Non thermal Activity in Clusters of Galaxies

Vahe' Petrosian

CSSA and KIPAC

Stanford University

With: Greg Madejski

Graduate students: Wel Liu, Yanwei Jiang,

Undergraduate students: Kevin Luli, and William East,

OUTLINE

1. Observations: General

2. Radiation Mechanisms

3. Acceleration Processes

1. OBSERVATIONS

A. DIFFUSE RADIO HALOS: since 1977

• *Schlickeiser et al.* 1987: COMA cluster: Synchrotron emission by electrons with Lorentz factors in the range

$$10^4(\mu G/B_\perp)^{1/2} < \langle\gamma_{radio}\rangle < 10^5(\mu G/B_\perp)^{1/2}. \quad (1)$$

• *Giovannini and Feretti 2000*: A Survey of NVSS Clusters. *Relic and Halo* emission in about 30 clusters.

• Rate of occurence and luminosity of diffuse radio emission increases with redshift and Soft X-ray luminosity (or temperature) of the cluster.

B. EXTREME UV

• *Lieu et al.* 1996: Observed by *EUVE* in the **0.07 to 0.4 keV** range. Coma, YES; Othe clusters, MAYBE.

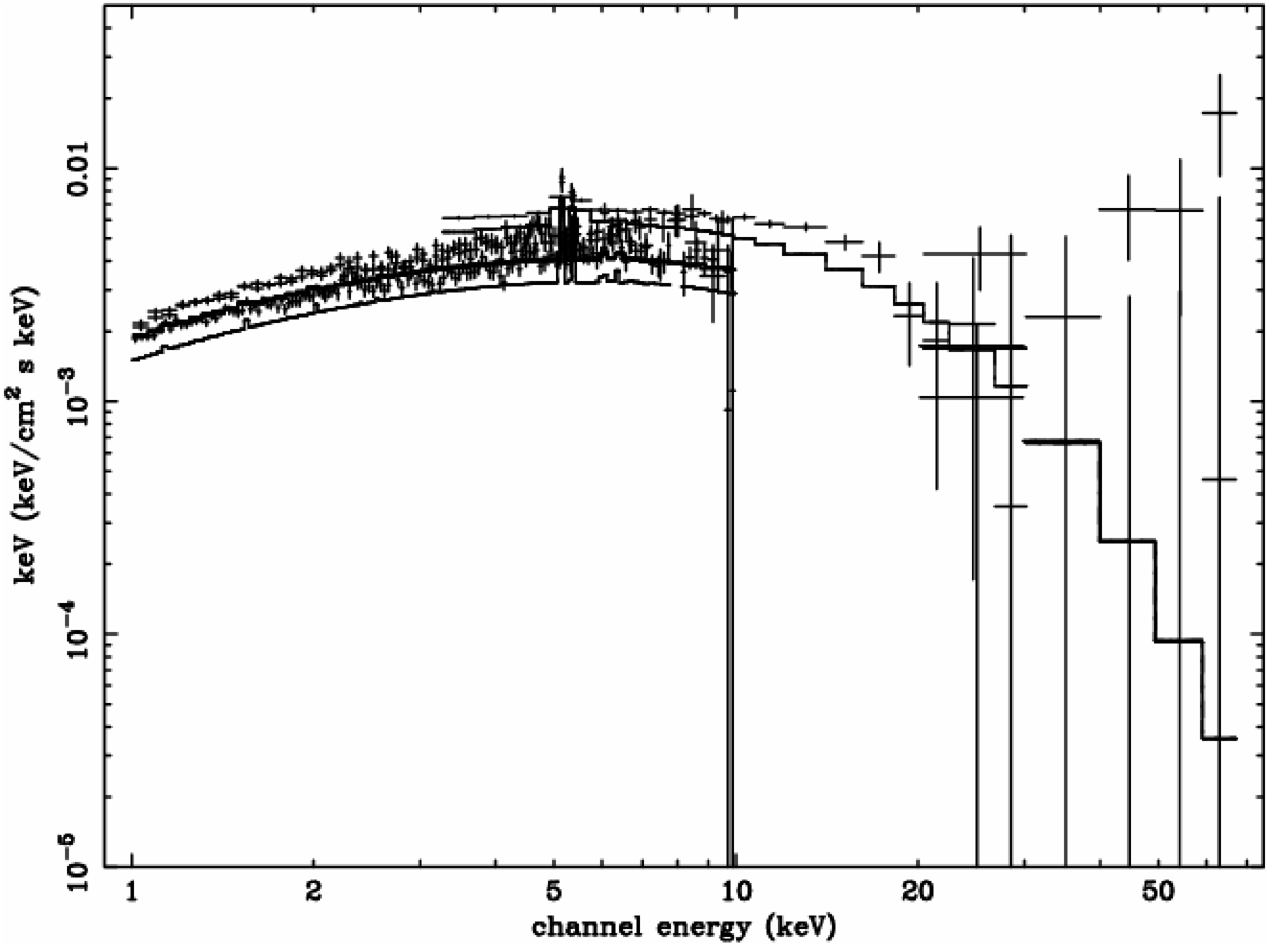
• Emisson Process: Cooler Thermal $kT \sim 2$ **keV** or Inverse Compton of CMB photons by electrons with $300 < \gamma < 750$.

B. HARD X-RAYS

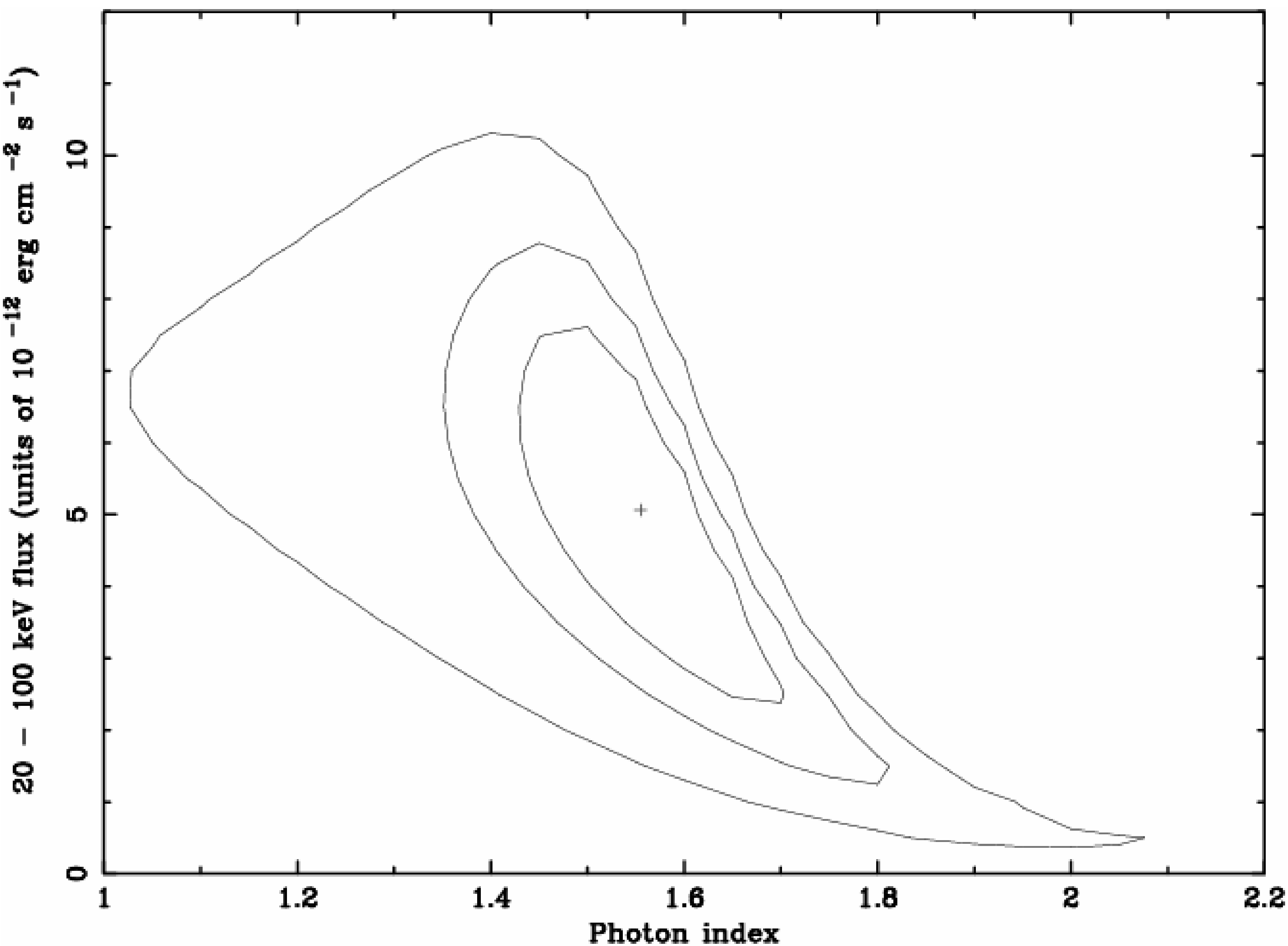
• *Fusco-Femiano et al.* 1999; *BeppoSAX* and *Rephaeli et al.* 1999: *RXTE*. Detection of power law tails in the hard X-ray (**20 to 80 keV**) range from Coma.

• Emisson Process: Nonthermal Bremsstrahlung by electrons with $10 < E <$ 100 keV or Inverse Compton by electrons with $5000 < \gamma < 10^4$.

RXTE Observations of Bullet Cluster



Final, corrected version of the Figure will appear in ApJ Dec. 1, 2006 issue Petrosian, Madejski & Luli 2006)



20 – 100 keV flux (units of 10 $^{-12}$ erg cm $^{-2}$ s $^{-1}$)
10
5
0
1
1.2
1.4
1.6
1.8
2
2.2
Photon index

Table 2

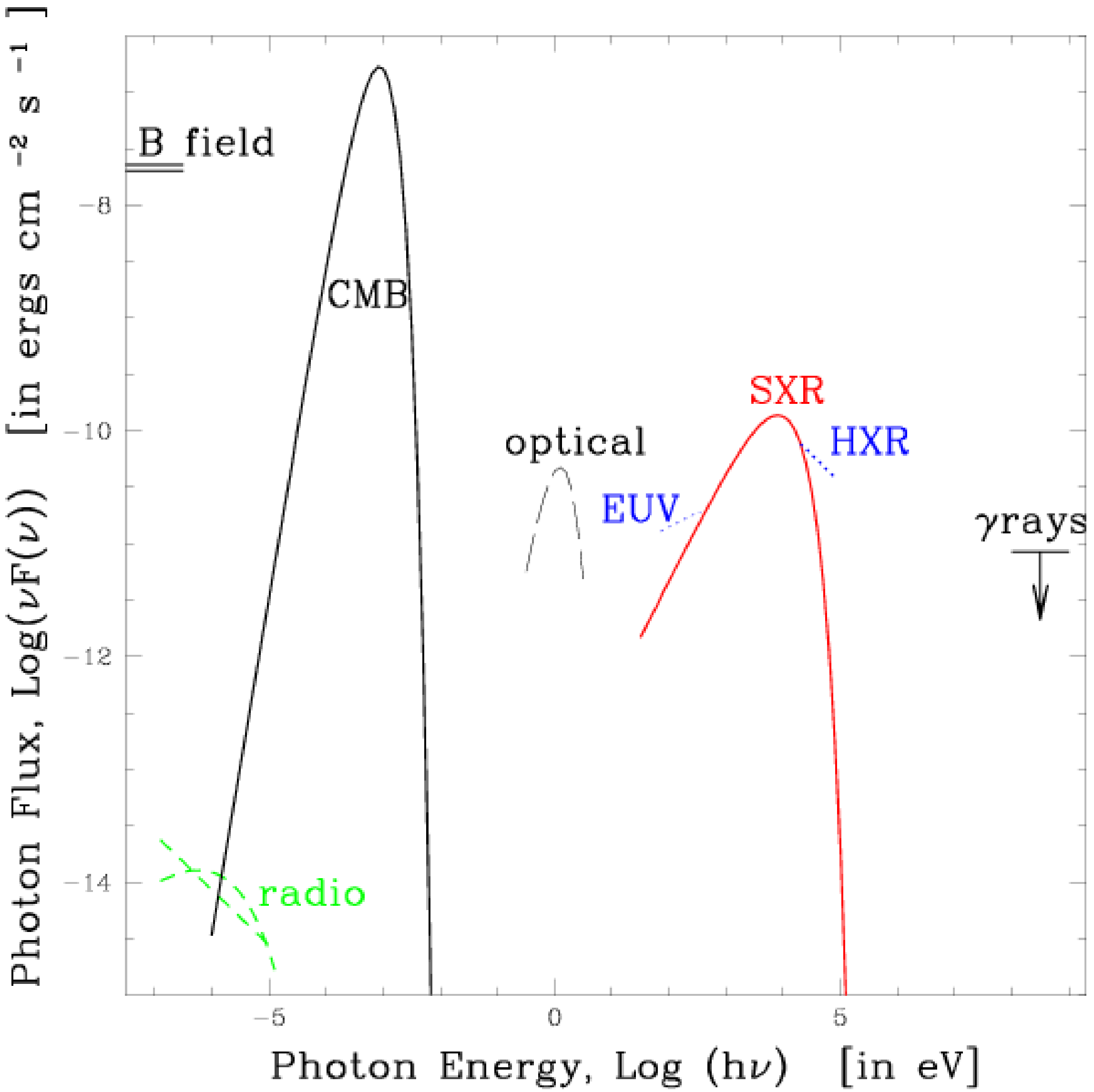
PARAMETERS FROM SPECTRAL FITTINGS

Data Set	Parameter	Single Thermal	Double-Thermal	Thermal+Power Law
RXTE	$(n_H/10^{20}\ \mathrm{cm}^{-2})$	4.6^f	–	4.6^f
	kT_1 (keV)	12.1 ± 0.4	–	11.7 ± 0.5
	kT_2 (keV)	–	–	–
	Abundance (Solar)	0.16 ± 0.04	–	0.25 ± 0.08
	Photon Index	–	–	> 2
	F_{20keV}^{100keV}/F_0	–	–	0.3 ± 0.2
	χ^2/dof	114/98	–	102/96
RXTE and XMM	$(n_H/10^{20}\ cm^{-2})$	2.8 ± 1.0	4.6^f	4.6^f
	kT_1 (keV)	12.1 ± 0.2	10.1 ± 0.9	11.2 ± 0.8
	kT_2 (keV)	–	50 (> 30)	–
	Abundance (Solar)	0.19 ± 0.03	0.19 ± 0.03	0.22 ± 0.04
	Photon Index	–	–	1.6 ± 0.2
	F_{20keV}^{100keV}/F_0	–	0.5 ± 0.3	0.5 ± 0.3
	χ^2/dof	1483/1508	1471/1506	1464/1506

$F_0 = 10^{-11}\ \mathrm{erg\ cm^{-2}\ s^{-1}}$

^{f} denotes parameter fixed at the given value

Electromagnetic Energy Spectrum in Coma



Hard X-Ray Emission Processes

1. Nonthermal Bremsstrahlung Emission

Inefficient compared to Coulomb losses.

