Physics 331 – Problem Set #7

Solutions

1.) a.) Recall that in QED, all particles massless,

$$\frac{d\sigma}{d\cos\theta}(e^-_L e^+_R \to \mu^-_L \mu^+_R) = \frac{d\sigma}{d\cos\theta}(e^-_R e^+_L \to \mu^-_R \mu^+_L)$$

$$= \frac{\pi\alpha^2}{2s}(1+\cos\theta)^2$$

$$\frac{d\sigma}{d\cos\theta}(e^-_L e^+_R \to \mu^-_R \mu^+_L) = \frac{d\sigma}{d\cos\theta}(e^-_R e^+_L \to \mu^-_L \mu^+_R)$$

$$= \frac{\pi\alpha^2}{2s}(1-\cos\theta)^2$$

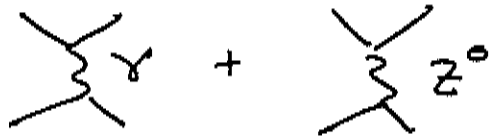
In all cases, the total cross section is $\sigma(\text{polarized } e^+e^- \to \text{pol. } \mu^+\mu^-)$

$$= \frac{4}{3}\frac{\pi\alpha^2}{s}$$

Check: $\sigma(e^+e^- \to \mu^+\mu^-) = \frac{1}{4} \times (\text{sum of 4 cross sect.})$

$$= \frac{4\pi\alpha^2}{3s}$$

In the electroweak thy, we must compute



$$i\mathcal{M}(\underbrace{e^-e^+}_{\text{chiral states}} \to f\bar{f}) = (ie)^2\, \bar{v}\gamma^\mu u\ \bar{u}\gamma_\mu v \cdot \frac{-i}{s}$$

$$\cdot \left[Q_e Q_f + \frac{s}{s-m_Z^2}\,\frac{(I_e^3 - s_w^2 Q_e)(I_f^3 - s_w^2 Q_f)}{s_w^2\, c_w^2} \right]$$

where $s_w^2 = \sin^2\theta_w \qquad c_w^2 = \cos^2\theta_w$

the QED result is modified by the expression in the brackets.

For $e^+e^- \to \mu^+\mu^-$ we then have:

$$\frac{d\sigma}{d\cos\theta}(e_L^- e_R^+ \to \mu_L^- \mu_R^+) = \frac{\pi\alpha^2}{2s}(1+\cos\theta)^2 \left| 1 + \frac{(\frac{1}{2}-s_w^2)^2}{s_w^2\, c_w^2}\,\frac{s}{s-m_Z^2} \right|^2$$

$$\frac{d\sigma}{d\cos\theta}(e_L^- e_R^+ \to \mu_R^- \mu_L^+) = \frac{\pi\alpha^2}{2s}(1-\cos\theta)^2 \left| 1 - \frac{(\frac{1}{2}-s_w^2)\, s_w^2}{s_w^2\, c_w^2}\,\frac{s}{s-m_Z^2} \right|^2$$

$$\frac{d\sigma}{d\cos\theta}(e_R^- e_L^+ \to \mu_L^- \mu_R^+) = \frac{\pi\alpha^2}{2s}(1-\cos\theta)^2 \left| 1 - \frac{s_w^2(\frac{1}{2}-s_w^2)}{s_w^2\, c_w^2}\,\frac{s}{s-m_Z^2} \right|^2$$

$$\frac{d\sigma}{d\cos\theta}(e_R^- e_L^+ \to \mu_R^- \mu_L^+) = \frac{\pi\alpha^2}{2s}(1+\cos\theta)^2 \left| 1 + \frac{(s_w^2)^2}{s_w^2\, c_w^2}\,\frac{s}{s-m_Z^2} \right|^2$$

the spin-averaged total cross section is then

$$\sigma(e^+e^-\to\mu^+\mu^-) = \frac{\pi\alpha^2}{3s}$$

$$\cdot\left\{ \left|1 + \frac{(\frac{1}{2}-s_W^2)^2}{s_W^2 c_W^2}\frac{s}{s-m_Z^2}\right|^2 + 2\left|1 - \frac{(\frac{1}{2}-s_W^2)}{c_W^2}\frac{s}{s-m_Z^2}\right|^2 \right.$$

