# 15. The Minimal Supersymmetric Standard Model

At the end of the previous lecture, we arrived at a possible prescription for constructing a realistic theory of supersymmetry: Write down a supersymmetric theory of scalars, fermions, and gauge bosons in 4 dimensions. Add the soft supersymmetry breaking terms suggested by Girardello and Grisaru, which might be generated by supersymmetry breaking in a "hidden sector". In this lecture, I would like to discuss the simplest model of this type and work out some of its properties.

We might as well start with the Standard Model of high-energy physics, the $SU(3) \times SU(2) \times U(1)$ gauge theory of quarks and leptons. This model contains 12 vector fields

$$
\begin{array}{ccc}
A_m^i & A_m^a & B_m \\
\text{gluons } (SU(3)) & SU(2) & U(1) \\
i = 1 \text{---} 8 & a = 1,2,3 &
\end{array}
$$

coupling to fermions with the covariant derivative

$$
D_m = (\partial_m - i g_s A_m^i \underbrace{t^i}_{\text{"color"}} - i g A_m^a \underbrace{\tau^a}_{\text{"isospin}} - i g' B_m \underbrace{Y}_{\text{hypercharge}})
$$

The fermions consist of 6 quarks and 6 leptons:

$$
\begin{array}{ccc}
\nu_e & \nu_\mu & \nu_\tau \\
e^- & \mu^- & \tau^-
\end{array}
\qquad\qquad
\begin{array}{ccc}
u & c & t \\
d & s & b
\end{array}
$$

The quantum numbers of these particles are such that their mass terms are forbidden by $SU(2) \times U(1)$ symmetry. All of the quark and lepton masses, and also the masses of the weak interaction gauge bosons $W^+ W^- Z^0$ (3 linear combinations of $A_m^a$ $B_m$), are created by the spontaneous breaking of $SU(2) \times U(1)$. The left-handed components of the fermions have the quantum numbers:

$$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix} \begin{pmatrix} \nu_{\tau L} \\ \tau_L^- \end{pmatrix} \quad \text{color } \underline{1} \quad I = \tfrac{1}{2} \quad Y = -\tfrac{1}{2}$$

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix} \begin{pmatrix} c_L \\ s_L \end{pmatrix} \begin{pmatrix} t_L \\ b_L \end{pmatrix} \quad \text{color } 3 \quad I = \tfrac{1}{2} \quad Y = \tfrac{1}{6}$$

The right-handed fermions have the quantum numbers:

$$e_R \ \mu_R \ \tau_R \qquad \text{color } 1 \quad I=0 \quad Y=-1$$

$$u_R \ c_R \ t_R \qquad \text{color } 3 \quad I=0 \quad Y=\tfrac{2}{3}$$

$$d_R \ s_R \ b_R \qquad \text{color } 3 \quad I=0 \quad Y=-\tfrac{1}{3}$$

After $SU(2) \times U(1)$ is spontaneously broken down to $U(1)$ = gauge symmetry of electromagnetism, the only conserved charge left from this group is

$$Q = I^3 + Y \qquad = \quad \text{electric charge}$$

Then $f_L$ and $f_R$ with the same $Q$ have symmetry-allowed mass terms.

To make this model supersymmetric, I will consider each gauge field as a member of an $N=1$ vector supermultiplet: $A_m \to (A_m, \lambda, D)$

and I will extend each fermion to a chiral supermultiplet

$$f_L \rightarrow \tilde{f}\ ,\ \psi_f = f_L\ ,\ F_f$$

Thus,

$$L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \rightarrow \begin{matrix} \tilde{\nu}_e & \nu_{eL} & F_{\nu e} \\ \tilde{e}^- & e^-_L & F_e \end{matrix}$$

"sleptons"

$$Q = \begin{pmatrix} u \\ d \end{pmatrix}_L \rightarrow \begin{matrix} \tilde{u} & u_L & F_u \\ \tilde{d} & d_L & F_d \end{matrix}$$

To identify the chiral multiplets for the right-handed fermions, we begin from their left-handed antiparticles!

$$e^-_R \rightarrow e^+_L \rightarrow \quad \tilde{\bar{e}} \quad \bar{e}_L = e^+_L \quad F_{\bar{e}} \quad \text{color } 1 \ I=0\ Y=+1$$

$$u_R \rightarrow \bar{u}_L \rightarrow \quad \tilde{\bar{u}} \quad \bar{u}_L \quad F_{\bar{u}} \quad \text{color } \bar{3} \ I=0\ Y=-\tfrac{2}{3}$$

$$d_R \rightarrow \bar{d}_L \rightarrow \quad \tilde{\bar{d}} \quad \bar{d}_L \quad F_{\bar{d}} \quad \text{color } \bar{3} \ I=0\ Y=+\tfrac{1}{3}$$

From here on, the subscript $L$ will be understood.

