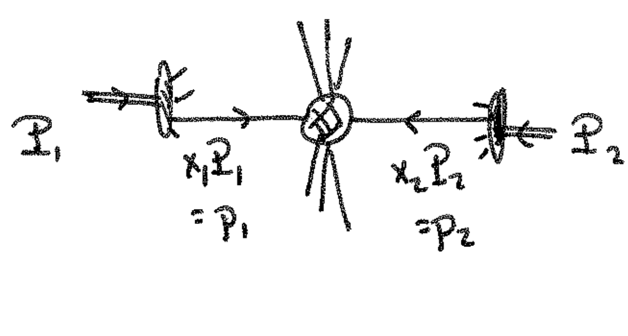
The Parton Model in Hadron-Hadron Collisions

In the previous lecture, I introduced the parton model of hadron structure. The proton or other hadron is treated as an assemblage of collinearly moving quarks and gluons, all of which share the proton's momentum. The probability of finding a quark or gluon of flavor f at a momentum fraction ξ $(0 \leq \xi \leq 1)$ is given by parton distribution functions (pdf's):

$$d\xi \quad f_f(\xi) \qquad \text{for} \quad p^\mu = \xi P^\mu$$

The model as I have defined it ignores initial-state transverse momentum of the parton; this is reasonable because we will apply the model to processes with momentum transfer $Q \gg 1$ GeV. I showed that the model gives a good description of $e^- p$ inelastic scattering.

In 1970, Drell and Yan proposed that this model should also be applied to hadron-hadron collisions. We can easily write the basic formulae, as always ignoring the proton and quark masses. A hard scattering process in pp scattering is viewed as a scattering process of partons with longitudinal fractions x_1, x_2 respectively.



$$\sigma(pp \to \underbrace{Y}_{} + X) = \sum_{f_1} \int dx_1\, f_{f_1}(x_1) \sum_{f_2} \int dx_2\, f_{f_2}(x_2)$$

result of the hard process

$$\cdot\ \sigma(\, f_1(p_1)\, f_2(p_2) \to Y\,)$$

For the full pp scattering, $s = (P_1 + P_2)^2 \approx 2P_1 \cdot P_2$. The invariants of the parton parton scattering are often written with hats for clarity. Then

$$\hat{s} = (p_1 + p_2)^2 = 2p_1 \cdot p_2 = x_1 x_2\, s$$

Drell and Yan applied the parton model to lepton pair production in pp collisions. It is simple and instructive to work out all of the formulae for this process. At leading order, the parton process is a QED cross section which is just the reverse of the one considered in the previous lecture: $q_f \bar{q}_f \to \mu^- \mu^+$:

$$\frac{d\sigma}{d\cos\theta_{cm}}(q_f \bar{q}_f \to \mu^- \mu^+) = \frac{\pi\alpha^2}{2\hat{s}}(1+\cos^2\theta) \cdot \frac{1}{3} Q_f^2$$

This is identical to the cross section quoted last time except that the factor $(3Q_f^2)$ has become $(\frac{1}{3}Q_f^2)$. Now the quark and antiquark are in the initial state, and so we must average over colors:

$$\frac{1}{3} \cdot \frac{1}{3}\, \mathrm{tr}[1] = \frac{1}{3} \cdot \frac{1}{3} \cdot 3 = \frac{1}{3}$$

The total cross section is

$$\sigma(q_f \bar{q}_f \to \mu^- \mu^+) = \frac{4\pi\alpha^2}{3\hat{s}} \cdot \frac{1}{3} Q_f^2$$

so the cross section for the Drell-Yan process is

$$\sigma(pp \to \mu^+\mu^- + X) = \int dx_1 dx_2 \sum_f \left[f_f(x_1) f_{\bar f}(x_2) + f_{\bar f}(x_1) f_f(x_2) \right] \cdot$$

$$\hat{s} = x_1 x_2 s \qquad \cdot \left(\frac{1}{3} Q_f^2\right) \cdot \left(\frac{4\pi\alpha^2}{3\hat{s}}\right)$$