$$\left. + \left|1 + \frac{s_W^2}{c_W^2}\frac{s}{s-m_Z^2}\right|^2 \right\}$$

the corresponding formulae for quarks are:

$$\sigma(e^+e^-\to u\bar{u}) = \frac{\pi\alpha^2}{3s}\cdot 3\cdot\left(1+\frac{\alpha_s}{\pi}\right)$$

$$\cdot\left\{ \left|\frac{2}{3} + \frac{(\frac{1}{2}-s_W^2)(\frac{1}{2}-\frac{2}{3}s_W^2)}{s_W^2 c_W^2}\frac{s}{s-m_Z^2}\right|^2 + \left|\frac{2}{3} - \frac{(\frac{1}{2}-s_W^2)\cdot\frac{2}{3}}{c_W^2}\frac{s}{s-m_Z^2}\right|^2 \right.$$

$$\left. + \left|\frac{2}{3} - \frac{(\frac{1}{2}-\frac{2}{3}s_W^2)}{c_W^2}\frac{s}{s-m_Z^2}\right|^2 + \left|\frac{2}{3} + \frac{\frac{2}{3}s_W^2}{c_W^2}\frac{s}{s-m_Z^2}\right|^2 \right\}$$

$$\sigma(e^+e^-\to d\bar{d}) = \frac{\pi\alpha^2}{3s}\cdot 3\cdot\left(1+\frac{\alpha_s}{\pi}\right)$$

$$\left\{ \left|\frac{1}{3} + \frac{(\frac{1}{2}-s_W^2)(\frac{1}{2}-\frac{1}{3}s_W^2)}{s_W^2 c_W^2}\frac{s}{s-m_Z^2}\right|^2 + \left|\frac{1}{3} - \frac{(\frac{1}{2}-s_W^2)\frac{1}{3}}{c_W^2}\frac{s}{s-m_Z^2}\right|^2 \right.$$

$$\left. + \left|\frac{1}{3} - \frac{(\frac{1}{2}-\frac{1}{3}s_W^2)}{c_W^2}\frac{s}{s-m_Z^2}\right|^2 + \left|\frac{1}{3} + \frac{\frac{1}{3}s_W^2}{c_W^2}\frac{s}{s-m_Z^2}\right|^2 \right\}$$