The supersymmetric Lagrangian of these fields is completely specified by the gauge quantum numbers, except that we can add a superpotential. The superpotential I would like to add is the one that gives the masses of the quarks and leptons through their coupling to the <u>Higgs</u> <u>field</u> whose vacuum expectation value spontaneously breaks $SU(2) \times U(1)$. To obtain the correct Yukawa coupling for leptons and d quarks,

$$- y_e^{ij}\, \bar{e}^i H_a \epsilon_{ab} L_b^j \; - y_d^{ij}\, \bar{d}^i H_a \epsilon_{ab} Q_b^j \; + h.c.$$

$a,b = 1,2$ , we should write

$$W_{4D} = y_e^{ij}\, \tilde{\bar{e}}\, H_a \epsilon_{ab} \tilde{L}_b + y_d^{ij}\, \tilde{\bar{d}}\, H_a \epsilon_{ab} \tilde{Q}_b$$

The Yukawa coupling is chosen so ~~that~~ set $\langle H_a \rangle = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix}_a$ and diagonalizing $y_e^{ij} = y_{ei}\,\delta^{ij}$ gives

$$- \frac{y_{ei} v}{\sqrt{2}}\, \bar{e}^i e^i \; - \frac{y_{di} v}{\sqrt{2}}\, \bar{d}^i d^i$$

e and d mass terms with $m_{ei} = \frac{y_{ei} v}{\sqrt{2}}$ $m_{di} = \frac{y_{di} v}{\sqrt{2}}$. To get a similar mass term for the u quarks, we would have to write the Yukawa interaction

$$- y_{u}^{ij}\, \bar{u}^i H_a^* Q_a$$

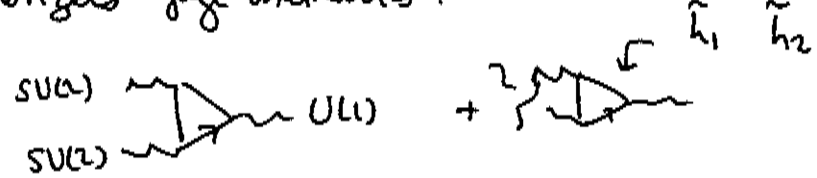
But this cannot be simply converted to a superpotential because a superpotential must be an analytic function of the bosonic fields. Instead we must introduce a second Higgs field $H_2$ with $\langle H_{2a} \rangle = \begin{pmatrix} 0 \\ v_2/\sqrt{2} \end{pmatrix}$ and write

$$W_U = - y_u^{ij}\, \tilde{\bar{u}}^i H_{2a} \epsilon_{ab} \tilde{Q}_b$$

The two Higgs fields have the quantum numbers (color $\underline{1}$)

$$H_1 \quad I = \tfrac{1}{2} \;\; Y = -\tfrac{1}{2} \qquad\qquad H_2 \quad I = \tfrac{1}{2} \;\; Y = +\tfrac{1}{2}$$

Each leads to a chiral supermultiplet. There we find another reason for two Higgs fields: The fermionic components (higgsinos) have nonzero gauge anomalies:




$\tilde h_1 = \psi_{H_1}$, $\tilde h_2 = \psi_{H_2}$

which cancel when we include both $\tilde h_1$ and $\tilde h_2$ with $Y(\tilde h_1) = -Y(\tilde h_2)$.

Some additional terms in the superpotential are allowed by the symmetries of the Standard Model. First

$$W_\mu = -\mu \, H_{1a} \epsilon_{ab} H_{2b}$$

$\mu$ is a new parameter, the first one we have added to the Standard Model. We could set $\mu = 0$, but then we would have a charged Higgsino with mass $< m_W$, which would already have been observed.