For Cold PLasma:

$$Y_{\rm brem} = (4/3\pi)(\alpha/\ln\Lambda)E_{in} = 7.7 \times 10^{-5} E_{in} < 3 \times 10^{-6}, \qquad (1)$$

$$\mathcal{E}_{input} \sim 10^{48} \text{ ergs/s}. \qquad (2)$$

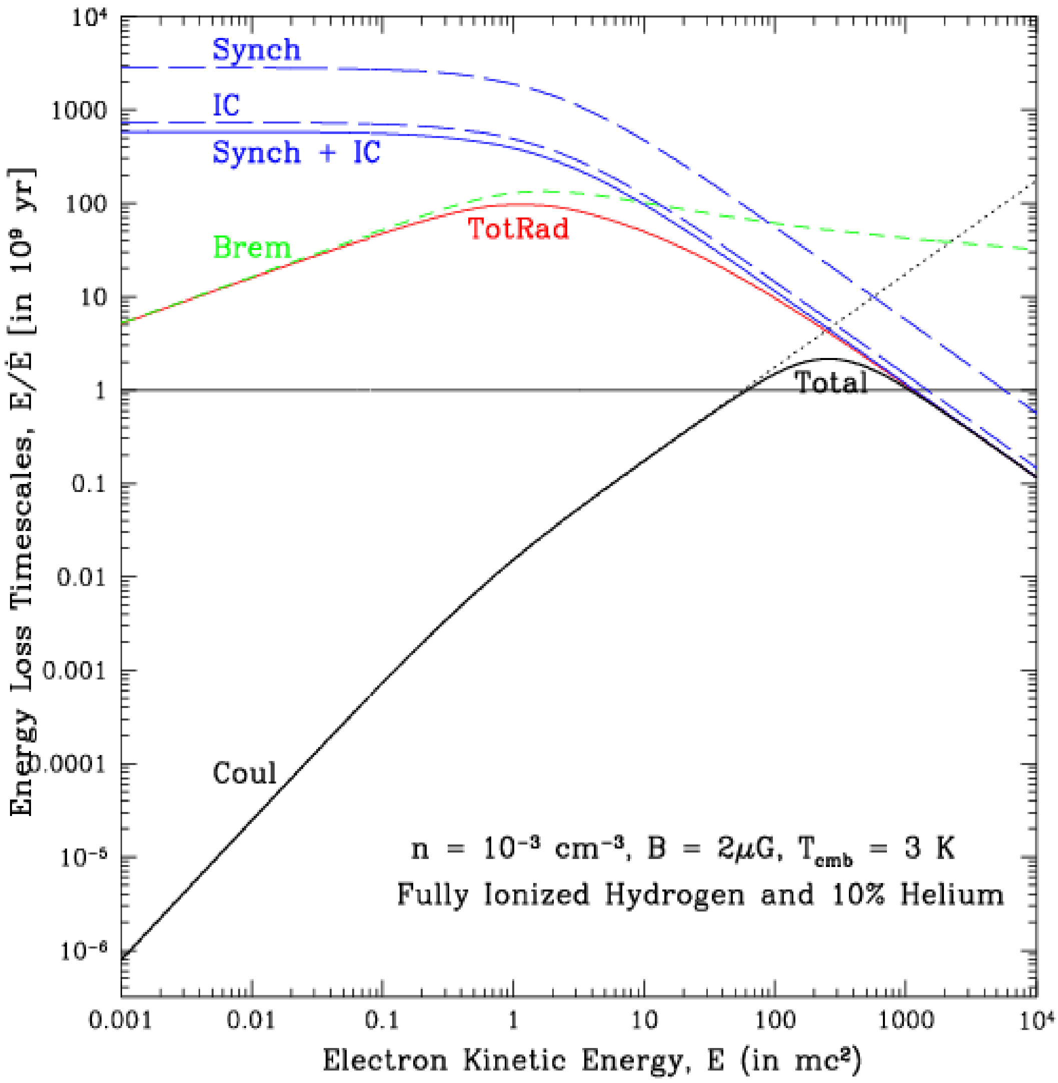
Therefore, *NTB emission as source of HXRs is not tenable, unless it is a short-lived phenomenon.*

$$DURATION < 10^{8} \text{ yr}. \qquad (3)$$

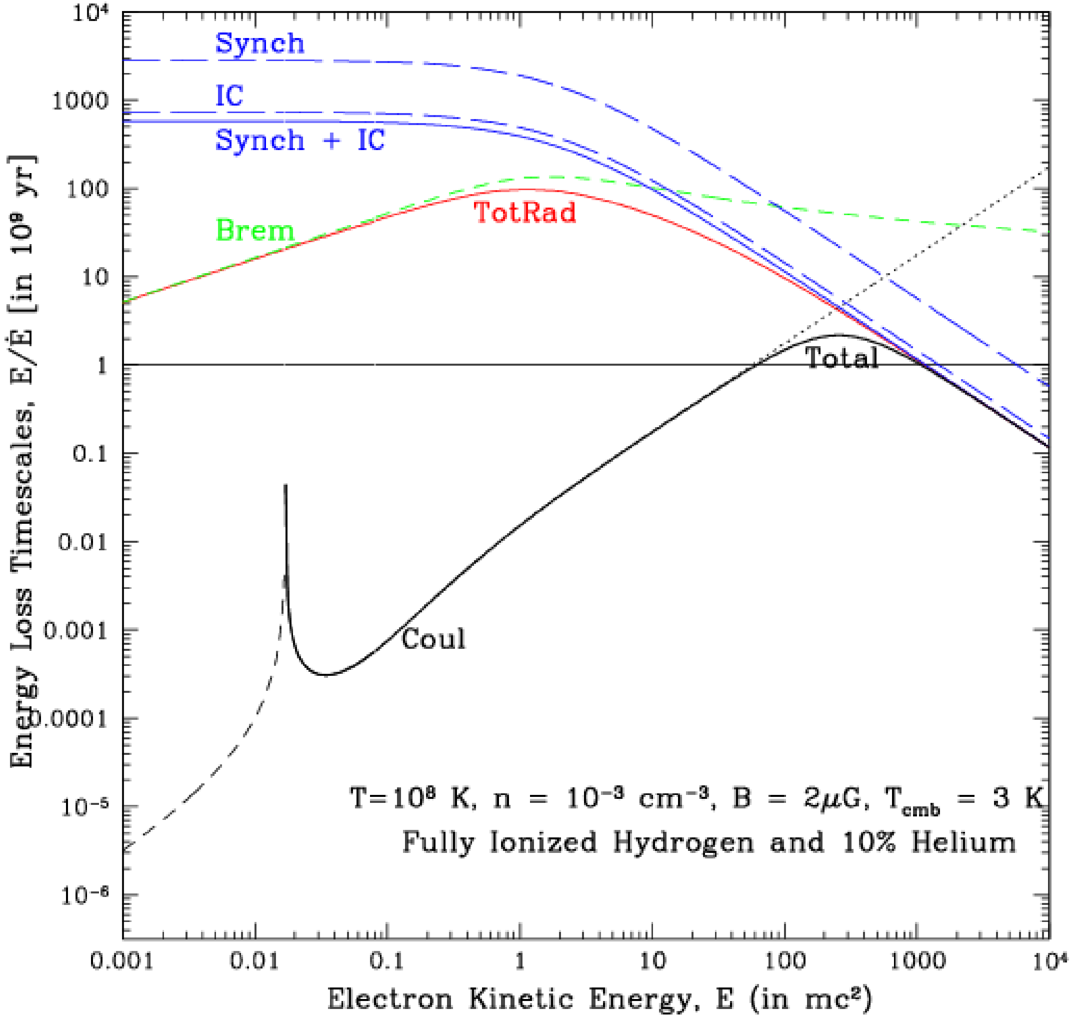
For Hot Plasma: Good approximation for $E > kT$
but YIELD is higher by factor of 2 to 3 for $E \sim (2\,\text{to}\,3) \times kT$ and

$$DURATION \sim (\text{Few}) \times 10^{8} \text{ yr}. \qquad (4)$$

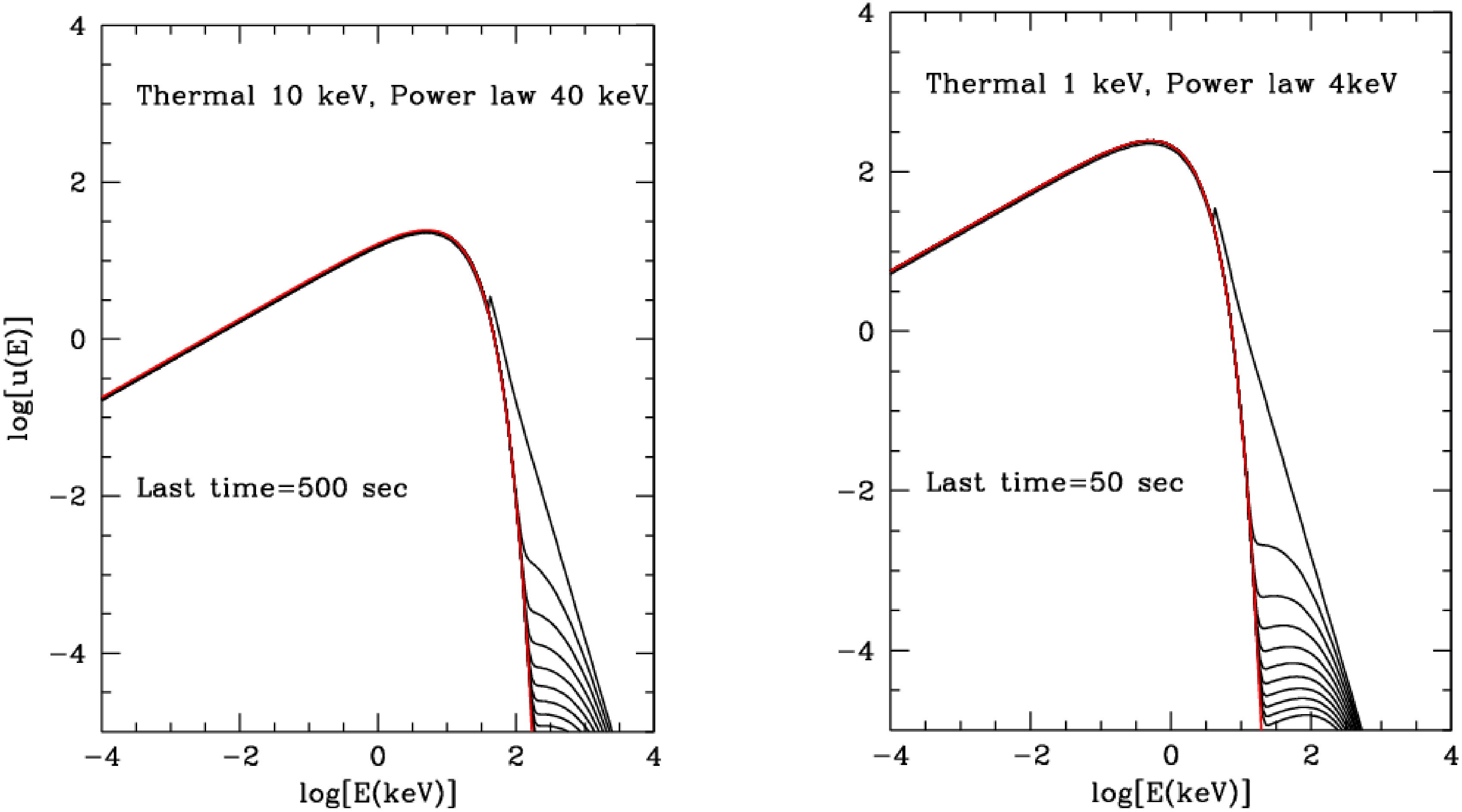
Energy Loss Timescale: Cold Plasma



Timescales For Hot Plasma



Thermalization Time: POWER LAW TAIL



$$\tau_{relax} = 1.5 \times 10^{8}\ \mathrm{yr}\,(10^{-3}/n)(kT/20\,\mathrm{keV})^{3/2}$$

Hard X-Ray Emission Processes

2. Inverse Compton Emission

Inefficient compared to synchrotron: For a simple power law:

