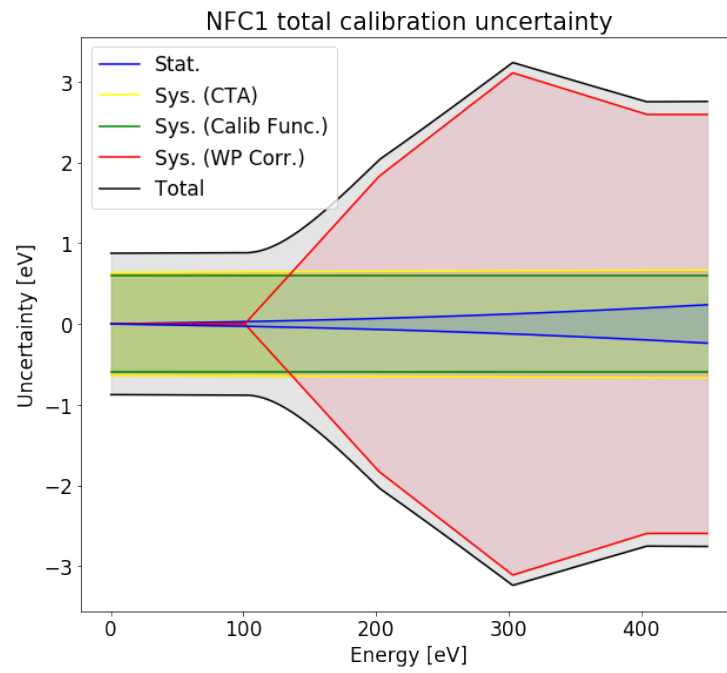


Figure 3.14: Contribution of various sources of uncertainty to the total uncertainty on the calibrated energies of the DM search data of NR13 (detector NFC1 at 100 V crystal bias).

c_1 and c_2 is propagated using the calibration function. All four uncertainties are added in quadrature, see Fig. 3.14. The total uncertainty on the phonon energy scale up to and including the $4e^-h^+$ peak is $\lesssim 1\%$.

3.3 Resolution Modelling

The phonon resolution (peak width) in the LED spectrum was 4.88 ± 0.03 eV from the fit of Fig. 3.5, significantly higher than the $1e^-h^+$ peak width of the NR13 DM search spectrum. This is mostly attributed to differing noise conditions and cross-talk effects. See Fig. 3.15 for peak-specific resolution values. It is seen in Fig. 3.15 that the resolution varies by < 0.5 eV between $1e^-h^+$ and $5e^-h^+$. In theory, the total phonon energy resolution should not differ much between the peaks because ‘position dependence’ [124, 213] should not be a large contributor to the resolution in NFC1 given the equal channel weighting and small (4 mm) thickness of the detector (Fig. 2.19(b), see also Ref. [125]).

Noise conditions affect the resolution value significantly, as we processed all these datasets with the NR13 PSD (black curve in Fig. 3.6(a)), so run-dependent noise was not taken into account. In end of run (NR14) LED datasets, additional noise peaks are found in the range 1-10 kHz (Fig. 3.6(b)), and these are also found background datasets taken around the same time (Fig. 3.6(a)), indicating a change in environmental noise unrelated to the operation of the LEDs. The beginning of run LED PSD on the other hand has a noise peak at ~ 500 Hz that was **not** present in the background series taken around the same time. The e^-h^+ peaks are also wider for the early run dataset with the additional ~ 500 Hz noise peak, as can be seen in Fig. 3.15. So it is clear that noise conditions and cross-talk are the main components to the excessive widths seen in LED data.

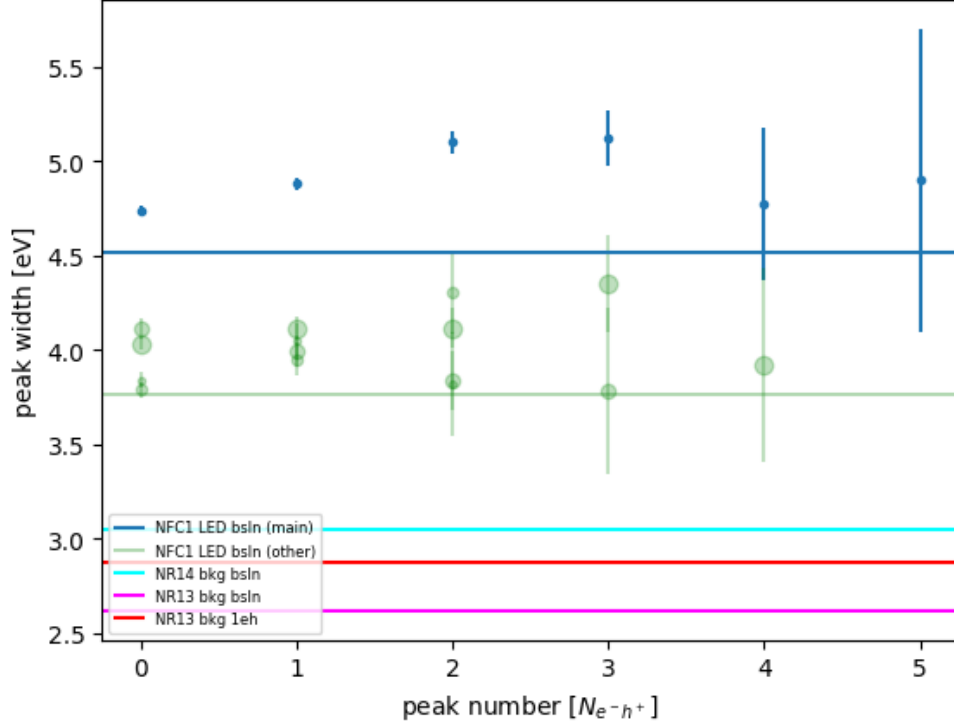


Figure 3.15: Various estimates of the energy resolution. Flat lines indicate benchmark values (baseline resolutions from random events labeled ‘bsln’) from LED and background/DM search (‘bkg’) data. Flat red line is the $1 e^-h^+$ peak resolution from the DM search data of NR13. The points are estimates from separate e^-h^+ peak fits in LED data-sets, with the blue points (corresponding to ‘main’ in the legend) indicating the early-run main calibration dataset shown in Fig. 3.5 (which suffered from an additional ~ 500 Hz noise source seen in Fig. 3.6(b)), while the green points are from subsequent datasets (corresponding to ‘other’ in the legend) of varying LED intensity when the ~ 500 Hz noise source was absent (datasets 2-5 in Fig. 3.9, larger green points are from datasets taken at larger LED intensity).

Chapter 4

HVeV Run 4 Analysis: Search for Dark Matter - Electron Interactions

In this chapter, an electron recoil dark matter (ERDM) search analysis of new data from HVeV detectors (the setup is described in section [2.2.2](#)) is outlined. We start by quantifying the triggering threshold, and move on to live-time selections, event quality cuts and estimating cut efficiencies. We end the chapter by setting exclusion bounds on the scattering cross-sections of sub-GeV dark matter with electrons, as well as the axion-electron coupling and the dark photon kinetic mixing parameter. This chapter will present results from detector NFC1 since it will be used for the dark matter search, while the other detectors will be used as vetoes as will be outlined in [4.4.1](#).

4.1 Calculating Trigger Efficiencies

The reconstruction of event quantities from raw traces was covered in [2.2.3](#), where a Gaussian-derivative filtered trace is used to identify pulses. A relevant question to ask for the dark matter search is, what percentage of dark matter energy depositions does a triggering algorithm identify? To answer this question, we can test the trigger on events of known energy and frequency, such as LED events. See [3.1.1](#) for details. Alternatively, one can run toy monte carlo experiments and apply the trigger algorithm on simulated events. We did this using a sample of randomly triggered events summed with average pulse shapes found at various pulse amplitudes, applying the triggering algorithm and finding the fraction of events identified by the Gaussian-derivative trigger. The templates and random

noise events are taken from 0V gamma source (^{137}Cs) data taken during NR13, which had very similar noise conditions to the DM-search data. Fig. 4.1 summarizes the result of both methods for finding the trigger efficiency for NFC1. It can be seen that at total phonon energies $\gtrsim 60$ eV. The trigger efficiency reaches 100%.

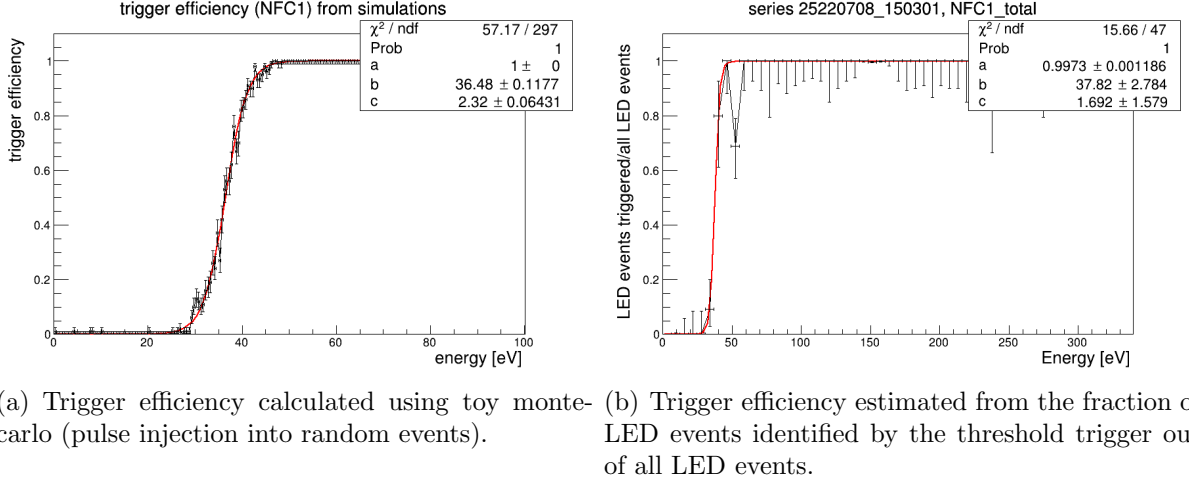


Figure 4.1: Estimates of the trigger efficiency for NFC1 detector

4.2 Data Selections

To ensure stable run conditions and maximize the signal-to-background ratio for a certain analysis, selection criteria (or ‘cuts’) are often devised, which are applied to enhance the signal-to-background ratio. Our 2-channel crystal bolometer in general has no ability to discriminate a single DM event from any other recoil (say, based on event topology), so optimizing the signal-to-background ratio will not play a significant role in the tuning of the following cuts. The cuts are instead devised primarily to select stable run conditions (live-time cuts) and ensure accurate reconstruction (event quality cuts).

4.2.1 Live-time Cuts

The total raw exposure of the DM search data was 10.80 gram days for each detector (70% blinded exposure = 7.56 gram days per detector) after the exclusion of the readout trace edges which suffer from incomplete filtering (shaded regions in Fig. 2.16). To select periods

of time where the detector is suitably stable for a dark matter search, a series of live-time cuts were developed. The standard fridge temperature cut and TES baseline cut (defined in Sec. 2.2.6) are preformed to ensure a constant TES response. While the fridge temperature cut has minimal effect on the pulse amplitude spectrum, Figure 4.4 shows that the event passage fraction after applying the TES baseline cut dips most significantly at an energy just below 100 eV; this is likely due to these events being improperly reconstructed $1e^-h^+$ events. The calibration of Ch. 3 suggests that at elevated baselines, an event with a given energy would produce a shorter pulse in the TES current. Two more analysis-specific selections are defined for the dark matter search. The trigger rate cut and coincidence cuts defined below deal with periods of time when a large number of events are time-correlated, indicating the taking place of a temporary background process unrelated to DM halo scatterings in the detector. In any case, these cuts remove live-time when our detector is not operating optimally for a DM search. The net exposure (after the cuts) will be evaluated after defining and applying the live-time cuts below.

Trigger rate cut

Periods of time when the event rate (specifically the $1e^-h^+$ event rate, see Fig. 4.4) surges suddenly and decays over a couple of hours have been identified in the DM search dataset. These events are contained within a single detector (although there seem to be close coincidences between different detectors with tens of minutes between elevated rate periods, see Ch. 6). These periods are initiated by a very high energy pulse. While Fig. 2.16 shows an example of this that was removed by the TES baseline live-time cut, the elevated rate continues long after the baseline stabilizes. After 10 hours, the rate was statistically consistent with background for all elevated rate periods in the 30% unblinded DM search data from all detectors. Therefore, a constant time cut of 10 hours was implemented each time the rate spiked (only in the detector experiencing the rate spike). The background rate was first estimated using a Poisson fit of the histogram of event counts within each 40 minutes of data-taking. Large spikes were identified any time the rate (seen on a histogram of event times with a bin-size of 10 minutes) exceeded the 9σ upper bound of the estimated background rate. This is a conservative criterion targeting only the main event rate spikes. Figure 4.2 shows the histogram of $1e^-h^+$ event times in NFC1 for the entire DM search data. After the full unblinding of the DM dataset, the spike identification algorithm was **re-run for the full dataset**, identifying fluctuations that cross the 9σ boundary of the **background rate found from the initially analyzed 30% of the DM search data**.

Interestingly, there is no sign of elevated rate periods in NR14 background datasets

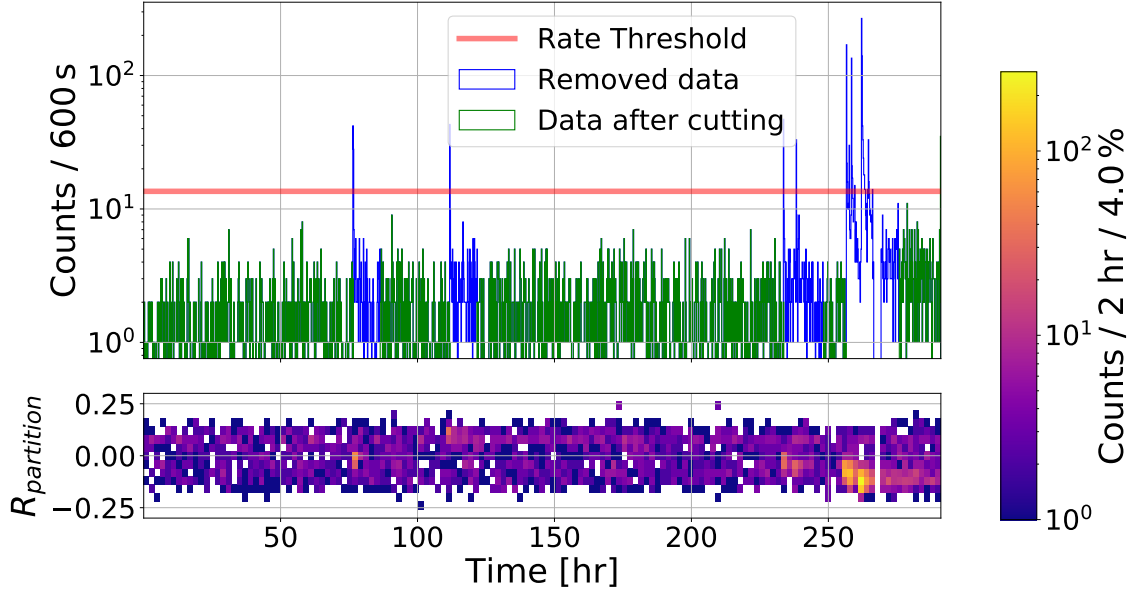


Figure 4.2: Top: Rate of $1e^-h^+$ events in detector NFC1 (DM search dataset after 100% unblinding). Spikes are identified if the rate crosses the horizontal line, which is taken as a 9σ upper bound of the background rate found from the 30% initially unblinded data. Each time a spike is identified, 10 hours of live-time is removed. Bottom: Event position information can be acquired by comparing the pulse amplitudes from inner and outer QET channels in $R_{\text{partition}} = \frac{A_{\text{OFL},\text{out}} - A_{\text{OFL},\text{in}}}{A_{\text{OFL},\text{out}} + A_{\text{OFL},\text{in}}}$ for the $1e^-h^+$ events. Clusters in $R_{\text{partition}}$ are seen coinciding with the elevated rate periods, indicating the localization of these currents.

(see Fig. 2.19(a), where the $1e^-h^+$ event rate was higher in NR13 compared to NR14 prior to live-time cuts), which points to the copper lids of the light-tight detector holder (Fig. 2.15(a)) as the likely source of this background (the lids were replaced with the LED boxes in NR14). The copper used in the detector holder was not of high radiopurity, so it could have trace amounts of ^{210}Pb implanted on its surfaces (a product of radon gas decay). The ^{210}Pb may produce high energy alpha particles along its decay chain $^{210}_{82}\text{Pb} \rightarrow ^{210}_{83}\text{Bi} + \beta^- (+\gamma) \rightarrow ^{210}_{84}\text{Po} + \beta^- \rightarrow ^{206}_{82}\text{Pb} + \alpha^{2+}$. An alpha originating in the copper lid may then hit a detector's back-side, getting implanted there close to the surface (an α^{2+} with a kinetic energy of 5 MeV would travel $\mathcal{O}(10\mu\text{m})$ in silicon before stopping [214]). The energy deposition (and the neutralization of the α^{2+}) would result in prompt charge carrier creation, which we see as a large pulse in the TES current. The latent $1e^-h^+$ events (elevated rate for hours) may be the result of a trail of vacancies and interstitial

atoms created in the α^{2+} 's wake. An interstitial is a donor and a vacancy is an acceptor which may act as a trap [192]. The subsequent $1e^-h^+$ events then likely result from holes traversing the crystal thickness (the back-sides are biased at +100 V). The hypothesis of hole propagation is supported by the precise clustering of the excess $1e^-h^+$ events in the channel partition quantity $R_{partition} = \frac{A_{OFL,out} - A_{OFL,in}}{A_{OFL,out} + A_{OFL,in}}$ in Figure 4.2, as holes are expected to experience a smaller transverse diffusion as they traverse the crystal thickness [215], producing NTL phonons along a more collimated path than an electron traversing the crystal. For more discussion on this background, see Ch. 6.

Coincidence cuts

Looking at the the time difference between adjacent events, we find that there are a number of time-correlated events (Figure 4.3) with characteristic times $< 1024 \cdot dt$, or half the pulse analysis window (so a single pulse analysis window will contain multiple pulses). These events survive all other live-time cuts. In fact, the times between these events is very precise, resulting in distinct peaks in the Δt_{min} distribution. These precise frequencies indicate an electronics source for the highly coincident events, in the readout equipment perhaps. Further, the events occur in all four detectors simultaneously. We label these events as *coincident burst events*, see Fig. 2.18 for an example of such an event (it shows only two detectors for brevity, but the pulses also occur in the other two detectors). Regardless of the source of these events (which persists at 0V crystal bias, see Ch. 6), this pattern does not correspond to DM particles from the galactic halo reacting randomly and independently in our Si substrate. We define a live-time cut such that: any 0.5s readout trace which contains a coincident pair (or more) of events, whether within a single detector or across different detectors, shall be eliminated as dead-time. This cut requires a threshold to identify events as being “too close” in time and thus coincident, so we choose half the pulse analysis window ($t_{thresh} = 1024 \cdot dt \sim 6.5$ ms). This is a natural choice as it ensures the proper reconstruction of each event, and it also includes the characteristic times of the coincident burst events.