Next,

$$W_{BL} = \eta_1^{pqr} \epsilon_{ijk} \bar u^{ip} \bar d^{jq} \bar d^{kr} \qquad \begin{array}{l} i,j,k = \text{colors} \\ p,q,r = \text{flavors} \end{array}$$

$$+ \eta_2^{pqr} \bar d^p \epsilon_{ab} L_a^q Q_b^r$$

$$+ \eta_3^{pqr} \bar e^p \epsilon_{ab} L_a^q L_b^r + \eta_4^{p} \epsilon_{ab} L_a^p H_{2b}$$

these terms violate Baryon # & lepton # with dimensionless couplings. Thus, they are forbidden — or extremely small. This completes the list of renormalizable superpotentials allowed by the symmetries of the Standard Model.

There is a nice advantage in removing the superpotential terms in the last group. If B and L are conserved, we can form a discrete symmetry

$$R = (-1)^{3B+L+2J}$$

R has the property that

$$R = 1 \text{ on } e \ \bar{e} \ u \ d \ \bar{u} \ \bar{d} \ A^i_m \ A^a_m \cdots$$

$$= -1 \text{ on their supersymmetric partners.}$$

So, if $B, L$ are conserved, the lightest SUSY partner is absolutely stable! If this particle was produced in the early universe, it should still be around — and thus a good candidate for cosmological dark matter.

Spontaneous SUSY breaking in a hidden sector can provide effective soft SUSY breaking terms in this theory. Keeping terms consistent with $SU(2) \times U(1)$:

$$\mathcal{L}_{soft} = -M_i^2 |\tilde{f}_i|^2 - \tfrac{1}{2} m_a \lambda^{Ta} C \lambda^a + h.c.$$

$$+ \{ A_u y_u \tilde{\bar{u}} H_{2a} \epsilon_{ab} \tilde{Q}_b - A_d y_d \tilde{\bar{d}} H_{1a} \epsilon_{ab} \tilde{Q}_b$$

$$- A_e y_e \tilde{\bar{e}} H_{1a} \epsilon_{ab} \tilde{L}_b + B\mu H_{1a} \epsilon_{ab} H_{2b} \}$$

In principle, there could be arbitrary flavor mixing in these terms, but I will ignore this for simplicity.

Sometimes people joke ~~that~~ supersymmetry is good because we have already discovered half of the particles. However, I think this is not such a joke. It is nontrivial that the particles we know are those whose masses are forbidden by $SU(2) \times U(1)$, while the particles ~~that~~ we have not discovered are those whose masses are forbidden only by supersymmetry and thus get mass from $\mathcal{L}_{soft}$. Setting $M_i \sim m_a \sim m_{3/2}$ at a few hundred GeV puts these new particles above the reach of current accelerators — for now, anyway.

A remarkable property of the model we have defined is its consistency with the idea of grand unification. The SM gauge group $SU(2)\times U(1)\times SU(3)$ embeds naturally in $SU(5)$: Generators of $SU(5)$ are $5\times 5$ traceless Hermitian matrices normalized to $\mathrm{tr}\, t^a t^b = \frac{1}{2}\delta^{ab}$. Among these, we can identify:

$$SU(3) \text{ generators}: \quad \begin{pmatrix} t^i & 0 \\ \hline 0 & 0 \end{pmatrix} \begin{matrix} \}\,3 \\ \}\,2 \end{matrix}$$

$$SU(2) \text{ generators}: \quad \begin{pmatrix} 0 & 0 \\ \hline 0 & \tau^a \end{pmatrix}$$

$$U(1) \text{ generator}: \quad \sqrt{\tfrac{3}{5}}\begin{pmatrix} -\frac{1}{3} & & & & \\ & -\frac{1}{3} & & & \\ & & -\frac{1}{3} & & \\ \hline & & & +\frac{1}{2} & \\ & & & & +\frac{1}{2} \end{pmatrix}$$

[ w. this identification $\bar{5} + 10$ of $SU(5)$ $\psi^a$ $\underbrace{\psi_{[ab]}}_{\text{antisym.}}$ give the $SU(3)\times SU(2)\times U(1)$ q. nos. of 1 generation of SM fermions! ]