$N(E) \propto E^{-p}$,

$$R = \frac{f_{\rm IC}}{f_{\rm synch}} \propto \left(\frac{B_\perp}{\mu{\rm G}}\right)^{-(p+1)/2} g(p). \tag{1}$$

For Coma the magnetic field is

$$B_\perp = 0.18\mu{\rm G},\ p = 3; \quad B_\perp = 0.5\mu{\rm G},\ p = 5. \tag{2}$$

On the other hand, for

$$N(E) = N_0(E/E_{\rm cr})^{-p}{\rm exp}\{-E/E_{\rm cr}\}; \quad B_\perp \simeq 1\mu{\rm G}, \quad E_{\rm cr} \sim 10^4 \tag{3}$$

Pitch Angle Distribution

- **Isotopic Distribution:**

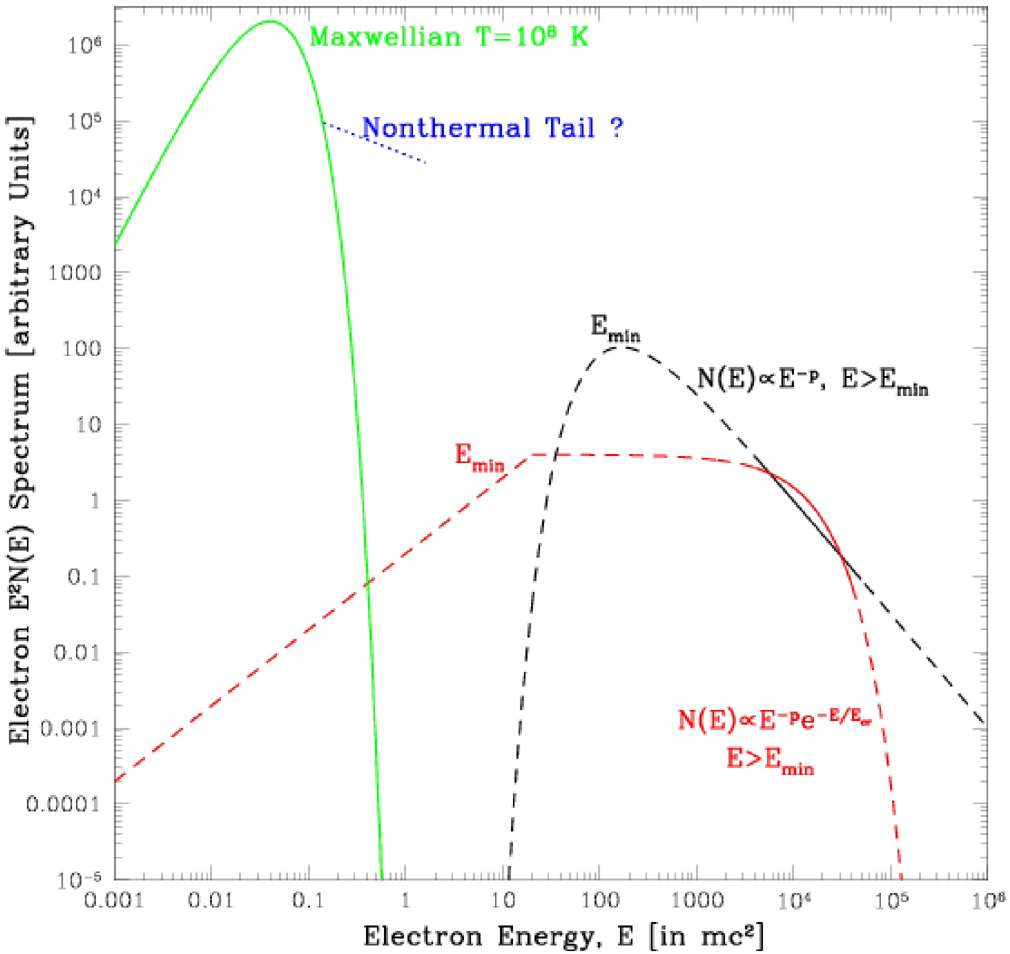
$$B = B_\perp \times < \sin\psi >^{(p+1)/2} \sim 2B_\perp. \tag{4}$$

- **Anisotropic Distributions:**

Small Pitch Angle Emission. If mean angle $\psi_0 \ll 1$, then

$$B \propto \psi_0^{-s}, \quad s > 1. \tag{5}$$

The Required Electron Spectrum



Inverse Compton Hard X-Ray Flux?

Basic Requirements

Diffuse Radio Emission: $F_{HXR} \propto F_{radio}(\nu)$

High Temerature or L_{SXR}: $F_{HXR} \propto f_1(T)$

High Redshift; $CMB \propto (1+z)^4$**:** $F_{HXR} \propto f_2(z)$

Turbulence; Substructure and Merger: $F_{HXR} \propto f_3(W_{turb})$

Equipartition: B **Field and Electrons,** $N \propto \gamma^{-p}$

$$B^2/8\pi = \zeta \mathcal{E}_e \tag{1}$$

$$F_{\rm synch} \propto B_\perp^\alpha g(p)/[r(z)^2(1+z)^\alpha] \tag{2}$$

$$R = \frac{F_{\rm IC}}{F_{\rm synch}} \propto B_\perp^{-\alpha}(1+z)^{(\alpha+2)} g(p). \tag{3}$$

For Example: $p = 3,\ \alpha = (1+p)/2 = 2$

$$F_{HXR} \propto \left(\frac{\nu\zeta}{F_{radio}(\nu)}\right)^{1/2} R^{3/2}(1+z)^3/(r(z))^2, \tag{4}$$

$$B_\perp \propto \left(\frac{\nu F_{radio}(\nu)}{\zeta}\right)^{1/4} R^{-3/4}(1+z)(r(z))^{1/2}, \tag{5}$$

where $r(z)$ is the comoving metric distance.

Table 1

OBSERVED AND ESTIMATED PROPERTIES OF CLUSTERS

Cluster	z	kT^a keV	$F^b_{1.4\rm GHz}$ mJy	$\theta^{c,b}$ arcmin	$F_{\rm SXR}$ F_0^f	B^d μG	$F^e_{\rm HXR}$ F_0^f
Coma	0.023	7.9	52	30	33	0.40	1.4(2.0)
A 2256	0.058	7.5	400	12	5.1	1.1	1.8(1.0)
1E0657-56	0.296	15.6	78	5	3.9	1.2	0.52(0.5)
A 2219	0.226	12.4	81	8	2.4	0.86	1.0
MACSJ0717	0.550	13	220	3	3.5	2.6	0.76
A 2163	0.208	13.8	55	6	3.3	0.97	0.51
A 2744	0.308	11.0	38	5	0.76	1.0	0.41
A 1914	0.171	10.7	50	4	1.8	1.3	0.22

[a] From Allen & Fabian (1998), except 1E0657-56 data from Liang et al. (2000)

[b] From Giovannini et al. (1999, 2000), except 1E0657-56 data from Liang et al. (2000)

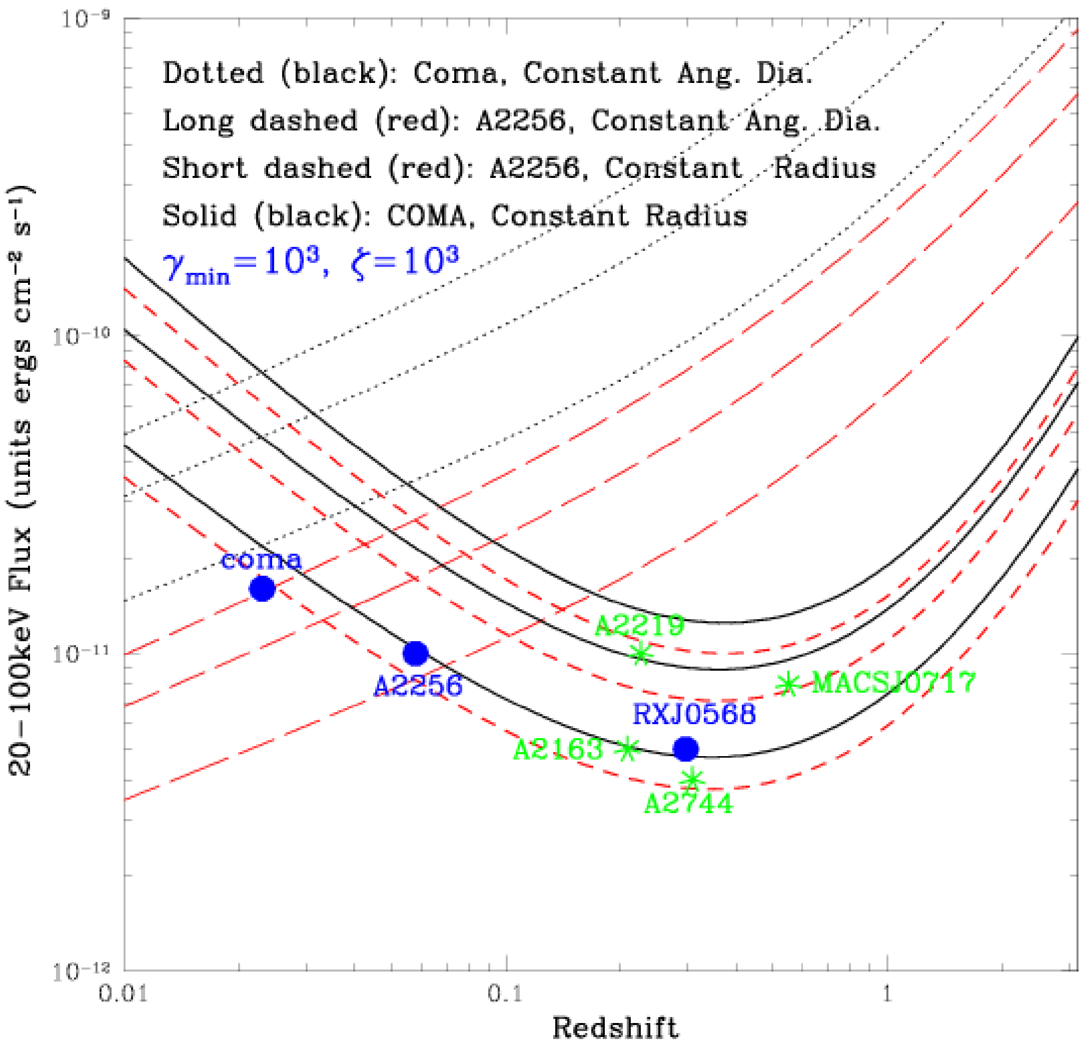
[c] Approximate largest angular extent.

[d] Estimates based on equipartition.

[e] Estimates assuming $\zeta\gamma_{\rm min} = 10^6$, with observed values in parentheses for Coma from Rephaeli et al. (1999; 2002), and Fusco-Femiano et al. (1999; 2004; 2005) and for Abell 2256 by Fusco-Femiano et al. (2000) and Rephaeli & Gruber (2003).

[f] $F_0 = 10^{-11}$ erg cm^{-2} s^{-1}

Predicted Variation of HXR Flux With Redshift



3. ACCELERATION MECHANISMS
GENERAL

A: Electric Fields: **Parallel to B Field**

Unstable leads to TURBULENCE

B: Fermi Acceleration

1. Shock or Flow Divergence: **First Order**

Shocks and Scaterers; i.e. TURBULENCE

2. Stochastic Acceleration: **Second Order**

Scat. and Acceleration by TURBULENCE

TURBULENCE

II. ACCELERATION MECHANISMS

A. ELECTRIC FIELDS: $\mathcal{E}$ (parallel to $\mathbf{B}$ field)

Acceleration Rate: $dp/dt = e\mathcal{E}$

Astrophysical Plasmas Highly Conductive: $\mathcal{E} \rightarrow 0$

Dricer Field: $\mathcal{E}_D = kT/(e\lambda_{\rm Coul})$

$\mathcal{E} < \mathcal{E}_D$: Energy Gain $\Delta E < kT(L/\lambda_{\rm Coul})$

For Clusters of Galaxies:

$kT \sim 10\,{\rm keV}\,, L \sim 1\,{\rm Mpc}\,, n \sim 10^{-3}\,{\rm cm}^{-3}$

$\lambda_{\rm Coul} \sim 0.03\,{\rm Mpc}$, $\Delta E \sim 300\,{\rm keV}$.

$\mathcal{E} > \mathcal{E}_D$: Runaway Unstable Distribution Leads to

PLASMA TURBULENCE

II. ACCELERATION MECHANISMS

B. FERMI ACCELERATION

Random scattering by moving scattering centers.

Diffusive Process: Why Acceleration?

More headon than trailing scatterings

Phase space availibility

$$\frac{1}{p^2}\frac{\partial}{\partial p}(p^2 D_{pp}\frac{\partial f}{\partial p}) \rightarrow \frac{\partial}{\partial E}(D(E)\frac{\partial N}{\partial E}) - \frac{\partial}{\partial E}(A(E)N) \quad (1)$$

a. SHOCK ACCELERATION: (First Order Fermi)

Energy Gain: $\dot{p} = \frac{p}{3}\frac{\partial u}{\partial x}$, $\delta p/p \sim U_{\rm shock}/v$

Need Scattering Agent *i.e.* **TURBULENCE**

Diffusive Shocks

Scattering Rate $D_{\rm scat}$, Acceleration Rate $\sim (U_{\rm sh}/v)^2 D_{\rm scat}$

Clusters of Galaxies: Weak shock; Mach number

$M < 2$, and $(U_{\rm sh}/c)^2 \sim (v_s/c)^2 \sim 10^{-5}$

So for accelerating to relativistic energies in

A BILLION YEAR we need scattering rate of

$D_{\rm scat} \sim 10^{-4}{\rm yr}^{-1}$

II. ACCELERATION MECHANISMS

B. FERMI ACCELERATION

b. STOCHASTIC ACCELERATION:

(Second Order Fermi)

Plasma Waves or **TURBULENCE**
Energy Gain; *e.g.* Alfven Waves: $\delta p/p \sim (V_{\rm Alfven}/v)^2$
Scattering Rate $\sim D_{\rm scat}$
Acceleration Rate $\sim D_{pp}/p^2 \sim (V_{\rm Alfven}/v)^2 D_{\rm scat}$

•For $V_{\rm Alfven} > V_{\rm sound}$ TURBULENCE more efficient than SHOCKS

•At low energies or high B fields $D_{pp}/p^2 \gg D_{\rm scat}$ and TURBULENCE efficient accelerator of ELECTRONS. But high densities and/or low B fields are more favotable for acceleration of PROTONS.

