Weak Interactions at High Energy

Polarization vectors of a massive vector boson — 3 d.o.f.

rest frame $\varepsilon_i^\nu = (0, \vec{\varepsilon}_i)$ $\quad i = 1,2,3$

$k \cdot \varepsilon_i = 0$ $\quad$ true in any frame.

$$\sum_i \varepsilon_i^\mu \varepsilon_i^{\nu *} = -\left(g^{\mu\nu} - \frac{k^\mu k^\nu}{m^2}\right)$$

W propagator (U gauge) $\quad \mu \underset{W}{\sim\sim\sim} \nu = \dfrac{-i}{k^2 - m_W^2}\left(g^{\mu\nu} - \dfrac{k^\mu k^\nu}{m_W^2}\right)$

oddly $\rightarrow$ k^0 as $k \to \infty$

does this threaten renormalizability?

rest frame polarization vectors:

$$\varepsilon_+^\mu = \frac{1}{\sqrt{2}}(0\ 1\ i\ 0)^\mu \qquad \text{helicity} = +1 \ \text{about}\ \hat{3}\ \text{axis}$$

$$\varepsilon_-^\mu = \frac{1}{\sqrt{2}}(0\ 1\ {-i},\ 0)^\mu \qquad \text{helicity} = -1 \ \text{about}\ \hat{3}\ \text{axis}$$

$$\varepsilon_0^\mu = \frac{1}{\sqrt{2}}(0\ 0\ 0\ 1)^\mu \qquad \text{helicity} = 0 \ \text{about}\ \hat{3}\ \text{axis}$$

boost along $\hat{3}$  $k^\mu = (E, 0, 0, k)^\mu$

$$\left.\begin{matrix}\varepsilon_+ \\ \varepsilon_-\end{matrix}\right\} \text{unchanged}$$

$$\varepsilon_0^\mu = \left(\frac{k}{m_W},\ 0\ 0\ \frac{E}{m_W}\right)$$

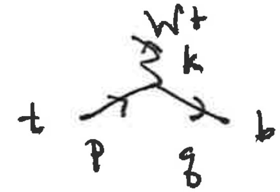
Note, $k \cdot \varepsilon_0 = 0$ but as $k \to \infty$ $\varepsilon_0^\mu \sim \dfrac{k^\mu}{m_W}$!

In previous examples in this course, k^μ terms in the propagator were irrelevant because

k^μ  $= 0$

But, this is not true for spontaneously broken symmetry currents.

e.g. t quark decay



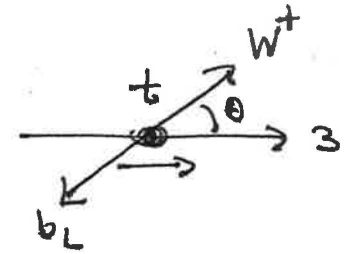
$$k^\mu \quad \bar{u}(b)\,\gamma^\mu \frac{(1-\gamma^5)}{2}\, u(t)$$

$$= \bar{u}(b)\,(\not{p} - \not{q})\left(\frac{1-\gamma^5}{2}\right) u(t)$$

$$= \bar{u}(b)\left[m_b\left(\frac{1-\gamma^5}{2}\right) - \left(\frac{1+\gamma^5}{2}\right) m_t\right] u(t)$$

$$\approx -m_t\, \bar{u}(b)\left(\frac{1+\gamma^5}{2}\right) u(t) \quad \neq 0 \; !$$

explicit computation: $(m_b = 0)$



$$i\mathcal{M}(t \to b_L W^+_+) = 0$$

$$i\mathcal{M}(t \to b_L W^+_-) = \sqrt{2}\, i g \sqrt{2 m_t E_b}\, \sin\theta/2$$

$$i\mathcal{M}(t \to b_L W^+_0) = i g \sqrt{2 m_t E_b}\, \cos\theta/2 \cdot \frac{m_t}{m_W}$$ largest contribution!

$$\frac{\Gamma(t \to W^+_0 b)}{\Gamma(t \to W^+ b)} = \frac{m_t^2/2m_W^2}{1 + m_t^2/2m_W^2} = 70\%$$ ✓ experiment

