Standard Model of Weak Interactions

β - decay $\quad n \to p\, e^-\, \bar{\nu}_e \qquad$ (today:) $d \to u\, e^-\, \bar{\nu}$

muon decay $\quad \mu \to \nu_\mu\, e^-\, \bar{\nu}_e$

π decay $\quad \pi^- \to \mu^-\, \bar{\nu}_\mu \qquad d\bar{u} \to \mu^-\, \bar{\nu}_\mu$

K decay $\quad K^- \to \mu^-\, \bar{\nu}_\mu \qquad s\bar{u} \to \mu^-\, \bar{\nu}_\mu$

effective 4-fermion interaction $\qquad \mathcal{L} = \frac{4G_F}{\sqrt{2}}\, \bar{\Psi}\Gamma^a\Psi\, \bar{\Psi}\Gamma_a\Psi$

Lee + Yang: Parity Violation!

Feynman Gell-Mann Marshak Sudarshan

<u>V - A model</u>

$$\mathcal{L}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \, J^{\mu +} J^{\mu -}$$

$$J^{\mu +} = u_L^{+} \bar{\sigma}^{\mu} d_L + \nu_{eL}^{+} \bar{\sigma}^{\mu} e_L + \nu_{\mu L}^{+} \bar{\sigma}^{\mu} \mu_L + \cdots$$

maximally violates P + C

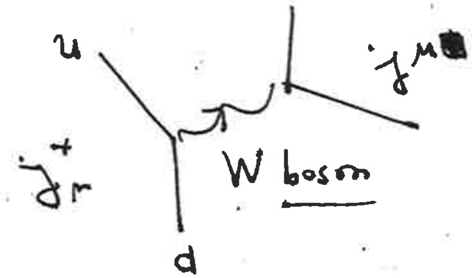
$d \to u e^- \bar{\nu}_e$ produces only <u>right</u>-handed antineutrinos.

$\bar{d} \to \bar{u} e^+ \nu_e$ produces only <u>left</u>-handed neutrinos

P: $\nu_L \leftrightarrow \nu_R$ C: $\nu_L \to \bar{\nu}_L$

Glashow + Schwinger, Umezawa, many others:

split the V-A $\mathcal{L}_{eff}$



spin 1 boson $\longrightarrow$ Yang Mills field.

couples to SU(2) doublets

$$\begin{pmatrix}u\\d\end{pmatrix}_L \quad \begin{pmatrix}\nu_e\\e\end{pmatrix}_L \quad \begin{pmatrix}\nu_\mu\\\mu\end{pmatrix}_L$$

<u>chiral</u> coupling

unified thy of weak and EM ?

$$Q_u + Q_d = \tfrac{1}{3}, \quad Q_\nu + Q_e = -1 \quad \neq 0 \quad !$$

Glashow: solve this problem using $G = \underbrace{SU(2)}_{\text{"isospin"}} \times \underbrace{U(1)}_{\text{"hypercharge"}}$

$$\mathcal{L} = Q_L^\dagger i\bar{\sigma}\cdot D\, Q_L + L^\dagger i\bar{\sigma}\cdot D\, L$$
$$+ u_R^\dagger i\sigma\cdot D\, u_R + d_R^\dagger i\sigma\cdot D\, d_R + e_R^\dagger i\sigma\cdot D\, e_R$$

$Q_L = \begin{pmatrix} u \\ d \end{pmatrix}$ $\quad u_R \quad d_R \quad$ $L = \begin{pmatrix} \nu \\ e \end{pmatrix}$ $\quad e_R$

$I = \frac{1}{2}$ $\quad I = 0 \quad$ $I = \frac{1}{2}$ $\quad I = 0$

$$D_\mu = \partial_\mu - ig A_\mu^a \underbrace{\tau^a}_{\text{of } I = 1/2} - ig' B_\mu Y$$

$\tau^a = \sigma^a/2$

2 simple groups $\rightarrow$ 2 independent coupling constants

$SU(2) \times U(1) \longrightarrow$ 4 vector particles

$W^+ \quad W^- \quad Z^0 \quad \gamma$

$m_W, m_Z \gg 1$ GeV $\qquad m_\gamma = 0$

Glashow: "We must ignore this problem."