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Scat. and Acceleration by TURBULENCE

TURBULENCE

3B. Particle Acceleration
ISOTROPIC AND HOMOGENEOUS

$$\frac{\partial N}{\partial t} = \frac{\partial^2}{\partial E^2}(D_{EE}N) + \frac{\partial}{\partial E}[(\dot{E}_{\mathrm{L}} - A)N] - \frac{N}{T_{\mathrm{esc}}} + Q$$

$$A(E) = \frac{\mathrm{d}D_{EE}}{\mathrm{d}E} + D_{EE}\frac{2\gamma^2 - 1}{(\gamma^2 - 1)\gamma m c^2} \quad + A_{shock}$$

$$T_{\mathrm{esc}} = \frac{L}{\sqrt{2}v}\left(1 + \frac{\sqrt{2}L}{v\tau_{\mathrm{sc}}}\right) \qquad \tau_{\mathrm{sc}} = \frac{1}{2}\int_{-1}^{1} \mathrm{d}\mu \frac{(1-\mu^2)^2}{D_{\mu\mu}}$$

Model Parameters

In principle:

Density	n
Temperature	T
Magnetic Field	B
Scale (geometry)	L
Level of Turbulence	

$(\delta B / B)^2$ *or* $(\delta v / v_{sound})^2$

Kinetic Equation Coefficients

Acceleration rate or time: τ_{ac}

Loss rate or time: τ_{loss}

Escape rate or time: T_{esc}

Characteristic Times:

$$\tau_p^{-1} \propto \Omega_e(\delta B / B)^2 \text{ and } T_{cross} \approx L / \sqrt{2}v$$

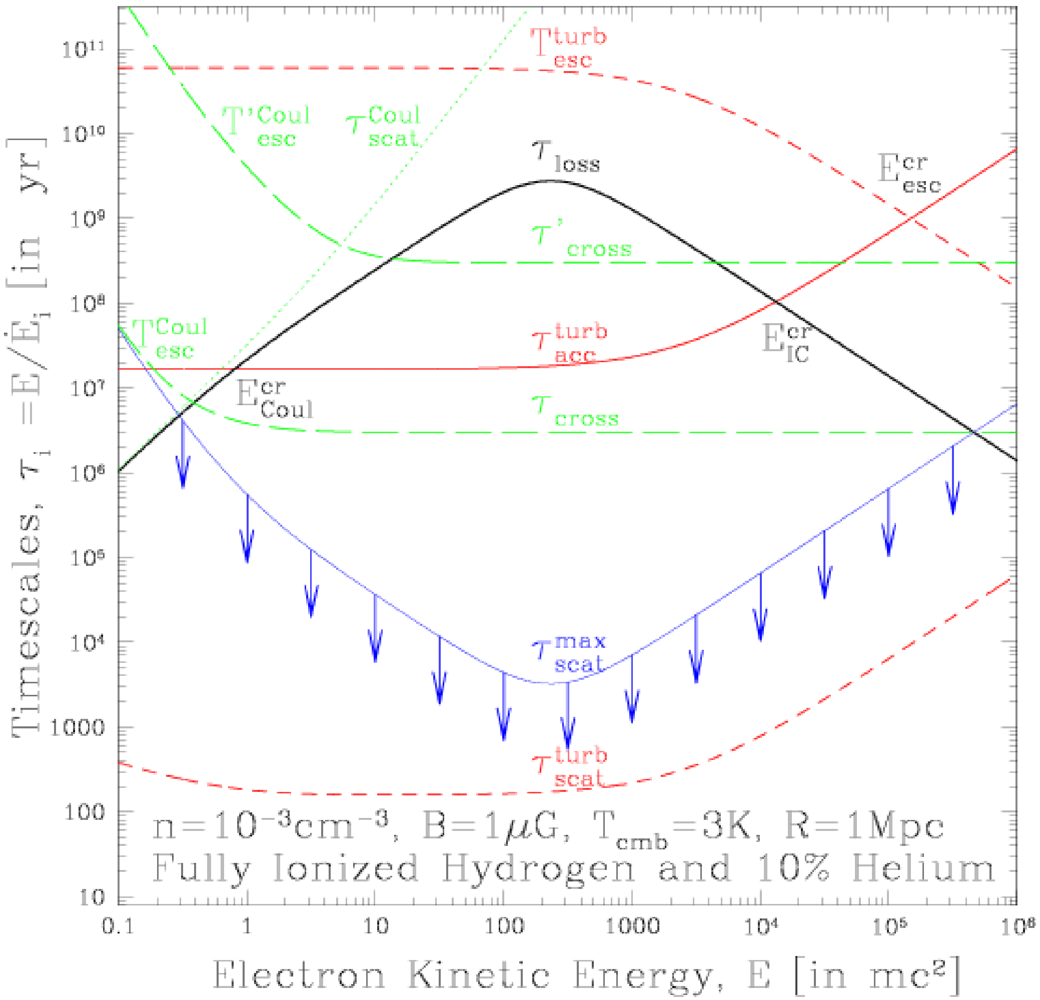
3. ACCELERATION IN CLUSTERS

1. Steady State Acceleration
    a. Background thermal particles
    b. Injected relativistic particles
2. Time Dependent or Episodic
    a. Background thermal particles
    b. Injected Relativistic Particles

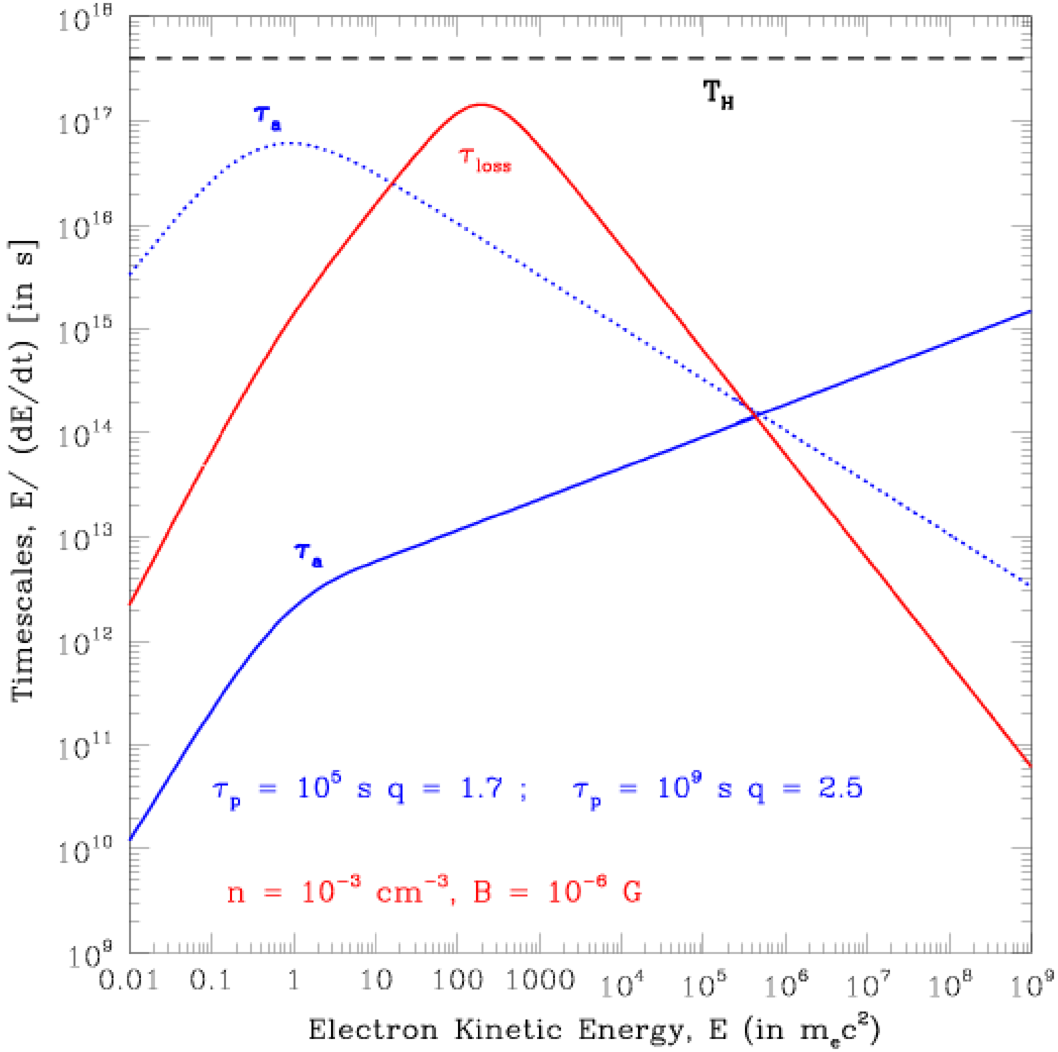
General requirements:

$$T_{esc} > \tau_{loss} \text{ for all } E, \;\; \tau_{\rm acc} < \tau_{\rm loss} \text{ for relevant } E$$

Loss, Scattering, Escape and Acceleration Times



Loss and Acceleration Times: Turbulence



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1. Steady State Acceleration
   - a. Background thermal particles
   - b. Injected relativistic particles
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Acceleration of Thermal Electrons

The Source Term

$$Q(E) = (\sqrt{\pi}/2)n(kT/E)^{3/2}\sqrt{E}e^{-E/kT} \qquad (1)$$

Many Problems

NEED	*OBSERVED*
$\beta_{Alfven} \sim 10^{-2}$	3×10^{-4}
$\lambda_{turb} \sim 10^{9}$ cm	few kpc
$\alpha = \omega_p/\Omega_e \sim 1$	2×10^{2}
$L_{input} \sim 10^{48}$	$< 10^{45}$

- **Acceleration of protons more likely**
- **Too much heating unless shotrtlived**

$$Duration < 10^{8} \text{ yrs.}$$

ELECTRON ACCELERATION

$$\frac{\partial f}{\partial t} = \frac{\partial^2}{\partial E^2}[D(E)f] - \frac{\partial}{\partial E}[(A(E) - |\dot{E}_L|)f] - \frac{f}{T_{esc}(E)} + Q(E,t). \quad (1)$$

Acceleration of Injected NonThermal Electron

Injected Spectrum $Q(E) = Q_0\delta(E - E_0)$

Example: Assume Parametric Forms

$$D(E) = \mathcal{D}E^{q'}, A(E) = a\mathcal{D}E^{q'-1}, \text{ and } T_{esc} = E^s/(\theta\mathcal{D}) \quad (2)$$

Special case of $s = 2 - q'$:

$$N(E) \propto Q_0 \begin{cases} (E/E_0)^{a-x+\sqrt{(x^2+\theta)}} & \text{if } E < E_0, \\ (E/E_0)^{a-x-\sqrt{(x^2+\theta)}} & \text{if } E > E_0, \end{cases} \quad (3)$$

$$x = (a - 1 + q')/2.$$

But we need

$$\theta \sim \tau_{ac}/T_{esc} \ll 1$$

So

$$N(E) \propto Q_0 \begin{cases} (E/E_0)^{a} & \text{if } E < E_0, \\ (E/E_0)^{-q'+1} & \text{if } E > E_0, \end{cases} \quad (4)$$

For $p = 3$ we need $q' = 4$!. For $q' < 2$, $p < 1$.
Too flat. Predicts HXR/EUV=200 while observed value is < 2.