$f_{\bar f}(x)$ is the pdf for an antiquark of flavor f in the proton. When we compute cross sections for the Tevatron, we need the pdf's for an antiproton. These are given by

$$f_f(x)\Big|_{\bar p} = f_{\bar f}(x)\Big|_p \quad \text{and vice versa.}$$

Just as in deep inelastic scattering, we can use the kinematics of the Drell-Yan process to infer the momentum fractions $x_1 x_2$ in each event. Let M be the mass of the $\mu^+\mu^-$ pair: $q^2 = \hat{s} = M^2$. The quark and antiquark have negligible transverse momentum, so the momentum q of the $\mu^+\mu^-$ pair is along the $\hat{3}$ axis. We can parametrize this as

$$q^\mu = (M\cosh y,\ \vec{0},\ M\sinh y)$$

where y is the rapidity. The initial momentum is

$$p_1 + p_2 = ((x_1+x_2)E,\ \vec{0},\ (x_1-x_2)E)$$

in the CM system $P_1 = (E, \vec{0}, E)$ $P_2 = (E, \vec{0}, -E)$. Now we can solve for x_1, x_2:

$$x_1 = \frac{M}{\sqrt{s}} e^{y} \qquad x_2 = \frac{M}{\sqrt{s}} e^{-y} \qquad \sqrt{s} = 2E$$

Change variables in the cross section formula: this requires the Jacobian

$$\frac{\partial(x_1,x_2)}{\partial(M,y)} = \begin{vmatrix} \frac{1}{\sqrt{s}} e^{y} & \frac{1}{\sqrt{s}} e^{-y} \\ \frac{M}{\sqrt{s}} e^{y} & -\frac{M}{\sqrt{s}} e^{-y} \end{vmatrix} = \frac{2M}{s} = \frac{2}{M}\frac{\hat{s}}{s} = \frac{2}{M} x_1 x_2$$

Then

$$\frac{d\sigma}{dM\,dy}(pp \to \mu^-\mu^+ + X) = \sum_f \left[x_1 f_f(x_1)\, x_2 f_{\bar{f}}(x_2) + x_1 f_{\bar{f}}(x_1)\, x_2 f_f(x_2) \right] \cdot \left(\frac{1}{3} Q_f^2\right) \cdot \frac{8\pi\alpha^2}{M^3}$$

with x_1, x_2 given by the formulae at the top of this page.

In the discussion above, I have used the QED formulae for $q\bar{q} \to \mu^+\mu^-$. At the energies of the Tevatron and the LHC, the weak interactions also come into play in this process. Including both the photon and the Z^0, the cross section on p. 2 is modified to:

$$\sigma(q_f \bar{q}_f \to \mu^-\mu^+) = \frac{\pi\alpha^2}{3\hat{s}} \cdot \frac{1}{3} \left\{ \left| (-Q_f) + \frac{\hat{s}}{\hat{s}-m_Z^2} \frac{(\pm\frac{1}{2} - Q_f s_w^2)(-\frac{1}{2}+s_w^2)}{c_w^2 s_w^2} \right|^2 \right.$$

$$+ \left| (-Q_f) + \frac{\hat{s}}{\hat{s}-m_Z^2} \frac{(\pm\frac{1}{2} - Q_f s_w^2)(s_w^2)}{c_w^2 s_w^2} \right|^2 + \left| (-Q_f) + \frac{\hat{s}}{\hat{s}-m_Z^2} \frac{(-Q_f s_w^2)(-\frac{1}{2}+s_w^2)}{c_w^2 s_w^2} \right|^2$$

$$\left. + \left| (-Q_f) + \frac{\hat{s}}{\hat{s}-m_Z^2} \frac{(-Q_f s_w^2)(s_w^2)}{c_w^2 s_w^2} \right|^2 \right\}$$

with $s_w^2 = \sin^2\theta_w$, $c_w^2 = \cos^2\theta_w$, and $\pm\frac{1}{2} = +\frac{1}{2}$ for u, c $\;-\frac{1}{2}$ for d, s, b

The interference factors refer to the four possible helicity amplitudes.

We have <u>constructive</u> interference in $q_L\bar{q}_R \to \mu^-_L\mu^+_R$, $q_R\bar{q}_L \to \mu^-_R\mu^+_L$ and <u>destructive</u> interference in the opposite two channels.

Of course, the most prominent feature of this formula is the presence of a narrow resonance at $\hat{s} = m_Z^2$. Figs. p. 2 shows CDF data on the Drell-Yan cross section. The Z^0 resonance is a prominent feature. Figs. p. 3 shows a beautiful $Z^0 \to e^+e^-$ event.