Give masses by the Higgs mechanism (Weinberg, Salam)

Higgs field Φ w. $I = \frac{1}{2}$ $Y = \frac{1}{2}$

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

vacuum manifold sphere S^3

$$\Phi = \frac{1}{\sqrt{2}} e^{i\,\alpha(x)\cdot\vec{\sigma}/2} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

U gauge $\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$

$$\langle \Phi(x) \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$\mathcal{L} = |D_\mu \Phi|^2$$ gives the mass matrix of gauge bosons

$$= \left| \left(\partial_\mu - i g A^a_\mu \frac{\sigma^a}{2} - i g' B \frac{1}{2}\right) \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+h(x) \end{pmatrix} \right|^2$$

$$= \frac{1}{2} (\partial_\mu h)^2 + \frac{1}{2} \left| -i g \left(\frac{A^1_\mu - i A^2_\mu}{2} \right) \begin{pmatrix} v \\ 0 \end{pmatrix} \right|^2$$

$$+ \frac{1}{2} \left| -i \left(\frac{g}{2} A^3_\mu (-1) - i \frac{g'}{2} B_\mu \right) \begin{pmatrix} 0 \\ v \end{pmatrix} \right|^2$$

$$= \frac{1}{2} (\partial_\mu h)^2 + \left(\frac{g^2}{4} v^2 \right) W^{+\mu} W^-_\mu \qquad W^\pm_\mu = \frac{1}{\sqrt{2}} (A^1_\mu \mp i A^2_\mu)$$

$$+ \frac{1}{2} \frac{v^2}{4} \left(g A^3_\mu - g' B_\mu \right)^2$$

Let $$Z_\mu = \frac{g A_\mu - g' B_\mu}{\sqrt{g^2 + g'^2}} \qquad A_\mu = \frac{g' A_\mu + g B_\mu}{\sqrt{g^2 + g'^2}}$$

we find

$$m_W^2 = \left(\frac{gv}{2}\right)^2 \qquad m_Z^2 = \left(\frac{v}{2}\right)^2 (g^2 + g'^2) \qquad m_A = 0$$

It is useful to define

$$\cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}} \qquad \sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}}$$

then

$$m_W = m_Z \cos\theta_W \qquad \text{(observed in nature !)}$$

why is $m_A = 0$? $\quad \Phi \to e^{i\vec{\alpha}\cdot\vec{\sigma}/2 + i\beta/2}\,\Phi$

the transformation $\alpha^3 = \beta$, $\underline{Q = I^3 + Y}$ is <u>not</u> broken.

$Q|\Omega\rangle = 0$ so the corresponding vector boson A_μ keeps zero mass.

Work out the covariant derivative on a general representation

$$D_\mu = \partial_\mu - i g A_\mu^a T^a - i g' B Y$$

$$A^1_\mu T^1 + A^2_\mu T^2 = \frac{1}{\sqrt{2}} W^+_\mu (\underbrace{T^1 + iT^2}_{T^+}) + \frac{1}{\sqrt{2}} W^-_\mu (\underbrace{T^1 - iT^2}_{T^-})$$

$$g A^3_\mu T^3 + g' B Y = \frac{g}{\sqrt{g^2+g'^2}}(g Z_\mu + g' A_\mu) T^3 + \frac{g'}{\sqrt{g^2+(g')^2}}(-g' Z_\mu + g A_\mu) Y$$

$$= \left(\frac{g^2 Z_\mu T^3 - (g')^2 Z_\mu Y}{\sqrt{g^2+g'^2}}\right) + \frac{g g'}{\sqrt{g^2+g'^2}} A_\mu (T^3 + Y)$$

$$= \sqrt{g^2+g'^2}\left[T^3 - \frac{(g')^2}{(g^2+g'^2)} T^3 + Y\right] Z_\mu + \frac{g g'}{\sqrt{g^2+g'^2}} (T^3 + Y) A_\mu$$

$$= \sqrt{g^2+g'^2}\,(T^3 - \sin^2\theta_w Q)\, Z_\mu + e\, Q\, A_\mu$$

electric charge

so

$$D_\mu = \partial_\mu - i \frac{g}{\sqrt{2}} \left(W_\mu^+ T^+ + W_\mu^- T^- \right)$$

$$- i e Q A_\mu \qquad\qquad e = \frac{g g'}{\sqrt{g^2 + g'^2}}$$

$$= \sin\theta_W \, g$$

$$- i \frac{g}{\cos\theta_W} \left(T^3 - \sin^2\theta_W Q \right) Z_\mu$$

Q - electric charge $\qquad Q_Z = T^3 - \sin^2\theta_W Q \qquad$ Z charge

note that the electric charge must be the same for f_L and f_R

but Q_Z may be different for f_L and f_R

"Generation" of fermions

$$Q = \begin{pmatrix} u \\ d \end{pmatrix}_L \quad u_R \quad d_R \quad L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \quad e_R \quad \nu_R$$