One may feel uncomfortable about devising a live-time cut based on the pulse content of a period of time. Shouldn't we calculate an efficiency for such a cut? Dark matter, however rarely, may double-scatter. More realistically and observably, our background rate (from Figure 2.19(a)) exhibits “normal coincidences” that do not have precise characteristic times, indicating truly random coincidences that are also removed by the coincidence live-time cut described (to a much smaller extent compared to coincident burst events, see Fig. 4.3). The smallness of the cut threshold compared to the inverse background rate ($R_{background} \cdot t_{thresh} \sim 3 \times 10^{-5}$) means that only a couple thousandths of a percent of

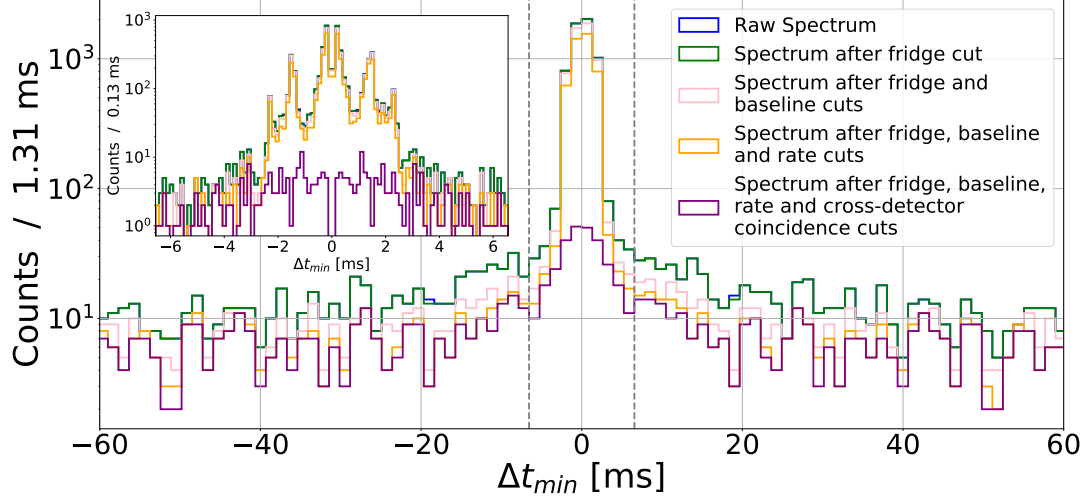


Figure 4.3: Distribution of minimal time to adjacent NFC1 event for each NFC1 event (Δt_{min}), after the successive application of various live-time cuts discussed in the text. The vertical grey dotted lines indicate the coincidence cut thresholds $\pm t_{thresh}$ (the width of a pulse analysis range). The inner plot shows events within that range using a tenth of the original bin-size. In this figure, only the cross-detector coincidence is applied (see Fig. 4.4). In the zoomed in plot, one can see clear features peaked at $\Delta t_{min} \simeq \pm (0.2, 1.5, 2.3)$ ms .

observed “true” (non-burst) background events will be identified as coincident because they happen to occur within t_{thresh} of another such background event, so a cut efficiency would be exceedingly close to 100%. This is an important consideration because in Section 4.4 we will attempt to set a (conservative) limit on dark matter interactions without having a background model, so we will consider the remainder of the background spectrum after all cuts are applied as being the result dark matter hits from the model under consideration (at least in the e^-h^+ peak regions). Further, a random background/DM event of any energy is just as likely to coincide with another event as any other random event with a different energy, so a signal passage fraction must be flat (absent the possibility of a cascade effect). And while 0.5s is arguably more time than we absolutely have to cut, we stick with the structure of previously discussed live-time cuts of passing/failing whole readout traces.

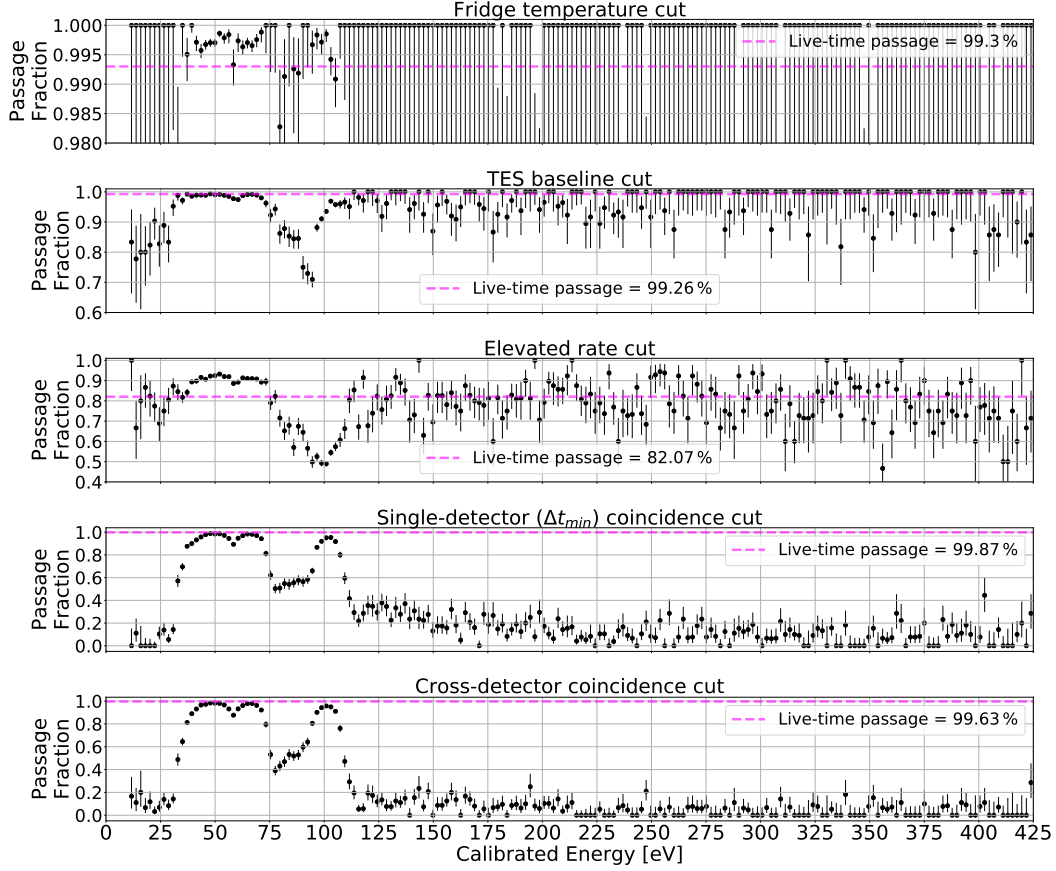


Figure 4.4: Event passage fractions after applying each live-time cut individually, as a function of event energy (calibration of Sec. 4.3). The fridge temperature cut has a constant passage fraction and does not change the relative rates at different energies. The TES baseline and elevated rate cuts remove a disproportionate number of $\sim 1 e^-h^+$ events, which is discussed in the text. The coincidence cut was divided into two cuts in this plot: the single-detector cut identifies all readout traces with at least two events **in any single detector** occurring within $\Delta t_{min} \leq t_{thresh} = 1024 \cdot dt \sim 6.5$ ms of each other, while the cross-detector cut identifies all readout traces with at least two events **across at least two different detectors** occurring within t_{thresh} of each other. The two cuts remove almost the same events, mostly the coincident burst events that trigger both. Coincidence cut passage fractions mirror the shape of the original spectrum because the burst events are proportionately less frequent when the rate at a certain energy is higher.

4.2.2 Event Quality Cuts

After applying the live-time cuts specified in 4.2.1, we are left with 8.70 gram days (6.08 gram days) of exposure for the 100% unblinded dark matter search dataset (70% initially blinded data) in the NFC1 detector (the one we will use in the dark matter limit setting of 4.4). This means $\sim 80.5\%$ of the live-time passes all live-time cuts (see Fig. 4.4). The following will outline a cut evaluating each triggered event using the χ_{red}^2 reduced quantity, along with an estimate for the cut’s signal efficiency.

Reduced- χ^2 Cut

An event quality cut based on the χ_{red}^2 from the OF pulse fits (2.2.4). The cut boundary is defined similarly to what was done for LED data in 3.1.2. Cs-137 source data sets from NR13 (same noise conditions as DM search data) taken at 0V bias provide a continuous spectrum of low energy ($E_{tot} \lesssim \text{keV}$) events, making them suitable for defining the cut boundary. LED datasets from NR14 had different noise and cross-talk conditions so a cut on NR13 data could not be reliably derived from them, see Fig 3.6. Events from the 0V Cs-137 dataset are first binned in pulse amplitudes, and the χ_{red}^2 distribution is fit with a Gaussian (+constant) for each OF amplitude bin. The 3σ upper boundaries at each bin determines the maximum χ_{red}^2 value passed by the cut. Fig. 4.5 shows the derivation of the cut boundary, below which events pass the cut.

Around $\sim 3\%$ of all triggered events in the DM search data are removed by this cut. This general event passage fraction is irrelevant when we’re interested in measuring the signal component of events: the cut does not only remove the occasional random mal-reconstructed signal events, it also removes other (non-signal) events. For example, if we had not applied the coincidence live-time cuts of Section 4.2.1, the event passage fraction at energies higher than $\sim 150 \text{ eV}$ would be dominated by the coincident burst events which are likely due to an electronics process. Another example is an event in NFC1 (DM search data) that passed all live-time cuts but failed the χ_{red}^2 cut due to subtle fluctuations of the baseline (Fig. 4.6). This was probably not a dark matter event as it was preceded and succeeded by sub-threshold events in quick succession, and baseline fluctuations are also seen in other detectors. This is then a non-signal event that is rejected by the χ_{red}^2 cut. Since we will be setting conservative limits on dark matter interaction strengths without having reliable background models, we will assume that all events within an e^-h^+ peak after all cuts are ionization events due to energy depositions in Section 4.4. In that case we do not need to estimate background rejection probabilities, only an energy-dependent signal efficiency.

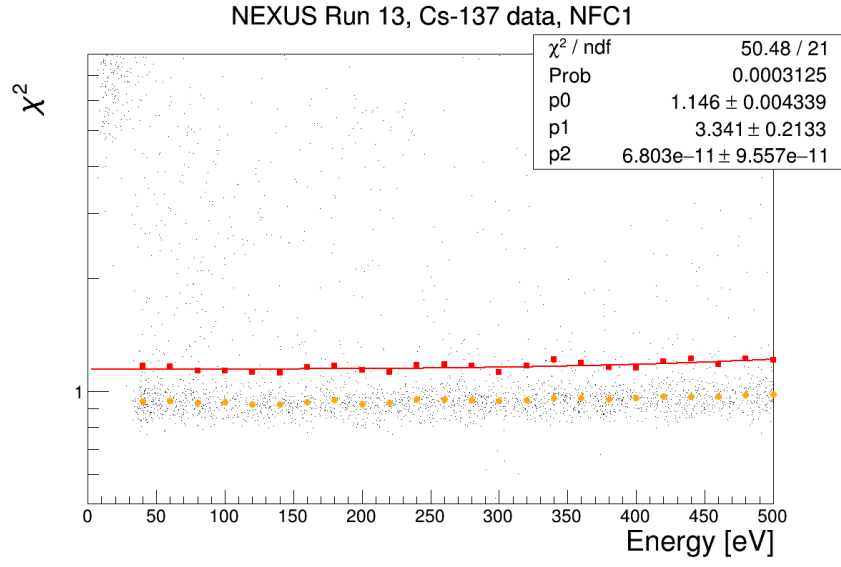


Figure 4.5: Events from NR13 0V Cs-137 x-ray data (grey dots) in (E, χ_{red}^2) , where E is the calibrated event energy. Yellow points indicate the mean χ_{red}^2 values derived from Gaussian fits at various energy ranges of width 20 eV, while the red points are the 3σ upper bounds from said fits. The cut boundary (red line) is parametrized by $\chi_{red}^2 < f_{cut}(E) = p_2(E)^{p_1} + p_0$, with parameters p_i optimized by fitting to 3σ upper boundaries of χ_{red}^2 .

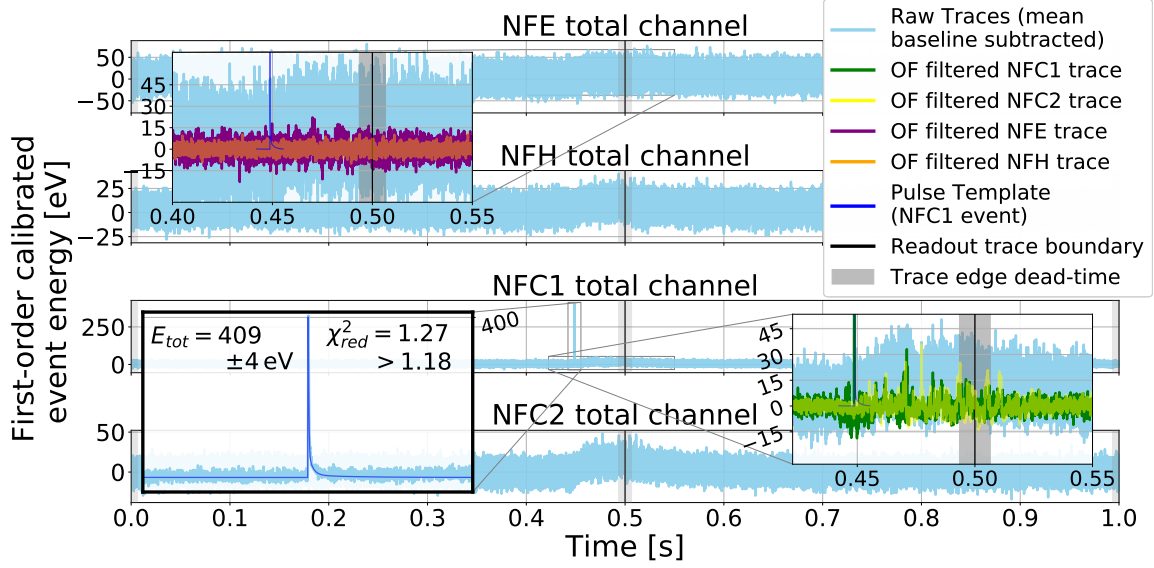


Figure 4.6: An event in detector NFC1 with a total energy of ~ 409 eV passing all live-time cuts but failing the χ_{red}^2 event quality cut (bottom left zoom in showing pulse analysis window, overlaid with the scaled template from 2.17(a)). Elevated baselines are seen in the readout traces in the vicinity of the NFC1 event across all detectors. The slight elevations do not trigger the TES baseline cut in any detector within the readout traces shown (the effect extends over two 0.5 s traces which are simply concatenated in the figure). The OF filtered traces for NFC1 and NFC2 (NFE and NFH) are shown in the lower right (upper left) zoomed-in plot, which indicate that the NFC1 event (the only event identified by the Gaussian derivative trigger within this 1 s of live-time) is preceded by baseline fluctuations in the same detector. Slight dips in the TES baselines are seen before the triggered event.

Calculating Cut Efficiencies

While it would have made sense to simply apply the derived χ_{red}^2 cut on a sample of NR14 LED events, and take the passage fraction as the signal efficiency, noise conditions in NR14 differed from NR13, causing a shift in the χ_{red}^2 distribution, necessitating that the cut be redefined using NR13 data. Further, cross-talk between readout and LED pulsing wires was observed in LED events. Therefore, we estimated the cut efficiency again using Cs-137 data taken at 0V. However, before applying the χ_{red}^2 cut and taking the passage fraction, we must select signal-like events first: i.e. events that pass a pile-up cut which

removes time-correlated events, a baseline slope cut removing events with non-zero slope in the pre-pulse region and a pulse fall time cut removing slow fall pulses unique to 0V datasets which are consistent with local TES saturation. The binned passage fraction of the remaining events after finally applying the reduced χ^2 cut is taken as an estimate of the energy-dependent efficiency. The efficiency is consistent with being energy-independent. We take fluctuations into account by assigning a conservative uncertainty that includes all fluctuations, producing $\epsilon_{\text{eff}} = 0.95 \pm 0.02$. See Fig. 4.7.

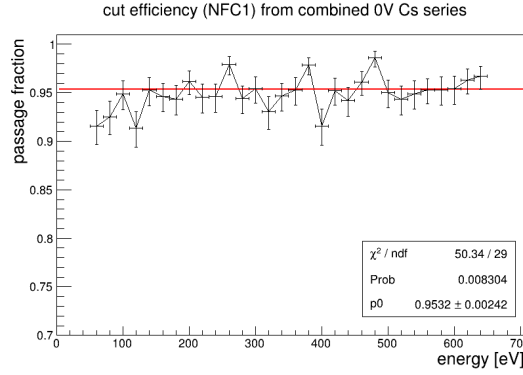


Figure 4.7: Estimates of the χ^2_{red} cut efficiency for NFC1 detector.

4.3 Calibration Using LEDs

While the quantized background spectrum provides a first order calibration in and of itself, our detector is not merely an ionization counter. Further, we do not expect the energy to scale linearly with the pulse amplitude, as the resistance-temperature curve of the TES is nonlinear. Thus, a known source of low energy events is needed to get a precise energy calibration. In addition, our detectors are susceptible to incomplete NTL amplification due to charge trapping and impact ionization (CTII) processes, and their corresponding parameters must be determined. While these detector response parameters have been measured in earlier studies, it is important to check for the possibility of dependence on run conditions. Also, with the development of more precise models for CTII processes, it could be possible to distinguish between surface events and bulk events based on the inter-peak spectrum. For these reasons, a dedicated LED run acquired a plethora of datasets for calibration and a robust evaluation of related uncertainties. This calibration was done in Sec. 3.2.4.

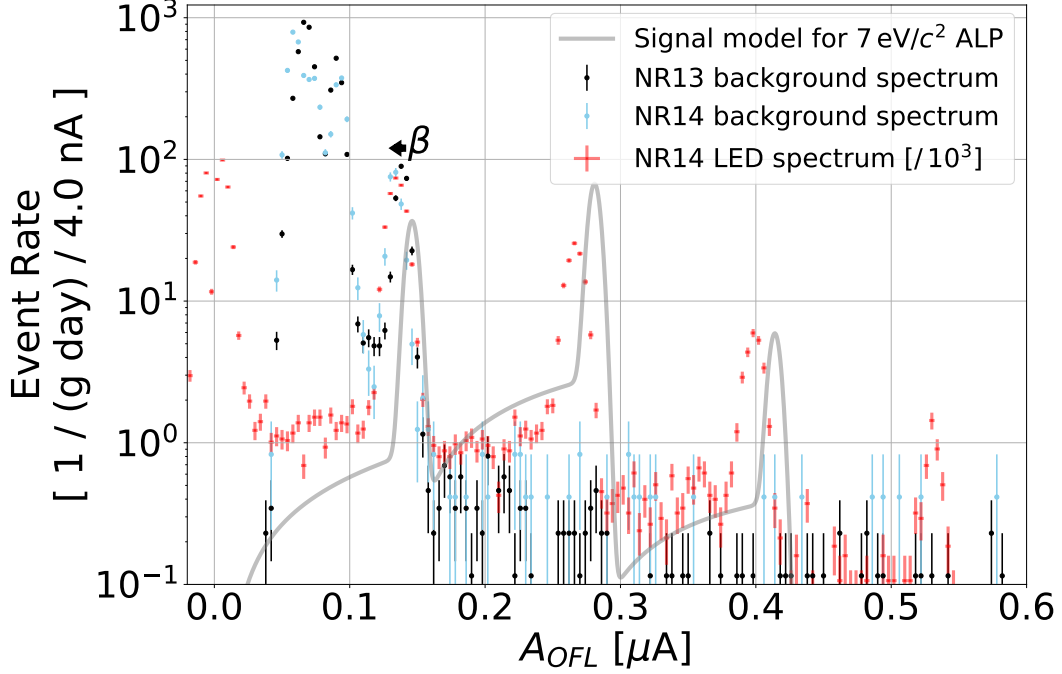


Figure 4.8: Various pulse amplitude spectra from NR13 and NR14 datasets. The ‘background’ spectra are shifted from NR13 to NR14, resulting in the largest contribution to the systematic uncertainty on the energy scale when applying the NR14 LED calibration on NR13 DM search data. The CTII model used in constructing the DM signal models is that of bulk charge pair production, as opposed to the surface event model used in the LED calibration [194].

The CTII fractions used in the following section are taken from the fit in Fig. 3.5, and are quoted in Table 4.1. We use the calibrated width of the $1e^-h^+$ peak width $\sigma_{res} = 3.24 \pm 0.12$ from the DM search data of NR13 as our estimate for the energy resolution of the detector in the following sections, as the resolution estimates from LED data of NR14 were inconsistent with NR13 (3.3). This is higher than the baseline phonon resolution estimated at $\sigma_{baseline} \sim 2.8$ eV. Since a change in resolution of 0.5 eV at most was observed within a single LED dataset (Sec. 3.3), the $1e^-h^+$ is a well-motivated upper bound on the phonon resolution. The higher resolution is also more conservative in the limit setting method described below as it includes more events in the analysis window while the signal model normalization remains constant. One reason why $\sigma_{res} > \sigma_{baseline}$ for the

$1e^-h^+$ peak of NR13 DM search data is the existence of the ‘elevated rate’ events discussed in Section 4.2.1 and other leakage events among bulk recoil events. While the estimated $1e^-h^+$ peak energy in NR13 DM search data was 101.1 ± 0.1 (stat.) eV, the elevated rate cut removed events peaked at 100 eV. For more details on calibration and detector response modelling using NR14 data, see Chapter 3. The statistical and systematic uncertainties are estimated, and the total calibration uncertainties for e^-h^+ peaks 1-4 were estimated in Fig. 3.14 and are shown in Table 4.1 (energy shift parameter).

4.4 Exclusion Bounds

Our experiment is most competitive in the low-mass region of dark matter parameter space, given the small recoil energy threshold $\sim E_{gap}$. It would not make sense to analyze the data for DM heavier than a GeV, as our exposure is much smaller than the ton-scale WIMP searches. In the sub-GeV range, a minimal simplified dark matter model would be the addition to the SM of a fermionic dark particle along with a bosonic force carrier (a dark photon) [75, 76]. In the following subsections, statistical methods for testing the data’s compatibility with DM models are described. The first case assumes a galactic halo made up completely of fermionic sub-GeV DM, which scatters off of electrons through a dark photon mediator, which we further divide into the light mediator case ($M_{med} \ll q$) and the heavy mediator case ($M_{med} \gg q$), where $q \sim \alpha m_e$ is the typical momentum transfer. Bosonic DM scenarios are also tested, where the halo is assumed to be purely made out of dark photons, or as a bonus we consider it being made out of pseudoscalar axion-like particles (ALPs).

