Parton-parton cross sections needed for the problem sets.

Here are the formulae for $d\sigma/d\cos\theta_*$ for various parton-parton reactions. In all cases, the formulae are appropriately polarization and color averaged over initial and final state particles. Cross sections with identical particles in the final state should be integrated over $\cos\theta_* > 0$ only, or multiplied by $\frac{1}{2}$ and integrated over all angles.

parton + parton → parton + parton:

$$u + d \to u + d \ : \qquad \frac{2}{9}\frac{\pi\alpha_s^2}{s}\frac{s^2+u^2}{t^2}$$

$$u + \overline{u} \to d + \overline{d} \ : \qquad \frac{2}{9}\frac{\pi\alpha_s^2}{s}\frac{t^2+u^2}{s^2}$$

$$u + u \to u + u \ : \qquad \frac{2}{9}\frac{\pi\alpha_s^2}{s}\left[\frac{s^2+u^2}{t^2} + \frac{s^2+t^2}{u^2} - \frac{2}{3}\frac{s^2}{tu}\right]$$

$$u + \overline{u} \to u + \overline{u} \ : \qquad \frac{2}{9}\frac{\pi\alpha_s^2}{s}\left[\frac{s^2+u^2}{t^2} + \frac{t^2+u^2}{s^2} - \frac{2}{3}\frac{u^2}{st}\right]$$

$$u + \overline{u} \to g + g \ : \qquad \frac{16}{27}\frac{\pi\alpha_s^2}{s}\left[\frac{u}{t} + \frac{t}{u} - \frac{9}{4}\frac{t^2+u^2}{s^2}\right]$$

$$g + g \to u + \overline{u} \ : \qquad \frac{1}{12}\frac{\pi\alpha_s^2}{s}\left[\frac{u}{t} + \frac{t}{u} - \frac{9}{4}\frac{t^2+u^2}{s^2}\right]$$

$$u + g \to u + g \ : \qquad \frac{2}{9}\frac{\pi\alpha_s^2}{s}\left[-\frac{u}{s} - \frac{s}{u} + \frac{9}{4}\frac{s^2+u^2}{t^2}\right]$$

$$g + g \to g + g \ : \qquad \frac{9}{4}\frac{\pi\alpha_s^2}{s}\left[3 - \frac{tu}{s^2} - \frac{ts}{u^2} - \frac{su}{t^2}\right]$$

Drell-Yan cross sections:

Let $Q_f = \frac{2}{3}$, $I_f = +\frac{1}{2}$ for u, c; $Q_f = -\frac{1}{3}$, $I_f = -\frac{1}{2}$ for d, s, b, $M_Z^2 = m_Z^2 - im_Z\Gamma_Z$, $M_W^2 = m_W^2 - im_W\Gamma_Z$. Let

$$\mathcal{Q}_{LL} = Q_f + \frac{(I_f - Q_f s_w^2)(\frac{1}{2} - s_w^2)}{c_w^2 s_w^2}\frac{s}{s - M_Z^2}, \quad \mathcal{Q}_{LR} = Q_f + \frac{(I_f - Q_f s_w^2)(-s_w^2)}{c_w^2 s_w^2}\frac{s}{s - M_Z^2}$$

$$\mathcal{Q}_{RL} = Q_f + \frac{(I_f - Q_f s_w^2)(\frac{1}{2} - s_w^2)}{c_w^2 s_w^2}\frac{s}{s - M_Z^2}, \quad \mathcal{Q}_{RR} = Q_f + \frac{(-Q_f s_w^2)(-s_w^2)}{c_w^2 s_w^2}\frac{s}{s - M_Z^2}$$

$$q_f + \overline{q}_f \to \mu^+\mu^- \ : \qquad \frac{1}{6}\frac{\pi\alpha^2}{s}\left[\frac{u^2}{s^2}(|\mathcal{Q}_{LL}|^2 + (|\mathcal{Q}_{RR}|^2) + \frac{t^2}{s^2}(|\mathcal{Q}_{LR}|^2 + (|\mathcal{Q}_{RL}|^2)\right]$$

$$u + \overline{d} \to \ell^+ \nu \ : \qquad \frac{1}{24} \frac{\pi \alpha_w^2}{s} \frac{t^2}{s^2} | \frac{s}{s - M_W^2} |^2$$

$$d + \overline{u} \to \ell^- \overline{\nu} \ : \qquad \frac{1}{24} \frac{\pi \alpha_w^2}{s} \frac{u^2}{s^2} | \frac{s}{s - M_W^2} |^2$$

parton + parton $\to \gamma$ + parton cross sections:

$$q_f + \overline{q}_f \to \gamma + g \ : \qquad \frac{4}{9} \frac{\pi \alpha \alpha_s}{s} Q_f^2 \left[\frac{u}{t} + \frac{t}{u} \right]$$

$$q_f + g \to \gamma + q_f \ : \qquad \frac{1}{6} \frac{\pi \alpha \alpha_s}{s} Q_f^2 \left[-\frac{s}{t} - \frac{t}{s} \right]$$