The cross section for production of the Z and W resonances in hadron-hadron collisions are very simple; let's work them out. For W^+:

$$i\mathcal{M}(u\bar{d} \to W^+)$$

$$= \frac{ig}{\sqrt{2}}\, \bar{v}_R(\bar{p})\gamma^\mu u_L(p) \cdot \epsilon^*_\mu(q)$$

The W couples only to $u_L\bar{d}_R$. In the previous lecture, we computed

$$\bar{v}_R(\bar{p})\gamma^\mu u_L(p) = \sqrt{2}\cdot 2E \cdot \epsilon_-^\mu(q)$$

This overlaps only with the $J^3 = -1$ (transverse) polarization state of the W^+. Then

$$i\mathcal{M}(u_L\bar{d}_R \to W^+_L) = -ig\sqrt{\hat{s}} = -igm_W$$

The cross section, from unpolarized $u\bar{d}$, is

$$\sigma(u\bar{d} \to W^+) = \frac{1}{2\hat{s}}\int \frac{d^4q}{(2\pi)^4} \cdot \overbrace{\frac{1}{3}}^{\text{color avg}} \cdot \overbrace{\frac{1}{4}}^{\text{pol. avg.}} \cdot |gm_W|^2\; 2\pi\delta(\hat{s}-m_W^2)\,(2\pi)^4\delta^{(4)}(q-p-\bar{p})$$

$$= \frac{\pi g^2}{12}\,\delta(\hat{s}-m_W^2)$$

or

$$\sigma(u\bar{d} \to W^+) = \frac{\pi^2}{3}\,\alpha_w\,\delta(\hat{s}-m_W^2)$$

where $\alpha_w = \frac{g^2}{4\pi} = \frac{1}{29.6}$. The similar formula for Z^0 is

$$\sigma(q_f\bar{q}_f \to Z^0) = \frac{2\pi^2\alpha_w}{3c_w^2}\left[\left(\pm\tfrac{1}{2} - Q_f s_w^2\right)^2 + \left(Q_f s_w^2\right)^2\right]\delta(\hat{s} - m_Z^2)$$

In these formulae, the delta functions reduce the integrals over longitudinal fractions

$$\int dx_1\, dx_2\; f_f(x_1) f_{\bar{f}}(x_2)\; \delta(\hat{s} - m_W^2) \qquad \hat{s} = x_1 x_2 s$$

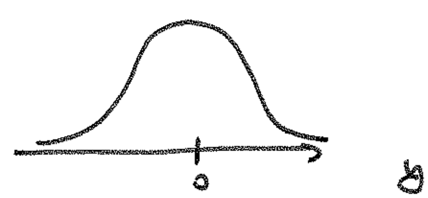
$$= \int \frac{dx_1}{x_1 s}\; f_f(x_1)\, f_{\bar{f}}\left(\frac{m_W^2}{x_1 s}\right)$$

$$= \int dy\; \frac{1}{s}\; f_f(x_1)\, f_{\bar{f}}\left(\frac{m_W^2}{x_1 s}\right)$$

so

$$\frac{d\sigma}{dy}(pp \to W^+ + X) = \frac{\pi^2}{3s}\alpha_w\left[f_u(x_1) f_{\bar{d}}\left(\frac{m_W^2}{x_1 s}\right) + f_c(x_1) f_{\bar{s}}\left(\frac{m_W^2}{x_1 s}\right) + f_{\bar{d}}(x_1) f_u\left(\frac{m_W^2}{x_1 s}\right) + f_{\bar{s}}(x_1) f_c\left(\frac{m_W^2}{x_1 s}\right)\right]$$

The product of pdf's. falls off as $x_1 \to 1$ and as $\frac{m_W^2}{x_1 s} \to 1$, giving a distribution in y peaked near $y = 0$:



For pp or $p\bar{p} \to Z^0$ this distribution is symmetric about $y = 0$.
The CDF data on the shape is shown in Figs. p4. (I will

discuss the normalization in a later lecture.) Figs p. 6 shows the rapidity distributions in $p\bar{p} \to W^+, W^-$.

There are a few more things to say about pp or $p\bar{p} \to W^\pm$.

W is visible at a hadron collider in its decay to $\ell\nu$. The ν is invisible, but it shows up as missing E_T. A beautiful CDF event of this type is shown in Figs. p. 5: the green tower in the lego plot is an isolated muon.