The expected event rates for each of these four scenarios are well documented in the literature [76, 128], and were described in detail in Ch. 2. In the fermionic case, scattering with an electron occurs through a light (heavy) mediator, with a corresponding dark matter form factor $F_{DM} \propto 1/q^2$ ($F_{DM} \propto 1$). In this case the interaction strength is captured by the cross section $\bar{\sigma}_e$. The mass range of interest is $1 \text{ MeV}/c^2 < m_{DM} < 10^3 \text{ MeV}/c^2$. The dark photons on the other hand are directly absorbed by electrons through the kinetic mixing parameter κ , which is the coefficient of the mixing term for the SM hypercharge field and the gauge field of the additional (broken) U(1) gauge symmetry. ALPs are also directly absorbed by electrons through a coupling constant g_{ae} , resulting in an “axio-electric” effect. In the case of absorption, the mass range of interest is $E_{gap}/c^2 < m_{DM} < 50 \text{ eV}/c^2$. The recoil spectrum is then convolved with detector response effects including the Ramanathan-Kurinsky ionization model [165], charge trapping and impact ionization model for a bulk event [194], and a Gaussian to smear the spectrum to include our finite phonon resolution.

The latter two effects have parameters that were measured from NR14 LED data (Section 4.3, and Ch. 3 for more details).

The expected event rate due to everything else is not known in the region of interest $50\text{ eV} < E_{\text{phonon}} < 650\text{ eV}$, which includes the expected first to sixth e^-h^+ peaks. We know that at least some of the events we observe are not galactic DM halo scattering events because of the varying event rates between different detectors, even after cuts. For example, the background $1e^-h^+$ rates for the 4 detectors of HVeV R4 are not statistically consistent. This is not surprising as we have already seen the external (hypothesized luminescence) background in previous runs which was eliminated in this run, and we know of leakage currents that traverse a semiconductor substrate under an electric field. There is also the non-ionizing low energy excess (0QLEE) which has been of interest in the field of direct detection. A leading theory suggests that detector holder strains and spontaneous relaxations are the cause of these phonon-only events. This lack of knowledge of the background precludes any discovery of dark matter with this device for now. While the superCDMS simulation infrastructure is mature and comprehensive [123], such low energy backgrounds have rarely been simulated [201]. And as the lack of quantized $\geq 2e^-h^+$ events suggests, it is easy to block external sources of ionization whose energy depositions are a couple of eVs (as we did with our light-tight detector holder).

4.4.1 Limit Setting Methods

Current leakage and the low energy excess appear to be the main backgrounds in this analysis, both of which are most likely in-detector effects which were not relevant for older, larger CDMS detectors with a higher energy threshold. The lack of a model for the backgrounds does not stop us from conservatively estimating what dark matter is *not* consistent with, given our data. The first choice we make is to analyze only NFC1 for DM signals, as it had the lowest $1e^-h^+$ event rate, and we use the other three detectors only as vetoes as was discussed in 4.2.1. Next, each e^-h^+ peak is analyzed separately, to avoid problems related to our signal model fitting the data poorly (having no background model). Our quantized expected signal spectrum provides natural energy bins to look for events within (the peaks), and within these narrow bins (of three resolutions width) the signal will be degenerate with any background. We pursue a frequentist approach, formally a Neyman construction to conduct statistical tests. As outlined in [216], we require an estimator (N_{events} for simple Poissonian counting, or profile likelihood ratio test statistic q_μ for maximum likelihood methods), a reference ensemble (N_{events} follows Poisson statistics for random independent events, q_μ has asymptotic formulae at high statistics),

an ordering rule (more on this below), and a confidence level (we choose 90% consistent with other analyses).

It may be argued that the number of events within each peak is already a sufficient statistic, and that there may be no need for a likelihood analysis. However, the likelihood function is a natural place to take into account systematic uncertainties. For a simple Poissonian counting upper bound on interaction strengths, one has to run Monte Carlo pseudo-analyses with varied detector response parameters to get the variation or ‘uncertainty’ on the limit, casting doubts on the meaning of the reported confidence interval. Further, the simple counting method removes some energy information (namely the distribution within the peak interval), and our detector has potential for background rejection through peak splitting (based on different initial energy depositions), especially as phonon resolution continues to improve in ongoing work. Therefore, we use a maximum likelihood approach to set our upper boundaries on interaction strengths of dark matter. However, we will make use of the simple Poissonian counting method in 4.4.2 to estimate the effect of signal attenuation in the rock above the experiment (overburden) on the exclusion region, so we shall define quantities for both methods. If we had any power for discovery (by having a reliable background model), then the p-value of 10% would be taken as $1 - CL_b$ to reject the background model. To exclude a hypothetical signal, one may keep increasing the signal strength until CL_{s+b} is below the desired p-value.

In the Poissonian counting case, we exclude a region of parameter space if

$$N_{expected}(\mu_{sig}) > N_{upper}(P_{CL}), \quad (4.1)$$

where $N_{expected}(\mu_{sig}) = \int_a^b \frac{dR_{model}}{dX dE_{total}}(\mu_{sig}, E) \cdot \epsilon_{efficiency}(E) \cdot X dE$ is the expected number of signal events and $N_{upper}(P_{CL})$ is the $P_{CL} = 90\%$ confidence upper bound of the number of observed events within a certain energy range (analysis window $[a, b]$), and X is the exposure after live-time cuts. The signal strength μ_{sig} is the parameter of interest ($\bar{\sigma}_e$ or κ or g_{ae}). The energy range for the calculation of both N_{upper} and $N_{expected}$ is centered at $n_{e-h^+} \times 100 \text{ eV}$ (NTL amplification energy) plus the weighted average of the energy depositions which generate each e^-h^+ peak in the model under consideration, with a width covering $\pm 3\sigma_{res}$ around the center, where σ_{res} is the estimated phonon resolution (value used in Table 4.1). This upper bound assumes all events within the peak window are dark matter events, but we report only the upper bound of the estimated “signal strength” that would account for all of the events. This produces a conservative upper bound, since if any of the events within the peak window are background non-DM events, the real 90% confidence boundary would be lower (a more exclusive result, $1 - CL_{s+b} \geq 1 - CL_{s+0} = 90\%$). The situation is similar when using the maximum likelihood approach without a

background model. The test statistic in this case is:

$$q(\nu_\chi) = \begin{cases} -2 \ln \left(\frac{\mathcal{L}(\nu_\chi, \hat{\theta}|m_{DM}, \mu_{sig})}{\mathcal{L}(\hat{\nu}_\chi, \hat{\theta}|m_{DM}, \mu_{sig})} \right) & \nu_\chi > N_{fit} \\ 0 & \nu_\chi < N_{fit} \end{cases} \quad (4.2)$$

where $\mathcal{L} = \mathcal{L}_{Poisson} \cdot \mathcal{L}_{model} \cdot \mathcal{L}_{constraint}$ is the extended likelihood function multiplied by a constraint term with:

$$\begin{aligned} \mathcal{L}_{Poisson} &= \left[\nu_\chi + \sum_b \nu_b \right]^N \times \frac{e^{-(\nu_\chi + \sum_b \nu_b)}}{N!}, \\ \mathcal{L}_{model} &= \prod_{i=1}^N \left[f_\chi(E_i) + \sum_b f_b(E_i) \right], \\ \mathcal{L}_{constraint} &= \prod_{i=1}^5 G_i(\theta_i, \mu_i, \sigma_i), \end{aligned} \quad (4.3)$$

where ν_χ , ν_b , are the signal and background strength fit parameters, N is the total number of events in the analysis range, f_χ , f_b are the signal and background total phonon energy PDFs, and G_i are five normalized Gaussian constraint terms for five nuisance parameters in Table 4.1 (cross-correlation between nuisance parameters is neglected). We do not have a background model f_b , so we take $\nu_b = 0$ to set our ‘ CL_{s+0} ’ limits.

Wilks’ theorem states that, as the number of observed events N increases, the distribution of observed test statistic $q(\nu_\chi)$ (upon repetition of the experiment) asymptotically approaches a χ^2 distribution with degrees of freedom equal to the number of parameters of interest (1 in our case). So an exclusion region on the DM parameter space can be found by finding ν_χ which gives $q(\nu_\chi) > 1.64$ for each dark matter mass, the threshold 1.64 being the 90% critical value for a χ^2 distribution with 1 degree of freedom. We tested the coverage of this method by running monte-carlo pseudo-experiments, generating ensembles of pseudo-datasets with randomly varied nuisance parameters within the values stated in Table 4.1 in addition to random Poisson statistics for total number of events, and finding the true distribution of the test statistic, given a signal strength $q(\nu_\chi = N_{expected})$. Signs of under-coverage are seen when the expected number of events is less than 20. This is due to the domination of Poisson statistics over any variation of the nuisance parameters, as summarized in Fig. 4.9. So, where the number of observed events is less than or equal to 20 (as is the case for higher e^-h^+ peaks), we use a larger (more conservative) threshold

Nuisance parameter θ_i	Mean value μ_i	Standard deviation σ_i
Phonon energy resolution (eV)	3.24	0.12
Charge trapping fraction (%)	12.3	0.5
Impact ionization fraction (%)	0.0	0.4
Energy shift parameter (eV)	0.0	$1 e^-h^+$: 0.7 $2 e^-h^+$: 2.0 $3 e^-h^+$: 3.3 $4 e^-h^+$: 3.2
χ^2_{red} cut efficiency (%)	95	2

Table 4.1: Table of nuisance parameters and the corresponding parameters that define their constraint terms in Eq. 4.3. The energy shift parameter depends on which peak is being analyzed.

value in the ordering rule which indicates the exclusion region:

$$q(\nu_\chi) = \begin{cases} q(\nu_\chi) > 1.64 & N > 20 \\ q(\nu_\chi) > c_1 e^{-c_2 N} + c_3 & N \leq 20 \end{cases} \quad (4.4)$$

where $c_1 = 5.80$, $c_2 = 0.38$ and $c_3 = 2.01$ come from a fit of the local maxima in 4.9 and N is the number of events in an analysis window.

As stated earlier, we analyze each peak separately. But it is customary to present one exclusion curve to compare with other experiments. We first run the maximum likelihood analysis to extract upper bounds on each model’s signal strength from each e^-h^+ peak in the 30% unblinded data. For each mass in the exclusion curves, the most stringent constraint is selected. This is called peak selection. We stick to this choice of peaks from 30% blinded data for each mass tested when we re-run the analysis on the initially blinded 70% of the data, to avoid any potential bias (look-elsewhere effect for example) when we “choose the best limit”. The final results for 70% data are shown in the Fig. 4.10, along with the most recent results from other experiments.

4.4.2 Including the Effect of DM Attenuation in Earth’s Crust

Before reaching the detector, the local dark matter halo particles must first pass through the Earth’s atmosphere, as well as through the rocks of the crust (overburden), or sometimes

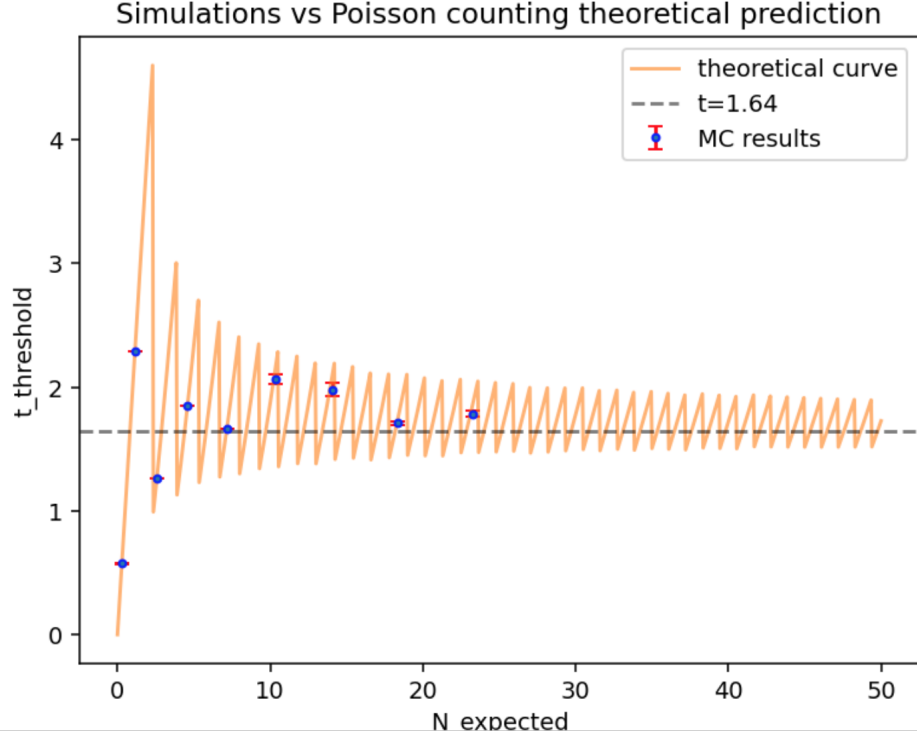


Figure 4.9: Threshold test statistic (Eqn. 4.4) which gives 90% statistical coverage, validated through pseudo-experiments (MC results).

a particle must pass through the entire earth to reach the detector. For direct boson absorption models (ALPs and dark photons), one may use a simple exponential attenuation model, as the interaction in those cases results in the complete loss of the particle. On the other hand, for electron recoiling dark matter, one must take into account the details of the kinematics of interaction with the overburden constituents. One then inputs the original local velocity distribution and estimates, using assumptions about the interaction type, the DM velocity distribution near the detector. This was previously done in the Inelastic DM Scattering in CDMSlite R3 analysis [223], where great efforts were taken to estimate the composition of the Soudan mine overburden, as well as taking into account the angular distribution of the attenuation.

In that previous analysis, however, the search was for GeV-scale dark matter, and that justified using the simple continuous stopping model. A public package that implements the continuous stopping model used in CDMS-CPD analyses is `verne` [224]. For sub-GeV dark matter (which we’re interested in for HVeV Run 4), deflections become important

to model. While an analytic modeling tool valid for low-mass DM (**EarthShadow** [225]) exists, it utilizes the EFT approach which does not include the light mediator case and does not include multiple scatters. Semi-analytic models have been explored, such as the one implemented in **DMprop** [226], which utilizes a probabilistic approach that recursively calculates probabilities of particle survival after an arbitrary number of scatterings, with the crucial simplifying assumption of isotropic scattering. While isotropic scattering with heavy nuclei approximately occurs in the case of light dark matter with masses no less than 100 MeV interacting through a point-like interaction (heavy mediator), that barely covers the HVeV use case which includes dark matter masses down to 1 MeV and also the light mediator case which exhibits highly anisotropic scattering [222].

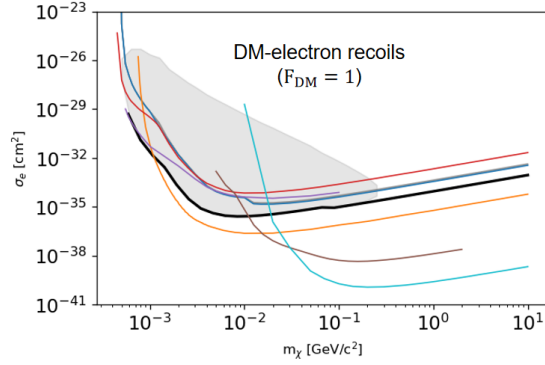
A Monte Carlo tool (**DaMaSCUS-CRUST** [227, 228, 222]) was used in HVeV Run 4 to estimate the extent of the exclusion region. This tool was also used in HVeV Run 3. It includes the light mediator case as well as the point-like interaction case, and also models multiple-scatters. It does so by simulating and tracking individual particles, as many particles as necessary to acquire sufficient statistics for a given cross-section. This is time consuming, so the authors of the software implemented geometric-importance sampling (similar to what is implemented in **Geant4** [229]), which requires tuning. The tuning of these parameters is especially important for the light mediator case at dark matter masses approaching ≥ 1 GeV, as in that case forward scattering is highly favored so higher statistics are needed to find the precise attenuation [222]. The ideal parameters could also vary for each dark matter mass tested.

When including the effect of overburden attenuation on fermionic dark matter, the local DM flux does not simply scale with the interaction DM cross-section: the shape of the speed spectrum also changes (particles scatter and lose energy), and the overall event rate at the experiment starts to decrease with increased interaction strength. Therefore, to produce a double sided exclusion region, one may have to test each (σ_e, m_{DM}) point in parameter space as a separate hypothesis instead of simply solving $q(\nu_\chi) = 1.64$ if utilizing a profile likelihood approach [223]. However, when using the Poissonian counting approach to set a limit, spectrum shape effects within the analysis window are neglected and we may simply find the roots of $N_{expected}(\mu_{sig}) - N_{upper}(P_{CL} = 90\%)$ from Equation 4.1 to find the boundaries of the exclusion region, after modifying our signal model $N_{expected}(\mu_{sig})$ to take in simulated velocity distributions from **DaMaSCUS-CRUST** instead of the standard halo model (SHO) [67]. We ignore the effect of the atmosphere and model our 107 m of rock as being mostly made out of SiO_2 , some aluminium and iron, and the rest is made out of common alkali and alkaline earth metals (period 3 and 4). The dark matter local density at the earth's surface is assumed to be $\rho_{DM} = 0.3 \text{ GeV}/cm^3$ [67].

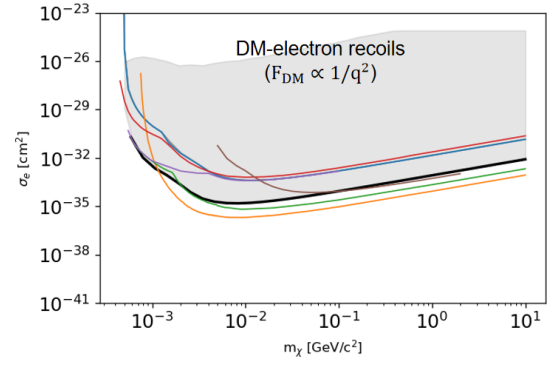
Some simplifications are made in the **DaMaSCUS-CRUST** simulations. First, since our

model has a dark photon as mediator, other charged particles are expected to have the same ability to exchange a dark photon as an electron. It follows that the most severe attenuation occurs due to DM interactions with nuclei in the overburden. Since $\frac{\bar{\sigma}_p}{\bar{\sigma}_e} \propto \frac{\mu_{p\chi}^2}{\mu_{e\chi}^2} \gg 1$ for DM masses $m_\chi \gtrsim 1$ MeV, and the nuclear recoil rate is coherent $\sigma_N \propto Z^2$ (where $\mu_{p\chi}$, $\mu_{e\chi}$ are the DM-proton and DM-electron reduced masses, and Z is the atomic number), so we neglect other processes like electron or atomic recoils [222]. Secondly, the layers of the inner earth were not modeled as was done in [225, 227, 223] where the approach angle of a dark matter particle could mean it had to pass through the entire earth to pass the detector. This level of detail is relevant to experiments looking for diurnal modulation due to earth shielding to detect dark matter. A simplification in DaMaSCUS-CRUST is to assume the overburden is flat with DM coming from ‘above’ in random angles relative to the zenith. At high cross-sections, this is reasonable as it would be prohibitively unlikely for a particle to approach from ‘below’ the experiment as there is too much material to pass through.

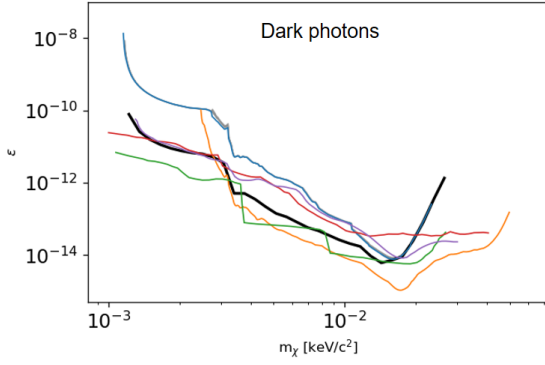
Velocity distributions were generated for every DM mass under consideration, in cross section intervals sampled in log space $\Delta \log(\bar{\sigma}_e) = 0.1$ in the range $10^{-38} \text{ cm}^2 < \bar{\sigma}_e < 10^{-24} \text{ cm}^2$. $f_{90}(\bar{\sigma}_e) \equiv N_{expected}(\mu_{sig} = \bar{\sigma}_e) - N_{upper}(P_{CL} = 90\%) = 0$ is linearly interpolated in $\log(\bar{\sigma}_e)$ space and solved for the boundaries of the exclusion region for that mass, as shown in Fig. 4.11. The mean values of the nuisance parameters in Table 4.1 are used. The results for all masses are shown in the shaded regions of Figures 4.10(a) and 4.10(b). As for the absorption models, we simply delineate the coupling constant that, given the mass of the boson, would result in a reduction of the signal strength by a factor of e (flux $\Phi = \Phi_0 e^{-\rho L \sigma}$ where $L = 107 \text{ m}$ is the depth of the experiment, $\rho = 2.7 \text{ g/cm}^3$ is the overall mass density of the overburden (same value used in DaMaSCUS-CRUST) and σ is the mass attenuation factor ($\sigma \rho \equiv \sigma_{cross-section} \rho_n$, ρ_n being the number density), which must be calculated in each model. The cross section $\sigma_{cross-section}$ can be calculated by dividing the flux $\rho_n v_{DM}$ from the rate calculated by [128], then finding κ or g_{ae} which give $\rho L \sigma = 1$. This line in parameter space would be shown in Figures 4.10(c) and 4.10(d) but is higher than the interaction ranges displayed.