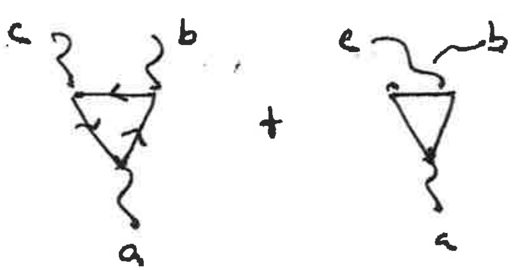
Q	u_R	d_R	L	e_R	ν_R
$T = \frac{1}{2}$	$T = 0$	$T = 0$	$T = \frac{1}{2}$	$T = 0$	$T = 0$
$Y = \frac{1}{6}$	$Y = \frac{2}{3}$	$Y = -\frac{1}{3}$	$Y = -\frac{1}{2}$	$Y = -1$	$Y = 0$

do not actually couple.

consistency condition:
because of the "axial vector anomaly", if we compute

$$\langle j_L^{\mu a} \, j_L^{\mu b} \, j_L^{\mu c} \rangle$$

in general. not all 3 currents can be conserved



$$\propto \operatorname{tr}\left[t^a, \{ t^b \, t^c \} \right]$$

the axial vector anomaly vanishes if $\mathrm{tr}\,[t^a\{t^b, t^c\}] = 0$

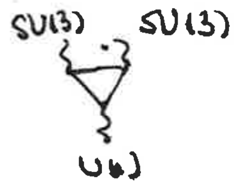
when all fermions are expressed as L-fermions e.g. $e^-_R \to e^+_L$

$T=0 \quad Y=+1$

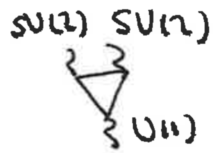
for any triangle with 1 non-Abelian factor

$\sim \mathrm{tr}\,\frac{\sigma^a}{2} = 0$

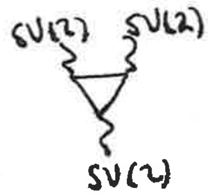
$\sim \mathrm{tr}\, t^a = 0$



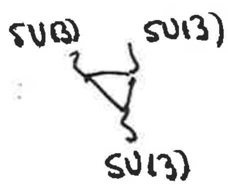
$\sim \mathrm{tr}[t^a t^b] \cdot \left[2 \cdot \frac{1}{6} - \frac{2}{3} + 1/3\right] = 0$



$\sim \mathrm{tr}\left[\frac{\sigma^a}{2}\frac{\sigma^b}{2}\right] \cdot \left[3 \cdot \frac{1}{6} + 2\left(-\frac{1}{2}\right)\right] = 0$



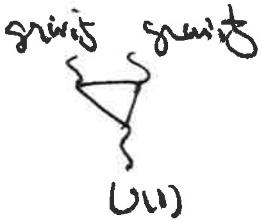
$\sim \mathrm{tr}\,\frac{\sigma^a}{2}\left\{\frac{\sigma^b}{2}, \frac{\sigma^c}{2}\right\} = \mathrm{tr}\left[\frac{\sigma^a}{2}\,\delta^{bc} \cdot \frac{1}{2}\right] = 0$