Quantization of a spontaneously broken gauge theory.
(Abelian case, for simplicity)

$$Z = \int \mathcal{D}A \, d\Phi \; e^{i\int \mathcal{L}} \int \mathcal{D}\alpha \, \mathcal{D}\omega \; e^{-i\int \omega_2^2} \, \delta(G[A_\alpha] - \omega) \det\left(\frac{\delta G}{\delta \alpha}\right)$$

choose $\quad G = \frac{1}{\sqrt{\xi}}\left(\partial_\mu A_\alpha^\mu - \xi e v \pi_\alpha\right)$

$$\det \left.\frac{\delta G}{\delta \alpha}\right|_{\alpha=0} = \det\left(-\frac{1}{e}\partial^2 - \xi e v (v+h)\right)$$

$$\mathcal{L} = -\tfrac{1}{4}(F_{\mu\nu})^2 - \tfrac{1}{2\xi}(\partial_\mu A^\mu - \xi\, ev\, \pi)^2 + |D_\mu \Phi|^2$$

$$= -\tfrac{1}{2} A_\mu \left[-\partial^2 g^{\mu\nu} + \partial^\mu \partial^\nu + \tfrac{1}{\xi}\partial^\mu\partial^\nu \right] A_\nu$$

$$+ \partial_\mu A^\mu\, ev\, \pi \quad - \tfrac{1}{2}\xi\, (ev)^2\, \pi^2$$

$$+ \tfrac{1}{2}(\partial_\mu h)^2 + \tfrac{1}{2}\left(\partial_\mu \pi + e A_\mu (v+h)\right)^2$$

by construction $\quad \partial_\mu A^\mu (ev)\pi \quad$ terms cancel $\qquad \underline{ev = m}$

$$\xrightarrow{h} \quad \frac{i}{k^2 - m_h^2} \qquad \xrightarrow{\pi} \quad \frac{i}{k^2 - \xi m^2} \qquad \overset{c}{\cdots\cdots} \quad \frac{i}{k^2 - \xi m^2}$$

$$\mu \overset{A}{\underset{k}{\sim\sim\sim}} \nu \quad = \quad -i \left\{ \frac{1}{k^2 - m^2}\left[g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2} \right] + \frac{k^\mu k^\nu}{k^2}\, \frac{\xi}{k^2 - \xi m^2} \right\}$$

$$= \quad -i \, \frac{1}{k^2 - m^2}\left[g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2 - \xi m^2}(1-\xi) \right]$$

R_ξ gauges. ξ = finite.

$\xi = 1$ all masses = m Feynman 't Hooft gauge

$\xi = 0$ Landau gauge all masses of unphysical particles = 0

U gauge $\xi \to \infty$

't Hooft + Veltman:

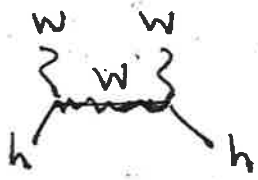
Ward identities $\Rightarrow$ S-matrix elements are independent of ξ

in the U gauge, the theory is manifestly unitary

(no unphysical states

in an R_ξ gauge, the theory is manifestly renormalizable.

simple example: $hh \to W^+W^-$

U gauge: 

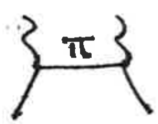
$$= \left(+i\,\frac{2m_W^2}{v}\right)^2(-i)\left(g^{\mu\nu} - \frac{k^\mu k^\nu}{m_W^2}\right)\frac{1}{k^2-m_W^2}$$

R_ξ gauge:

$$= \left(+i\,\frac{2m_W^2}{v}\right)^2(-i)\left[\frac{g^{\mu\nu}}{k^2-m_W^2} - \frac{k^\mu k^\nu(1-\xi)}{(k^2-m^2)(k^2-\xi m_W^2)}\right]$$

$$= \left(+i\,\frac{2m_W^2}{v}\right)^2(-i)\left[\frac{g^{\mu\nu}}{k^2-m^2} - \frac{k^\mu k^\nu}{m_W^2}\left(\frac{1}{k^2-m_W^2} - \frac{1}{k^2-\xi m_W^2}\right)\right]$$

and, there is another diagram



$$= (2ek^\mu)(2ek^\nu)\frac{i}{k^2-\xi m_W^2} \quad \leftarrow \text{cancels}$$

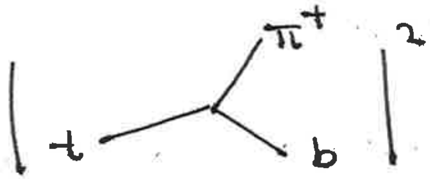
insight into $t \to W^+ b$

in Feynman 't Hooft gauge, we have $\sim\!\sim\!\sim = \dfrac{-i g^{\mu\nu}}{k^2 - m_W^2}$

$$\sum \varepsilon^\mu \varepsilon^{*\nu} = -g^{\mu\nu}$$

there is no $(k/m_W)^2$ enhancement.