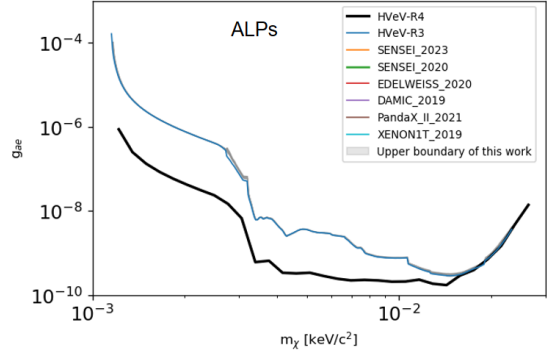
(a) Fermion with heavy mediator



(b) Fermion with light mediator



(c) Dark photons



(d) Axion-like particles

Figure 4.10: Exclusion bounds derived from HVeV Run 4 (black lines, all e^-h^+ peaks combined) for various DM models discussed in the text. Limits from other experiments are shown (colored lines) [142, 119, 217, 218, 219, 220, 221]. The gray shaded regions in the upper two plots indicate the exclusion boundaries when including overburden attenuation [222].

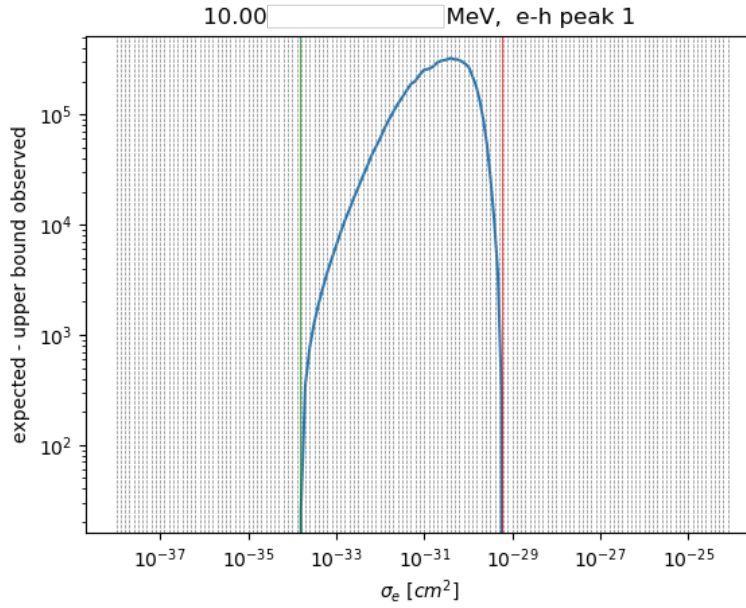


Figure 4.11: The function $f_{90}(\bar{\sigma}_e) \equiv N_{expected}(\mu_{sig} = \bar{\sigma}_e) - N_{upper}(P_{CL} = 90\%)$ (see Eq. 4.1) used for defining exclusion bounds with the effect of overburden scattering (10 MeV fermionic DM with heavy mediator, $F_{DM} \propto 1$). Velocity distributions from DaMaSCUS-CRUST for each of the vertical dotted lines. The green and red vertical lines indicate the range of excluded cross-sections for this mass ($f_{90}(\bar{\sigma}_e) = 0$).

Chapter 5

Detection Prospects for Solar Axions in SuperCDMS SNOLAB

In this chapter, we present the potential of detecting ultra-light $m \lesssim \mathcal{O}(\text{eV}/c^2)$ solar axions using SuperCDMS SNOLAB’s initial payload. This search is usually exposure-limited, making the kg-scale SuperCDMS SNOLAB detectors a suitable choice. A brief review of the event rate calculation of axion-to-photon conversion in a semiconducting crystal is presented. The SuperCDMS SNOLAB is currently in the final stages of commissioning, so this analysis will be applied to real data in the coming years. For a review of the theory and detection techniques of bosonic DM, see Sec. 1.2.2 and Sec. 2.1.2.

5.1 SuperCDMS SNOLAB as a Helioscope

As outlined in [158], the SuperCDMS SNOLAB experiment will be sensitive to a variety of light (sub-GeV) dark matter candidates, down to $\mathcal{O}(10 \text{ eV}/c^2)$ in the case of bosonic cold dark matter (CDM) candidates that would be directly absorbed by the substrates from the galactic DM halo. Lighter particles that could make up the CDM halo, such as the QCD axion, are therefore out of (direct detection) reach for our experiment (but may be probed using haloscopes [149, 150]). We may still use the CDMS detector to look for boosted light axions from astrophysical sources, rather than the axions making up the halo. This type of search may be characterized as an ‘indirect’ probe of a DM candidate. The most significant potential source of light axions available in the solar system is the Sun. In case the axions do not interact with electrons ($g_{ae} = 0$, hadronic axion), the main production mechanism

is the conversion of thermal photons in the solar plasma into axions of the same energy through the Primakoff coupling $g_{a\gamma\gamma}$ (see Sec. 1.2.2 for Lagrangian) [99, 230]. Helioscope experiments such as CAST [231, 152] use magnetic fields to convert these solar axions back into detectable photons via the inverse Primakoff effect. Our crystals have strong local electric fields in them that can cause the same effect, leading to energy depositions at the $\mathcal{O}(\text{keV})$ scale. The keV scale is special in our crystal because it also specifies the inverse scale of the inter-atomic spacings in the crystals, or the reciprocal lattice vectors. This causes a highly time-dependent signal as solar axions may satisfy Bragg conditions depending on the angle of incidence and their wavelengths.

Other (non-helioscope) indirect probes of the ‘invisible’ axion utilizing the Primakoff process exist, including the measurement of a ‘light-shining-through-wall’ (LSW) effect, where light from optical lasers or x-ray sources [232, 233] is used to produce axions and detect deviations in an opaque ‘wall’s transmittance due to an intermediate axion state. A broad-band LSW search can be performed in the ForwArD Search ExpeRiment (FASER) at CERN’s LHC [234]. (LSW measurements may also be done using astrophysical objects as ‘walls’ [235]). While the LSW approach is inapplicable to SuperCDMS, we introduce it here as the event rate calculations for LSW are closely related to that of our crystals, as will be outlined below. For a review of experimental probes for axions, see Ref. [98].

5.2 Solar Axion Event Rate

To search for these solar axions in SuperCDMS, we need to calculate the event rate in the detector as a function of energy. This was done in [236] for a crystal detector with energy resolution Δ :

$$R(E_1, E_2) = \int_{E_1}^{E_2} dE_{ee} \int_0^\infty dE_\gamma \frac{dR}{dE_\gamma}(E_\gamma) \times \frac{1}{\Delta\sqrt{2\pi}} e^{-(E_{ee}-E_\gamma)^2/2\Delta^2}, \quad (5.1)$$

where $R(E_1, E_2)$ is the event rate in an energy bin spanning $[E_1, E_2]$, and the rate of photon production $\frac{dR}{dE_\gamma}$ is convolved with a Gaussian of width Δ to include resolution effects. The rate of photon production is taken found by combining the flux and conversion cross-section σ :

$$\frac{dR_\gamma}{dE_\gamma} = \int_0^\infty dE_a \frac{d\phi_a}{dE_a} \int d\Omega \frac{d\sigma}{d\Omega dE_\gamma}, \quad (5.2)$$

where the integral over solid angle Ω comes from the phase space integration and $d\phi_a/dE_a$ is the solar axion differential flux. The differential cross section $\frac{d\sigma}{d\Omega dE_\gamma}$ is taken from an

axio-electrodynamic wave calculation [237]:

$$\frac{d\sigma}{d\Omega dE_\gamma} = \frac{d\sigma}{d\Omega} \delta(E_a - E_\gamma) = \left[\frac{g_{a\gamma\gamma}^2}{16\pi^2} F_a^2(\vec{q}) \sin^2(2\theta) \right] \delta(E_a - E_\gamma), \quad (5.3)$$

where θ is the scattering angle, F_a is the axion form factor of the target evaluated at momentum transfer $\vec{q}$, and the energy conservation δ function is shown in the limit of a fixed heavy target (negligible amount of energy is transferred to the crystal at conversion, compared to the energy of the final state x-ray). The periodic electric field is encoded in F_a , which can be further decomposed into structure function $S(\vec{G}) = \sum_i e^{i\vec{a}_i \cdot \vec{G}}$ and single-target form factor $F_a^0 = \frac{Ze}{\lambda_D^{-2} + q^2}$ with Debye radius λ_D and atomic number Z . The substitution of these components into Eq. 5.1 yields a simple sum over reciprocal lattice vectors $\vec{G}$ (in place of the integral over momentum transfer $\vec{q}$), which we compute to find the event rate in our crystals. When $\vec{q} = \vec{G}$, energy conservation demands that $E_a = \frac{\hbar c |\vec{G}|^2}{2(\hat{s} \cdot \vec{G})}$, where $\hat{s}$ is the sun-earth unit vector (direction of incoming solar axions). The conservation of energy equation $E_a = \frac{\hbar c |\vec{G}|^2}{2(\hat{s} \cdot \vec{G})}$ is the conjugate of the familiar Bragg condition $n\lambda = 2d \sin(\theta)$ for constructive interference of radiation diffracted by a lattice. An advantage to using a crystal as a helioscope rather than a haloscope using a magnetic field is the sensitivity to a wide range of axion masses, as Eqn. 5.3 does not depend on mass. This is not the case for helioscopes utilizing magnets such as CAST, as those experiments must keep pressurized gas in which a photon would have an effective mass equal to the axion mass under consideration [238].

Figure 5.1 shows the event rate of solar axion-to-photon conversion in two SuperCDMS SNOLAB Ge iZIP detectors at various times. We assume that all solar axions are produced through the Primakoff process from thermal photons in the sun's core, and use a parametrization of the flux used by the CAST helioscope [239] (for a discussion on various possible solar axion production channels and the associated uncertainties, see [240, 241]). Of course, many approximations are made in such rate calculations. Calculations which provide higher-order corrections are part of the recent literature. In the case of *haloscope* experiments, pure QFT descriptions of axion-photon dynamics may be used [242]. In the case of laboratory-based LSW experiments, the microscopic modeling of in-medium interactions of the initial and final state photons, including Thomson scattering and absorption as the photon passes successive lattice planes [237, 243]. In our case of solar axion conversion and detection of the final state x-ray within a crystal, the strongest assumption in the differential cross-section of Eq. 5.3 is that the conservation of energy delta function $\delta(E_a - E_\gamma)$ assumed *asymptotic* initial and final states (eigenstates of the free Hamiltonian). As [244] notes, the inclusion of the effect of a finite absorption length λ for final

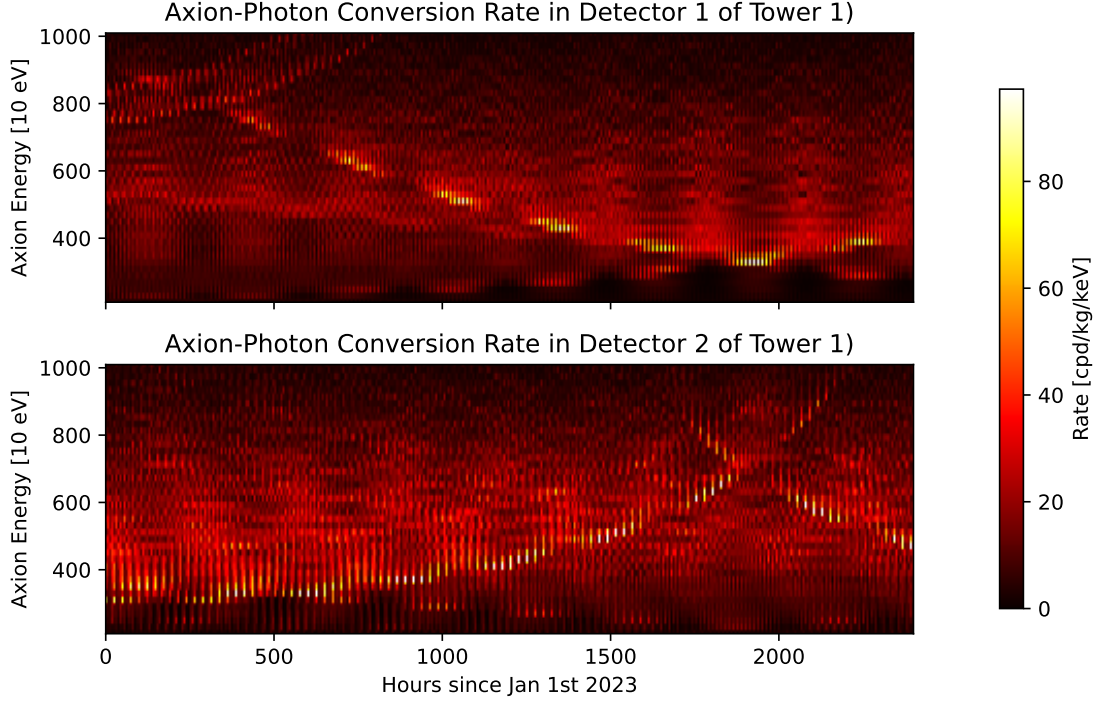


Figure 5.1: The calculated event rate for SuperCDMS SNOLAB Germanium iZIP detectors based on [236] (starting from Eq. 5.1).

state photons will result in the widening of the delta function in Eq. 5.3 by $\frac{\hbar c}{\lambda}$, and since the absorption length of 1-10 keV photons in germanium and silicon is of $\mathcal{O}(\mu\text{m})$, x-ray absorption is neglected as $\frac{\hbar c}{\lambda} \sim \mathcal{O}(\text{eV}) \ll \Delta$, with Δ being the energy resolution in Eq. 5.1.

5.2.1 A tension in the literature

A recent study concluded that the finite absorption length results in an attenuated overall solar axion conversion rate in a crystal detector, rather than simply adding negligible smearing of the energy spectrum of events [245]. That study was done to reconcile a perceived discrepancy between different methods of rate calculations between the calculated event rate in LSW experiments vs. crystal experiments. The study did not perform a calculation from first principles; instead, the authors start from the Born approximation for the matrix element of a scattering process involving N scattering sites $\mathcal{M}(\vec{k}_a, \vec{k}) =$

$\sum_{i=1}^N f_i(\vec{k}_a, \vec{k}) e^{i(\vec{k}-\vec{k}_a) \cdot \vec{r}_i}$, where plane wave solutions for initial and final states were assumed, f_i is the matrix element for lattice site i at position $\vec{r}_i$ in space, $\vec{k}_a, \vec{k}$ being the axion and photon wave-vectors, respectively. The outgoing photon wave-vector is modified for the effects of absorption by including a complex index of refraction, leading to an axion conversion probability ($|\mathcal{M}|^2$) scaling with $\propto V^n$ where V is the volume of the crystal, with $n < 2$. From there, the authors state that the form of the event rate found in [236] (Eqs. 5.1 and 5.3) assumes that $n = 2$. Next, an ‘attenuation factor’ < 1 is derived, and an attenuated rate is found by multiplying Eq. 5.1 with that factor. In effect, the study claims that the axion rate scales with V^m where $m < 1$, rather than simple scaling directly with volume. While the study may be relevant for LSW, it is not clear that it has been applied correctly to crystal detectors. Both the conversion and detection are performed in our crystal detectors, in contrast to LSW. Therefore, in the following section we calculate the the axion conversion rate in a semiconductor crystal by solving Maxwell’s modified equations in the presence of an axion field, including the effect of final state photon absorption. The steps are laid out and quantities are defined clearly, with careful consideration of the various length scales involved.

5.2.2 A semi-classical treatment

To calculate the differential rate of axion conversion in a crystal, we solve Maxwell’s equations with the addition of an axion component. The electromagnetic current density generated by an axion field φ_a in a static electric field $\mathcal{E}$ as [237]:

$$\mathbf{j}(\mathbf{x}, t) = \frac{\eta}{M} \mathcal{E}(\mathbf{x}) \times \nabla \varphi_a(\mathbf{x}, t), \quad (5.4)$$

where $\eta = 1$ gives an equation in natural units (Lorentz-Heaviside and $\hbar = c = 1$), and $\eta = \frac{1}{\sqrt{4\pi}}$ gives the Gaussian units equation. In the following, we explicitly define initial configurations, find the energy deposition rate by finding the Green’s function of the process, and then modify it to include the effect of final state photon absorption.

Initial Field Configurations

If a static electric field is generated by a crystal, then it may be written as:

$$\mathcal{E}(\mathbf{x}) = W(\mathbf{x}) \sum_{\mathbf{G}} \mathcal{E}_{\mathbf{G}} e^{-i\mathbf{G} \cdot \mathbf{x}}, \quad (5.5)$$

where $\mathbf{G}$ are reciprocal lattice vectors of the crystal, and $W(\mathbf{x}) = 1$ where the crystal exists, and $W(\mathbf{x}) = 0$ otherwise. From [236], $\mathcal{E}_{\mathbf{G}}$ can be written as:

$$\mathcal{E}_{\mathbf{G}} = \frac{n_{\mathbf{G}}}{k_a^2} \mathbf{G} = \frac{F_a^0(\mathbf{G})S(\mathbf{G})}{k_a^2 v_c} \mathbf{G}, \quad (5.6)$$

for one target-nucleus type. Here $F_a^0(\mathbf{G})$ is the (unitless) scattering form factor of the crystal, $S(\mathbf{G})$ is the structure function of the crystal, and v_c is the volume of the unit cell. Note that $\mathcal{E}_{\mathbf{G}}$ is independent of the axion wavenumber k_a . The Fourier transform of the electric field of the crystal is given by:

$$\mathcal{E}(\mathbf{k}) \equiv \int d^3\mathbf{x} e^{-i\mathbf{k}\cdot\mathbf{x}} \mathcal{E}(\mathbf{x}) = \sum_{\mathbf{G}} W(\mathbf{k} + \mathbf{G}) \mathcal{E}_{\mathbf{G}} \quad (5.7)$$

The real scalar axion field may be written as:

$$\varphi_a(\mathbf{x}, t) = \int \frac{d^3\mathbf{k}_a}{(2\pi)^3} \frac{1}{\sqrt{2E_a}} [a(\mathbf{k}_a) e^{i\mathbf{k}_a\cdot\mathbf{x}} e^{-i\omega_a t} + a^*(\mathbf{k}_a) e^{-i\mathbf{k}_a\cdot\mathbf{x}} e^{i\omega_a t}], \quad (5.8)$$

where $E_a \equiv \hbar c k_a$. This is a classical field. At quantization, one would promote $a \rightarrow A$ and $a^* \rightarrow A^\dagger$ to the annihilation and creation operators, which follow the canonical commutation relations. For the classical field, one may find a relation between $|a(\mathbf{k}_a)|^2$ and the energy dependent flux of solar axions in the vicinity of the crystal $\frac{d\phi}{dE_a}(E_a)$ by defining the field in a confining space of volume V_{box} . The relation is:

$$|a(\mathbf{k}_a)|^2 = V_{box} \frac{(2\pi)^3}{E_a^2} \frac{d\phi}{dE_a}(E_a) \delta(\cos\theta - \cos\theta_{\hat{\mathbf{s}}}) \delta(\phi - \phi_{\hat{\mathbf{s}}}), \quad (5.9)$$

where (θ, ϕ) determine the direction of $\mathbf{k}_a$, and $(\theta_{\hat{\mathbf{s}}}, \phi_{\hat{\mathbf{s}}})$ determine the direction of the sun-earth unit vector $\hat{\mathbf{s}}$. Equation 5.9 is derived in Appendix A.1. The analogous quantity to this amplitude had we been dealing with a quantum field is:

$$\langle \Psi | A^\dagger(\mathbf{k}_a) A(\mathbf{k}'_a) | \Psi \rangle = (2\pi)^3 \delta^3(\mathbf{k}_a - \mathbf{k}'_a) \frac{|a(\mathbf{k}_a)|^2}{V_{box}} \quad (5.10)$$

for a dimensionless mixed state of axion field excitations $|\Psi\rangle$. For example, to construct such a state with one particle of momentum $\mathbf{k}_0$, we take $|\Psi\rangle = \frac{A^\dagger(\mathbf{k}_0)}{\sqrt{V_{box}}} |0\rangle \equiv |1\rangle$, $|0\rangle$ being the Fock vacuum. The shape of $\frac{d\phi}{dE_a}$ is determined by the state $|\Psi\rangle$. As a check, one can

calculate:

$$\begin{aligned}\langle 1|A^\dagger(\mathbf{k}_a)A(\mathbf{k}'_a)|1\rangle &= \frac{1}{V_{box}} \langle 0|A(\mathbf{k}_0)A^\dagger(\mathbf{k}_a)A(\mathbf{k}'_a)A^\dagger(\mathbf{k}_0)|0\rangle \\ &= \frac{(2\pi)^6}{V_{box}} \delta^3(\mathbf{k}_0 - \mathbf{k}'_a) \delta^3(\mathbf{k}_a - \mathbf{k}_0) = (2\pi)^3 \delta^3(\mathbf{k}_a - \mathbf{k}'_a) \frac{(2\pi)^3 \delta^3(\mathbf{k}_a - \mathbf{k}_0)}{V_{box}},\end{aligned}$$

where the canonical commutation relations have been utilized. It can be seen that if $|\Psi\rangle = |1\rangle$, then the corresponding classical amplitude is $|a(\mathbf{k}_a)|^2 = (2\pi)^3 \delta^3(\mathbf{k}_a - \mathbf{k}_0)$.