we can write these formulae as

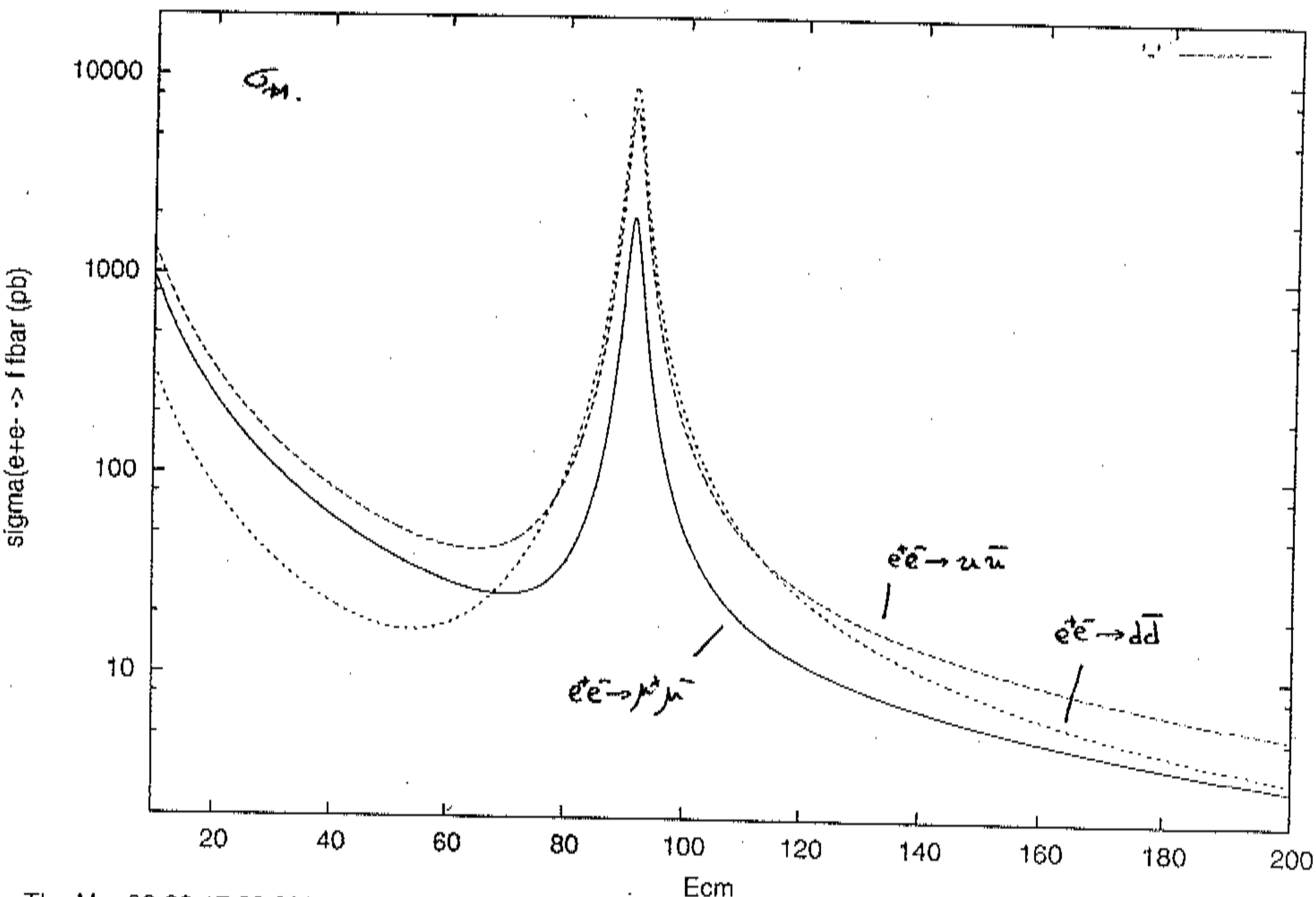
$$\sigma(e^-e^+ \to f\bar{f}) = \frac{\pi\alpha^2}{3s}\left\{ |G_{LL}|^2 + |G_{LR}|^2 + |G_{RL}|^2 + |G_{RR}|^2 \right\}$$

where
G_{LL} is the amplitude correction for $e^-_L \to f_L$
G_{LR} is the amplitude correction for $e^-_L \to f_R$
etc.

to treat the Z^0 resonance more correctly,
we should replace

$$\frac{s}{s-m_Z^2} \to \frac{s}{s-m_Z^2 + i m_Z \Gamma_Z}$$

A plot of the three cross-sections is given
on the next page $\longrightarrow$



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b.) To compute forward-backward asymmetries, we need:

$$\int_{-1}^{1} d\cos\Theta \; (1+\cos\Theta)^2 = 2 + 2\cdot 0 + \tfrac{2}{3} = \tfrac{8}{3}$$

$$\int_{0}^{1} d\cos\Theta \; (1+\cos\Theta)^2 = 1 + 2\left(\tfrac{1}{2}\right) + \tfrac{1}{3} = \tfrac{7}{3}$$

$$\int_{-1}^{0} d\cos\Theta \; (1+\cos\Theta)^2 = 1 + 2\left(-\tfrac{1}{2}\right) + \tfrac{1}{3} = \tfrac{1}{3}$$