$SU(5)$ will break spontaneously to $SU(3)\times SU(2)\times U(1)$ if a boson field in the adjoint rep acquires an expectation value

$$\langle \Phi \rangle = a \cdot \begin{pmatrix} -\frac{1}{3} & & & & \\ & -\frac{1}{3} & & & \\ & & -\frac{1}{3} & & \\ \hline & & & \frac{1}{2} & \\ & & & & \frac{1}{2} \end{pmatrix}$$

then $[t^a, \langle\Phi\rangle] = 0$ only for the 12 generators above. We obtain then an $SU(3)\times SU(2)\times U(1)$ thy with coupling constants

$$g_s = g_5 \qquad g = g_5 \qquad g' = \sqrt{\tfrac{3}{5}}\, g_5$$

These relations are not correct for the observed couplings, in particular

$$\alpha_s(m_Z) \cong 1/8.5 \qquad \alpha_w \cong 1/30.$$

But these couplings have nontrivial $\beta$ function and run from scale to scale. Write

$$g_3 = g_s \qquad g_2 = g \qquad g_1 = \sqrt{\tfrac{5}{3}}\, g' \qquad \alpha_i = \frac{g_i^2}{4\pi}$$

and write their 1 loop renormalization group equations as

$$\frac{d g_i}{d \log Q} = - \frac{b_i}{(4\pi)^2}\, g_i^3$$

then

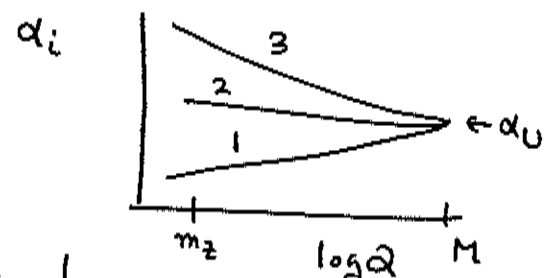
$$g_i^2(Q) = \frac{g_i^2(M)}{1 + b_i \frac{g_i^2(M)}{8\pi^2} \log Q/M}$$

or

$$\alpha_i^{-1}(Q) = \alpha_i^{-1}(M) + \frac{b_i}{2\pi} \log \frac{Q}{M}$$

If $M$ is the symmetry-breaking scale, grand unification predicts that all of the couplings are equal at $M$ and diverge for $Q \ll M$. The $\beta$ function is more negative for larger non-Abelian groups, so for $Q \ll M$

$$\alpha_3 > \alpha_2 > \alpha_1$$




as observed; at $Q = m_Z$

$$\alpha_3 \cong \frac{1}{8.5} \qquad \alpha_2 \cong \frac{1}{30} \qquad \alpha_1 \cong \frac{1}{60}.$$

Does this work quantitatively? Eliminating the two parameters $\alpha_U = \alpha(M_U)$ and $M_U$, we find the following relation between the $\alpha_i$:

$$\alpha_3^{-1} = (1+B)\,\alpha_2^{-1} - B\,\alpha_1^{-1}$$

where $$B = \left(\frac{b_3 - b_2}{b_2 - b_1}\right)$$

Putting in the measured values: $B = 0.719 \pm .008$

up to corrections from 2-loop $\beta$ functions, threshold corrections etc.

In the Standard model:

$$b_i = \frac{11}{3} N_i - \frac{2}{3} \sum_{\substack{\text{chiral}\\\text{fermions}}} C(r) - \frac{1}{3} \sum_{\substack{\text{complex}\\\text{bosons}}} C(r)$$

so for

$$\begin{aligned} SU(3): \quad & b_3 = 11 - \tfrac{4}{3} n_g \\ SU(2): \quad & b_2 = \tfrac{22}{3} - \tfrac{4}{3} n_g - \tfrac{1}{6} n_H \\ U(1): \quad & b_1 = \quad - \tfrac{4}{3} n_g - \tfrac{1}{10} n_H \end{aligned}$$

$n_g = \#$ of generations $= 3$

$n_H = \#$ of Higgs doublets.

for $n_H = 1$, $$B = \frac{11 + \frac{1}{2}}{22 - \frac{1}{5}} = 0.53$$

Now just add the superpartners:

$$b_i = 3 N_i - \sum_{\substack{\text{chiral}\\\text{super mults.}}} C(r)$$

$$\begin{aligned} SU(3) \quad & b_3 = 9 - 2n_g \\ SU(2) \quad & b_2 = 6 - 2n_g - \tfrac{1}{2} n_H \\ U(1) \quad & b_1 = \quad -2n_g - \tfrac{3}{10} n_H \end{aligned}$$

for $n_H = 2$, as we require $$B = \frac{4}{28/5} = \frac{5}{7} = \underline{\underline{0.714}}$$

so our SUSY model is consistent with grand unification and predicts just the right relation between $\alpha_s$, $\alpha$, and $\alpha'$.