ELECTRON ACCELERATION

$$\frac{\partial f}{\partial t} = \frac{\partial^2}{\partial E^2}[D(E)f] - \frac{\partial}{\partial E}[(A(E) - |\dot{E}_L|)f] - \frac{f}{T_{esc}(E)} + Q(E,t).$$

TIME DEPENDENT MODELS

$$Q(E,t) = Q(E)\delta(t - t_0)$$

1. Transport Effects

$D = A = 0$; T_{esc} and $\dot{E}_L$ constants in time.

$$f(E,t) = \exp\{-t/T_{esc}\}Q(E'(E,t))\dot{E}_L(E'(E,t))/\dot{E}_L(E),$$

$E'(E,t) = \tau^{inv}(\tau(E) - t)$ and τ^{inv} is the inverse function of

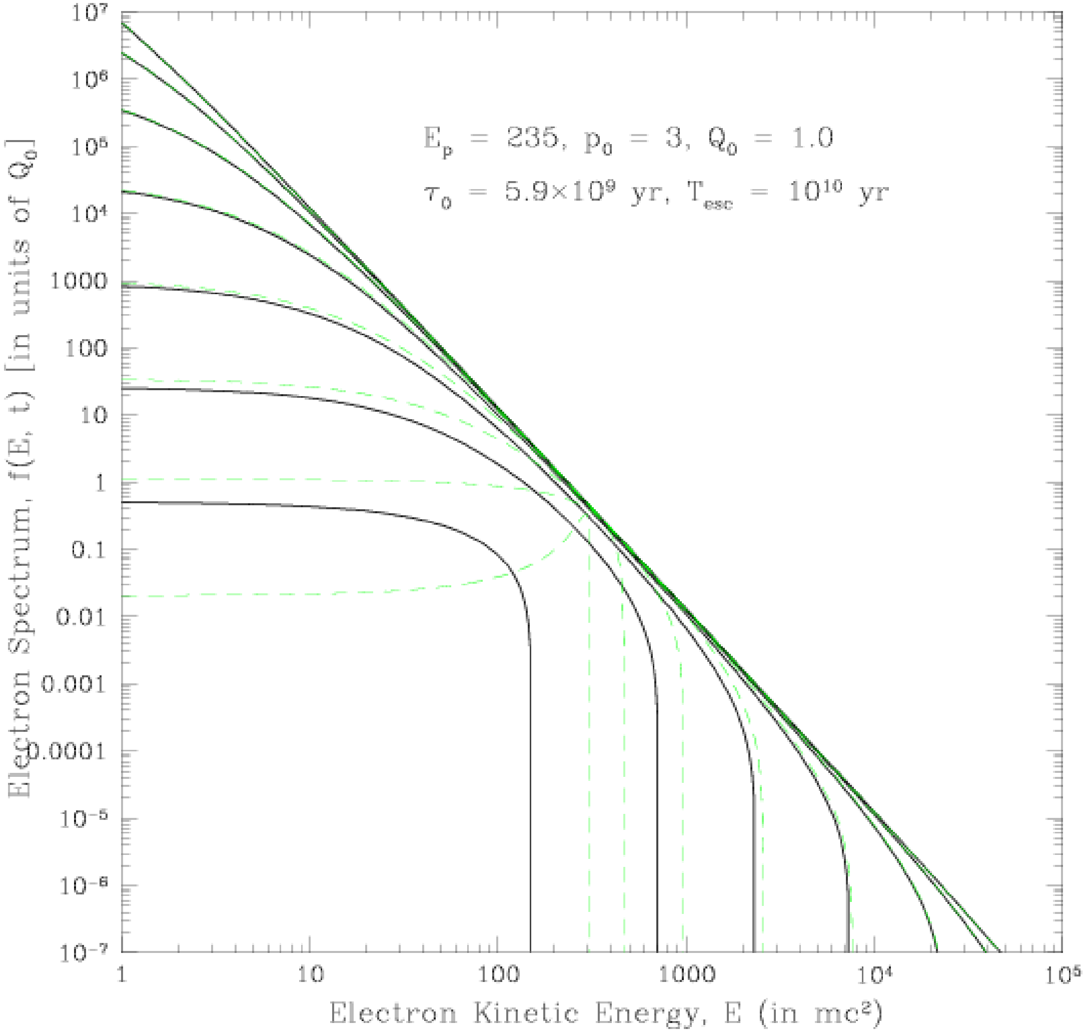
$$\tau(E) = \int_E^\infty dE/\dot{E}_L(E) = \pi/2 - \tan^{-1}(E/E_p), \quad \tau^{inv}(x) = \coth x,$$

$$E'/E_p = (E/E_p + \tan(t/\tau_0))/(1 - (E/E_p)\tan(t/\tau_0)).$$

For power law injection: $Q(E) = Q_0(E/E - p)^{-p_0}$, $p_0 > 2$

$$f(E,t) = \exp\{-t/T_{esc}\}Q_0\frac{[1 - (E/E_p)\tan(t/\tau_0)]^{p_0 - 2}}{\cos^2(t/\tau_0)[E/E_p + \tan(t/\tau_0)]^{p_0}}.$$

Spectral Evolution of Injected Power-law: Loss Only



ELECTRON ACCELERATION

$$\frac{\partial f}{\partial t} = \frac{\partial^2}{\partial E^2}[D(E)f] - \frac{\partial}{\partial E}[(A(E) - |\dot{E}_L|)f] - \frac{f}{T_{\rm esc}(E)} + Q(E,t).$$

TIME DEPENDENT MODELS

2. Acceleration Plus Transport

No Diffusion: $D(E) = 0$

For power law injection: $Q(E) = Q_0(E/E-p)^{-p_0}, \;\; p_0 > 2$

$$\dot{E}_L(E)/E_p = (1 + (E/E_p)^2 - b(E/E_p)^{q'-1})/\tau_0,$$

$$b = a\mathcal{D}\tau_0 E_p^{q'} = \tau_0/\tau_{\rm ac}(E_p) \sim 10^2 \text{ or } 1$$

For shock and stochastic acceleration, respectively.

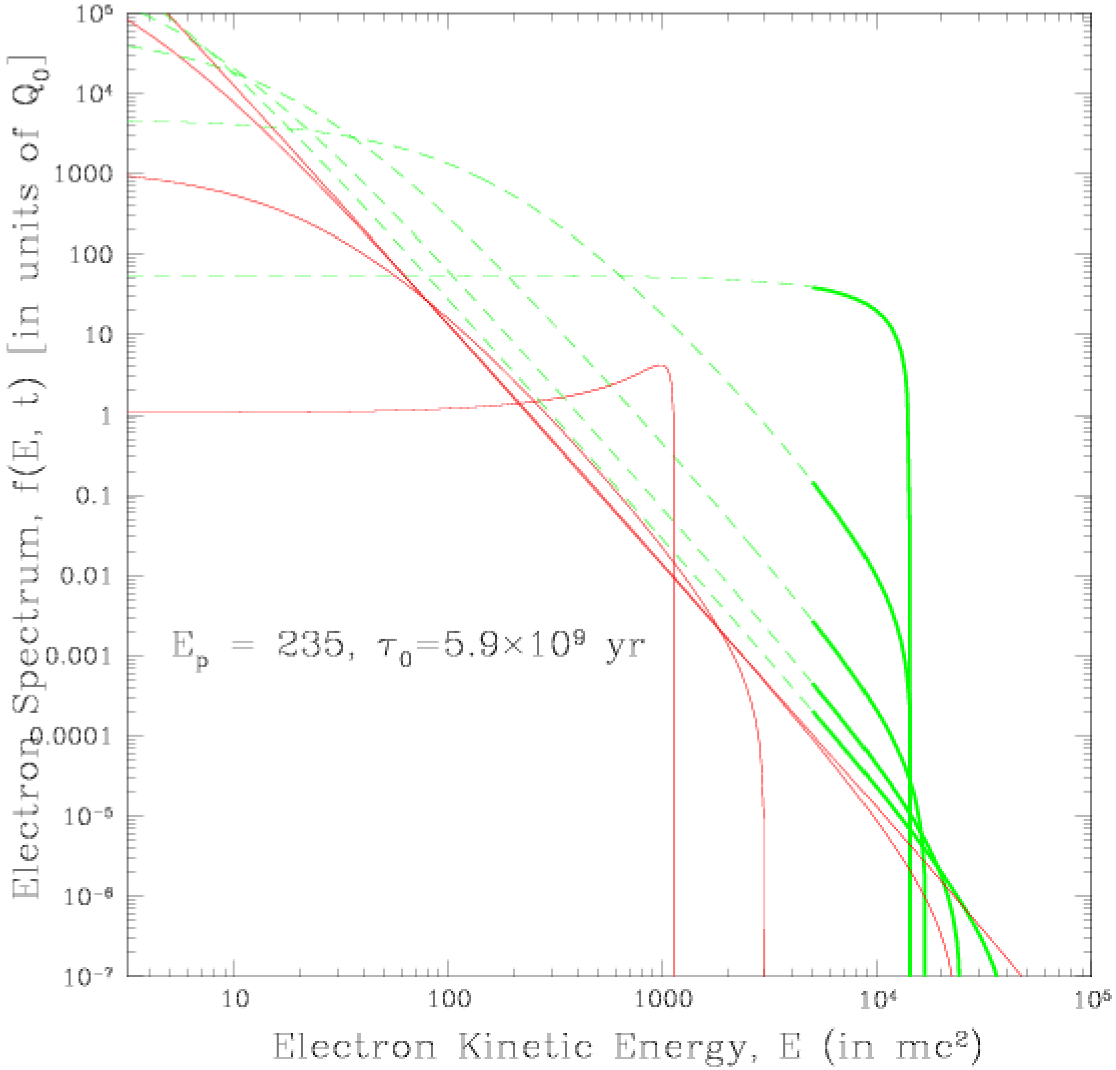
For $q' = 2$

$$f(E,t) = \exp\{-t/T_{\rm esc}\}Q_0 \frac{[T_+ - (E/E_p)\tan(\delta t/\tau_0)/\delta]^{p_0-2}}{\cos^2(\delta t/\tau_0)[T_-(E/E_p) + \tan(\delta t/\tau_0)/\delta]^{p_0}},$$

$$\delta^2 = 1 - b^2/4 \text{ and } T_\pm = 1 \pm b\tan(\delta t/\tau_0)/(2\delta).$$

For $\delta = 0$ or $b = 2$

$$f(E,t) = \exp\{-t/T_{\rm esc}\}Q_0 \frac{[1-(E/E_p-1)t/\tau_0]^{p_0-2}}{[E/E_p-(E/E_p-1)t/\tau_0]^{p_0}}.$$



Electron Spectrum, f(E, t) [in units of Q_0]
Electron Kinetic Energy, E (in mc^2)
E_p = 235, τ_0=5.9×10^9 yr
10^5
10^4
1000
100
10
1
0.1
0.01
0.001
0.0001
10^{-5}
10^{-6}
10^{-7}
10
100
1000
10^4
10^5