so

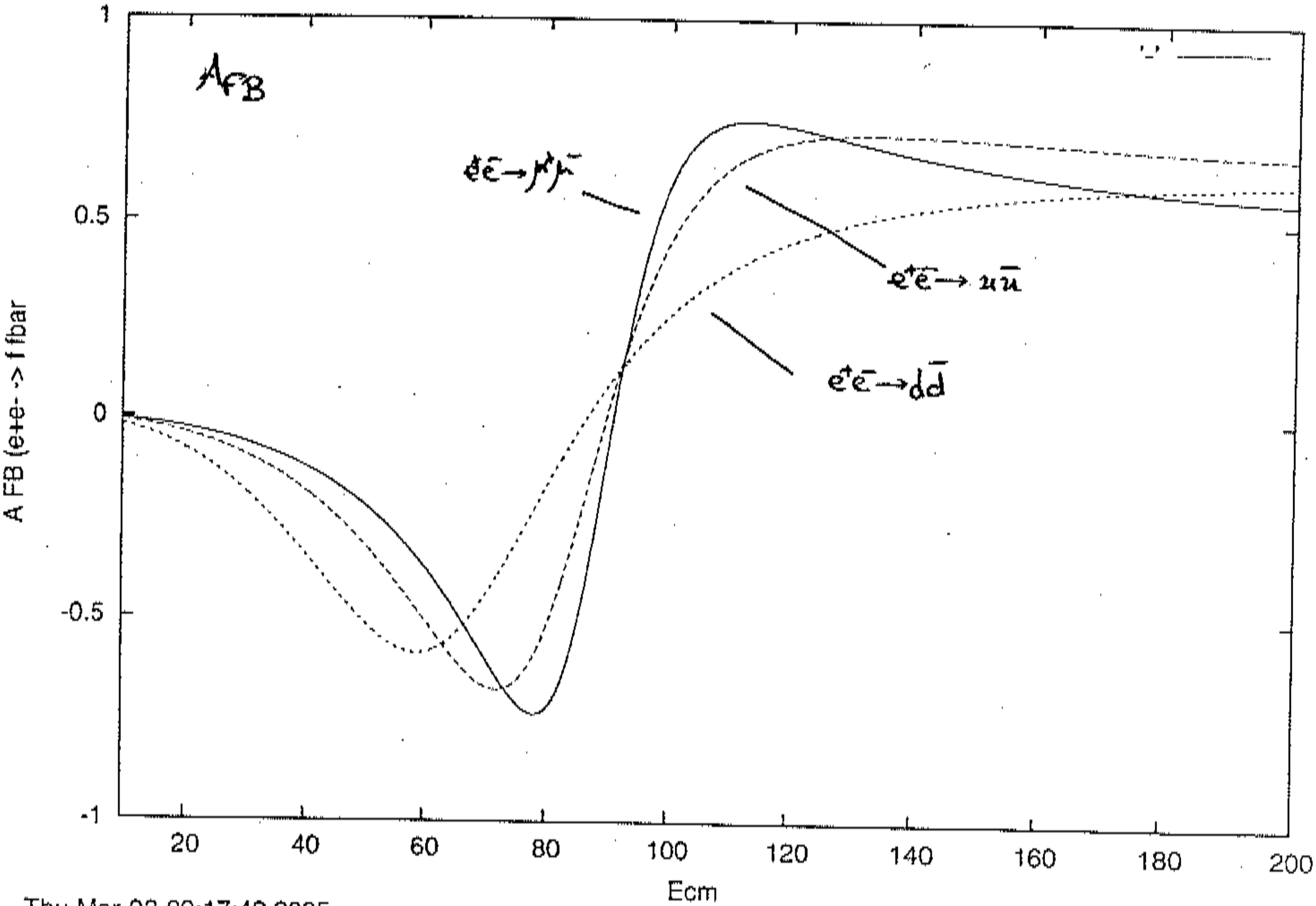
$$\frac{\left(\int_0^1 d\cos\Theta - \int_{-1}^0 d\cos\Theta\right)(1+\cos\Theta)^2}{\left(\qquad + \qquad\right)(1+\cos\Theta)^2} = \frac{7/3 - 1/3}{8/3} = \frac{3}{4}$$

then for any species:

$$A_{FB}(e^-e^+ \to f\bar{f}) = \frac{3}{4}\,\frac{|G_{LL}|^2 + |G_{RR}|^2 - |G_{LR}|^2 - |G_{RL}|^2}{|G_{LL}|^2 + |G_{RR}|^2 + |G_{LR}|^2 + |G_{RL}|^2}$$

where G_{ij} are the factors on pp. 2, 3

Plots of the asymmetries for μ, u, d are given on the next page $\longrightarrow$



