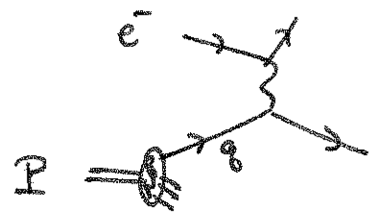
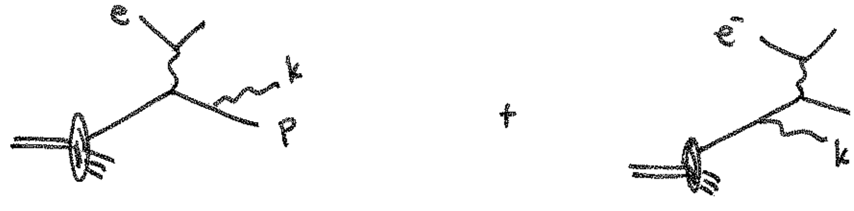
Evolution and Measurement of Parton Distributions

In the previous lecture, we analyzed the leading terms in the emission of quarks and gluons in QCD hard scattering processes. We saw that these terms are described by collinear emissions and can be accounted for by integrating the Altarelli-Parisi equations. I would now like to show that this same logic applies to quark and gluon emission from quarks and gluons in the initial state. This leads to an evolution of parton distribution functions with Q.

To see this, analyze the real gluon emission corrections to deep inelastic scattering. In a previous lecture, we computed

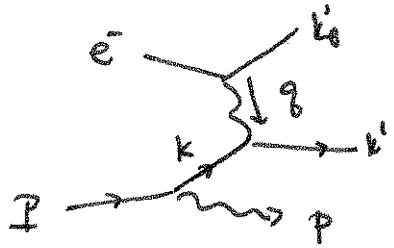


The leading QCD corrections come from



The first of these diagrams can be analyze just like those in the previous lecture. It leads to evolution of the fragmentation function associated with the development of the final quark into hadrons. The second diagram is a new effect.

To compute this effect, we apply the parton model to the diagram



This diagram has a singularity when the emitted gluon is <u>collinear</u> with the initial quark. Write

$$P = (P, 0, 0, P)$$

$$p = \left((1-z)P,\ -p_T,\ 0,\ (1-z)P - \frac{p_T^2}{2(1-z)P}\right)$$

$$k = \left(zP,\ p_T,\ 0,\ zP + \frac{p_T^2}{2(1-z)P}\right)$$

I have put P and p on shell: $P^2 = p^2 = 0$. Then k^2 is given by

$$k^2 = -p_T^2 - \frac{p_T^2}{(1-z)} z = -\frac{p_T^2}{(1-z)}$$

The matrix element is

$$i\mathcal{M} = i\mathcal{M}_0(e^- q(k) \to e^- q) \cdot \frac{i}{\left(-\frac{p_T^2}{(1-z)}\right)} \cdot \left[ig t^a \bar{u}(k) \cdot \gamma \varepsilon^*(p)\, u(P)\right]$$

The gluon emission matrix element in brackets is exactly the one that we analyzed in the previous lecture. Turn this into a cross section:

$$|\mathcal{M}(\bar{e}q \to \bar{e}qg)|^2 = |\mathcal{M}_0(\bar{e}q(k) \to \bar{e}q)|^2 \cdot \frac{(1-z)^2}{p_T^4} \cdot |\mathcal{M}(q(P) \to qg)|^2$$

$$\sigma(\bar{e}q \to \bar{e}qg) = \frac{1}{2s}\int \frac{d^3k' \, d^3k_e' \, d^3p}{(2\pi)^3 \, 2k' \, 2k_e' \, 2p} (2\pi)^4 \delta(\;) \cdot |\mathcal{M}|^2$$

p is collinear with the initial momentum P, so write

$$\int \frac{d^3p}{(2\pi)^3 2p} = \int \frac{dp_\parallel \, d^2p_T}{(2\pi)^3 \, 2p} \approx \int \frac{dz}{(1-z)} \frac{\pi \, dp_T^2}{16\pi^3}$$

so

$$\sigma(\bar{e}q(P) \to \bar{e}qg) = \frac{1}{2s}\int \frac{dz}{1-z} \frac{dp_T^2}{(4\pi)^2} \underbrace{\int d\Pi_2}_{\text{eq final phase space}} |\mathcal{M}_0|^2 \cdot \frac{(1-z)^2}{p_T^4} |\mathcal{M}(q \to qg)|^2$$

