Physics 332 – Problem Set # 7

(due Wednesday, May 31)

1. Peskin and Schroeder, Problem 18.1.

2. Compute the anomalous dimension of the twist-2 quark-antiquark operators:

$$\overline{q}_f \gamma^{\{\mu_1}(iD^{\mu_2})\cdots(iD^{\mu_n\}})q_f - \text{traces} \tag{1}$$

   The calculation is sketched in Peskin and Schroeder, pp. 636-8. You need to track the 1-loop corrections that yield matrix elements of the form of eq. (18.161). Ignore any contributions proportional to $g^{\mu_i\mu_j}$, since these correspond to operators of lower spin. The final result is given in (18.172):

$$\gamma_f^n = \frac{8}{3}\frac{g^2}{(4\pi)^2}\left[1 + 4\sum_2^n \frac{1}{j} - \frac{2}{n(n+1)}\right] . \tag{2}$$

3. In class, I stated that the expression (2) for γ_f^n is the Mellin transform of the kernel in the Altarelli-Parisi equations. Verify this explicitly as follows: Begin from the equation

$$\frac{d}{d\log Q} f(x,Q) = \frac{\alpha_s(Q)}{\pi}\int_x^1 \frac{dz}{z} P_{q\leftarrow q}(z)\, f(\frac{x}{z}) \tag{3}$$

   where

$$P_{q\leftarrow q}(z) = \frac{4}{3}\left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(z-1)\right] . \tag{4}$$

   Work out the equation for the Mellin transform of $f(x)$

$$M_n = \int dx x^{n-1} f(x) \tag{5}$$

   Show explicitly that $M_n(Q)$ obey

$$\frac{d}{d\log Q} M_n(Q) = -\gamma_f^n M_n(Q) . \tag{6}$$

   You will find the calculation sketched in Peskin and Schroeder, pp. 644-646.