Radiation Energy Rate

Now that we have defined the initial field configurations, we may use Maxwell's equations to find the radiation energy deposited in the crystal. Combining 5.4, 5.7 and 5.8 yields:

$$\begin{aligned}\mathbf{j}(\mathbf{x}, t) &= \frac{i\eta}{M} \sum_{\mathbf{G}} \int \frac{d^3\mathbf{k}_a}{(2\pi)^3} \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{W(\mathbf{k} + \mathbf{G})\mathcal{E}_{\mathbf{G}} \times \mathbf{k}_a}{\sqrt{2E_a}} a(\mathbf{k}_a) e^{i(\mathbf{k}_a + \mathbf{k}) \cdot \mathbf{x}} e^{-i\omega_a t} + c.c. \\ &= \int \frac{d^3\mathbf{k}_a}{(2\pi)^3} \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{\mathbf{j}_+(\mathbf{k}, \mathbf{k}_a)}{\sqrt{2E_a}} a(\mathbf{k}_a) e^{i\mathbf{k} \cdot \mathbf{x}} e^{-i\omega_a t} + c.c.\end{aligned}\tag{5.11}$$

where

$$\mathbf{j}_+(\mathbf{k}, \mathbf{k}_a) \equiv \frac{i\eta}{M} \sum_{\mathbf{G}} W(\mathbf{k} - \mathbf{k}_a + \mathbf{G})(\mathcal{E}_{\mathbf{G}} \times \mathbf{k}_a),\tag{5.12}$$

The current density 5.11 induces an electric field $\mathbf{E}(\mathbf{x}, t)$ (and an accompanying magnetic field $\mathbf{B}$). In Appendix A.2, a Green's function is found which relates the current density to each Fourier mode of the induced electric field. Given a background dielectric constant $\varepsilon(\omega')$, and a permeability close to that of the vacuum, the relevant Fourier space Maxwell's equations used were

$$\begin{aligned}i\mathbf{k}' \times \mathbf{E}(\mathbf{k}', \omega') - i\omega' \mathbf{B}(\mathbf{k}', \omega') &= 0, \\ i\mathbf{k}' \times \mathbf{B}(\mathbf{k}', \omega') + i\omega' \varepsilon(\omega') \mathbf{E}(\mathbf{k}', \omega') &= 4\pi \mathbf{j}(\mathbf{k}', \omega'),\end{aligned}$$

where $(\omega', \mathbf{k}')$ are the angular frequency and wave-vector of the Fourier mode of the induced electric field which corresponds to amplitude $\mathbf{E}(\mathbf{k}', \omega')$, namely:

$$\mathbf{E}(\mathbf{k}', t) = \int \mathbf{E}(\mathbf{k}', \omega') e^{-i\omega' t} d\omega' = \int \mathbf{E}(\mathbf{x}, t) e^{-i\mathbf{k}' \cdot \mathbf{x}} d^3\mathbf{x}\tag{5.13}$$

and similarly for the current density:

$$\mathbf{j}(\mathbf{k}', t) = \int \mathbf{j}(\mathbf{k}', \omega') e^{-i\omega' t} d\omega' = \int \mathbf{j}(\mathbf{x}, t) e^{-i\mathbf{k}' \cdot \mathbf{x}} d^3\mathbf{x} \quad (5.14)$$

Plugging equation 5.11 into the right hand side of 5.14:

$$\mathbf{j}(\mathbf{k}', \omega') = \int dt e^{i\omega' t} \left[\int \frac{d^3\mathbf{k}_a}{(2\pi)^3} \frac{\mathbf{j}_+(\mathbf{k}', \mathbf{k}_a)}{\sqrt{2E_a}} a(\mathbf{k}_a) e^{-i\omega_a t} + c.c. \right]. \quad (5.15)$$

The Green's function found in Appendix A.2 is:

$$\mathbb{G}(\mathbf{k}', \omega') = -\frac{4\pi i k'^2 (\mathbb{U} - \hat{\mathbf{k}}' \hat{\mathbf{k}}')}{\omega' \epsilon(\omega') (\omega'^2 \epsilon(\omega') - k'^2)} - \frac{4\pi i}{\omega' \epsilon(\omega')} \mathbb{U}, \quad (5.16)$$

where $\mathbb{U}$ is the 3×3 identity matrix. $\mathbb{G}$ relates the current density to the induced electric field as $\mathbf{E}(\mathbf{k}', \omega') = \mathbb{G}(\mathbf{k}', \omega') \cdot \mathbf{j}(\mathbf{k}', \omega')$, so that the relation for $\mathbf{E}(\mathbf{x}, t)$ is equivalent to the final line in 5.11 with the replacement $\mathbf{j}_+(\mathbf{k}, \mathbf{k}_a) \rightarrow \mathbb{G}(\mathbf{k}', \omega') \cdot \mathbf{j}_+(\mathbf{k}, \mathbf{k}_a)$. Here, $\mathbf{k}' = \mathbf{k}$ since the first argument of the Green's function is the same as the first argument in $\mathbf{j}(\mathbf{k}', \omega')$, and from the right hand side of 5.15, that's also the first argument of $\mathbf{j}_+(\mathbf{k}, \mathbf{k}_a)$. The angular frequency ω' appearing in $\mathbb{G}$ is set to $\omega' = \omega_a$ by enforcing boundary conditions similar to those relating the frequencies of refracted waves to those of incident ones. The induced electric field in spacetime is then

$$\mathbf{E}(\mathbf{x}, t) = \int \frac{d^3\mathbf{k}_a}{(2\pi)^3} \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{\mathbb{G}(\mathbf{k}, \omega_a) \cdot \mathbf{j}_+(\mathbf{k}, \mathbf{k}_a)}{\sqrt{2E_a}} a(\mathbf{k}_a) e^{i\mathbf{k} \cdot \mathbf{x}} e^{-i\omega_a t} + c.c. \quad (5.17)$$

The rate at which energy appears as radiation is then the negative of the work done by the induced electric field on the current density

$$\frac{d\mathbb{E}}{dt} = - \int \mathbf{j}(\mathbf{x}, t) \cdot \mathbf{E}(\mathbf{x}, t) d^3\mathbf{x}. \quad (5.18)$$

plugging 5.11 and 5.17 into 5.18 yields

$$\begin{aligned}
\mathbf{j}(\mathbf{x}, t) \cdot \mathbf{E}(\mathbf{x}, t) = & \int \frac{d^3 \mathbf{k}_a}{(2\pi)^3} \frac{d^3 \mathbf{k}'_a}{(2\pi)^3} \frac{d^3 \mathbf{k}}{(2\pi)^3} \frac{d^3 \mathbf{k}'}{(2\pi)^3} \times \left[Q(\mathbf{k}_a, \mathbf{k}'_a, \mathbf{k}, \mathbf{k}') \right. \\
& + \frac{\mathbf{j}_+(\mathbf{k}, \mathbf{k}_a)}{\sqrt{2\omega_a}} \cdot \mathbb{G}^*(\mathbf{k}', \omega'_a) \cdot \frac{\mathbf{j}_+^*(\mathbf{k}', \mathbf{k}'_a)}{\sqrt{2\omega'_a}} a(\mathbf{k}_a) a^*(\mathbf{k}'_a) e^{i(\mathbf{k}-\mathbf{k}') \cdot \mathbf{x}} e^{-i(\omega_a - \omega'_a)t} \\
& \left. + \frac{\mathbf{j}_+^*(\mathbf{k}', \mathbf{k}'_a)}{\sqrt{2\omega'_a}} \cdot \mathbb{G}(\mathbf{k}, \omega_a) \cdot \frac{\mathbf{j}_+(\mathbf{k}, \mathbf{k}_a)}{\sqrt{2\omega_a}} a^*(\mathbf{k}_a) a(\mathbf{k}'_a) e^{-i(\mathbf{k}-\mathbf{k}') \cdot \mathbf{x}} e^{i(\omega_a - \omega'_a)t} \right]
\end{aligned} \tag{5.19}$$

where $Q(\mathbf{k}_a, \mathbf{k}'_a, \mathbf{k}, \mathbf{k}')$ contains terms proportional to $a(\mathbf{k}_a)a(\mathbf{k}'_a)$ and $a^*(\mathbf{k}_a)a^*(\mathbf{k}'_a)$. We now canonically quantize the quantity 5.19 by promoting $a \rightarrow A$ and $a^* \rightarrow A^\dagger$ and normal-order the creation and annihilation operators. So equation 5.18 is modified as:

$$\frac{d\mathbb{E}}{dt} \rightarrow - \int \langle \Psi | \mathbf{j}(\mathbf{x}, t) \cdot \mathbf{E}(\mathbf{x}, t) | \Psi \rangle d^3 \mathbf{x},$$

where $|\Psi\rangle$ is the mixed state of incident axions. Note that $\langle \Psi | Q | \Psi \rangle = 0$. Then using equation 5.10, the total energy conversion rate becomes:

$$\frac{d\mathbb{E}}{dt} = - \int \frac{d^3 \mathbf{k}_a}{(2\pi)^3} \frac{d^3 \mathbf{k}}{(2\pi)^3} \frac{|a(\mathbf{k}_a)|^2}{V_{box} E_a} \text{Re}(\mathbf{j}_+^*(\mathbf{k}, \mathbf{k}_a) \cdot \mathbb{G}(\mathbf{k}, \omega_a) \cdot \mathbf{j}_+(\mathbf{k}, \mathbf{k}_a)). \tag{5.20}$$

Before we plug the Green's function 5.16 into 5.20, we must make some assumptions and simplifications.

Including Photon Absorption

Suppose we characterized x-ray propagation by an index of refraction $n(\omega)$, where $\varepsilon(\omega) = n^2(\omega)$, and at a fixed frequency put $n = n' + in''$, n' and n'' respectively the real and imaginary parts of the index of refraction. Then a beam of x-rays would propagate with k given by $\omega n(\omega)/c$ (where c is the speed of light), according to

$$e^{ikz} = e^{i(\frac{\omega}{c})(n' + in'')z} = e^{i(\frac{\omega}{c})n'z} e^{-(\frac{\omega}{c})n''z},$$

so the intensity $I(z)$ follows

$$I(z) \propto e^{-2(\frac{\omega}{c})n''z} \equiv e^{-\mu z},$$

where $1/\mu$ is the "skin depth" which relates to the imaginary index of refraction as $n'' = \frac{c\mu}{2\omega} = \frac{\lambda\mu}{4\pi}$, λ being the vacuum wavelength.

Of course, the simple use of a kind of macroscopic dielectric constant here is not reasonable, for two reasons. First, the wavelength ($\lambda = 1.24 \text{ nm}$ at 1 keV and $\lambda = 0.124 \text{ nm}$ at 10 keV) is on the order of or less than the lattice constant (0.543 nm for silicon and only slightly higher for germanium). Secondly, the current density source is peaked near the nuclei, and it is unreasonable to consider it "embedded" in a background dielectric constant. In particular, putting a dielectric constant with an imaginary component "in between" the separated charges associated with the current density near a nucleus would lead to an unphysically huge absorption. A full theoretical treatment would involve taking local field corrections into account in how the current density sources radiated.

However, since the absorption length $1/\mu$ of x-rays in the energy range of $1 \text{ keV} < E_\gamma < 10 \text{ keV}$ is no less than $O(1\mu\text{m}) \gg O(\text{lattice constant})$ for materials such as germanium and silicon, and assuming that these energy ranges lie above all major resonances of the material, we can put in the Green's function 5.16 $\varepsilon(\omega) = 1$ everywhere except in the resonance denominator, where we put $\varepsilon(\omega) = n^2(\omega)$ with $n(\omega) = 1 + in''(\omega) = 1 + \frac{ic}{2\omega}\mu(\omega)$. That is, we take $\mathbb{G} \rightarrow \bar{\mathbb{G}}$, where

$$\bar{\mathbb{G}}(\mathbf{k}, \omega) = -\frac{4\pi i k^2 (\mathbb{U} - \hat{\mathbf{k}}\hat{\mathbf{k}})}{\omega(\omega^2 n^2(\omega) - k^2)} - \frac{4\pi i}{\omega} \mathbb{U}.$$

Only the real part of $\bar{\mathbb{G}}$ is relevant, since 5.20 takes the real part of the integrand. We show in Appendix A.2 that the real part of $\bar{\mathbb{G}}$ is

$$\bar{\mathbb{G}}_R(\mathbf{k}, \omega) = -\frac{4\pi^2 k^2}{\omega(k + \frac{\omega}{c})} L(k, \omega) (\mathbb{U} - \hat{\mathbf{k}}\hat{\mathbf{k}}), \quad (5.21)$$

where

$$L(k, \omega) = \frac{1}{\pi} \frac{\frac{\mu(\omega)}{2}}{(k^2 - \frac{\omega}{c})^2 + \frac{\mu^2(\omega)}{4}}$$

is a Lorentzian function of k , showing that indeed a range of k is associated with each ω . We show in Appendix A.2 that the vacuum Green's function can be obtained using the replacement $L(k, \omega) \rightarrow \delta(k - \frac{\omega}{c})$. Of course, using such a Green function formally assumes that the detector medium is infinite, but since the skin depth is much less than the size of the detector that should not introduce serious error.

After these simplifications to the Green's function, we can find

$$\begin{aligned} \text{Re}(\mathbf{j}_+^*(\mathbf{k}, \mathbf{k}_a) \cdot \bar{\mathbf{G}}(\mathbf{k}, \omega_a) \cdot \mathbf{j}_+(\mathbf{k}, \mathbf{k}_a)) &= -\frac{4\pi^2 k^2}{E_a(k + \frac{E_a}{\hbar c})} L(k, E_a) \times \\ \frac{\eta^2}{M^2} \sum_{\mathbf{G}, \mathbf{G}'} W^*(\mathbf{k} - \mathbf{k}_a + \mathbf{G}') W(\mathbf{k} - \mathbf{k}_a + \mathbf{G}) (\mathcal{E}_{\mathbf{G}'}^* \times \mathbf{k}_a) \left(\mathbb{U} - \hat{\mathbf{k}} \hat{\mathbf{k}} \right) (\mathcal{E}_{\mathbf{G}} \times \mathbf{k}_a) \end{aligned}$$

Recall that the W function in momentum space is the Fourier transform of $W(\mathbf{x})$ in 5.5, which defines the extent of the crystal. Supposing the crystal in question is at least 1cm wide in any direction, the width of $W(\mathbf{k})$ will be of order 10^{-7}nm^{-1} . Differences between reciprocal lattice vectors may only have magnitudes as small as $(\text{lattice constant})^{-1} \sim 1\text{nm}^{-1}$. Therefore, due to the crystal being much larger than the unit cell, the sum over two reciprocal lattice vectors above gives a nonzero result only when the reciprocal lattices are the same, so inside the sum over reciprocal lattice vectors one may add a Kronecker $\delta_{\mathbf{G}, \mathbf{G}'}$ without changing the result. Now, 5.20 becomes:

$$\begin{aligned} \frac{d\mathbb{E}}{dt} &= \frac{4\pi^2 \eta^2}{M^2} \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \int_0^\infty \sum_{\mathbf{G}} \frac{k^2}{(k + \frac{E_a}{\hbar c})} \frac{d\phi}{dE_a}(E_a) \left| W(\mathbf{k} - \frac{E_a}{\hbar c} \hat{\mathbf{s}} + \mathbf{G}) \right|^2 L(k, E_a) \\ &\quad \times (\mathcal{E}_{\mathbf{G}}^* \times \hat{\mathbf{s}}) \left(\mathbb{U} - \hat{\mathbf{k}} \hat{\mathbf{k}} \right) (\mathcal{E}_{\mathbf{G}} \times \hat{\mathbf{s}}) dE_a \quad (5.22) \end{aligned}$$

where 5.9 was used. Note that the width of the Lorentzian $L(k, E_a)$ is at most $\mu(E_a) \sim 10^{-3}\text{nm}^{-1}$ which is much larger than the width of $W(\mathbf{k})$ quoted above at around 10^{-7}nm^{-1} . Thus, we may approximate

$$\left| W(\mathbf{k} - \frac{E_a}{\hbar c} \hat{\mathbf{s}} + \mathbf{G}) \right|^2 \rightarrow V_{\text{xtal}} (2\pi)^3 \delta^3(\mathbf{k} - \frac{E_a}{\hbar c} \hat{\mathbf{s}} + \mathbf{G}), \quad (5.23)$$

where V_{xtal} is the volume of the crystal. This is the "infinite crystal" limit, which means the crystal is much larger than the absorption length $1/\mu$. With the delta function of 5.23, one can evaluate the integral $d^3 k$. The delta function sets $\mathbf{k} = \mathbf{k}_{\mathbf{G}, \hat{\mathbf{s}}} \equiv \frac{E_a}{\hbar c} \hat{\mathbf{s}} - \mathbf{G}$. So the factor in 5.22 with the cross products becomes

$$(\mathcal{E}_{\mathbf{G}}^* \times \hat{\mathbf{s}}) \left(\mathbb{U} - \hat{\mathbf{k}}_{\mathbf{G}, \hat{\mathbf{s}}} \hat{\mathbf{k}}_{\mathbf{G}} \right) (\mathcal{E}_{\mathbf{G}} \times \hat{\mathbf{s}}) = \left| \frac{F_a^0(\mathbf{G}) S(\mathbf{G})}{k_a^2 v_c} \right|^2 (|\mathbf{G} \times \hat{\mathbf{s}}|^2 - |\hat{\mathbf{k}}_{\mathbf{G}, \hat{\mathbf{s}}} \cdot (\mathbf{G} \times \hat{\mathbf{s}})|^2),$$

where 5.6 was used. We can also write $|\mathbf{G} \times \hat{\mathbf{s}}|^2 = G^2 \sin^2(\theta)$, where θ is the scattering angle.

Replacing the Lorentzian with a delta function $L(k, \omega) \rightarrow \delta(k - \frac{\omega}{c})$ reproduces the previous result in [236], as shown in Appendix A.3. Using delta function properties, one can show

$$\delta(k_{\mathbf{G}, \hat{\mathbf{s}}} - E_a/\hbar c) = \delta(E_a/\hbar c - |\hat{\mathbf{s}}E_a/\hbar c - \mathbf{G}|) = \frac{1}{2\cos^2(\theta)} \delta\left(\frac{E_a}{\hbar c} - \frac{|\mathbf{G}|^2}{2(\hat{\mathbf{s}} \cdot \mathbf{G})}\right), \quad (5.24)$$

which is the Bragg condition.

The extraction of the differential photon production rate $\frac{dR}{dE_\gamma}(E_\gamma)$ from equation 5.22, which is used in equation 5.1 turns out to be non-trivial in the non-vacuum limit, and is also outlined in Appendix A.3. The extraction of that rate depends upon the interpretation of $\mathbf{k}$ in 5.22 as the wave-vector of induced photons, with the relation $E_\gamma = \hbar ck$.

5.3 Projected Sensitivity of SuperCDMS SNOLAB to Solar Axions

Since we found in the previous section that the inclusion of final state photons affects the event rate negligibly, we stick to the original calculation from Ref. [236], with the results for SuperCDMS SNOLAB germanium iZIP detectors shown in Fig. 5.1. To find the maximum reach of the experiment, we calculate event rates for all SuperCDMS detectors in the initial payload (documented in Table. 2.1), and find the zero-background discovery reach (calculate $g_{a\gamma\gamma}$ which gives 4 events given a net live-time of 4 years). The result is shown in Fig. 5.2. An analysis of the effect of orientation uncertainty on the solar axion reach of multiple crystal detectors was done by [246]. While the ‘zero background’ reach represents a best case scenario, it is an important benchmark to motivate the analysis of SuperCDMS SNOLAB data for axions.