The cross section for $q(p)\bar{e}$ scattering is

$$\sigma(\bar{e}q(p) \to \bar{e}q) = \frac{1}{2(\underbrace{zs}_{=\hat{s}})}\int d\Pi_2 \, |\mathcal{M}_0|^2$$

so

$$\sigma(\bar{e}q(P) \to \bar{e}qg) = \int \frac{dz \; z}{(1-z)} \frac{d p_T^2}{(4\pi)^2} \cdot \frac{(1-z)^2}{p_T^4} |\mathcal{M}(q \to qg)|^2$$

$$\cdot \; \sigma(\bar{e}q(p = zP) \to \bar{e}q)$$

Now represent $|\mathcal{M}|^2$, as we did in the previous lecture, in terms of the splitting function:

$$|\mathcal{M}(q \to q+g)|^2 = \frac{2 g_s^2 p_T^2}{z(1-z)} \, P_{q \leftarrow q}(z)$$

Then

$$\sigma(e^- q(P) \to e^- q g) = \int dz \, \frac{\alpha_s}{2\pi} \int \frac{dp_T^2}{p_T^2} \, P_{q \leftarrow q}(z) \cdot \sigma(e^- q(zP) \to e^- q)$$

Now integrate over the parton distribution of $q(P)$ and combine with the leading term:

$$\sigma(e^- p \to e^- + X) = \int d\xi \, f_q(\xi) \int dz \left[\delta(z-1) + \frac{\alpha_s}{\pi} \int \frac{dp_T}{p_T} \, P_{q \leftarrow q}(z) \right]$$

$$\cdot \quad \sigma^0(e^- q(p = \xi z P_p) \to e^- q)$$

This has the form of a parton-model cross section

$$\sigma(e^- p \to e^- X) = \int dx \, \bar{f}_q(x) \, \sigma(e^- q(p = x P_p) \to e^- q)$$

where

$$\bar{f}_q(x) = f_q(x) + \frac{\alpha_s}{\pi} \int \frac{dz}{z} \int \frac{dp_T}{p_T} \, P_{q \leftarrow q}(z) \, f_q\!\left(\frac{x}{z}\right)$$

so the collinear emission can be interpreted as a correction to the parton distribution function. p_T is integrated up to values as large as the momentum Q transferred from the electron. We can treat the Q-dependence just as we did in the previous lecture:

$$\frac{d}{d\log Q} \left(f_q(x, Q) \right) = \frac{\alpha_s(Q)}{\pi} \int \frac{dz}{z} \, P_{q \leftarrow q}(z) \, f_q\!\left(\frac{x}{z}\right)$$

This analysis can be applied generally to collinear radiation in the initial state of a hadron scattering process. Each collinear radiation gives a factor

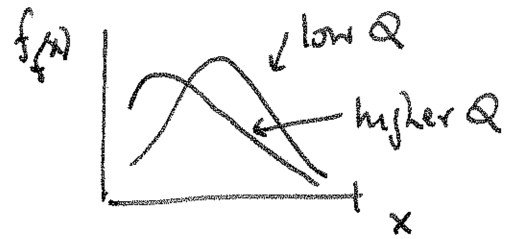
$$\alpha_s \log Q$$

and we must sum all of these factors. To do this, absorb the emission factors into the parton distribution functions. These functions will then depend on Q, with the Q-dependence given by the Altarelli-Parisi equations

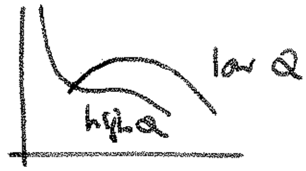
$$\frac{d}{d\log Q} f_f(x,Q) = \frac{\alpha_s(Q)}{\pi} \int_0^1 \frac{dz}{z} \sum_{\hat{f}} P_{f \leftarrow \hat{f}}(z)\, f_{\hat{f}}\left(\frac{x}{z}\right)$$

In our earlier discussion of the parton model, I described the pdf's as being independent of Q. However, we now see that, in QCD, this is just a first approximation. Still, the Q-dependence is slow, on the scale of $\underline{\log Q}$.

It is easy to visualize the form of the Q-dependence. As Q increases, there is more phase space for quarks to emit gluons. Then the bulk of $f_f(x)$ moves down to lower values of x



However, there is another effect. Gluons in the proton split to $q\bar{q}$. This produces new quarks and antiquarks at very low values of x



In all, we find the evolution of quark distribution functions in the proton shown in <u>Figs. p.2</u>.

<u>Figs. p.3</u> shows the evolution of

$$F_2(x,Q) = \sum_{f=u,d,s,c}' x\, Q_f^2 \left[f_f(x,Q) + f_{\bar{f}}(x,Q)\right]$$

from measurements of deep inelastic scattering. You can see a slow evolution, on a log scale, of just the form ~~that~~ I have described: $F_2(x)$ <u>decreases</u> at large x and in<u>creases</u> more and <u>more</u> <u>rapidly</u> as $x \to 0$.