I should say that the agreement gets worse if we include 2-loop corrections to the $\beta$ functions. Of course the agreement also requires that the SUSY particle masses are not much larger than $m_Z$.

We have now seen that our minimal SUSY extension of the Standard Model has a natural place for dark matter and for grand unification. It also has one more important phenomenological feature, related to $SU(2)\times U(1)$ breaking. But before discussing this, I would like to discuss the mass spectrum of the SUSY particles.

The masses of SUSY particles typically come from several terms in the Lagrangian, and we have to account carefully to get them all.

Consider first the squarks and sleptons. Their masses come from:

→ soft SUSY-breaking masses: $\quad - M_i^2 \tilde{f}_i^* \tilde{f}_i$

→ A terms in $\mathcal{L}_{soft}$!

$$\mathcal{L}_{soft} \cong - A_u y_u \bar{\tilde{u}} \frac{v_2}{\sqrt{2}} \tilde{u} - A_d y_d \bar{\tilde{d}} \frac{v_1}{\sqrt{2}} \tilde{d} - A_e y_e \bar{\tilde{e}} \frac{v_1}{\sqrt{2}} \tilde{e} + h.c.$$

$$\cong - A_u m_u \bar{\tilde{u}} \tilde{u} - A_d m_d \bar{\tilde{d}} \tilde{d} - A_e m_e \bar{\tilde{e}} \tilde{e} + h.c.$$

→ term from $-V_F = -(F_{H1}^* F_{H1} + F_{H2}^* F_{H2}) - F_f^* F_f$

$$= -\left(-\mu^* \frac{v_2}{\sqrt{2}}\right)\left(y_d \bar{\tilde{d}} \tilde{d} + y_e \bar{\tilde{e}} \tilde{e}\right) - \left(-\mu^* \frac{v_1}{\sqrt{2}}\right)\left(y_u \bar{\tilde{u}} \tilde{u}\right)$$

$$+ h.c. - m_d^2 (\tilde{d}^* \tilde{d} + \bar{\tilde{d}}^* \bar{\tilde{d}}) + \cdots$$

→ terms from the $SU(2)\times U(1)$ D potential

$$-V_D = -\frac{g^2}{2} \cdot (H_1^* \tau^3 H_1 + H_2^* \tau^3 H_2)(\tilde{f}^* I^3 \tilde{f}) \cdot 2$$

$$- \frac{g'^2}{2} \left( H_1^* H_1 (-\tfrac{1}{2}) + H_2^* H_2 (\tfrac{1}{2})\right) (\tilde{f}^* Y \tilde{f}) \cdot 2$$

$$= \; -\, g^2 \left(\tfrac{1}{2} v_1^2/2 - \tfrac{1}{2} v_2^2/2\right)(\tilde{f}^* I^3 \tilde{f})$$
$$- g'^2 \left(-\tfrac{1}{2} v_1^2/2 + \tfrac{1}{2} v_2^2/2\right) \tilde{f}^* Y \tilde{f} \qquad \frac{g'}{g} = \tan\Theta_w$$

let $v_1^2 + v_2^2 = v^2$ $\quad \tan\beta = v_2/v_1$ $\quad$ then this is

$$= \; - \frac{g^2}{4} (\cos^2\beta - \sin^2\beta)\, v^2 \;\; \tilde{f}^* (I^3 - \tan^2\Theta_w Y)\, \tilde{f}$$

$$= \; - \left(\frac{g^2 v^2}{4\cos^2\Theta_w}\right) \cos 2\beta \;\; \tilde{f}^* \left[ I^3 - \sin^2\Theta_w (I^3 + Y) \right] \tilde{f}$$

$$= \; - \; m_Z^2 \cos 2\beta \; (I^3 - Q \sin^2\Theta_w)\, \tilde{f}^* f$$

so we find the mass ~~matrix~~

$$m_e^2 = \begin{pmatrix} M_e^2 + m_e^2 + \Delta_e & m_e(A_e^* - \mu \tan\beta) \\ m_e(A_e - \mu^* \tan\beta) & M_{\bar{e}}^2 + m_e^2 + \Delta_{\bar{e}} \end{pmatrix} \quad \begin{pmatrix} \tilde{e} \\ \tilde{\bar{e}}^* \end{pmatrix}$$