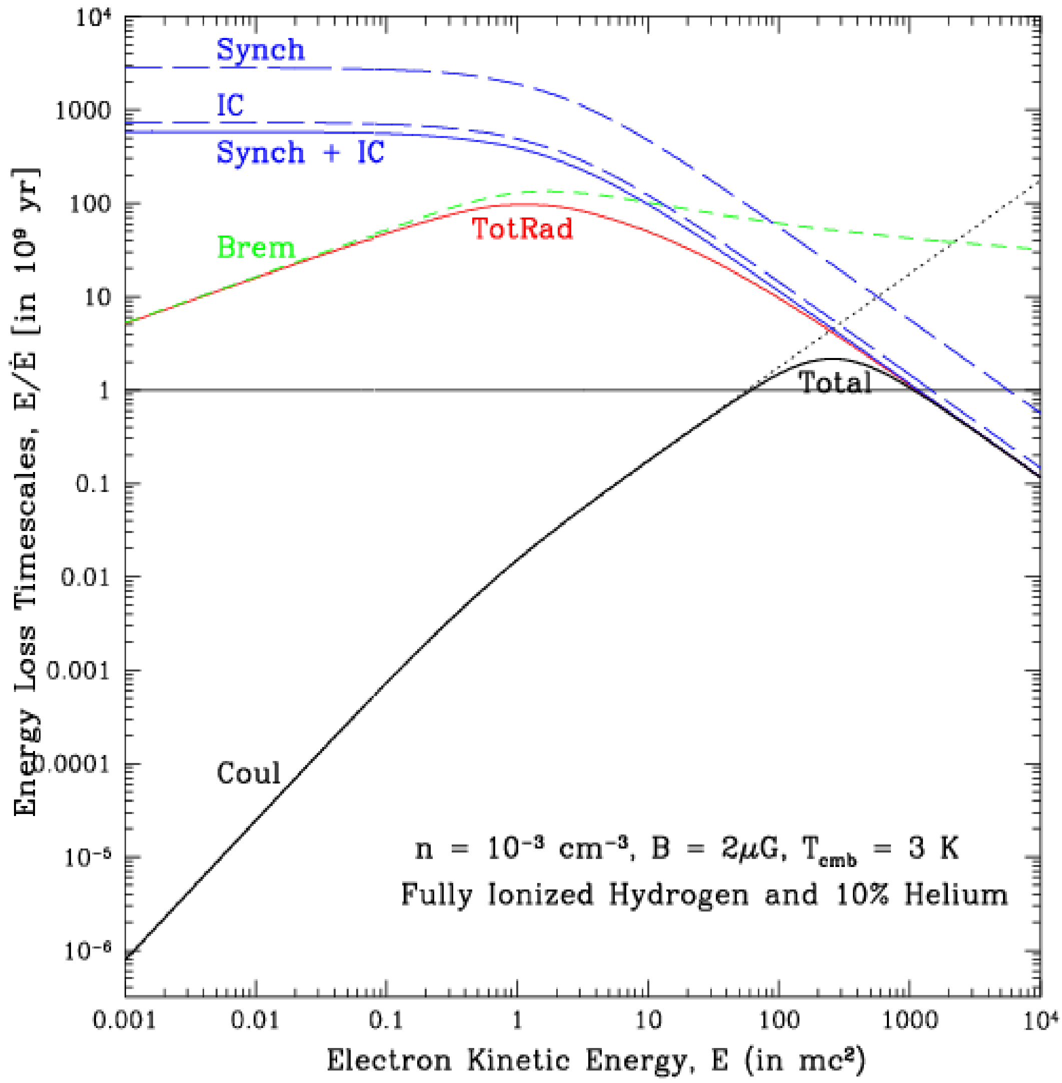
GAMMA-RAY EMISSION: GLAST

Mechanisms

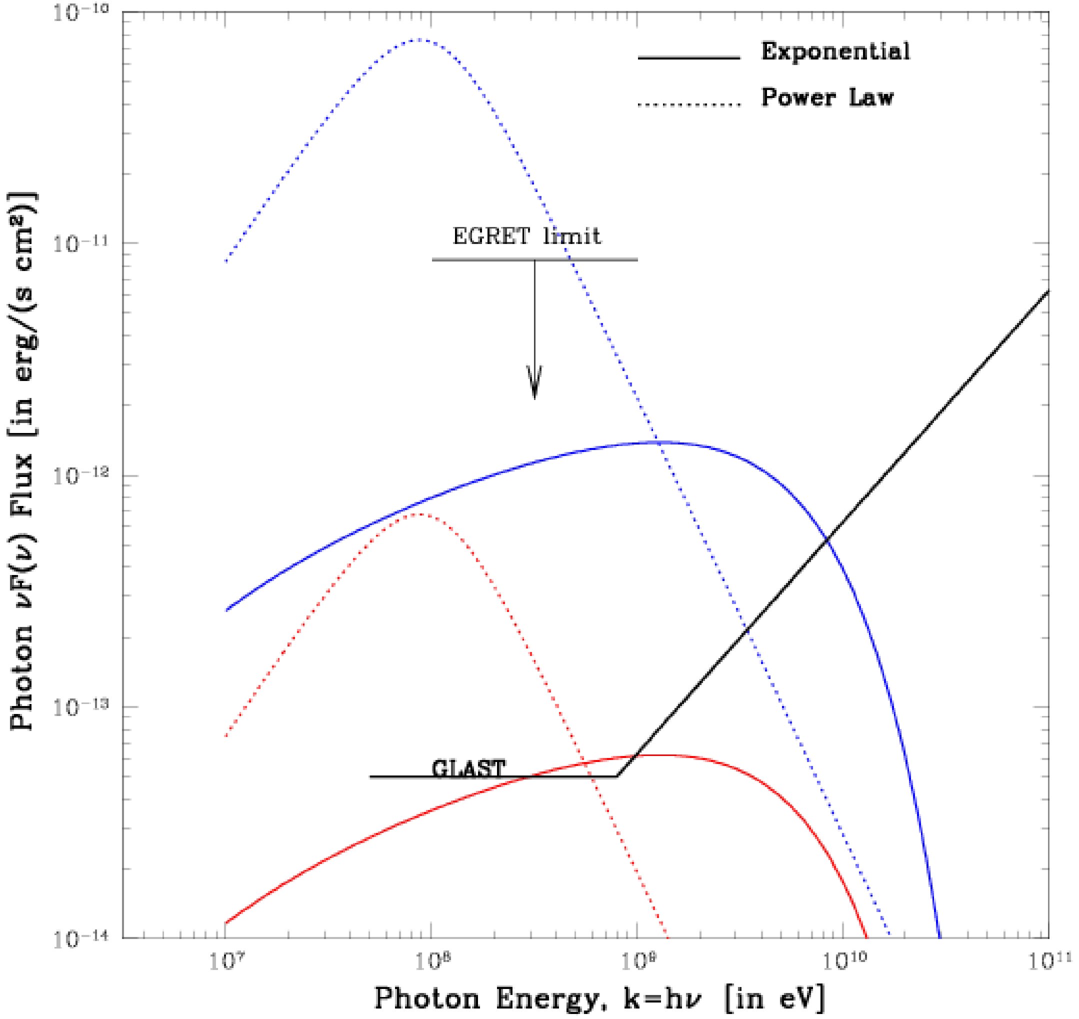
1. Non-Thermal Bremsstrahlung

2. Inverse Compton of Infrared-Optical Photons (Klein-Nishina)

Energy Loss Timescale: Cold Plasma



Gamma-ray Emission: Bremsstrahlung



SUMMARY and CONCLUSIONS

Radio and Hard X-ray(?) Observations
indicate that there are relativistic electrons in several clusters.

This Can Be Explained by

episodic acceleration of injected relativistic electrons by turbulence and shocks

GLAST (and more hard X-ray) Observations
can constrain the radiative and acceleration mechanisms

2. PLASMA TURBULENCE AND STOCHASTIC ACCELERATION

1. Generation $R_e = LV/\nu >> 1,$ $R_m = LV/\eta >> 1$

2. PLASMA TURBULENCE AND STOCHASTIC ACCELERATION

1. Generation $R_e = LV/\nu >>> 1,$ $R_m = LV/\eta >>> 1$

2. Cascade: *Nonlinear wave-wave int.*

$$\omega(k_1) + \omega(k_2) = \omega(k_3); \quad k_1 + k_2 = k_3$$

2. TURBULENCE CASCADE

HD: Large eddies breaking into small ones

Eddy turnover or *cascade* time $\tau_{cas} \approx 1/kv(k) < L/V_{sound}$

MHD: Nonlinear wave-wave interactions

$$\omega(k_1) = \omega(k_2) + \omega(k_3); \quad k_1 = k_2 + k_3$$

$$\tau_{cas} \leq L/V_{Alfven} \quad \text{OR} \quad \tau_{cas} \leq L/V_{Sound}$$

Dispersion Relation*:* *(For Low and High Beta Plasmas)*

For Alfven, Fast and Slow Modes

$$\omega(k) = k_{\parallel}V_{Alfven},\ kV_{Alfven},\ k_{\parallel}V_{Sound},\ \text{For } V_{Alfven} > V_{sound}$$

$$\omega(k) = k_{\parallel}V_{Alfven},\ kV_{Sound},\ k_{\parallel}V_{Alfven},\ \text{For } V_{Alfven} < V_{sound}$$

2. PLASMA TURBULENCE AND STOCHASTIC ACCELERATION

1. Generation $R_e = LV/\nu >>> 1, \quad R_m = LV/\eta >>> 1$

2. Cascade: *Nonlinear wave-wave int.*

$$\omega(k_1) + \omega(k_2) = \omega(k_3); \quad k_1 + k_2 = k_3$$

3. Interactions with Particles: *Resonant int.*

$$\omega = k_{\|} v \mu + n\Omega_i/\gamma$$

3. Wave-Particle Interactions

- Dominated by Resonant Interactions

$$D_{ij} = \pi e^2 \sum_{n=-\infty}^{+\infty} \int d^3k \langle d_{ij} \rangle \delta\left(\boldsymbol{k} \cdot \boldsymbol{v} - \omega + \frac{n\eta_0}{\gamma} \Omega_0 \right),$$

- Lower energy particles interacting with higher wavevectors or frequencies

2. PLASMA TURBULENCE AND STOCHASTIC ACCELERATION

1. Generation $R_e = LV/\nu >>> 1,\quad R_m = LV/\eta >>> 1$

2. Cascade: *Nonlinear wave-wave int.*

$$\omega(k_1) + \omega(k_2) = \omega(k_3);\quad k_1 + k_2 = k_3$$

3. Interactions with Particles: *Resonant int.*

$$\omega = k_{\|} v\mu + n\Omega_i/\gamma$$

A. Damping of Waves

B. Acceleration of Particles

Dispersion Relation for the Waves
(Propagating Along Field Lines)

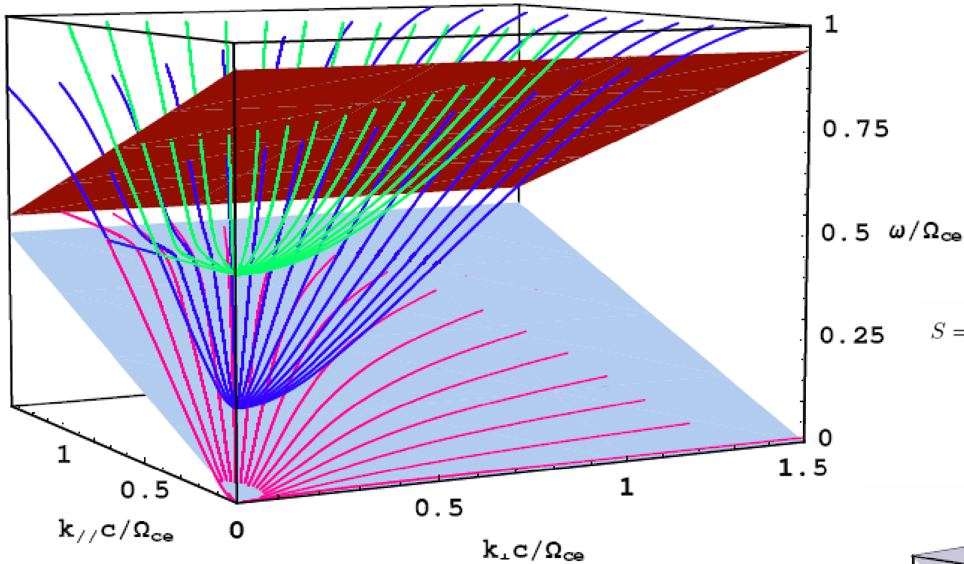
$$(ck)^2 = \omega^2 \left[1 - \sum_i \frac{\omega_{pi}^2}{\omega(\omega - q_i/|q_i|\Omega_i)} \right].$$

Plasma Parameter:

$$\alpha = \frac{\omega_{pe}}{\Omega_e} = 1.0 \left(\frac{n}{10^9 \mathrm{cm}^{-3}} \right)^{1/2} \left(\frac{B_0}{100\mathrm{G}} \right)^{-1}$$

Abundances: Electrons, protons and alpha particles

General Dispersion Relation



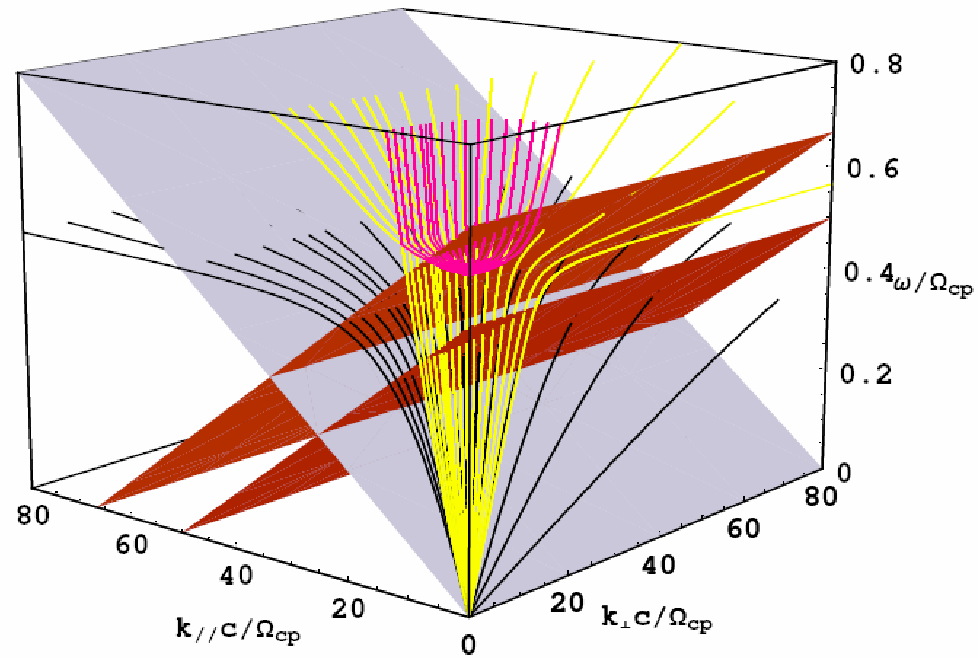
$$\tan^2\theta = \frac{-P(n^2-R)(n^2-L)}{(Sn^2-RL)(n^2-P)}$$

$$S=\frac{1}{2}(R+L),\qquad D=\frac{1}{2}(R-L),\qquad P=1-\sum_i\frac{\omega_{pi}^2}{\omega^2},\qquad R=1-\sum_i\frac{\omega_{pi}^2}{\omega^2}\left(\frac{\omega}{\omega+\epsilon_i\Omega_i}\right),$$

$$L=1-\sum_i\frac{\omega_{pi}^2}{\omega^2}\left(\frac{\omega}{\omega-\epsilon_i\Omega_i}\right),\qquad \omega_{pi}^2=\frac{4\pi n_i q_i^2}{m_i},\qquad \Omega_i=\frac{|q_i|B}{m_i c},\qquad \epsilon_i=\frac{q_i}{|q_i|}$$

Resonance Condition

$$\omega = k_\parallel v\mu + n\Omega_i/\gamma$$



3A. TURBULENCE DAMPING

Viscous or Collisional Damping: $l = k^{-1} >> \lambda_{Coul}$

Collisonless Damping: $k^{-1} << \lambda_{Coul}$

Thermal: *Heating of Plasma*

Nonthermal*: Particle Acceleration*

$$\text{Turbulence is damped} \quad \text{for} \quad k > k_{\max}$$

$$\text{where} \quad \tau_{damp}(\propto k^{-1}) = \tau_{cas}(\propto k^{-1/2})$$

$$\textit{Inertial Range} \quad k_{\min} < k < k_{\max}$$

Damping Rate: *Fast Mode*

General Non-thermal Rate

$$\Gamma_{nonth}(\mathbf{k}) = \frac{\pi}{8}\frac{\Omega^2 m}{n m_p c k \eta}\left(1-\frac{\beta_A^2}{\eta^2}\right)\int_{E_0}^{\infty} dE N(E)\Theta(E-E_c)/(\beta\gamma)$$

$$\left[2J_1^2(x) + xJ_1(x)\left(J_0(x)-J_2(x)\right)\right], \quad x \equiv \beta\gamma c k \Omega^{-1}\sqrt{1-\eta^2}\sqrt{1-\beta_A^2/(\beta\eta)^2}.$$