The ultimate reach of the experiment to solar axions would depend on the background rate in the region of interest ($4\text{ keV}_{\text{ee}} < E_r < 10\text{ keV}_{\text{ee}}$), and the possibility of an unknown/irreducible background. An analysis of SNOLAB data would utilize the time modulation information in the solar axion event rate by using an unbinned maximum likelihood analysis of the 2-dimensional dataset of event times and energies, as was done in [153]. This allows for the statistical ‘subtraction’ of relatively stable background sources. Therefore, this type of analysis tends to be exposure-limited, as increasing exposure is a more important factor in increasing sensitivity than reducing the residual background rate. The

exposure of the HVeV detectors discussed in Ch. 4 would be too small to obtain a competitive result. Thus we have demonstrated the potential of the large exposure of SuperCDMS crystals in searching for ultra-light dark matter.

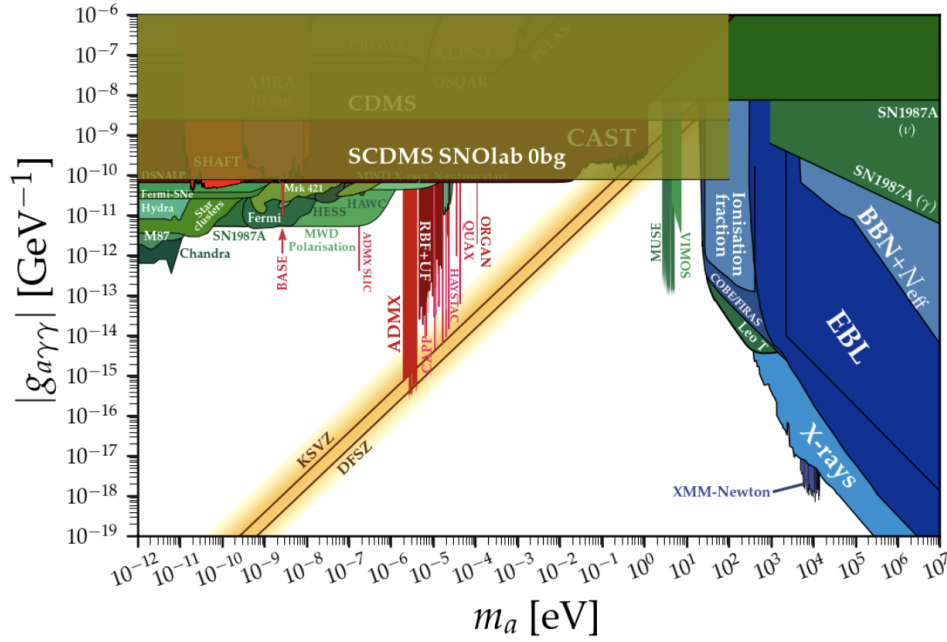


Figure 5.2: Zero-background benchmark sensitivity of SuperCDMS SNOLAB’s initial payload to solar axions with net exposure of 4 years (yellow box). The yellow sloped line indicates the parameter space for the QCD axion. Unlike the CAST helioscope, sensitivity to QCD axions of mass $1\text{ eV} < m < 100\text{ eV}$ is attained by SuperCDMS down to $g_{a\gamma\gamma} \sim 10^{-10}\text{ GeV}^{-1}$. Blue exclusion regions are cosmological bounds, while green regions are stellar bounds. Not shown are the solar neutrino [247] and horizontal branch [109] bounds, which would cover much of the reach of SuperCDMS (as well as CAST). The remainder of the red exclusion bounds come from haloscope experiments assuming the galactic DM halo is made up of axions. The previous CDMS limit is taken from [153]. An open source plotting tool was used to generate this figure [248].

Chapter 6

Conclusions

Here, we draw conclusions from what was observed. We start by investigating low-energy events in HVeV detectors, motivating a fiducial cut, and devising background control procedures for future runs.

6.1 On the background rate of HVeV detectors

In Chapter 4, exclusion regions were produced for various light dark matter candidates. That analysis rested on a key assumption: that the measured total event rate in any expected peak region of the discrete spectrum is due to the DM particles under consideration. The upper bounds reported in Figure 4.10 are the 90% one sided upper bounds on the interaction strengths of dark matter with electrons under that assumption. While this does provide a “conservative limit” on the dark matter interaction strength (i.e. we are over-covering the true interaction rate of dark matter with electrons), the true potential of the HVeV detector may not have been tapped yet. And the fact that the true interaction rate differs from the measured one (under the analysis scheme) is not exactly satisfying. Without the background modeling, we begin each run of the experiment knowing there will be ‘no discovery’, only a possible exclusion of parameter space. So the study and modeling of the background in HVeV detectors is of the essence if one is to continue using the detectors for DM search.

Unfortunately, the low energy background is difficult to predict or model. When one is looking at keV energy depositions, as in the case of the SuperCDMS SNOLAB kilogram scale detectors, the backgrounds of concern are those of gammas or other neutrons, and

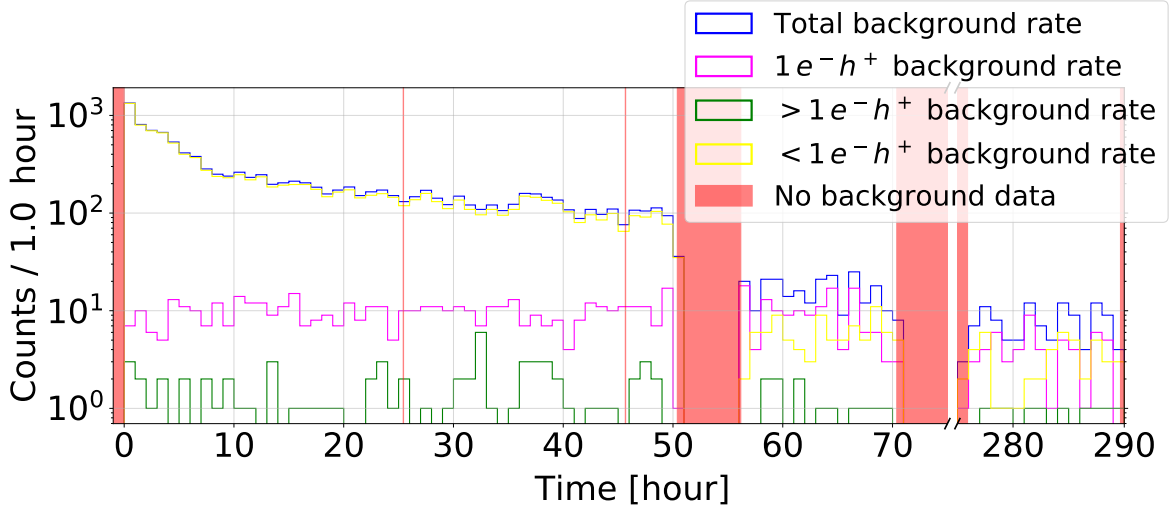


Figure 6.1: Event rate in background data sets from NR14 (after applying data selections described in Ch. 4). The broken x-axis represents times when most of the LED data was taken and crystal voltage was varied, indicating potential for detector ‘baking’ (inputting energy to release charge carriers from shallow traps caused by impurities) to reduce the background rate.

using simulations in **Geant4**, combined with the assaying of all line-of-sight materials, the event spectra from these radioactive processes can be modeled. In HVeV gram-scale detectors, our total energy scale of interest is $E_t \sim \mathcal{O}(100 \text{ eV})$ while the primary energy deposition scale is $E_r \sim \mathcal{O}(1 \text{ eV})$, and the number of charge carriers is $\mathcal{O}(1)$ per event. This means the main background sources are: 1) optical luminescence; 2) leakage current across the crystal; and 3) crystal relaxation from the various strains from the detector holder, as well as thermal contraction. Needless to say, these are hard to model. Their stochastic nature and dependence on the history of the substrate are the main limiting factors. Figure 6.1 shows the event rates for various regions of the spectrum in an HVeV detector at different times.

Not only do we have event rates, our HVeV detector provides energy information, calibrated using LED events of known energy. Perhaps the best avenue for characterizing some of the backgrounds in HVeV are the detectors themselves. We identify an important energy scale: the $1 e^- h^+$ peak energy. At $V_{bias} = 100 \text{ V}$, this energy is found to be $101.1 \pm 0.1(\text{stat.}) \pm 0.7(\text{sys.}) \text{ eV}$ in DM search data, where the systematic uncertainty was dominated

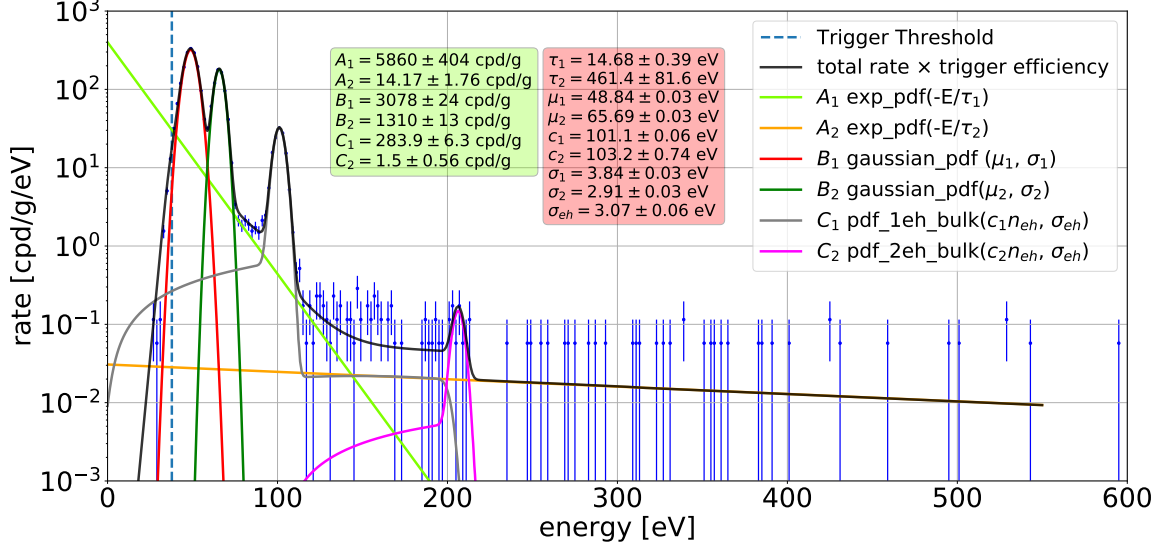


Figure 6.2: Result of an unbinned extended maximum likelihood fit of the DM search spectrum of Ch. 4, and histogram of the data (blue points with error bars). The first and second e^-h^+ peaks (around 100 eV and 200 eV) are fit with CTII PDFs described in Ch. 3, with benchmark trapping and ionization parameters taken from LED data calibration in that chapter. The events included are those with calibrated energies < 1.5 keV that pass all data selections (4.2). The sub- e^-h^+ peaks are fit with simple Gaussians, and two non-peak event ‘excesses’ are fit with two exponentials.

by that associated with cross-talk subtraction in LED calibration data. Other features in the energy spectrum also have energy scales. A first step in characterizing the spectrum would therefore entail identifying event rates and characteristic energy scales. We find that the event rate and spectrum of an HVeV Run 4 DM search/background dataset can be empirically characterized using 6 rates and 9 energy scales when using the charge trapping and impact ionization (CTII) models of Ch. 3 for e^-h^+ peaks (only the first and second are visible; nominal values for charge trapping and impact ionization $f_{CT} = 12\%$ and $f_{II} = 0.4\%$ are used), in addition to Gaussians and exponential functions for the various ‘excesses’ as shown in the fit of the DM search data Fig. 6.2.

Our HVeV detector also provides position information thanks to its two (inner and outer) phonon readout channels. Fig. 6.3 shows the distribution of pulse amplitude across different channels in the DM search data analyzed in Ch. 4 after all data selections. A

band of events can be identified with $A_{out} \simeq 1.2 A_{in}$. This band has a distinct thickness because for events in this band, energy fluctuation between inner and outer channels is mostly independent of the total amplitude. By removing this ‘outer band’ of events, the empirical fit only requires 3 rates and 4 energy scales as seen in Fig. 6.4, as various low-energy excesses are contained within the outer band. The sub- e^-h^+ peaks, as well as the non-ionizing low energy excess (0QLEE) of characteristic energy ~ 15 eV are eliminated from the ‘inner’ pool of events. The ‘inner events’ may then be modeled with e^-h^+ peak PDFs, with only one exponential ‘excess’ component. While the modeling of the source of these inner e^-h^+ events remains an open task, a partition cut of the sort demonstrated in Fig. 6.4 may almost completely eliminate the lowest energy background components, which are usually the most difficult to model. It is also interesting that the ‘inner’ sub- e^-h^+ events may be accounted for using the charge trapping model with $f_{CT} = 12\%$ (fixed value from results of Ch. 3). Thus the potential for HVeV fiducialization is demonstrated. This cut must first be evaluated for efficiency before being used in a DM search analysis, using a pointed laser as done in [215].

When the bias across HVeV detector is set to $V_{bias} = 0$ V, it is said to be in ‘0VeV’ mode. In the low energy ($E_t \lesssim 600$ eV) range, a ‘low-energy excess’ (LEE) is observed. See Fig. 6.5 for comparison of event rates between HV and 0V modes. Since the LEE observed in 0V mode does not seem to be affected by varying HV, it is called non-ionizing (0Q). This type of background is common among low-threshold detectors. Leading hypotheses include these events being due to crystal relaxation after cooling. As for the sub- e^-h^+ peaks, they may indicate a leakage current along the sidewalls, with specific scratches on the substrate sidewalls being the cause of the precise energies of these ‘excess’ events.

6.2 On the elevated rate periods in HVeV Run 4

In section 4.2.1, we discussed a live-time selection criterion (the ‘elevated rate cut’) that was derived for the DM search data of NR13 which removed extended periods of live-time (up to several hours at a time) experiencing an elevated rate of $1 e^-h^+$ events. These elevated event rate periods were preceded by high-amplitude (energy) pulses. We noted how, in NR14 (where the copper lids of the detector holders were replaced with boxes containing LEDs and filters) this type of elevated rate period was not observed. The elevated rate cut applied to NR13 DM search data reduced the $1 e^-h^+$ to similar levels observed in NR14. We noted in the discussion of section 4.2.1 that alpha particles from the decay $^{210}_{82}\text{Pb} \rightarrow ^{210}_{83}\text{Bi} + \beta^- (+\gamma) \rightarrow ^{210}_{84}\text{Po} + \beta^- \rightarrow ^{206}_{82}\text{Pb} + \alpha^{2+}$ were a likely candidate for the cause of the large pulses and the subsequent ‘train’ of $1 e^-h^+$ events. These would implant

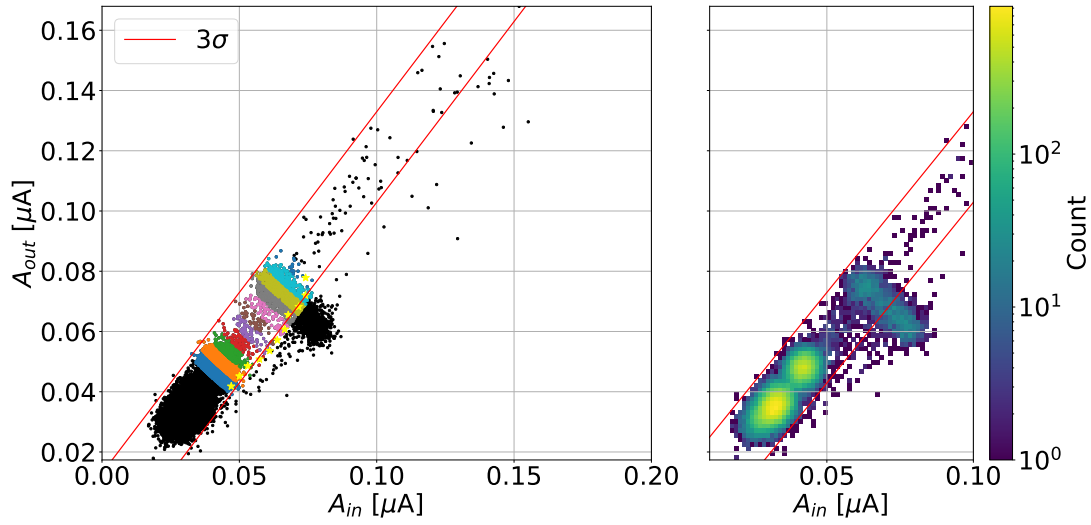


Figure 6.3: Pulse amplitude distributions in the inner and outer channels from DM search data analyzed in Ch. 4 (after applying data selections). Left: A series of Gaussian fits of events within the colored discrete slices of the ‘outer band’ ($A_{out} \simeq 1.2 A_{in}$) are used to derive $3 - \sigma$ boundaries (yellow stars). These discrete points are fit with lines (shown in red), which indicate a natural partition boundary. Right: 2D histogram of points on the left.

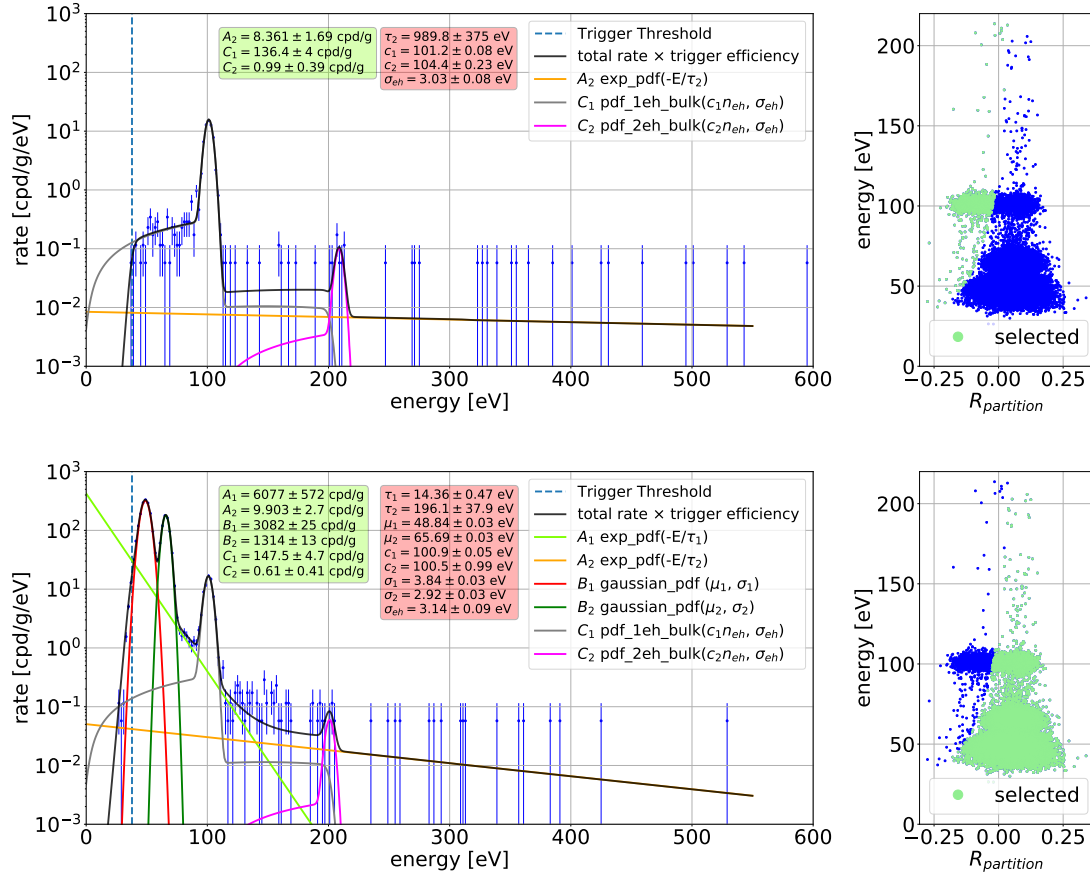


Figure 6.4: Same fits as in Fig. 6.2 but with ‘partition cuts’ (red lines from Fig. 6.3) applied. Upper left: the spectrum of events outside the outer band of events. Upper right: the distribution of events in energy, $R_{partition} = \frac{A_{out}-A_{in}}{A_{out}+A_{in}}$ space, with green points indicating events that are under consideration in the fit on the left. Lower plots: same as above but selecting events falling within the ‘outer band’.