$$m_d^2 = \begin{pmatrix} M_d^2 + m_d^2 + \Delta_d & m_d(A_d^* - \mu \tan\beta) \\ m_d(A_d - \mu^* \tan\beta) & M_{\bar{d}}^2 + m_d^2 + \Delta_{\bar{d}} \end{pmatrix} \quad \begin{pmatrix} \tilde{d} \\ \tilde{\bar{d}}^* \end{pmatrix}$$

$$m_u^2 = \begin{pmatrix} M_u^2 + m_u^2 + \Delta_u & m_u(A_u^* - \mu \cot\beta) \\ m_u(A_u - \mu^* \cot\beta) & M_{\bar{u}}^2 + m_u^2 + \Delta_{\bar{u}} \end{pmatrix} \quad \begin{pmatrix} \tilde{u} \\ \tilde{\bar{u}}^* \end{pmatrix}$$

where $\quad \Delta_f = m_Z^2 \cos 2\beta \, (I^3 - Q \sin^2\Theta_w)$

Next, consider the gauginos. For the SU(3) gaugino, the gluino, we have only the soft mass term. But for the $SU(2)\times U(1)$ gauginos, there are additional ~~terms that~~

mix them with the higgsinos. For the charged states, we have

→ soft mass term

$$\mathcal{L}_{soft} = -m_2 \tilde{w}^{-T} c\, \tilde{w}^{+} \qquad \tilde{w}^{+} = \text{gaugino of } W^{+}$$

→ Higgs mass term

$$\mathcal{L}_F = -\mu\, \tilde{h}^{-T} c\, \tilde{h}^{+} \qquad H_1 = \begin{pmatrix} v_1/\sqrt{2} + \cdots \\ h^- \end{pmatrix} \quad H_2 = \begin{pmatrix} h^+ \\ v_2/\sqrt{2} + \cdots \end{pmatrix}$$

→ gaugino-matter terms.

$$\mathcal{L} = -\sqrt{2}\, g \tfrac{1}{\sqrt{2}} \left( v_1/\sqrt{2}\; \tilde{w}^{+T} c\, \tilde{h}^{-} + v_2/\sqrt{2}\; \tilde{w}^{-T} c\, \tilde{h}^{+} \right) + \text{h.c.}$$

in all, we find

$$\mathcal{L} = -(\tilde{w}^-, \tilde{h}^-)^T c\; m_c \begin{pmatrix} \tilde{w}^+ \\ \tilde{h}^+ \end{pmatrix}$$

with

$$m_c = \begin{pmatrix} m_2 & \sqrt{2}\, m_W \sin\beta \\ \sqrt{2}\, m_W \cos\beta & \mu \end{pmatrix}$$

For the neutral states, the same three sources give the elements of a 4x4 mixing matrix between $\tilde{A}^3 = \tilde{w}^0$ $\tilde{B} = \tilde{b}$ $\tilde{h}_1^0$ $\tilde{h}_2^0$ :

$$\mathcal{L} = -\tfrac{1}{2} (\tilde{b}^0, \tilde{w}^0, \tilde{h}_1^0\; \tilde{h}_2^0)\; m_N \begin{pmatrix} \tilde{b}^0 \\ \tilde{w}^0 \\ \tilde{h}_1^0 \\ \tilde{h}_2^0 \end{pmatrix}$$

$$m_N = \begin{pmatrix} m_1 & 0 & -m_Z \cos\beta \sin\theta_w & m_Z \sin\beta \sin\theta_w \\ 0 & m_2 & m_Z \cos\beta \cos\theta_w & -m_Z \sin\beta \cos\theta_w \\ -m_Z \cos\beta \sin\theta_w & m_Z \cos\beta \cos\theta_w & 0 & -\mu \\ m_Z \sin\beta \sin\theta_w & -m_Z \sin\beta \cos\theta_w & -\mu & 0 \end{pmatrix}$$

The eigenstates of the charged fermion problem are called "charginos". The eigenstates of the neutral fermion problem are "neutralinos". The lightest neutralino is the most probably dark matter candidate.

The mass matrix of the scalar Higgs bosons is more complicated and I will omit it, except for one point that is coming later.

