Physics 451 – Problem Set # 2

(due Tuesday, March 11)

1. Starting from the formulation of Einstein's gravity in terms of vierbein and spin connection, show explicitly that the constraint $T^a_{mn} = 0$ leads to

$$\Gamma^p_{mn} = \frac{1}{2} g^{pq} \left[\partial_m g_{nq} + \partial_n g_{mq} - \partial_q g_{mn} \right] \tag{1}$$

Introduce the linearization

$$g_{mn} = \eta_{mn} + h_{mn} \tag{2}$$

and show that, at the linearized level, h_{mn} obeys

$$\Delta_{kl;mn} h_{mn} = 0 \ , \tag{3}$$

where

$$\begin{aligned} \Delta_{kl;mn} \ &= \ -\partial^2 \left(\eta_{km}\eta_{ln} + \eta_{km}\eta_{ln} \right) + \left(\partial_k \partial_m \eta_{ln} + \partial_l \partial_m \eta_{kn} + \partial_k \partial_n \eta_{lm} + \partial_l \partial_n \eta_{km} \right) \\ &\quad -2 \left(\partial_k \partial_l \eta_{mn} + \partial_m \partial_n \eta_{kl} \right) + 2 \eta_{kl} \eta_{mn} \partial^2 \end{aligned} \tag{4}$$

2. From the expression for the torsion in $4d$ supergravity and the transformation laws for e^a_m and ψ_m, show explicitly that

$$\begin{aligned} \delta\omega_{mbc} \ &= \ \frac{\kappa}{2} e_{ma} \Big[\xi^\dagger \overline{\sigma}_c \psi_{ab} + \xi^\dagger \overline{\sigma}_b \psi_{ca} - \xi^\dagger \overline{\sigma}_a \psi_{bc} \\ &\qquad + \psi^\dagger_{ab} \overline{\sigma}_c \xi + \psi^\dagger_{ca} \overline{\sigma}_b \xi - \psi^\dagger_{bc} \overline{\sigma}_a \xi \Big] \end{aligned} \tag{5}$$

where $\psi_{ab} = e^m_a e^n_b (D_m \psi_n - D_n \psi_m)$. Account explicitly for the SUSY variations of all of the vierbeins that appear in the analysis.

3. Solve the Rarita-Schwinger equation in d dimensions

$$\gamma^{klm} \partial_l \psi_m = 0 \tag{6}$$

for waves propagating in the $\hat{3}$ direction: $p = (p, 0, 0, p, 0, \ldots, 0)$. Show that there are exactly $(d-3)$ solutions.

4. In 11-dimensional supergravity, the SUSY transformations are given, at the linearized level, by

$$\begin{aligned} \delta_\xi e^a_m \ &= \ -i\kappa \overline{\xi} \gamma^a \psi_m \\ \delta_\xi \psi_m \ &= \ \frac{2}{\kappa} D_m \xi + \frac{\sqrt{2} i}{144} [\gamma^{pqrs}{}_m - 8 \gamma^{qrs} e^p_m] G_{pqrs} \xi \\ \delta_\xi C_{mnp} \ &= \ \frac{3}{\sqrt{2}} \overline{\xi} \gamma_{mn} \psi_{p]} \end{aligned} \tag{7}$$

where G_{pqrs} is the field strength of C_{pqr}. Using an analysis similar that given in the lecture notes for the case of 5 dimensions, show that the SUSY commutators close at the linearized level.