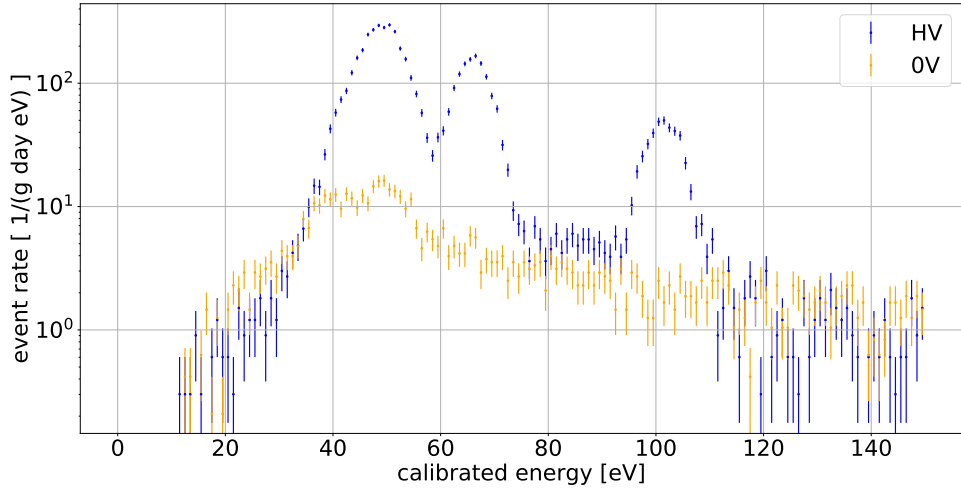


Figure 6.5: Low energy event rates from NR13, at crystal bias of 100 V and 0 V. A decaying ‘excess’ of events independent of voltage is thus called the non-ionizing low energy excess. The HV dataset shown is the 30% unblinded DM search data. No data selections are applied in this plot.

on the backside of an HVeV detector (which faces the copper lid). It was hypothesized in Ch. 4 that the latent $1e^-h^+$ events may indicate the relaxation of the impact sites of the alpha particle, which has a limited ($\mathcal{O}(10\mu\text{m})$) mean free path in silicon. Under the alpha particle hypothesis, the holes are the charge carrier that causes NTL gain in these $1e^-h^+$ events in NR13 data, given the polarity of the crystal voltage bias (positive near backside where alphas would be incident).

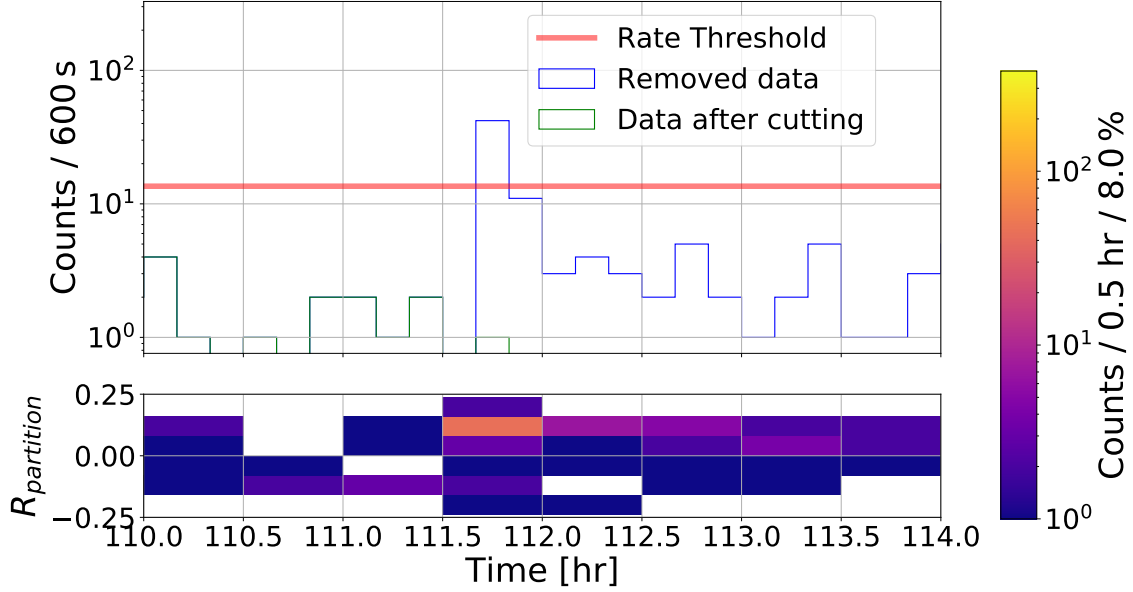


Figure 6.6: Top: Rate of $1e^-h^+$ events in detector NFC1 (NR13 DM search dataset after 100% unblinding, same as Fig. 4.2). Spikes are identified if the rate crosses the horizontal line, which is taken as a 9σ upper bound of the background rate. Each time a spike is identified, 10 hours of live-time is removed. Bottom: Clusters in $R_{\text{partition}} = \frac{A_{\text{out}} - A_{\text{in}}}{A_{\text{out}} + A_{\text{in}}}$ are seen coinciding with the elevated rate periods, indicating the localization of these currents.

The study of this background source is important as it can cause significant experimental dead-time. The controls implemented for materials used in SuperCDMS SNOLAB from copper scrubbing to radon purging would likely help in reducing this background compared to HVeV Run 4. Other experimental facilities may not have such strict protocols. Further, these high-energy surface events could be used to study crystal defects, which in turn may be a major cause of ‘leakage current’ events, which limit the reach of HVeV detectors. Figure 6.6 shows a spike in the $1e^-h^+$ event rate of NR13 DM search data of Ch. 4. Recall

that strings of $1 e^- h^+$ events were also observed following LED events (see Ch. 3). Fig. 6.7 shows the time distribution of LED events relative to the nearest non-LED events. This burst effect in LED data may be a microcosm of what occurs when a much more energetic particle strikes the surface (like the hypothesized α^{2+} of kinetic energy ~ 5 MeV which caused the spike in Figure 6.6).

Because it is unlikely that a 2 eV photon causes defects in the crystal’s surface, this indicates that the surface defects involved in these latent bursts of single- $e^- h^+$ pair may not only be caused by the most recent incident particles, but a surface energy deposition is enough to re-activate a long-lived process in a pre-existing crystal defect on the surface. Thus reducing surface defects by shielding the substrate throughout its production and transportation to a clean experimental site may prove valuable. Alternatively, reducing potential low-energy activation sources (e.g. luminescence sources) which could trigger secondary long-lived surface leakage from a pre-existing defect. The 2 eV photon seems to result in a long decaying tail of characteristic time of a couple milliseconds, while the hypothetical ~ 5 MeV alpha results in a decay tail of order kiloseconds. The effect of defects in silicon caused by H^+ ion implantation (kinetic energy $\mathcal{O}(10 \text{ keV})$) was considered in [249] using simulations and comparing with analytic predictions. The study showed that (at a temperature of $T=300 \text{ K}$) it would take around 0.1 s to half the number of defects. A detailed study of the dependence of burst time scales on primary surface deposition energy may be warranted to shine light on the physics involved. Dependence on bias voltage and substrate temperature may also be investigated.

6.3 Looking Forward

HVeV detectors are currently the most sensitive detectors for 1 MeV-scale electron-recoil dark matter (ERDM), as demonstrated by HVeV Run 4. HVeV Run 5 data is currently undergoing analysis, with great potential to widen that lead. The run included detectors of improved resolution operated in SNOLAB’s Cryogenic Underground TEsting (CUTE) facility. These new detectors also have an insulating layer on detector faces, intended to reduce leakage current. Future background mitigation measures have been proposed in this chapter. While detector ‘baking’ (Fig. 6.1) shows potential for reducing much of the background rate (particularly low-energy excesses), the fiducialization proposed in Section 6.1 also has the potential to be a powerful background mitigation tool. Studies on the signal efficiency of a partition cut similar to that in Fig. 6.3 are also underway. Another promising avenue is the study of causes of surface leakage discussed in Section 6.2. Surface defects and energy depositions may result in a prolonged period of charge leakage within

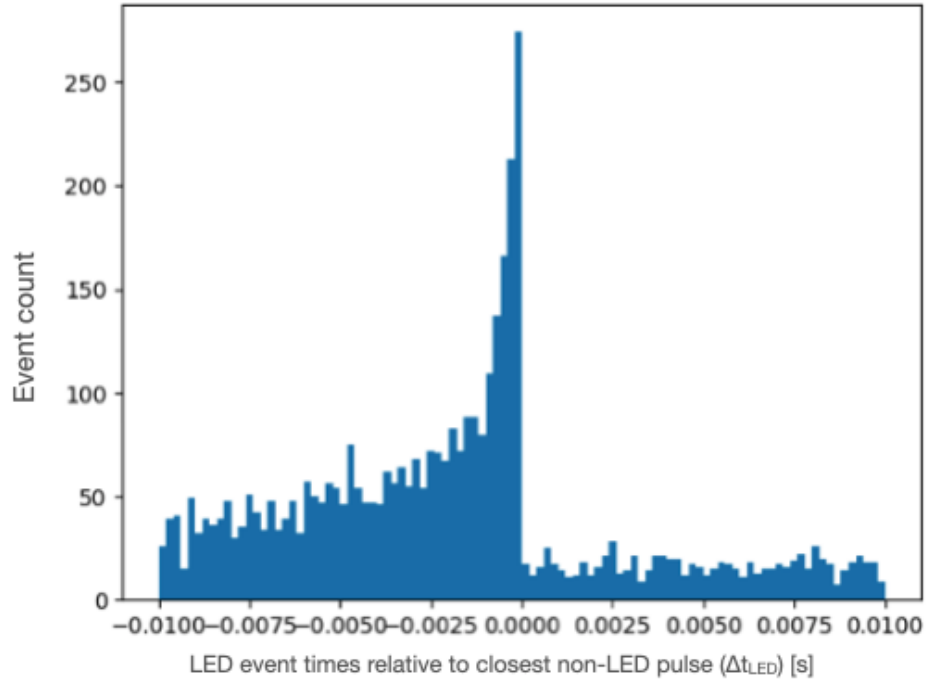


Figure 6.7: LED event times relative to the closest non-LED event within the LED calibration data set used in the analysis of Ch. 4. At least two different time-scales may be identified by inspection, reminiscent of the two time scales observed in the elevated event rate periods (Fig. 6.6), albeit being much shorter. This shows that pulses are more likely to occur after an LED event.

the semiconductor. Mitigating these defects might help reduce the overall leakage rate. As HVeV detectors achieve optical (sub-eV) scale resolutions, the detectors may also be sensitive to effects relevant to optics, materials sciences, and semiconductor physics. HVeV detectors may also be used to study coherent elastic neutrino-nucleus scattering (CE ν NS), as is done in the RICOCHET experiment [250].

In this thesis, we considered the evidence for various low-mass particle dark matter candidates introduced in Ch. 1, from MeV-scale ERDM and eV scale dark bosons considered in Ch. 4, down to sub-eV axions investigated in Ch. 5. To do so, we also investigated the detector response modeling of gram-scale semiconducting bolometers (HVeV) in Ch. 3. Our studies resulted in new regions of parameter space being excluded in the few-MeV dark ERDM range. The potential of SuperCDMS SNOLAB as an indirect probe of sub-eV solar axions was demonstrated. Finally, this concluding chapter focused on newly derived selection criteria to help study and further reduce low-energy background event rates in HVeV detectors. To extend the reach of HVeV detectors to lower cross-sections for higher ($\gtrsim 1$ MeV) mass ERDM, a larger exposure (kg-days rather than g-days) is needed. While the residual ‘background’ rate of HVeV detectors is already very competitive with the latest CCD experiments which also search for low-mass ERDM [251], the CCD experiments are comprised of many modules providing a larger exposure. The CCD experiments are fast approaching the freeze-in/out cross-section benchmarks for light ERDM with mass ranges of $\gtrsim 100$ MeV, as they are limited in the lower mass sensitivity due to a large $1 e^- h^+$ ‘dark current’. HVeV detectors, with their precision calorimetry and lower event rates, will continue to complement CCD experiments in the $\lesssim 100$ MeV mass range for ERDM.

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APPENDICES

Appendix A

Calculating the solar axion rate using semi-classical field theory

A.1 Relation between field density and axion flux

Figure 1 shows the sun-earth system. For $R \gg r_{Earth}$, the volume of a box surrounding the Earth can be approximated by $V_{box} = cR^2 \sin(\theta) \Delta\theta \Delta\varphi \Delta t$, for an arbitrary small period of time Δt . The number of axions emitted from the sun in time Δt is related to the differential flux of axions a distance R from the sun by:

$$N_{esc} = \left(\int_0^\infty \frac{d\phi}{dE_a}(E_a) dE_a \right) (4\pi R^2 \Delta t) \equiv \Phi_{tot}(4\pi R^2 \Delta t). \quad (\text{A.1})$$

Thus the number of axions in the box surrounding the Earth is:

$$N_{box} = \frac{cR^2 \sin(\theta) \Delta\theta \Delta\varphi \Delta t}{4\pi c R^2 \Delta t} N_{esc} = \frac{\sin(\theta) \Delta\theta \Delta\varphi}{4\pi} \Phi_{tot}(4\pi R^2 \Delta t) = V_{box} \frac{\Phi_{tot}}{c}. \quad (\text{A.2})$$

N_{box} is also given by the "number operator" (a number for a classical field):

$$N_{box} = V_{box} \frac{\Phi_{tot}}{c} = \int |a(\mathbf{k}_a)|^2 \frac{d^3 \mathbf{k}_a}{(2\pi)^3} \quad (\text{A.3})$$

Assuming again that $R \gg r_{Earth}$, the axions in the box, to a good approximation, all point in one direction $\hat{\mathbf{s}}$. This is even a better approximation if we consider only the axions

around our experiment. So we expand $|a(\mathbf{k}_a)|^2 = F(|\mathbf{k}_a|) \delta(\cos\theta - \cos\theta_{\mathbf{s}}) \delta(\phi - \phi_{\mathbf{s}})$.

$$(2\pi)^3 V_{box} \frac{\Phi_{tot}}{c} = \int k_a^2 F(|\mathbf{k}_a|) dk_a \quad (\text{A.4})$$

So, ignoring factors of c and $\hbar$, $F(|\mathbf{k}_a|) = V_{box} \frac{(2\pi)^3}{E_a^2} \frac{d\phi}{dE_a}(E_a)$ and

$$|a(\mathbf{k}_a)|^2 = V_{box} \frac{(2\pi)^3}{E_a^2} \frac{d\phi}{dE_a}(E_a) \delta(\cos\theta - \cos\theta_{\mathbf{s}}) \delta(\phi - \phi_{\mathbf{s}}), \quad (\text{A.5})$$

which is equation 5.9.

A.2 Green's functions derivation

We start with a general derivation of the Green function relating the generated electric field to a current density embedded in an infinite medium of background dielectric constant $\varepsilon(\omega)$. Two of the Maxwell equations (in Gaussian units) are

$$\begin{aligned} \nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} &= 0 \\ c \nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} &= 4\pi \mathbf{j} \end{aligned}$$

and their Fourier transforms follow from taking

$$\begin{aligned} \frac{\partial}{\partial t} &\rightarrow -i\omega \\ \nabla &\rightarrow i\mathbf{k} \end{aligned}$$

This gives us

$$\begin{aligned} i\mathbf{k} \times \mathbf{E} - i\omega \mathbf{B} &= 0 \\ i\mathbf{k} \times \mathbf{H} + i\omega \mathbf{D} &= 4\pi \mathbf{j} \end{aligned} \quad (\text{A.6})$$

Note that assuming charge conservation

$$i\mathbf{k} \cdot \mathbf{j} - i\omega \rho = 0$$

from A.6 we immediately get the other two Maxwell equations,

$$\begin{aligned} i\mathbf{k} \cdot \mathbf{B} &= 0 \\ i\mathbf{k} \cdot \mathbf{D} &= 4\pi\rho \end{aligned}$$

Now putting

$$\begin{aligned} \mathbf{H} &= \mathbf{B} \\ \mathbf{D} &= \varepsilon(\omega)\mathbf{E} \end{aligned}$$

together with $\mathbf{k} = k\hat{\mathbf{k}}$, we can write A.6 as

$$\begin{aligned} ck\hat{\mathbf{k}} \times \mathbf{E} &= \omega\mathbf{B} \\ ck\hat{\mathbf{k}} \times \mathbf{B} &= -\omega\varepsilon(\omega)\mathbf{E} - 4\pi i\mathbf{j} \end{aligned} \tag{A.7}$$

From the first of A.7 we have

$$c^2k^2(\hat{\mathbf{k}} \times \mathbf{E}) = \omega(ck\mathbf{B})$$

so

$$\begin{aligned} c^2k^2\hat{\mathbf{k}} \times (\hat{\mathbf{k}} \times \mathbf{E}) &= \omega(ck\hat{\mathbf{k}} \times \mathbf{B}) \\ &= \omega(-\omega\varepsilon(\omega)\mathbf{E} - 4\pi i\mathbf{j}) \\ &= -\omega^2\varepsilon(\omega)\mathbf{E} - 4\pi i\omega\mathbf{j} \end{aligned}$$

where in the second line we have used the second of A.7. Using a vector identity this gives us

$$c^2k^2(\hat{\mathbf{k}}(\hat{\mathbf{k}} \cdot \mathbf{E}) - \mathbf{E}) = -\omega^2\varepsilon(\omega)\mathbf{E} - 4\pi i\omega\mathbf{j}$$

or

$$c^2k^2\hat{\mathbf{k}}(\hat{\mathbf{k}} \cdot \mathbf{E}) + (\omega^2\varepsilon(\omega) - c^2k^2)\mathbf{E} = -4\pi i\omega\mathbf{j} \tag{A.8}$$

Now we want to solve for $\mathbf{E}$, but we are bothered by the $\hat{\mathbf{k}}(\hat{\mathbf{k}} \cdot \mathbf{E})$ term. To get rid of it, first dot $\hat{\mathbf{k}}$ into and then multiply the result by $\hat{\mathbf{k}}$. The result is

$$c^2k^2\hat{\mathbf{k}}(\hat{\mathbf{k}} \cdot \mathbf{E}) + (\omega^2\varepsilon(\omega) - c^2k^2)\hat{\mathbf{k}}(\hat{\mathbf{k}} \cdot \mathbf{E}) = -4\pi i\omega\hat{\mathbf{k}}(\hat{\mathbf{k}} \cdot \mathbf{j})$$

or

$$\omega^2\varepsilon(\omega)\hat{\mathbf{k}}(\hat{\mathbf{k}} \cdot \mathbf{E}) = -4\pi i\omega\hat{\mathbf{k}}(\hat{\mathbf{k}} \cdot \mathbf{j})$$

or

$$c^2k^2\hat{\mathbf{k}}(\hat{\mathbf{k}} \cdot \mathbf{E}) = -\frac{4\pi ic^2k^2}{\omega\varepsilon(\omega)}\hat{\mathbf{k}}(\hat{\mathbf{k}} \cdot \mathbf{j})$$

Using this in A.2 we have

$$\begin{aligned}
(\omega^2 \varepsilon(\omega) - c^2 k^2) \mathbf{E} &= -4\pi i \omega \mathbf{j} + \frac{4\pi i c^2 k^2}{\omega \varepsilon(\omega)} \hat{\mathbf{k}}(\hat{\mathbf{k}} \cdot \mathbf{j}) \\
&= -4\pi i \omega \left(\mathbb{U} - \frac{c^2 k^2}{\omega^2 \varepsilon(\omega)} \hat{\mathbf{k}} \hat{\mathbf{k}} \right) \cdot \mathbf{j} \\
&= -4\pi i \omega \left(\frac{c^2 k^2}{\omega^2 \varepsilon(\omega)} (\mathbb{U} - \hat{\mathbf{k}} \hat{\mathbf{k}}) + \left(1 - \frac{c^2 k^2}{\omega^2 \varepsilon(\omega)} \right) \mathbb{U} \right) \cdot \mathbf{j} \\
&= -\frac{4\pi i c^2 k^2}{\omega \varepsilon(\omega)} (\mathbb{U} - \hat{\mathbf{k}} \hat{\mathbf{k}}) \cdot \mathbf{j} - 4\pi i \omega \left(\frac{\omega^2 \varepsilon(\omega) - c^2 k^2}{\omega^2 \varepsilon(\omega)} \right) \mathbf{j}
\end{aligned}$$

Dividing by $(\omega^2 \varepsilon(\omega) - c^2 k^2)$ and where necessary adding $i\varepsilon$, where ε is a positive infinitesimal to ω to impose causality, we have

$$\mathbf{E} = -\frac{4\pi i c^2 k^2 (\mathbb{U} - \hat{\mathbf{k}} \hat{\mathbf{k}}) \cdot \mathbf{j}}{\omega \varepsilon(\omega) ((\omega + i\varepsilon)^2 \varepsilon(\omega) - c^2 k^2)} - \frac{4\pi i}{\omega \varepsilon(\omega)} \mathbf{j} \quad (\text{A.9})$$