Now I would like to discuss the relative sizes of the mass parameters in the soft SUSY breaking Lagrangian. I will assume that these parameters are generated by SUSY breaking in a hidden sector that is coupled to the Standard Model particles by heavy particles with mass $M$ (the "messenger scale"). In the supergravity scenario discussed in the previous lecture, $M = m_{planck}$ or $m_{string}$. In this case, the parameters in $\mathcal{L}_{soft}$ will undergo substantial renormalization to their values at $m_{3/2} \sim 1\,\text{TeV}$ (which are the physically observed values). In principle, the SUSY breaking could lay down any pattern at $M$, and then renormalization effects will alter it as we go to lower energies. However, it is simplest to begin with case of universal parameters in $\mathcal{L}_{soft}(M)$. Then renormalization generates an interesting pattern at the TeV scale.

Let's begin by discussing the gaugino masses $m_1\, m_2\, m_3$. To understand the renormalization of these masses, write the gauge Lagrangian as

$$\int d^2\theta \left\{ \frac{1}{4g^2} W^T c W + \Phi\, W^T c W \right\} + h.c.$$

where $\Phi$ is a chiral field. If $\langle F_\Phi \rangle \neq 0$, this will generate gaugino masses. If $M$ is above the grand unification scale and $\Phi$ is an SU(5) singlet, these masses will be universal: $m_1 = m_2 = m_3$ at $M$.

As we go to lower energies, the first term in this Lagrangian receives an additive correction from the scale and axial anomalies:

$$+ \int d^2\theta \left( \frac{b_i}{32\pi^2} \log \frac{Q}{M} \right) W_i^T C W_i$$

while the second term is not renormalized. If we then scale out $\frac{1}{g(Q)}$ to the usual place, the second term gets a coefficient proportional to $g_i^2(Q) / g_i^2(M)$. Thus:

$$m_i(Q) / m_i(M) = \alpha_i(Q) / \alpha_i(M)$$

In particular, grand unification at $M$ implies at $Q \sim$ few 100 GeV

$$m_1 : m_2 : m_3 = \underset{\text{at } m_Z}{\alpha_1 : \alpha_2 : \alpha_3} = \frac{1}{2} : 1 : 3.5$$

The gluino is heavy; the "bino" $\tilde{b}^0$ is light.

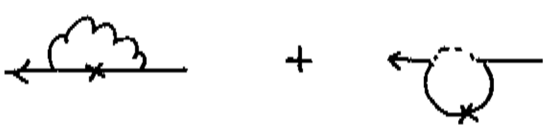
What about the scalar mass parameters $M_i^2$? First of all, SUSY implies ~~that~~ these parameters receive no additive quadratically divergent renormalization. In particular, for the Higgs boson, this solves the "gauge hierarchy problem" that, in the Standard Model without SUSY, the Higgs boson mass

term not be delicately tuned against the radiative corrections.
In SUSY, the $M^2$ term receive only corrections proportional to

$$\alpha_i \text{ (mass parameter)}^2 \times \log Q/M$$

and so naturally stay of order $m_{3/2}$.

We can then try to compute the 1-loop renormalization of $M_i^2$.
From gauge and D-term interactions:

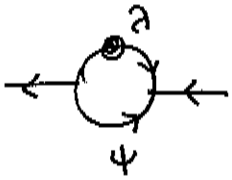


$$= (ig)^2 C_2(r) \int \frac{d^4k}{(2\pi)^4} k^m \frac{i}{k^2}\left(-M^2 \frac{1}{k^2}\right) k_m \frac{-i}{k^2}$$

$$+ (ig)^2 C_2(r) \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2}\left(-M^2 \frac{1}{k^2}\right) \cdot i$$

$$= 0 \quad !$$

From interaction with $\psi$ and gauginos: expand to order $m^2_{\text{gaugino}}$




$$= (-\sqrt{2} i g_i)(\sqrt{2} i g_i) \int \frac{d^4k}{(2\pi)^4} \mathrm{tr}\left[\frac{i\sigma\cdot k}{k^2}\left(\frac{+m_i^2}{k^2}\right) c \left(\frac{-i\sigma\cdot k}{k^2}\right)^T c\right]$$

$$= -4 g_i^2 C_2(r)\, m_i^2 \frac{i}{(4\pi)^2} \log \Lambda^2/M^2$$

so

$$\beta_{M_i^2} = -i M \frac{\partial}{\partial M} \text{ (above)}$$

(recall that, for scalars, the field strength renormalization vanishes at 1 loop in Feynman gauge) or $$\frac{d}{d\log Q} M_f^2(Q) = -\frac{2}{\pi} \sum_{i=1,2,3} \alpha_i(Q)\, C_2(r_i)\, m_i^2$$