We can find evidence for the W – and measure the W mass – in two ways. First, as we consider the reaction

$$u\bar{d} \to W^+ \to \ell^+\nu$$

concentrate on the charged lepton and convert $\cos\theta_{cm}$ in the parton-parton system to lepton p_T:

$$p_T(\ell^+) = \frac{m_W}{2}\sin\theta$$

$$dp_T = \frac{m_W}{2}\cos\theta\, d\theta = \frac{m_W}{2}\frac{\cos\theta}{\sin\theta}\, d\cos\theta$$

so

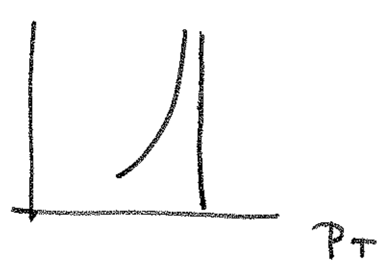
$$\frac{d\sigma}{d\cos\theta} = \frac{m_W}{2}\frac{\cos\theta}{\sin\theta}\frac{d\sigma}{dp_T}$$

or

$$\frac{d\sigma}{dp_T} = \frac{2}{m_W}\frac{p_T}{\sqrt{m_W^2/4 - p_T^2}}\frac{d\sigma}{d\cos\theta}$$

$d\sigma/d\cos\theta$ is a smooth distribution, so $\frac{d\sigma}{dp_T}$ has a peak at $p_T = \frac{m_W}{2}$.

This is called the Jacobian peak: it is located at the maximum value that p_T can take in the simple parton process.



In the real situation, the Jacobian peak is rounded by the W width and the initial-state p_T of the quarks.

Figs p. 7 shows the p_T distributions for the lepton and the $\not{E}_T$ from CDF.

If we observe both a lepton and $\not{E}_T$, we can combine these to make a more effective estimate of the W mass. Write the lepton and neutrino vectors, ignoring masses, as

$$p(\ell^+) = (E_{T\ell} \cosh y_\ell,\ \vec{p}_T(\ell),\ E_{T\ell} \sinh y_\ell) \qquad E_{T\ell} = |\vec{p}_{T\ell}|$$

$$p(\nu) = (E_{T\nu} \cosh y_\nu,\ \vec{p}_T(\nu),\ E_{T\nu} \sinh y_\nu) \qquad E_{T\nu} = |\vec{p}_{T\nu}|$$

If we could measure both vectors, the W mass would be

$$m_W^2 = (p(\ell^+) + p(\nu))^2 = 2p(\ell^+) \cdot p(\nu)$$

$$= 2\left\{E_{T\ell} E_{T\nu} (\cosh y_\ell \cosh y_\nu - \sinh y_\ell \sinh y_\nu) - \vec{p}_T(\ell) \cdot \vec{p}_T(\nu)\right\}$$

or

$$m_W^2 = 2\left\{|\vec{p}_{T\ell}||\vec{p}_{T\nu}| \cosh(y_\ell - y_\nu) - \vec{p}_{T\ell} \cdot \vec{p}_{T\nu}\right\}$$

Since we do not observe the neutrino vector or the longitudinal momentum balance, we cannot observe y_ν. However, we can observe the missing p_T, and this should be approximately $\vec{p}_{T\nu}$. Then define the transverse mass

$$m_{TW}^2 \equiv 2\left\{|\vec{p}_{T\ell}||\vec{p}_{T\nu}| - \vec{p}_{T\ell} \cdot \vec{p}_{T\nu}\right\}$$

we see that

$$m_{TW}^2 \leq m_W^2$$

for every event. So, if the distribution of m_{TW} has a sharp edge at the high end, it is a good estimator of m_W. Figs p. 8 shows recent CDF data on m_{TW}, using $\not{E}_T$ to estimate $\vec{p}_{T\nu}$. The cutoff at the high end is sharp; the rounding is mainly due to the width of the W. Correcting for detector effects, and calibrating to the m_{TZ} distribution and the accurately known Z mass, it is possible to measure the W mass very precisely. The current (2008) W mass from the Tevatron experiments is

$$m_W = 80.430 \pm .040 \text{ GeV}$$

to be compared to the determination from the LEP 2 experiments

$$m_W = 80.376 \pm .033 \text{ GeV}.$$

In the case that we have missing energy due to massive particles, the transverse mass technique is still effective. The formula above generalizes as follows:

$$E_{T1} = (m_1^2 + p_{T1}^2)^{1/2} \qquad E_{T2} = (m_2^2 + p_{T2}^2)^{1/2}$$

$$m^2(12) = m_1^2 + m_2^2 + 2(E_{T1}E_{T2}\cosh(y_1 - y_2) - \vec{p}_{T1}\cdot\vec{p}_{T2})$$

$$m_T^2(12) \equiv m_1^2 + m_2^2 + 2(E_{T1}E_{T2} - \vec{p}_{T1}\cdot\vec{p}_{T2})$$

and again $\quad m_T^2(12) \leq m^2(12)$.

We can use the same methods we have applied to the Drell-Yan process to discuss parton-parton scattering in QCD. Parton-parton elastic scattering with large momentum transfer leads to events with two jets at large p_T. Using the formulae on p. 7, we can write the cross section for jet production as

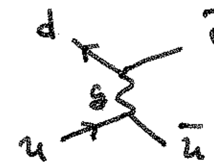
$$\frac{d\sigma}{dp_T(\text{jet})} = \sum_{1,2,3,4} \int dx_1\, dx_2\; f_1(x_1) f_2(x_2) \cdot \frac{2p_T}{\sqrt{\hat{s}}\,(\hat{s}/4 - p_T^2)}$$

$$\cdot \frac{d\sigma}{d\cos\theta_*}(1+2 \to 3+4)$$

where 1,2,3,4 are quark, antiquark, or gluon flavors and $\cos\theta_*$ is the parton-parton CM scattering angle. To use this formula, we need to go through the list of possible parton parton scattering processes and work out the relevant cross sections.

The easiest cross sections to work out are those involving two different quark species. Begin with

$$u\bar{u} \to d\bar{d}$$



This process is closely analogous to the QED process $e^-e^+ \to q\bar{q}$; the amplitude is the same except for color factors:

$$i\mathcal{M}(u_L\bar{u}_R \to d_L\bar{d}_R) = i\mathcal{M}(u_R\bar{u}_L \to d_R\bar{d}_L) = 2ig_s^2\; t^a t^a \left(\frac{\hat{u}}{\hat{s}}\right)$$

$$i\mathcal{M}(u_L\bar{u}_R \to d_R\bar{d}_L) = i\mathcal{M}(u_R\bar{u}_L \to d_L\bar{d}_R) = 2ig_s^2\; t^a t^a \left(\frac{\hat{t}}{\hat{s}}\right)$$

where t^a is an SU(3) representation matrix.

The square of these amplitudes contains the color factor

$$\underbrace{\frac{1}{3}\cdot\frac{1}{3}}_{\text{color avg.}} \;\operatorname{tr} t^a t^b \;\operatorname{tr} t^b t^a$$

$$= \frac{1}{9}\;\frac{1}{2}\delta^{ab}\;\frac{1}{2}\delta^{ab}$$

$$= \frac{1}{4\cdot 9}\cdot 8 = \frac{2}{9}$$

so

$$\frac{d\sigma}{d\cos\theta_*}(u\bar{u}\to d\bar{d}) = \frac{1}{2\hat{s}}\;\frac{1}{16\pi}\;4g_s^4\cdot\underbrace{\frac{2}{9}}_{\text{color}}\cdot\underbrace{\frac{2}{4}}_{\text{pol. avg.}}\cdot\frac{\hat{u}^2+\hat{t}^2}{s^2}$$

$$\frac{d\sigma}{d\cos\theta_*}(u\bar{u}\to d\bar{d}) = \frac{2}{9}\,\frac{\pi\alpha_s^2}{\hat{s}}\left(\frac{\hat{u}^2+\hat{t}^2}{s^2}\right)$$

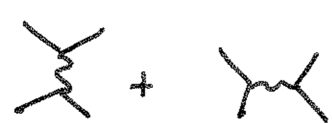
By crossing, we obtain

$$\frac{d\sigma}{d\cos\theta_*}(ud\to ud) = \frac{d\sigma}{d\cos\theta_*}(\bar{u}\bar{d}\to\bar{u}\bar{d})$$

$$= \frac{2}{9}\,\frac{\pi\alpha_s^2}{\hat{s}}\left(\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\right)$$