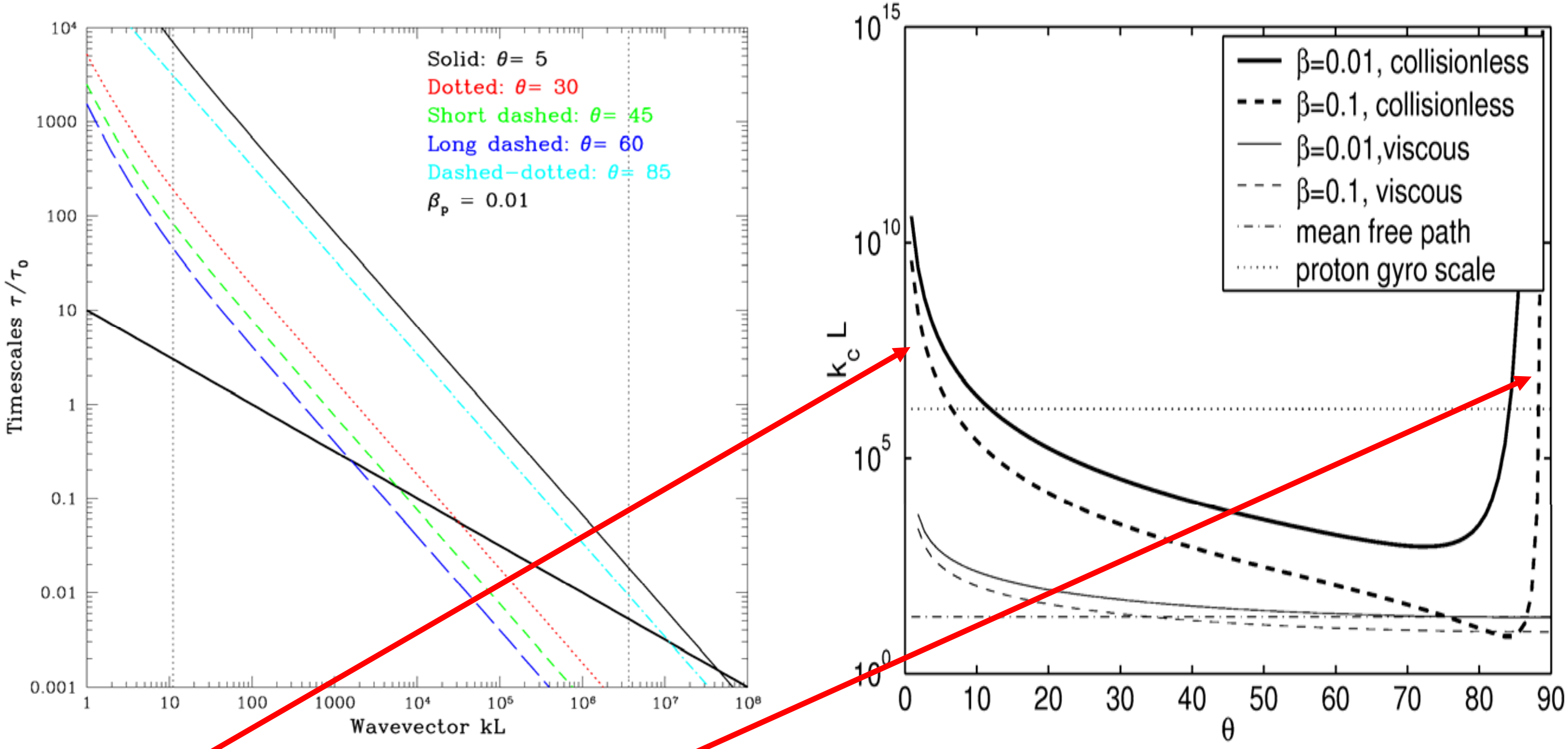
Non-relativistic Limit:

$$\Gamma_{nonth}(\mathbf{k}) = \frac{\pi}{8}\frac{\delta}{n\beta_A}\frac{kL}{\tau_0}\left(1-\frac{\beta_A^2}{\eta^2}\right)\left(\frac{1-\eta^2}{\eta}\right)\int_{E_m}^{\infty} dE N(E)\beta\gamma\left(1-\frac{\beta_A^2}{\beta^2\eta^2}\right);$$

Thermal:

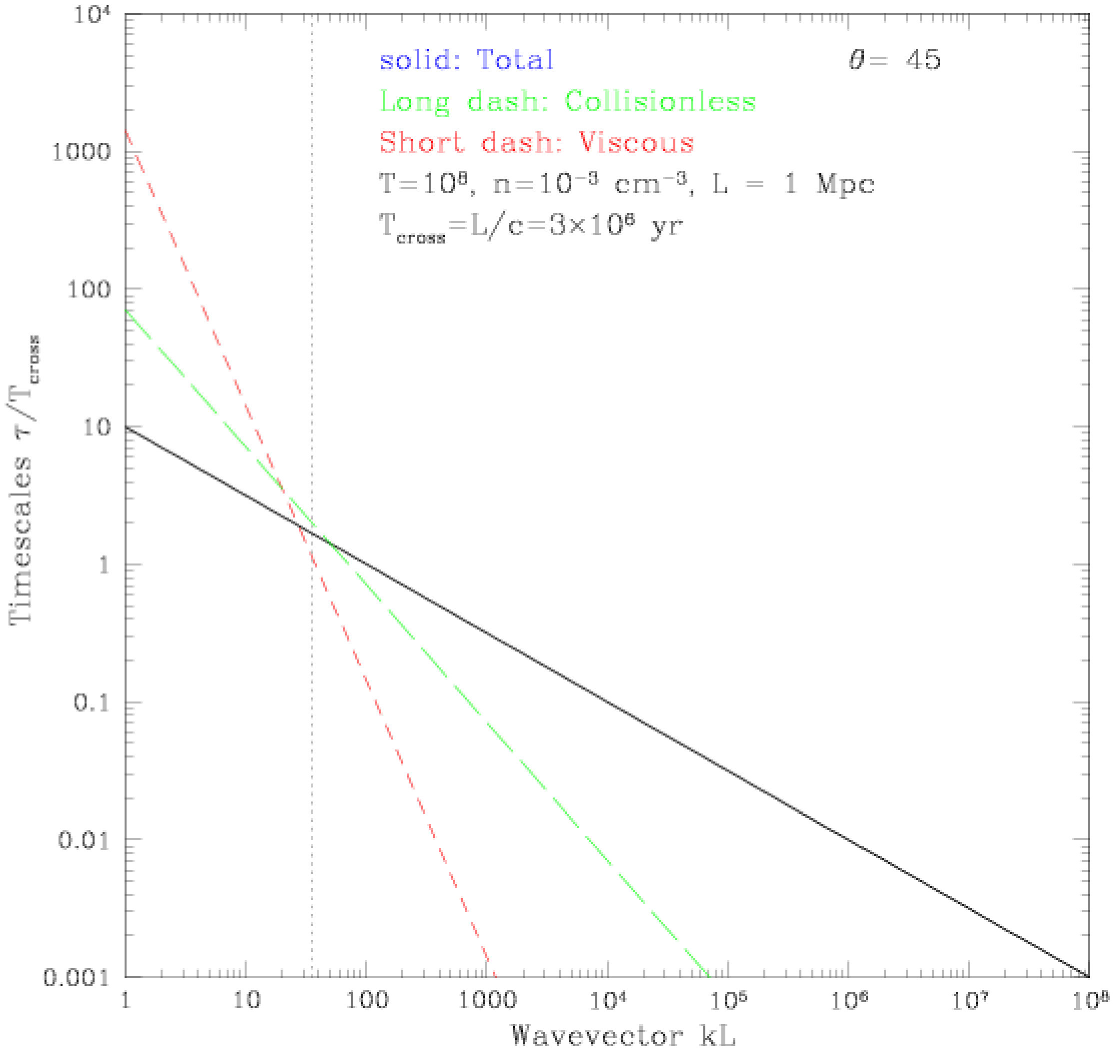
$$\Gamma_{th}(k,\theta) = \Gamma_0 \times \left[\exp\left(-\frac{\delta}{\beta_p \cos^2\theta}\right) + \frac{5}{\sqrt{\delta}}\exp\left(-\frac{1}{\beta_p\cos^2\theta}\right)\right] g(\theta)$$

3A. Turbulence Damping: Low Beta

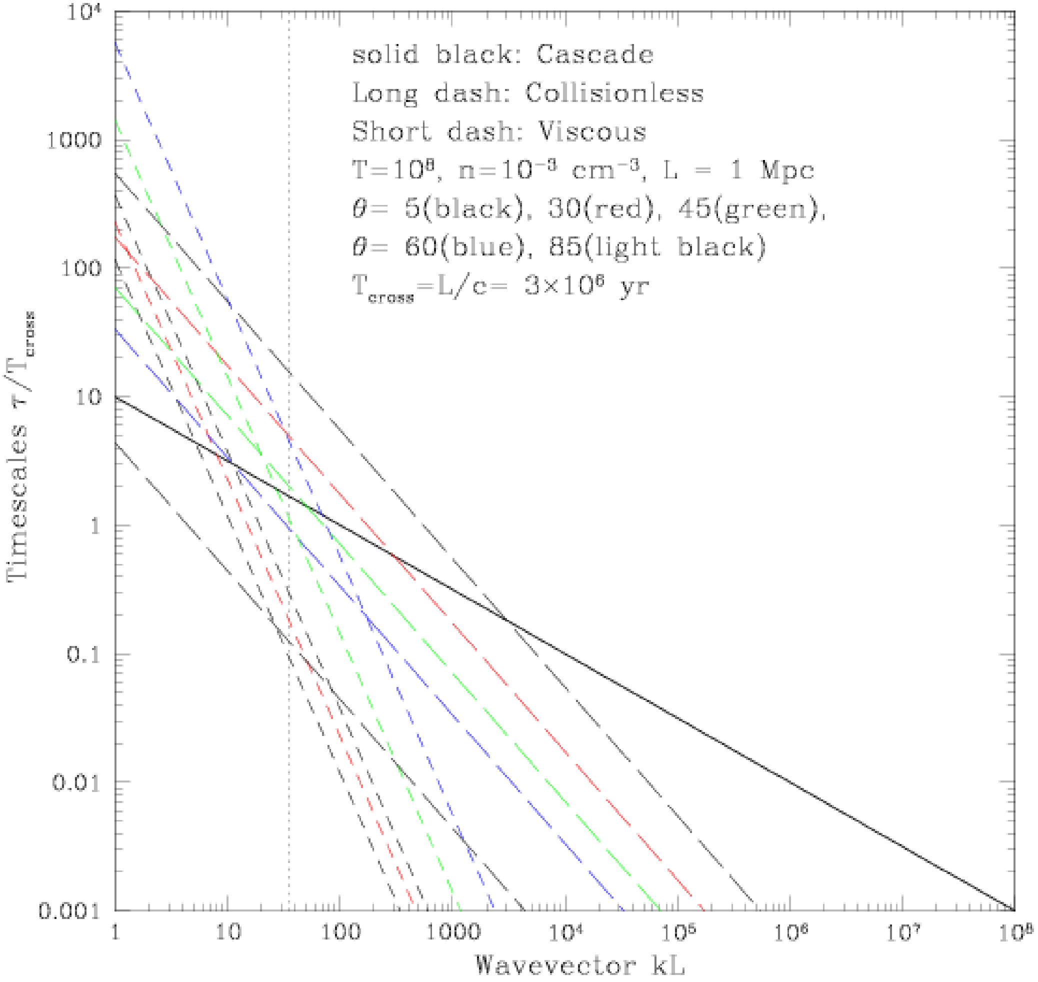


Parallel (and perpendicular) waves are not damped

3A. Turbulence Damping: High Beta



3A. Turbulence Damping High Beta

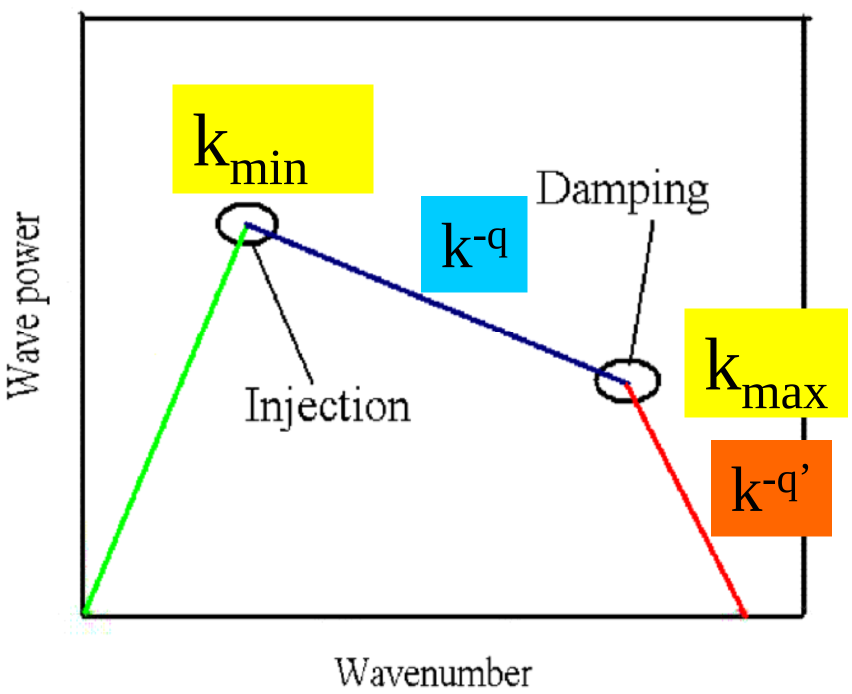


Turbulence Spectrum

$W(k) = ?$

General Features:

- Injection scale: $k_{\rm min}$
- Cascade and index q
- Damping scale or $k_{\rm max}$



Kinetic Equation:

$$\frac{\partial W(\mathbf{k},t)}{\partial t} = \dot{Q}_p(\mathbf{k},t) - \gamma(\mathbf{k})W(\mathbf{k},t) + \nabla_i\left[D_{ij}\nabla_j W(\mathbf{k},t)\right] - \frac{W(\mathbf{k},t)}{T^W_{\rm esc}(\mathbf{k})}$$

