Physics 332 – Final Exam

This exam is due at noon on Wednesday, June 8. Please hand it in to Alex Giryavets in Varian 361.

If you have any questions about the exam, please contact me at mpeskin@slac.stanford.edu or 926-3250. If errata are reported, I will announce them on the course Web page.

Please do not collaborate on this exam. Please return the exam in a blue book (or multiple blue books) with the honor code acknowledgement signed. Alex Giryavets has blue books, if you need one.

The exam is worth a total of 100 points. The distribution of points is indicated below.

I recommend that you try simply to answer the questions using methods from Physics 332. However, the exam is open-book. If you find a useful reference other than the class textbook and notes, feel free to use it, as long as you cite the reference in your solution.

The exam concerns various aspects of quark masses in the Standard Model.

1. (20 points) Consider the heavy top quark. For $Q \gg m_t$, the running QCD coupling constant has the form
$$\alpha_s(Q) = \frac{2\pi}{b_0(6)\log(Q/\Lambda_6)} \tag{1}$$
where
$$b_0(n_f) = 11 - \frac{2}{3}n_f \tag{2}$$
For $Q \ll m_t$, the running coupling constant has the form
$$\alpha_s(Q) = \frac{2\pi}{b_0(5)\log(Q/\Lambda_5)} \tag{3}$$
Find Λ_5 in terms of Λ_6 and m_t. You will need to use the value of the top quark vacuum polarization diagram to get the numerical coefficient. Using the same method, compute Λ_4 for α_s below the b quark threshold.

2. (20 points) Compute the one-loop QCD correction to the top quark mass. Regularize the diagram using $\overline{MS}$ subtraction at the scale $m_t(\overline{MS})$. Compute the position of the pole in the top quark propagator, *i.e.*, the kinematic mass of the top quark. In this way, find the relation between the top quark "pole" mass and the $\overline{MS}$ mass. The pole mass is measured to be 175 GeV. What is the numerical value of the $\overline{MS}$ mass, assuming $\alpha_s = 0.11$ at m_t?

3. (20 points) The $\overline{MS}$ masses of the lighter quarks can be found from QCD sum rules and the values of the pseudoscalar meson masses. The results are: For u, d, s, m_q(2 GeV) = 3 MeV, 5 MeV, 100 MeV. For c, $m_c(m_c) = 1.2$ GeV. For b, $m_b(m_b) = 4.2$ GeV. Express all six quark masses at a common scale $Q = m_t(m_t)$. What are the true ratios of the quark masses?

4. (20 points) Compute the β function for the t quark Yukawa coupling. Use the interaction

$$\Delta\mathcal{L} = -\lambda_t \epsilon^{ab} \overline{Q}_{La} \phi_b^\dagger t_R + h.c. \ , \tag{4}$$

where $Q_L = (t_L, b_L)$. This coupling is invariant to all of the Standard Model gauge symmetries. Show that

$$\beta_{\lambda_t} = \frac{\lambda_t}{(4\pi)^2}\left[\frac{9}{2}\lambda_t^2 - 8g_s^2 - \frac{9}{4}g^2 - \frac{17}{12}g'^2\right] \tag{5}$$

If λ_t has a very large value at a high scale (*e.g.*, at $Q = 10^{16}$ GeV), what value of m_t would be observed in experiments?

5. (20 points) Because of the answer to question 1, the value of the proton mass depends on the value of the top quark mass. Compute

$$\frac{\partial}{\partial m_t} m_p \tag{6}$$

holding fixed the parameters of QCD at very high Q. The Higgs boson coupling to the proton should be given by

$$\frac{\lambda_{hpp}}{\sqrt{2}} = \frac{\partial}{\partial v} m_p \tag{7}$$

where $m_q = \lambda_q v/\sqrt{2}$. Compute λ_{hpp}, including contributions from t, b, c. Compare these contributions to the direct contributions from the mass of the valence quarks u and d.