$$g + \overline{q}_f \to \gamma + \overline{q}_f \ : \qquad \frac{1}{6} \frac{\pi \alpha \alpha_s}{s} Q_f^2 \left[-\frac{s}{u} - \frac{u}{s} \right]$$

parton + parton $\to W$ + parton cross sections:

$$u + \overline{d} \to W^+ + g \ : \qquad \frac{1}{9} \frac{\pi \alpha_w \alpha_s}{s} (1 - \frac{m_W^2}{s}) \left[\frac{u}{t} + \frac{t}{u} + \frac{2 m_W^2 s}{ut} \right]$$

$$u + g \to W^+ + d \ : \qquad \frac{1}{24} \frac{\pi \alpha_w \alpha_s}{s} (1 - \frac{m_W^2}{s}) \left[-\frac{s}{t} - \frac{t}{s} - \frac{2 m_W^2 u}{st} \right]$$

$$g + \overline{d} \to W^+ + \overline{u} \ : \qquad \frac{1}{24} \frac{\pi \alpha_w \alpha_s}{s} (1 - \frac{m_W^2}{s}) \left[-\frac{s}{u} - \frac{u}{s} - \frac{2 m_W^2 t}{su} \right]$$

$$d + \overline{u} \to W^- + g \ : \qquad \frac{1}{9} \frac{\pi \alpha_w \alpha_s}{s} (1 - \frac{m_W^2}{s}) \left[\frac{u}{t} + \frac{t}{u} + \frac{2 m_W^2 s}{ut} \right]$$

$$d + g \to W^- + u \ : \qquad \frac{1}{24} \frac{\pi \alpha_w \alpha_s}{s} (1 - \frac{m_W^2}{s}) \left[-\frac{s}{t} - \frac{t}{s} - \frac{2 m_W^2 u}{st} \right]$$

$$g + \overline{u} \to W^- + \overline{d} \ : \qquad \frac{1}{24} \frac{\pi \alpha_w \alpha_s}{s} (1 - \frac{m_W^2}{s}) \left[-\frac{s}{u} - \frac{u}{s} - \frac{2 m_W^2 t}{su} \right]$$

parton + parton $\to t + \overline{t}$ cross sections:

Let m be the mass of the top quark or, below, the mass of a heavy pair-produced particle. $E = \frac{1}{2} s^{1/2}$, $p = (E^2 - m^2)^{1/2}$. Except in the prefactor, let $c = \cos\theta_*$ and $s = \sin\theta_*$.

$$q + \overline{q} \to t + \overline{t} \ : \qquad \frac{1}{9} \frac{\pi \alpha_s^2}{s} \frac{p}{E} \left[1 + \frac{m^2}{E^2} + \frac{p^2}{E^2} c^2 \right]$$

$$g + g \to t + \overline{t} \;:\qquad \frac{1}{12}\frac{\pi\alpha_s^2}{s}\frac{p}{E}\left[\frac{p^2 s^2(E^2 + m^2 + p^2c^2) + m^2(E^2 + p^2)}{E^2(E^2 - p^2c^2)}\right]\left[\frac{E^2 + p^2c^2}{E^2 - p^2c^2} - \frac{1}{8}\right]$$

parton + parton $\to S\overline{S}$, $F\overline{F}$ cross sections:

where S is a heavy scalar, F is a heavy spin $\frac{1}{2}$ particle. In each case, the particle belongs to the color representation X, with dimension d_X and Casimir $C_2(X)$. For color **3**, $d_3 = 3$, $C_2(3) = \frac{4}{3}$. For color **8**, $d_G = 8$, $C_2(G) = 3$. The rest of the notation is as for the top quark above.

$$q + \overline{q} \to S + \overline{S} \;:\qquad \frac{d_X C_2(X)}{72}\frac{\pi\alpha_s^2}{s}(\frac{p}{E})^3 s^2$$

$$g + g \to S + \overline{S} \;:\qquad \frac{d_X C_2^2(X)}{128}\frac{\pi\alpha_s^2}{s}\frac{p}{E}\left[\frac{p^4 s^4 + m^4}{E^2(E^2 - p^2c^2)}\right]\left[\frac{E^2 + p^2c^2}{E^2 - p^2c^2} + \frac{C_2(X) - \frac{1}{2}C_2(G)}{C_2(X)}\right]$$

$$q + \overline{q} \to F + \overline{F} \;:\qquad \frac{d_X C_2(X)}{36}\frac{\pi\alpha_s^2}{s}\frac{p}{E}\left[1 + \frac{m^2}{E^2} + \frac{p^2}{E^2}c^2\right]$$

$$g + g \to F + \overline{F} \;:\qquad \frac{d_X C_2^2(X)}{64}\frac{\pi\alpha_s^2}{s}\frac{p}{E}\left[\frac{p^2 s^2(E^2 + m^2 + p^2c^2) + m^2(E^2 + p^2)}{E^2(E^2 - p^2c^2)}\right]$$
$$\cdot\left[\frac{E^2 + p^2c^2}{E^2 - p^2c^2} + \frac{C_2(X) - \frac{1}{2}C_2(G)}{C_2(X)}\right]$$