The vacuum limit is when $\varepsilon(\omega) = 1$, or

$$\mathbf{E} = -\frac{4\pi i c^2 k^2 (\mathbb{U} - \hat{\mathbf{k}} \hat{\mathbf{k}}) \cdot \mathbf{j}}{\omega ((\omega + i\varepsilon)^2 - c^2 k^2)} - \frac{4\pi i}{\omega} \mathbf{j}$$

A.2.1 The vacuum Green function

In the vacuum we can write

$$\mathbf{E}(\mathbf{k}, \omega) = \mathbb{G}^0(\mathbf{k}, \omega) \cdot \mathbf{j}(\mathbf{k}, \omega)$$

where

$$\mathbb{G}^0(\mathbf{k}, \omega) = -\frac{4\pi i c^2 k^2 (\mathbb{U} - \hat{\mathbf{k}} \hat{\mathbf{k}})}{\omega ((\omega + i\varepsilon)^2 - c^2 k^2)} - \frac{4\pi i}{\omega} \mathbb{U}$$

To decompose this into real and imaginary parts we write

$$\begin{aligned}
\frac{1}{(\omega + i\varepsilon)^2 - c^2 k^2} &= \frac{1}{(\omega + i\varepsilon) + ck} \frac{1}{(\omega + i\varepsilon) - ck} \\
&\rightarrow \frac{1}{\omega + ck} \left(\frac{\mathcal{P}}{\omega - ck} - i\pi \delta(\omega - ck) \right) = \frac{1}{\omega + ck} \left(\frac{\mathcal{P}}{\omega - ck} - i\pi \frac{\delta(k - \frac{\omega}{c})}{c} \right) \\
&= -\frac{1}{c^2} \frac{1}{k + \frac{\omega}{c}} \left(\frac{\mathcal{P}}{k - \frac{\omega}{c}} + i\pi \delta\left(k - \frac{\omega}{c}\right) \right)
\end{aligned}$$

and so

$$\mathbb{G}^0(\mathbf{k}, \omega) = \frac{4\pi i k^2}{\omega} \frac{1}{k + \frac{\omega}{c}} \left(\frac{\mathcal{P}}{k - \frac{\omega}{c}} + i\pi \delta\left(k - \frac{\omega}{c}\right) \right) (\mathbb{U} - \hat{\mathbf{k}}\hat{\mathbf{k}}) - \frac{4\pi i}{\omega} \mathbb{U}$$

which we can write as

$$\mathbb{G}^0(\mathbf{k}, \omega) = \mathbb{G}_R^0(\mathbf{k}, \omega) + i\mathbb{G}_I^0(\mathbf{k}, \omega),$$

where

$$\begin{aligned} \mathbb{G}_I^0(\mathbf{k}, \omega) &= \frac{4\pi k^2}{\omega} \frac{1}{k + \frac{\omega}{c}} \frac{\mathcal{P}}{k - \frac{\omega}{c}} (\mathbb{U} - \hat{\mathbf{k}}\hat{\mathbf{k}}) - \frac{4\pi}{\omega} \mathbb{U} \\ \mathbb{G}_R^0(\mathbf{k}, \omega) &= -\frac{4\pi^2 k^2}{\omega} \frac{1}{k + \frac{\omega}{c}} \delta\left(k - \frac{\omega}{c}\right) (\mathbb{U} - \hat{\mathbf{k}}\hat{\mathbf{k}}) \\ &= -\frac{2\pi^2 c k^2}{\omega^2} \delta\left(k - \frac{\omega}{c}\right) (\mathbb{U} - \hat{\mathbf{k}}\hat{\mathbf{k}}) = -\frac{2\pi^2}{c} \delta\left(k - \frac{\omega}{c}\right) (\mathbb{U} - \hat{\mathbf{k}}\hat{\mathbf{k}}) \end{aligned}$$

A.2.2 Including absorption

We now return to the full Green function, where we can omit the $i\varepsilon$ because we will include an imaginary part of $\varepsilon(\omega)$. From we have

$$\mathbb{G}(\mathbf{k}, \omega) = -\frac{4\pi i c^2 k^2 (\mathbb{U} - \hat{\mathbf{k}}\hat{\mathbf{k}})}{\omega \varepsilon(\omega) (\omega^2 \varepsilon(\omega) - c^2 k^2)} - \frac{4\pi i}{\omega \varepsilon(\omega)} \mathbb{U}$$

We now put $\varepsilon(\omega) \rightarrow 1$ in most places except in the resonant denominator, where taking $\varepsilon(\omega) = n^2(\omega)$ and we take

$$n(\omega) = 1 + \frac{ic}{2\omega} \mu(\omega)$$

where we take the imaginary part to be very small; here $\mu(\omega)$ is the absorption coefficient. In this limit we write $\mathbb{G}(\mathbf{k}, \omega) \rightarrow \overline{\mathbb{G}}(\mathbf{k}, \omega)$, where

$$\overline{\mathbb{G}}(\mathbf{k}, \omega) = -\frac{4\pi i c^2 k^2 (\mathbb{U} - \hat{\mathbf{k}}\hat{\mathbf{k}})}{\omega (\omega^2 n^2(\omega) - c^2 k^2)} - \frac{4\pi i}{\omega} \mathbb{U}$$

Now

$$\begin{aligned} \frac{1}{\omega^2 n^2(\omega) - c^2 k^2} &= \frac{1}{\omega n(\omega) + ck} \frac{1}{\omega n(\omega) - ck} \rightarrow \frac{1}{\omega + ck} \frac{1}{\omega n(\omega) - ck} \\ &= \frac{1}{\omega + ck} \frac{1}{\omega + i \frac{c\mu(\omega)}{2} - ck} = -\frac{1}{c^2} \frac{1}{k + \frac{\omega}{c}} \frac{1}{\left(k - \frac{\omega}{c} - i \frac{\mu(\omega)}{2}\right)} \\ &= -\frac{1}{c^2} \frac{1}{k + \frac{\omega}{c}} \frac{\left(k - \frac{\omega}{c}\right) + i \frac{\mu(\omega)}{2}}{\left(k - \frac{\omega}{c}\right)^2 + \frac{\mu^2(\omega)}{4}} \end{aligned}$$

So

$$\overline{\mathbb{G}}(\mathbf{k}, \omega) = \frac{4\pi i k^2}{\omega} \left(\frac{1}{k + \frac{\omega}{c}} \frac{\left(k - \frac{\omega}{c}\right) + i \frac{\mu(\omega)}{2}}{\left(k - \frac{\omega}{c}\right)^2 + \frac{\mu^2(\omega)}{4}} \right) (\mathbb{U} - \hat{\mathbf{k}}\hat{\mathbf{k}}) - \frac{4\pi i}{\omega} \mathbb{U}$$

Thus we can write

$$\overline{\mathbb{G}}(\mathbf{k}, \omega) = \overline{\mathbb{G}}_R(\mathbf{k}, \omega) + i\overline{\mathbb{G}}_I(\mathbf{k}, \omega)$$

where

$$\begin{aligned}\overline{\mathbb{G}}_I(\mathbf{k}, \omega) &= \frac{4\pi k^2}{\omega} \frac{1}{k + \frac{\omega}{c}} \frac{\left(k - \frac{\omega}{c}\right)}{\left(k - \frac{\omega}{c}\right)^2 + \frac{\mu^2(\omega)}{4}} (\mathbb{U} - \hat{\mathbf{k}}\hat{\mathbf{k}}) - \frac{4\pi}{\omega} \mathbb{U} \\ \overline{\mathbb{G}}_R(\mathbf{k}, \omega) &= -\frac{4\pi k^2}{\omega \left(k + \frac{\omega}{c}\right)} \left(\frac{\frac{\mu(\omega)}{2}}{\left(k - \frac{\omega}{c}\right)^2 + \frac{\mu^2(\omega)}{4}} \right) (\mathbb{U} - \hat{\mathbf{k}}\hat{\mathbf{k}}) \\ &= -\frac{4\pi^2 k^2}{\omega \left(k + \frac{\omega}{c}\right)} L(k, \omega) (\mathbb{U} - \hat{\mathbf{k}}\hat{\mathbf{k}})\end{aligned}$$

with

$$L(k, \omega) = \frac{1}{\pi} \frac{\frac{\mu(\omega)}{2}}{\left(k - \frac{\omega}{c}\right)^2 + \frac{\mu^2(\omega)}{4}}$$

Note that

$$\int dk L(k, \omega) = 1$$

where we formally let integral range from $-\text{inf}$ to inf , while if we let $\mu(\omega)/2 \rightarrow \varepsilon$, an infinitesimally small positive number, we have the limit

$$L(k, \omega) \rightarrow \delta\left(k - \frac{\omega}{c}\right)$$

and $\overline{\mathbb{G}}_R(\mathbf{k}, \omega)$ reduces to $\mathbb{G}_R^0(\mathbf{k}, \omega)$, as we would expect.

A.3 Differential rate of photon production

In order to find the differential photon production rate $\frac{dR}{dE_\gamma}(E_\gamma)$ from equation 5.22, which is needed for the calculation of an event rate 5.1, one must evaluate the integral in 5.22 over axion energies E_a while preserving the integral over photon wave-number $k = E_\gamma/\hbar c$. First, we must convert the 3-dimensional delta function of 5.23 in cartesian coordinates to one in spherical coordinates via:

$$\delta^{(3)}\left(\mathbf{k} - \frac{E}{\hbar c} \hat{\mathbf{s}} + \mathbf{G}\right) = \frac{1}{k^2} \delta\left(k - \left|\frac{E}{\hbar c} \hat{\mathbf{s}} - \mathbf{G}\right|\right) \delta(\Omega - \Omega_{\mathbf{G}, \hat{\mathbf{s}}}(E)), \quad (\text{A.10})$$

where the second delta function on the right hand side selects the correct solid angle for the photon wave vector $\mathbf{k}$, and the infinitesimal photon phase space volume was taken as $d^3\mathbf{k} = k^2 dk d\Omega$. The first delta function on the right hand side can be written as a sum of

delta functions over axion energies E :

$$\delta(k - |\frac{E}{\hbar c} \hat{\mathbf{s}} - \mathbf{G}|) = \frac{\hbar c k \Theta((\hat{\mathbf{s}} \cdot \mathbf{G})^2 - G^2 + k^2)}{\sqrt{(\hat{\mathbf{s}} \cdot \mathbf{G})^2 - G^2 + k^2}} [\delta(E - E_+) + \delta(E - E_-)], \quad (\text{A.11})$$

where

$$E_{\pm}(k) = \hbar c(\hat{\mathbf{s}} \cdot \mathbf{G}) \pm \hbar c \sqrt{(\hat{\mathbf{s}} \cdot \mathbf{G})^2 - G^2 + k^2}, \quad (\text{A.12})$$

and $\Theta(x)$ is one when $x \geq 0$ and zero otherwise. We can see from A.12 that if the radicand is zero, $E_{\pm}(k) = \hbar c(\hat{\mathbf{s}} \cdot \mathbf{G})$, and as k increases, $E_+ > \hbar c(\hat{\mathbf{s}} \cdot \mathbf{G})$ increases and $E_- < \hbar c(\hat{\mathbf{s}} \cdot \mathbf{G})$ decreases. Unless $\frac{\hbar c |\mathbf{G}|^2}{2(\hat{\mathbf{s}} \cdot \mathbf{G})} = \hbar c(\hat{\mathbf{s}} \cdot \mathbf{G})$, only one of the solutions A.12 could satisfy the delta function on the right hand side of 5.24.

After evaluating the integral of equation 5.22 over $d\Omega$, the total energy conversion rate becomes:

$$\frac{d\mathbb{E}}{dt} = \Gamma \sum_{\mathbf{G}} \int_0^\infty \int_0^\infty \frac{k^2}{\hbar c k + E} \mathcal{M}(E, k, \mathbf{G}, \hat{\mathbf{s}}) \delta(k - |\frac{E_a}{\hbar c} \hat{\mathbf{s}} - \mathbf{G}|) dE_a dk, \quad (\text{A.13})$$

for some constant Γ and $\mathcal{M}(E, k, \mathbf{G}, \hat{\mathbf{s}}) \equiv \frac{d\phi}{dE_a}(E_a) L(k, E_a) (\mathcal{E}_{\mathbf{G}}^* \times \hat{\mathbf{s}}) \left(\mathbb{U} - \hat{\mathbf{k}}_{\mathbf{G}, \hat{\mathbf{s}}} \hat{\mathbf{k}}_{\mathbf{G}, \hat{\mathbf{s}}} \right) (\mathcal{E}_{\mathbf{G}} \times \hat{\mathbf{s}})$ for convenience. We then evaluate the inner integral in A.13 using A.11, and we get:

$$\frac{d\mathcal{E}}{dt} = \Gamma \sum_{\mathbf{G}} \int_{G \sin \theta}^\infty \left[\frac{\mathcal{M}(E_+) \Theta(E_+)}{1 + (E_+ / \hbar c k)} + \frac{\mathcal{M}(E_-) \Theta(E_-)}{1 + (E_- / \hbar c k)} \right] \frac{k^2}{\sqrt{(\hat{\mathbf{s}} \cdot \mathbf{G})^2 - G^2 + k^2}} dk. \quad (\text{A.14})$$

Since $dk = dE_\gamma / \hbar c$, the differential conversion rate can be read off as:

$$\frac{dR}{dE_\gamma} = \frac{\Gamma}{\hbar c} \sum_{\mathbf{G}} \Theta((\hat{\mathbf{s}} \cdot \mathbf{G})^2 - G^2 + k^2) \left[\frac{\mathcal{M}(E_+) \Theta(E_+)}{\hbar c k + E_+} + \frac{\mathcal{M}(E_-) \Theta(E_-)}{\hbar c k + E_-} \right] \frac{k^2}{\sqrt{(\hat{\mathbf{s}} \cdot \mathbf{G})^2 - G^2 + k^2}}. \quad (\text{A.15})$$

Does this reduce to the result in [236] if we take the Lorentzian L to be a delta function?

Yes. Let $g(k) = k - \frac{E_-(k)}{\hbar c}$ and $h(k) = k - \frac{E_+(k)}{\hbar c}$. Then using $\delta(g/h(k))$, we may replace each instance of $E_{\pm}$ by $\hbar ck$, using the identity $\int_0^{\infty} f(x)\delta(x-x_0)dx = \int_0^{\infty} f(x_0)\delta(x-x_0)dx$ for positive x_0 . We can now factor the other factors of $\mathcal{M}$ (other than L , which are now the delta functions) out of the square brackets of A.15, along with $\Theta(\hbar ck)$ and $1/2\hbar ck$. Next, we note that $\delta(g(k)) = \frac{\delta(k-k_0)\Theta((\hat{\mathbf{s}} \cdot \mathbf{G})-k_0)}{|dg/dk(k_0)|}$ and $\delta(h(k)) = \frac{\delta(k-k_0)\Theta(k_0-(\hat{\mathbf{s}} \cdot \mathbf{G}))}{|dh/dk(k_0)|}$, where $k_0 = G^2/2(\hat{\mathbf{s}} \cdot \mathbf{G})$. Equation A.15 then becomes:

$$\begin{aligned} \frac{dR}{dE_{\gamma}} = \frac{\Gamma}{\hbar c} \sum_{\mathbf{G}} \Theta((\hat{\mathbf{s}} \cdot \mathbf{G})^2 - G^2 + k^2) \Theta(\hbar ck) \frac{d\phi}{dE_a}(\hbar ck) \left| \frac{F_a^0(\mathbf{G})S(\mathbf{G})}{k_0^2 v_c} \right|^2 G^2 \sin^2 \theta \frac{\delta(k-k_0)}{2\hbar ck} \\ \times \left[\frac{\Theta((\hat{\mathbf{s}} \cdot \mathbf{G}) - k_0)}{|dg/dk(k_0)|} + \frac{\Theta(k_0 - (\hat{\mathbf{s}} \cdot \mathbf{G}))}{|dh/dk(k_0)|} \right] \frac{k^2}{\sqrt{(\hat{\mathbf{s}} \cdot \mathbf{G})^2 - G^2 + k^2}}. \end{aligned} \quad (\text{A.16})$$

The delta function sets all k in equation A.16 to k_0 , $(\hat{\mathbf{s}} \cdot \mathbf{G})^2 - G^2 + k_0^2 = (k_0 - (\hat{\mathbf{s}} \cdot \mathbf{G}))^2 \geq 0$ means $\Theta((\hat{\mathbf{s}} \cdot \mathbf{G})^2 - G^2 + k_0^2) = 1$. One can confirm that:

$$\left[\frac{\Theta((\hat{\mathbf{s}} \cdot \mathbf{G}) - k_0)}{|dg/dk(k_0)|} + \frac{\Theta(k_0 - (\hat{\mathbf{s}} \cdot \mathbf{G}))}{|dh/dk(k_0)|} \right] = \frac{2k_0 \sqrt{(\hat{\mathbf{s}} \cdot \mathbf{G})^2 - G^2 + k_0^2}}{G^2}, \quad (\text{A.17})$$

and equation A.16 reduces to:

$$\frac{dR}{dE_{\gamma}} = \frac{\Gamma}{\hbar^2 c^2} \sum_{\mathbf{G}} \frac{\sin^2(2\theta)}{G^2} \frac{d\phi}{dE_a}(\hbar ck_0) \Theta(\hbar ck_0) \frac{|S(\mathbf{G})F_a^0(\mathbf{G})|^2}{v_c^2} \delta(k-k_0), \quad (\text{A.18})$$

which is of the same form as the conversion rate from Cebrian et al.

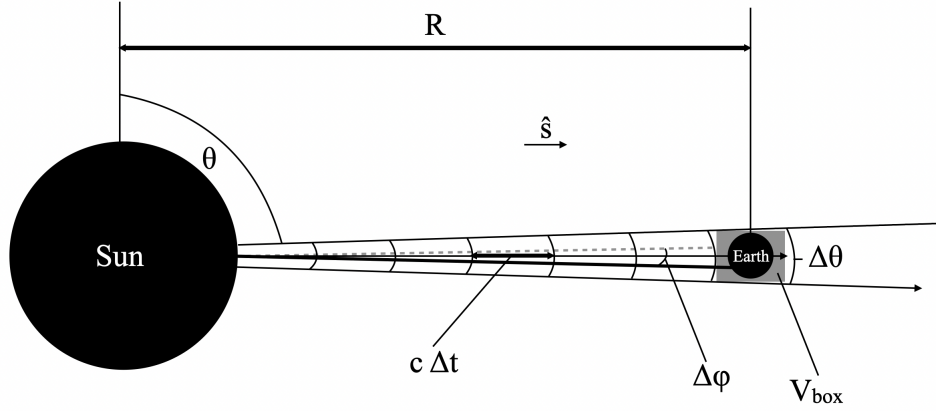


Figure A.1: The Sun-Earth system. Sun is shown emitting solar axions.

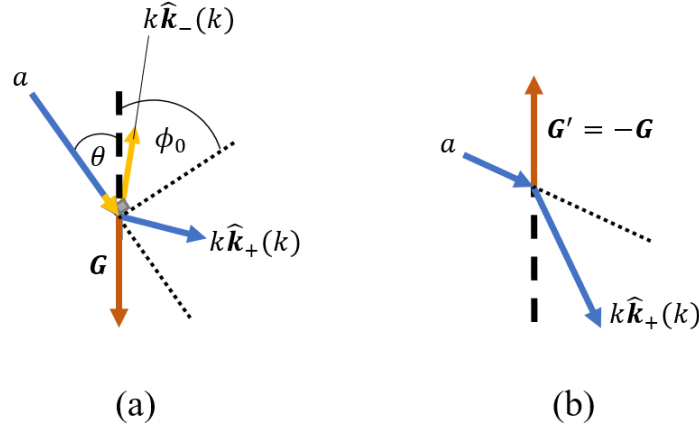


Figure A.2: (a): $(\hat{\mathbf{s}} \cdot \mathbf{G}) > 0$ case. Two possible directions for final state photons $\hat{\mathbf{k}}_{\pm}(k) = \frac{(E_{\pm}(k)/\hbar c)\hat{\mathbf{s}} - \mathbf{G}}{|(E_{\pm}(k)/\hbar c)\hat{\mathbf{s}} - \mathbf{G}|}$ are shown, given photon wavevector magnitude k , reciprocal lattice vector $\mathbf{G}$, and incident axion a of direction $\hat{\mathbf{s}}$. The incident axion with energy $E_{+}(k)$ is represented by the blue arrow, and the axion with energy $E_{-}(k)$ is the smaller yellow arrow in the same direction. (b): $(\hat{\mathbf{s}} \cdot \mathbf{G}) < 0$ case for a different axion. It can be seen that in this case, $\Theta(E_{-}) = 0$ in A.14 for all k , so we only have the E_{+} term contributing to the rate.