Once we understand that parton distributions run with Q, we can set up a procedure for determining the pdf's experimentally. We parametrize the various pdf's of the proton at a fixed, low value of Q, use the Altarelli-Parisi equations to compute the pdf's at any other value of Q, and fit to available cross sections. I will describe the sources of data for pdf's in a moment. Let's first define the problem. For the proton, there are 11 pdf's:

$$f_u(x,Q) \quad f_d(x,Q) \quad f_s(x,Q) \quad f_c(x,Q) \quad f_b(x,Q)$$

$$f_{\bar{u}}(x,Q) \quad f_{\bar{d}}(x,Q) \quad f_{\bar{s}}(x,Q) \quad f_{\bar{c}}(x,Q) \quad f_{\bar{b}}(x,Q)$$

<u>and</u> $\quad f_g(x,Q)$

For a neutron target (in D or a heavy nucleus), we can take

$$f_{u\leftarrow p}(x) = f_{d\leftarrow n}(x) \qquad f_{d\leftarrow p}(x) = f_{u\leftarrow n}(x)$$

to the accuracy (~1%) of isospin symmetry. For a $\bar{p}$ (in the Tevatron collider experiments, we can take

$$f_{u\leftarrow p}(x) = f_{\bar{u}\leftarrow \bar{p}}(x) \qquad f_{\bar{u}\leftarrow p}(x) = f_{u\leftarrow \bar{p}}(x) \quad \text{etc.}$$

Since c,b are heavy, it is a good approximation to assume that there are no c,b in the proton at low Q, and that all heavy quarks are generated by gluon splitting

This implies $f_c = f_{\bar{c}}$, $f_b = f_{\bar{b}}$. It is often also assumed that $f_s = f_{\bar{s}}$.

Now let's go through the sources of information on pdf's. Deep inelastic scattering from a proton measures:

$$F_2(x) = \left(\tfrac{2}{3}\right)^2 \times f_u(x) + \left(-\tfrac{1}{3}\right)^2 \times f_d(x) + (\text{antiquarks}) + (s\,c\,b)$$

Deep inelastic scattering from a neutron measures:

$$F_2^{(n)}(x) = \left(\tfrac{2}{3}\right)^2 \times f_d(x) + \left(-\tfrac{1}{3}\right)^2 \times f_u(x) + \ldots$$

by the relation at the top of this page. So, by measuring deep inelastic scattering on Hydrogen and Deuterium or Iron targets, we can separate the u and d contributions.

An even more effective way to do this separation is with neutrino scattering. The charged-current neutrino reactions pick out specific initial-state quarks. In addition, the weak interactions act only on left-handed fermions and right-handed antifermions. So neutrino scattering accesses

$$\nu_{\mu L}\, d_L \to \mu^-_L\, u_L \qquad \nu_{\mu L}\, \bar{u}_R \to \mu^-_L\, \bar{d}_R$$

and antineutrino scattering accesses

$$\bar{\nu}_{\mu R}\, u_L \to \mu^+_R\, d_L \qquad \bar{\nu}_{\mu R}\, \bar{d}_R \to \mu^+_R\, \bar{u}_R$$

The $\nu_L \bar{u}_R$ process and the $\bar{\nu}_R u_L$ process have mismatched helicities; in our analysis of deep inelastic scattering, these channels got the factor $(1-y)^2$. So the cross section formulae are:

$$\frac{d^2\sigma}{dx\,dy}(\nu_\mu p \to \mu^- X) = \frac{G_F^2 s}{\pi}\left[\, x f_d(x) + x f_{\bar{u}}(x)\,(1-y)^2 \,\right]$$

$$\frac{d^2\sigma}{dx\,dy}(\bar{\nu}_\mu p \to \mu^+ X) = \frac{G_F^2 s}{\pi}\left[\, x f_u(x)\,(1-y)^2 + x f_{\bar{d}}(x) \,\right]$$

These are wonderful formulae. Since there are more quarks than antiquarks in the proton at high x, we expect

$$\frac{d^2\sigma}{dx\,dy}(\nu p) \sim 1 \qquad \frac{d^2\sigma}{dx\,dy}(\bar{\nu} p) \sim (1-y)^2$$

Figs. p. 4 shows the data. It really works, except at $x < 0.1$, where we have more or less equal numbers of quarks and antiquarks. Figs. p. 5 shows the Q-dependence of the pdf's contributing to deep inelastic neutrino scattering.