$\sim$ (L-quarks) $-$ (L-antiquarks) $= 0$

$$= 3\cdot 2\left(\tfrac{1}{6}\right)^3 + 3\left(-\tfrac{2}{3}\right)^3 + 3\left(\tfrac{1}{3}\right)^3 + 2\left(-\tfrac{1}{2}\right)^3 + (+1)^3$$

$$\left.\begin{aligned} \frac{1-32+4}{36} &= -\frac{27}{36} \\ \frac{-1+4}{4} &= 3/4 \end{aligned}\right) = 0$$



$$\propto \operatorname{tr} Y = 6\,\tfrac{1}{6} + 3\left(-\tfrac{2}{3}\right) + 3\left(\tfrac{1}{3}\right) + 2\left(-\tfrac{1}{2}\right) + 1\cdot 1 = 0$$

many of these cancellations require cancellations between quarks and leptons

An important aspect of the phenomenology:

$$A_f = \frac{(Q_Z)_L^2 - (Q_Z)_R^2}{((Q_Z)_L^2 + (Q_Z)_R^2)} = \text{polarization of } f \text{ fermions in } Z \text{ decay}$$

$$e, \mu, \tau \qquad \frac{(\frac{1}{2} - \sin^2\theta_W)^2 - (\sin^2\theta_W)^2}{(\quad)^2 + (\quad)^2} \approx 15\%$$

$$u, c, t \qquad \frac{(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W)^2 - (\frac{2}{3}\sin^2\theta_W)^2}{(\quad)^2 + (\quad)^2} \approx 67\%$$

$$d, s, b \qquad \frac{(\frac{1}{2} - \frac{1}{3}\sin^2\theta_W)^2 - (\frac{1}{3}\sin^2\theta_W)^2}{(\quad)^2 + (\quad)^2} \approx 94\%$$

confirmed experimentally in precision studies of $e^+e^- \to Z^0 \to f\bar{f}$

If e_L, e_R have different quantum numbers, how can we write a mass term

$$\delta\mathcal{L} = -m_e \, e_L^\dagger e_R + e_R^\dagger e_L \qquad ?$$

Use the Higgs field

$$\delta\mathcal{L} = -y_e \, L_L^\dagger \cdot \Phi \, e_R + h.c.$$

$$Y = +\tfrac{1}{2} + \tfrac{1}{2} - 1 = 0 \checkmark$$

$$\Phi \to \langle \Phi \rangle$$

$$-\frac{y_e}{\sqrt{2}} v \; e_L^\dagger e_R + h.c. \qquad !$$

more generally

$$\delta\mathcal{L} = -y_e\, L^{\dagger}\cdot\Phi\, e_R - y_d\, Q^{\dagger}\cdot\Phi\, d_R - y_u\, Q^{\dagger}_{ab}\,\varepsilon_{ab}\,\Phi^{*}_{b}\, u_R$$

Y: $\quad +\tfrac{1}{2}+\tfrac{1}{2}+-1=0 \qquad -\tfrac{1}{6}+\tfrac{1}{2}-\tfrac{1}{3}=0 \qquad -\tfrac{1}{6}+-\tfrac{1}{2}+\tfrac{2}{3}=0$

fit to observations:

$\sin^2\theta_W = 0.231 \qquad \alpha(m_Z) = \frac{1}{129}.$

$m_W = 80.4$ GeV $\qquad m_Z = 91.18$ GeV

$\frac{g^2}{4\pi} = \frac{1}{29.6} \qquad \frac{(g')^2}{4\pi} = \frac{1}{96}. \qquad v = 246$ GeV

Values of y range from

$y_t \approx 1$ to $y_e \approx 3\times10^{-6}$

The model is renormalizable if the potential for Φ is dimension 4 or lower. So

$$\mathcal{L} = \quad \ldots \quad - V(\Phi)$$

$$V(\Phi) = -\mu^2 |\Phi|^2 + \lambda (|\Phi|^2)^2$$

Parameters of the theory:

$$g \quad g' \quad \mu^2 \quad \lambda \qquad \text{or} \qquad g \quad g' \quad v \quad \lambda$$

Remove infinities by fixing to experiment

$$\alpha(m_Z) \qquad \frac{G_F}{\sqrt{2}} = \frac{g^2}{8 m_W^2} \qquad m_Z \ \text{or} \ m_W \qquad m_h$$

one conceptual problem

μ^2 receives additive Λ^2 divergences.

so (1) the theory does not predict the <u>sign</u> of μ^2

(2) $|\mu^2| \ll \Lambda^2$ requires delicate cancellation

Most probably, the simple Higgs model is an effective theory representing a more fundamental, more predictive theory

But we have no evidence for that theory, and no idea of what is should be