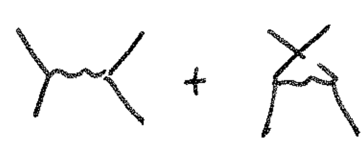
There are many more processes to consider:

identical quark scattering $\quad u\bar{u}\to u\bar{u}$



$uu\to uu$

$\bar{u}\bar{u}\to\bar{u}\bar{u}$



quark-gluon scatt'g: $u\bar{u} \to gg$

$gg \to u\bar{u}$

$ug \to ug$

$\bar{u}g \to \bar{u}g$

gluon-gluon scatt'g: $gg \to gg$

It is not so hard to compute all of the diagrams, but I will show you some more streamlined methods to do these calculations later in the course. So, for now, I would just like to quote the results: (dropping $\hat{}$ for convenience)

$$\frac{d\sigma}{d\cos\theta_*}(u\bar{u} \to d\bar{d}) = \frac{2}{9}\frac{\pi\alpha_s^2}{s}\left(\frac{u^2+t^2}{s^2}\right)$$

$$\frac{d\sigma}{d\cos\theta_*}(u\bar{u} \to u\bar{u}) = \frac{2}{9}\frac{\pi\alpha_s^2}{s}\left[\frac{u^2+t^2}{s^2} + \frac{u^2+s^2}{t^2} - \frac{2}{3}\frac{u^2}{st}\right]$$

$$\frac{d\sigma}{d\cos\theta_*}(ud \to ud) = \frac{2}{9}\frac{\pi\alpha_s^2}{s}\left[\frac{u^2+s^2}{t^2}\right]$$

$$\frac{d\sigma}{d\cos\theta_*}(uu \to uu) = \frac{2}{9}\frac{\pi\alpha_s^2}{s}\left[\frac{u^2+s^2}{t^2} + \frac{t^2+s^2}{u^2} - \frac{2}{3}\frac{s^2}{ut}\right]$$

$$\frac{d\sigma}{d\cos\theta_*}(u\bar{u}\to gg) = \frac{16}{27}\,\frac{\pi\alpha_s^2}{s}\left[\frac{u}{t}+\frac{t}{u}-\frac{9}{4}\,\frac{u^2+t^2}{s^2}\right]$$

$$\frac{d\sigma}{d\cos\theta_*}(gg\to u\bar{u}) = \frac{1}{12}\,\frac{\pi\alpha_s^2}{s}\left[\frac{u}{t}+\frac{t}{u}-\frac{9}{4}\,\frac{u^2+t^2}{s^2}\right]$$

$$\frac{d\sigma}{d\cos\theta_*}(ug\to ug) = \frac{2}{9}\,\frac{\pi\alpha_s^2}{s}\left[-\frac{u}{s}-\frac{s}{u}+\frac{9}{4}\,\frac{u^2+s^2}{t^2}\right]$$

$$\frac{d\sigma}{d\cos\theta_*}(gg\to gg) = \frac{9}{4}\,\frac{\pi\alpha_s^2}{s}\left[3-\frac{su}{t^2}-\frac{st}{u^2}-\frac{tu}{s^2}\right]$$

Formulae with identical particles in the final state (uu, gg) should be integrated over $\cos\theta_* > 0$ only (or, if integrated over all angles, divided by 2).

I would like to show some results obtained by using these cross sections in the formula on p. 10. Figs. p. 9 shows a computation of the 2-jet invariant mass $\hat{s}$, compared to CDF data. The normalization is slightly off — this is, after all, only a lowest order computation, but the shape tracks the data over **6** orders of magnitude in the cross section. The figure indicates the relative contributions from gg scattering, most important at lower p_T, qg and $\bar{q}g$ scattering, and $q(\bar{q}) - q(\bar{q})$ scattering, which dominates at the highest p_T values. Figs. p. 10 shows a current comparison of CDF data to the parton model formulae, including higher-order QCD corrections.

Figs. p 11+12 show the analogous results for the LHC. Along with the cross sections, I have indicated the actual rates for a luminosity of $10^{33}/\mathrm{cm}^2\,\mathrm{sec}$. You see ~~that~~ events with $p_T > 400$ GeV per jet occur at a rate greater than 1/second and that, already at $p_T \sim 150$ GeV per jet, not all events can be recorded into permanent storage.