Gluons are invisible in deep inelastic lepton scattering, but any rise in the $f_q(x)$ with Q is due to gluon splitting

$$g \rightsquigarrow \begin{smallmatrix} q \\ \bar{q} \end{smallmatrix}$$

Thus, the rate of change of F_2 with Q measures the gluon distribution. The splitting function $P_{g \leftarrow g}(x)$ has a large coefficient, so gluons multiply in Altarelli Parisi evolution. This effect is visible as a large rise in $F_2(x)$ with Q for $x \ll 0.1$. Figs. p. 6 compares $F_2(x)$ at $Q = 1.8$ GeV and $Q = 9.5$ GeV at very low values of x.

Additional information on pdf's comes from collider data. The Drell-Yan cross section measures the quark and antiquark distributions. The Tevatron collider is a $p\bar{p}$ collider, so high-energy Drell-Yan events measure mainly f_u, f_d. However, there are fixed-target Drell Yan measurements from Fermilab that give evidence on $f_{\bar{u}}, f_{\bar{d}}$ in the proton at large x. In particular, Figs p. 7, from the experiment E866, shows that

$$\frac{\sigma(\text{Drell-Yan},\ pd)}{\sigma(\text{Drell-Yan},\ pp)} > 2$$

so there are more $\bar{d}$ than $\bar{u}$ in the proton. This is expected if the proton can turn, by a fluctuation, into $p \to \pi^+ n$. However, it excludes the simplifying assumption $f_{\bar{u}}(x) = f_{\bar{d}}(x)$ in the pdf fit.

Finally, look again at the comparison of the jet cross section at Tevatron collider energies with the parton model. Figs. p. 8 shows the comparison given in a previous lecture. Actually, the CDF and DØ experiments found difficulties with this comparison in the highest p_T bins. The CTEQ group (see below) argued that the discrepancy could be fixed by making the gluon distribution $f_g(x)$ slightly harder, without affecting other constraints on the pdf's. This is now the strongest constraint on the gluon pdf at large x. The comparison of the CDF and DØ data to pdf sets with and without this adjustment is shown in Figs. p. 9,10. The comparison of theory and experiment for the best current set of pdf's is shown in Figs. p. 11.

Now we can put the ingredients together. This has been done by several groups and, most notably, by

MRST (Martin, Roberts, Stirling, Thorne - Durham)

CTEQ (Huston Tung (Michigan State) Kuhlman (Argonne) Morfin (Fermilab), et al.

The basic strategy of both groups is to take the basic pdf's to be

$$f_u, \quad f_d \quad f_{\bar u} \quad f_{\bar d} \quad f_s = f_{\bar s} \quad f_g$$

at low Q, and parametrize each of these by a simple form

$$f_a(x, Q_0) = x^{-\alpha} (1-x)^{\beta} \, P_a(x)$$

where $P_a(x)$ is a polynomial. For example, CTEQ uses $\alpha = 1.5$, $\beta = 3$, $Q_0 = 1$ GeV. They evolve these functions to higher Q using the Altarelli Parisi equations — and including all $O(\alpha_s)$ corrections in the formulae for evolution and cross sections. They then fit to the data I have described, subject to the sum rules:

$$2 = \int_0^1 dx \, [f_u(x) - f_{\bar u}(x)] \qquad \text{net } 2 \; u\text{'s in } p$$

$$1 = \int_0^1 dx \, [f_d(x) - f_{\bar d}(x)] \qquad \text{net } 1 \; d \text{ in } p$$

$$1 = \int dx \; x \, [f_g(x) + f_u(x) + f_d(x) + \dots + f_{\bar u}(x) + f_{\bar d}(x) + \dots]$$

(sum over partons gives to total momentum of the p)

The technical details of this procedure, including the treatment of the various data sets, is nicely described in the CTEQ paper hep-ph/9903282. Figs p. 12 shows a result of this process, the most recent MRST fit, evaluated at $Q = 3.$[redacted] GeV.

Although these fits do not include c and b at low Q, the Altarelli Parisi equations generate substantial amounts of $c\bar{c}$ and $b\bar{b}$ at high Q and low x. The HERA experiments have recently verified this by measuring the crosssections for charm and bottom production in deep inelastic scattering. The comparison to the MRST and CTEQ distributions is shown in Figs. p. 13.

The <u>Durham</u> group has put together a Web site that collects all global fits to parton distributions. The site offers computer programs that generate these distributions, and a graphical display that allows easy comparison of the various fits. See: http://durpdg.dur.ac.uk/HEPDATA/PDF/. There is a link to this page on the course Web page.