but we also need to include



$$y_t \, \bar{u}(b) \left(\frac{1+\gamma^5}{2} \right) u(t)$$

$$= \underbrace{\frac{m_t}{\sqrt{2}\, m_W}} g \, \bar{u}(b) \left(\frac{1+\gamma^5}{2} \right) u(t)$$

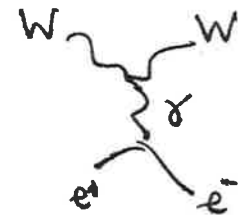
this is the m_t^2/m_W^2 enhancement in $\Gamma(t \to Wb)$

This is part of a general result:

$$|\mathcal{M}(X \to Y + W_0^+)|^2 \xrightarrow[E_W/m_W \to \infty]{} |\mathcal{M}(X \to Y + \pi^+)|^2 \left(1 + \frac{m_W}{E_W}\right)$$

the Goldstone boson equivalence theorem

a different example: $e^+e^- \to W^+W^-$



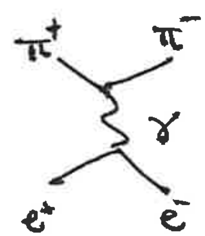
Estimate $\quad \sigma \sim \dfrac{\pi\alpha^2}{3s} \cdot \left|\varepsilon(+)\cdot\varepsilon(-)\right|^2$

but $\quad \left|\varepsilon_0(+)\cdot\varepsilon_0(-)\right|^2 \sim \left|\dfrac{k_+\cdot k_-}{m_W^2}\right|^2 \cong \left|\dfrac{s}{2m_W^2}\right|^2$

badly violates unitarity in the S-wave!

In the Standard Model, this cannot happen. for $W_0^+W_0^-$

$\sigma \xrightarrow[s\gg m_W^2]{}$



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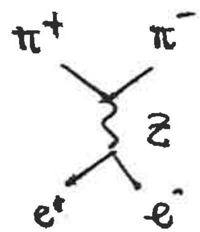
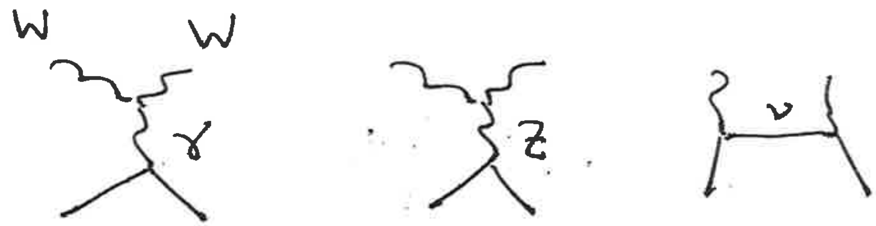
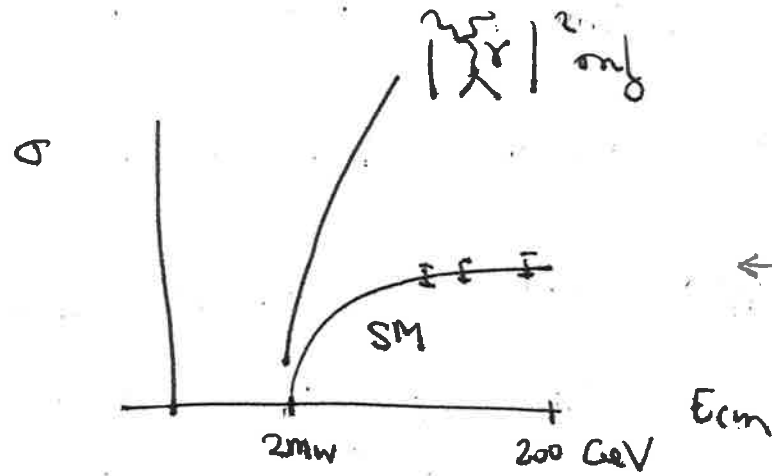


$\sim \dfrac{\pi\alpha_W^2}{s}$

Indeed, there are 3 diagrams



and these nicely cancel at high energy



agrees w. data from LEP 2