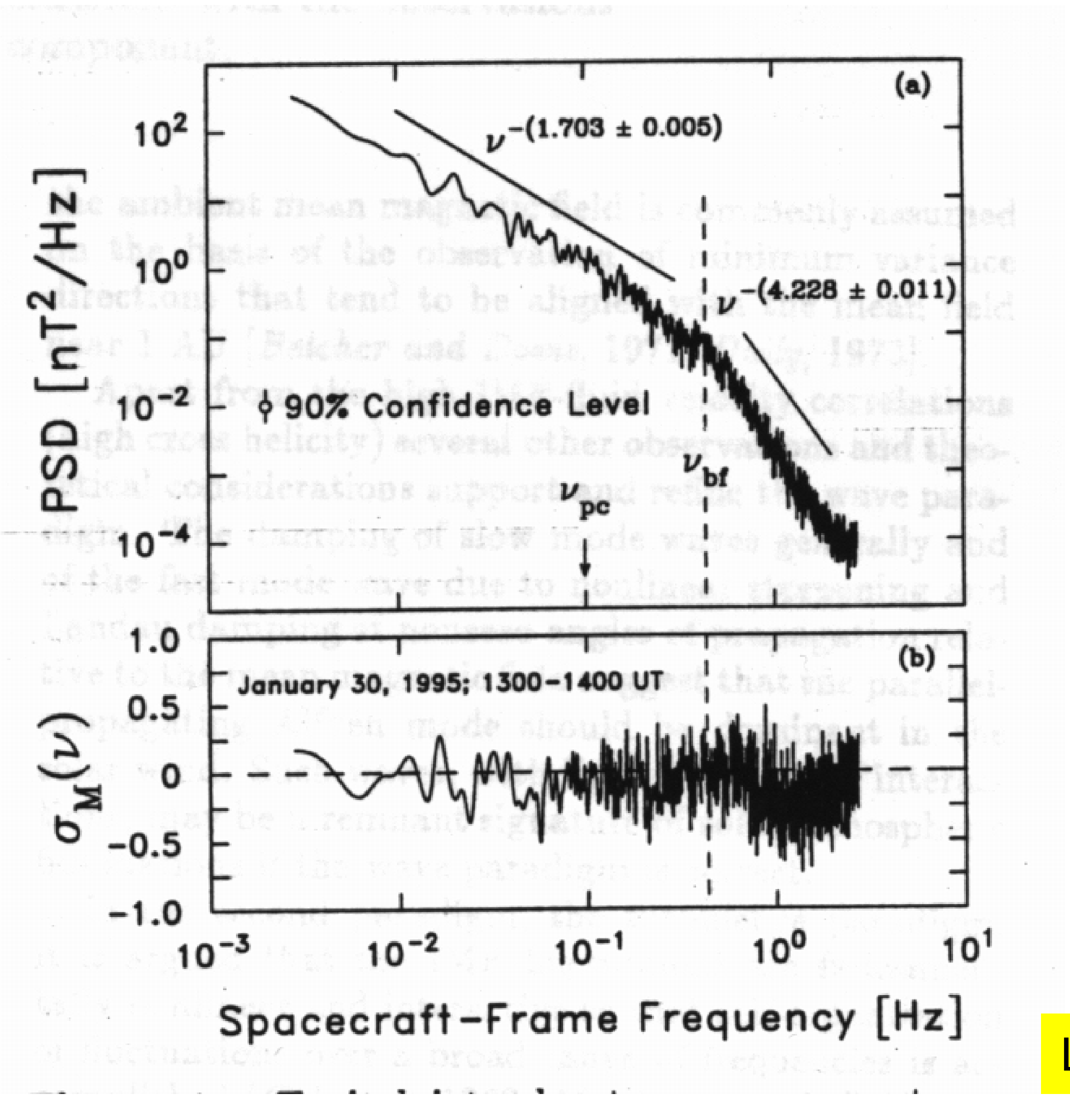
$\dot{Q}_p(\mathbf{k})$: Rate of wave generation.

$T^W_{\rm esc}$: Wave leakage timescale.

$\gamma(k) = \gamma_e + \gamma_p$: The damping coefficients.

D_{ij}: Wave diffusion tensor.

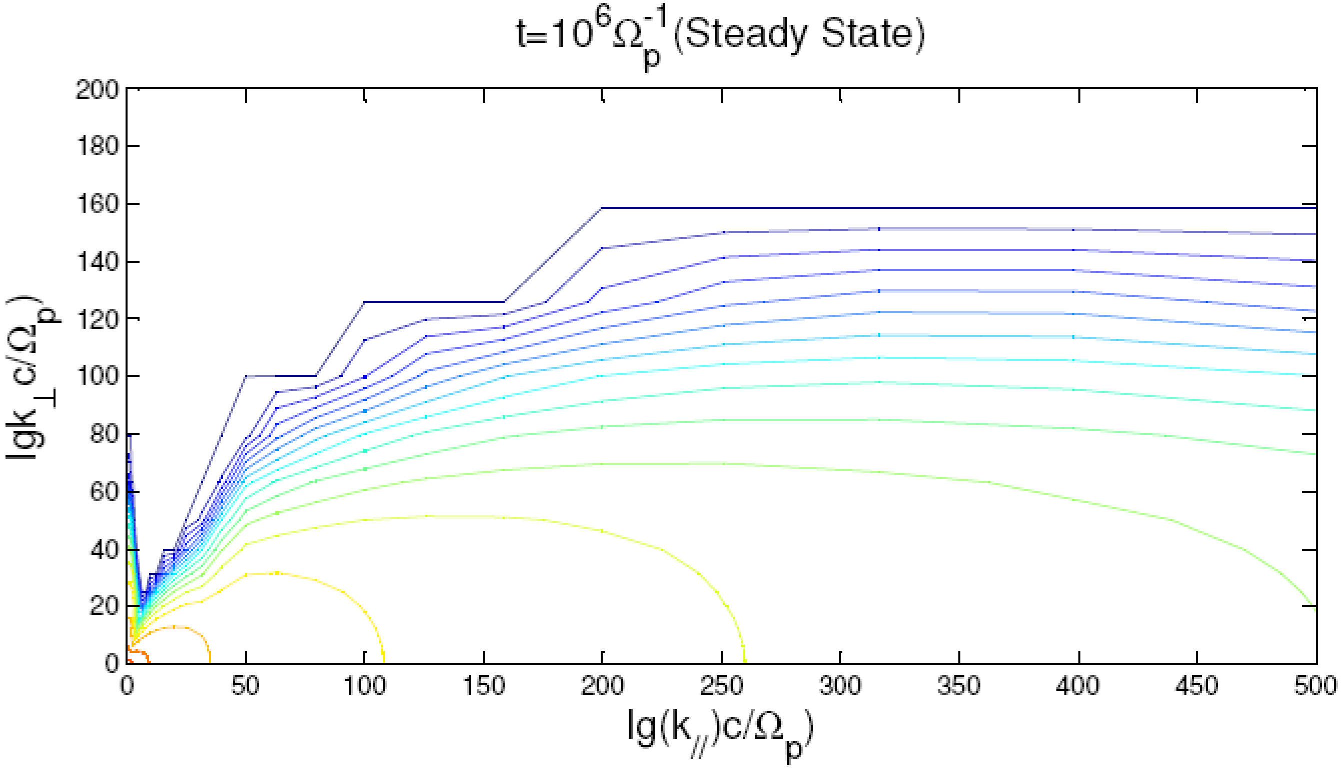
Magnetic fluctuations in Solar wind

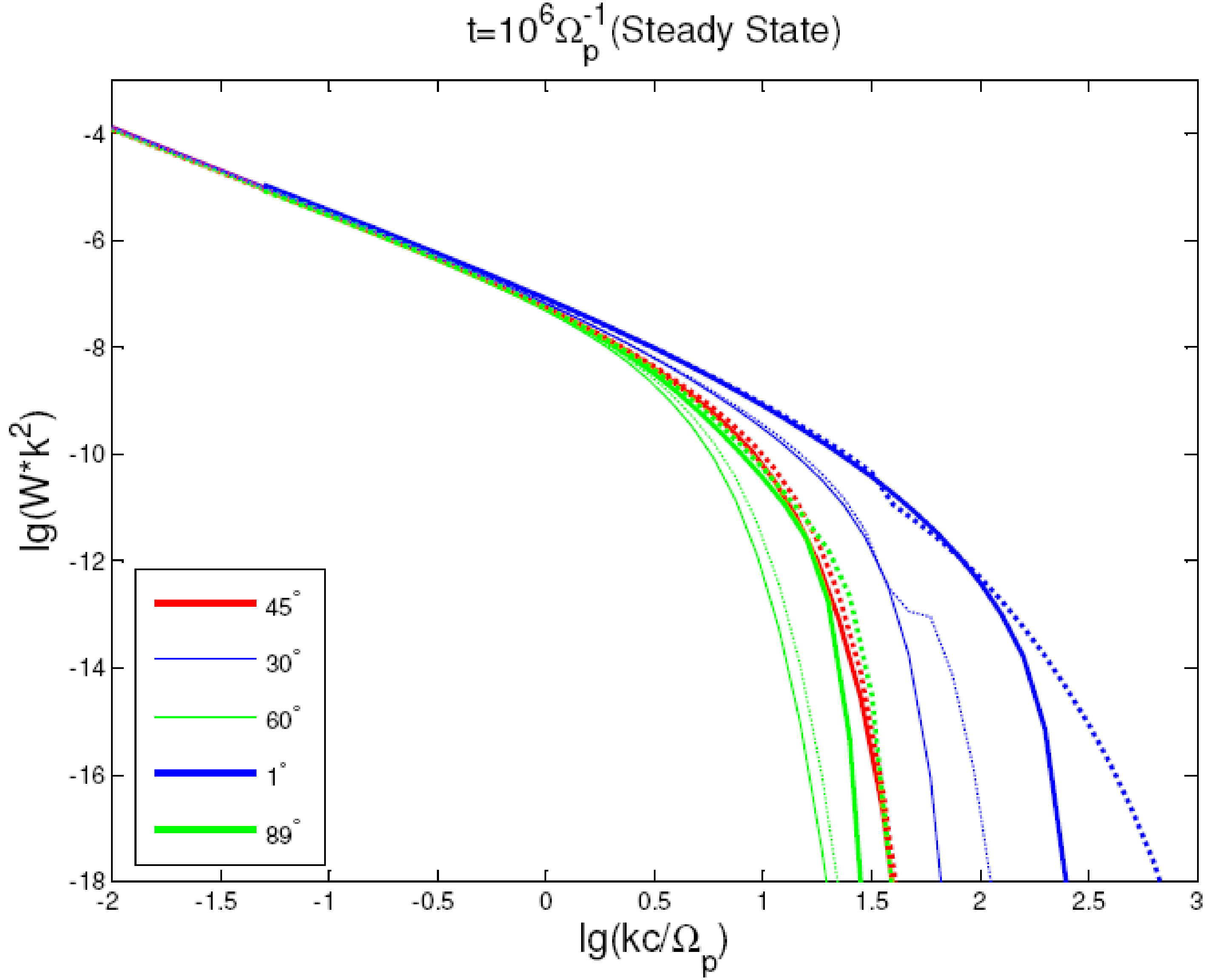


Magnetic fluctuations in Solar wind

Leamon et al (1998)

Solution of the Wave Equation





t=10^6Ω_p^-1(Steady State)
lg(W*k^2)
lg(kc/Ω_p)
-4
-6
-8
-10
-12
-14
-16
-18
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
3
45°
30°
60°
1°
89°

3B. Particle Acceleration

ISOTROPIC AND HOMOGENEOUS

$$\frac{\partial N}{\partial t} = \frac{\partial^2}{\partial E^2}(D_{EE}N) + \frac{\partial}{\partial E}[(\dot{E}_{\mathrm{L}} - A)N] - \frac{N}{T_{\mathrm{esc}}} + Q$$

$$A(E) = \frac{\mathrm{d}D_{EE}}{\mathrm{d}E} + D_{EE}\frac{2\gamma^2 - 1}{(\gamma^2 - 1)\gamma m c^2} \quad + A_{shock}$$

$$T_{\mathrm{esc}} = \frac{L}{\sqrt{2}v}\left(1 + \frac{\sqrt{2}L}{v\tau_{\mathrm{sc}}}\right) \qquad \tau_{\mathrm{sc}} = \frac{1}{2}\int_{-1}^{1}\mathrm{d}\mu\frac{(1-\mu^2)^2}{D_{\mu\mu}}$$

COUPLED EQUATIONS

1. Kinetic Equations

$$\frac{\partial N}{\partial t} = \frac{\partial}{\partial E}\left[D_{EE}\frac{\partial N}{\partial E} - (A - \dot{E}_L)N\right] - \frac{N}{T^p_{\rm esc}} + \dot{Q}^p$$

$$\frac{\partial W}{\partial t} = \frac{\partial}{\partial k_i}\left[D_{ij}\frac{\partial}{\partial k_j}W\right] - \Gamma(\mathbf{k})W - \frac{W}{T^W_{\rm esc}(\mathbf{k})} + \dot{Q}^W$$

2. Energy Balance

$$\dot{\mathcal{W}}_{nonth} \equiv \int \Gamma_{nonth}(\mathbf{k})W(\mathbf{k})d^3k = \dot{\mathcal{E}} \equiv \int A(E)N(E)dE$$

3. Rate Coefficients

$$A(E) = \frac{d[vp^2D(p)]}{4p^2dp} = \int_{k_{min}}^{\infty} d^3kW(\mathbf{k})\Sigma(\mathbf{k},E)$$

$$\Gamma_{nonth}(\mathbf{k}) = \int_{E_0}^{\infty} dEN(E)\Sigma(\mathbf{k},E)$$