From $Q = m_{planck}$ to $Q = \mathrm{TeV}$, the gluino mass typically gives

self-renormalization : $m_{\tilde g}^2 \sim 12\, m_2^2$

renorm of squark mass: $\Delta M_{\tilde q}^2 \sim +10\, m_2^2$

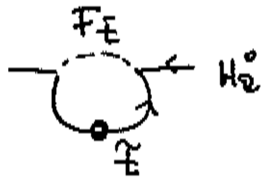
The $\tilde w$ and $\tilde b$ terms make smaller contributions. In particular

$$\Delta M_L^2 \sim 0.8\, m_2^2 \qquad\qquad \Delta M_{\bar e}^2 \sim 0.2\, m_2^2$$

What about the Higgs bosons? For these, there is another important renormalization effect, due to the top quark Yukawa coupling. From

$$W = y_t\, \tilde{\bar t}\, H_2^0\, \tilde t$$

we have




$$= (-iy_t)^2 \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2} \frac{M_{\tilde t}^2}{k^2}\, i \cdot \overbrace{3}^{\text{color}}$$

$$= +3iy_t^2\, M_{\tilde t}^2 \cdot \frac{i}{(4\pi)^2} \log \Lambda^2/M^2$$

Notice that this has the opposite sign from the contribution on p.15! Adding terms from $M_{\tilde t}^2$ and the $A$ term, one finds

$$\frac{d}{d\log Q} M_2^2 = (\text{previous}) + \frac{6 a_t^2}{(4\pi)^2}\left(M_2^2 + M_{\bar t}^2 + M_t^2 + A_t^2\right)$$

Actually, this term appears for $M_2^2$, $M_{\bar t}^2$ and $M_t^2$. But the factor in front is different, depending on the number of states summed over in the loop.

$$\frac{d}{d\log Q} M_t^2 = + \frac{2}{(4\pi)^2} \cdot 1 \cdot y_t^2 \left(M_2^2 + M_t^2 + M_{\bar t}^2 + A_t^2\right)$$

$$\frac{d}{d\log Q} M_{\bar t}^2 = \frac{2}{(4\pi)^2} \cdot \underset{\substack{\uparrow \\ (\text{isospin sum})}}{2} \cdot y_t^2 \left(M_2^2 + M_t^2 + M_{\bar t}^2 + A_t^2\right)$$

$$\frac{d}{d\log Q} M_2^2 = \frac{2}{(4\pi)^2} \cdot \underset{\substack{\uparrow \\ (\text{color sum})}}{3} \cdot y_t^2 \left(M_2^2 + M_t^2 + M_{\bar t}^2 + A_t^2\right)$$

These corrections potentially drive the $M^2$'s negative. Because of the relative coefficients, $M_2^2(Q)$ is the first to go negative as we decrease $Q$.

This is a very ~~interesting~~ conclusion. At first sight, supersymmetry seems very ~~interesting~~ for building a theory of Higgs fields, because it requires scalars and thus gives the Higgs field a raison d'être. Unfortunately, because if we supersymmetrize anything we must supersymmetrize everything, SUSY requires a very large number of scalar fields. So, why should any particular one get a vacuum expectation value? We have now seen that the minimal supersymmetrization of the Standard Model has — at least if the top quark is heavy — a definite reason why it is the Higgs field $H_2$, rather than any other field in the model, that is driven to get a vacuum expectation value. If $\mu \neq 0$, the v.e.v. of $H_2$ drives a v.e.v. of $H_1$ and we then get the full symmetry breaking pattern of the Standard Model.

In this lecture, then, we have seen that the supersymmetrization of the Standard Model has some very interesting consequences for phenomenological particle physics.

— It provides a candidate for cosmological dark matter.

— It allows a grand unification of the Standard Model couplings

— It solves the "gauge hierarchy problem" and — even better — gives a <u>reason</u> why $\langle H \rangle \neq 0$.

Of course, SUSY is still a hypothesis. But the successes above work only if SUSY particles lie close to the current range of accelerators. Thus, we will soon know if these ideas are correct or not. If they are correct, the theory extends to deep connections with generalized notions of gravity and spacetime. It will be an exciting road to travel.