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c.) as $s \to m_Z^2$

$$G_{LL} \to \frac{(-\frac{1}{2}+s_w^2)(I^3 - s_w^2 Q)}{s_w^2 c_w^2} \frac{s}{s-m_Z^2}$$

$$G_{LR} \to \frac{(-\frac{1}{2}+s_w^2)(-s_w^2 Q)}{s_w^2 c_w^2} \frac{s}{s-m_Z^2}$$

$$G_{RL} \to \frac{s_w^2 (I^3 - s_w^2 Q)}{s_w^2 c_w^2} \frac{s}{s-m_Z^2}$$

$$G_{RR} \to \frac{s_w^2 (-s_w^2 Q)}{s_w^2 c_w^2} \frac{s}{s-m_Z^2}$$

then

$$A_{FB} \to \frac{3}{4} \frac{(-\frac{1}{2}+s_w^2)^2 (I^3 - s_w^2 Q)^2 - (-\frac{1}{2}+s_w)^2(-s_w Q)^2 - (s_w^2)^2 (I^3 - s_w^2 Q)^2 + (s_w^2)^2(-s_w^2 Q)^2}{(\qquad + \qquad + \qquad + \qquad)}$$

$$= \frac{3}{4} \frac{\left[(-\frac{1}{2}+s_w^2)^2 - (s_w^2)^2\right]\left[(I^3 - s_w^2 Q)^2 - (-s_w^2 Q)^2\right]}{\left[(-\frac{1}{2}+s_w^2)^2 + (s_w^2)^2\right]\left[(I^3 - s_w^2 Q)^2 + (-s_w^2 Q)^2\right]}$$

$$= \frac{3}{4} A_{LRe} A_{LRf}$$

d.) At the peak of the Z^0

$$\frac{s}{s - m_Z^2 + i m_Z \Gamma_Z} \rightarrow \frac{m_Z^2}{i m_Z \Gamma_Z}$$

$$G_{LL} \longrightarrow \frac{(-\frac{1}{2}+s_w^2)(I^3 - s_w^2 Q)}{s_w^2 c_w^2} \cdot \left(\frac{-i m_Z}{\Gamma_Z}\right)$$

$$G_{LR} \longrightarrow \frac{(-\frac{1}{2}+s_w^2)(-s_w^2 Q)}{s_w^2 c_w^2} \left(\frac{-i m_Z}{\Gamma_Z}\right)$$

etc.

$$\sigma(e^+e^- \to f\bar{f})\Big|_{s=m_Z^2} = \frac{\pi\alpha^2}{3s} \cdot \overbrace{3\cdot\left(1+\frac{\alpha_s}{\pi}\right)}^{\text{for quarks.}} \cdot \frac{m_Z^2}{\Gamma_Z^2}$$

$$\cdot \frac{[(-\frac{1}{2}+s_w^2)^2 + s_w^4]}{s_w^2 c_w^2} \cdot \frac{(I^3 - s_w^2 Q)^2 + (s_w^2 Q)^2}{s_w^2 c_w^2}$$

now $$\Gamma(Z^0 \to f\bar{f}) = \frac{\alpha m_Z}{6 s_w^2 c_w^2}\left[(I^3 - s_w^2 Q)^2 + (s_w^2 Q)^2\right] \cdot \underbrace{3\cdot(\ \ )}_{\text{quarks.}}$$

so

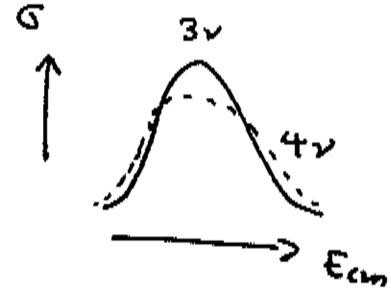
$$\sigma(e^+e^- \to f\bar{f}) = \frac{\pi}{3 m_Z^2} \cdot 36 \frac{1}{\Gamma_Z^2} \cdot \Gamma(Z\to e^+e^-)\,\Gamma(Z\to f\bar{f})$$

$$= \frac{12\pi}{m_Z^2} \frac{\Gamma(Z\to e^+e^-)\,\Gamma(Z\to f\bar{f})}{\Gamma_Z^2}$$

According to the previous problem set

$\Gamma_Z = 2.5$ GeV $\qquad \Gamma(Z \to \nu\bar{\nu}) = 0.165$ GeV / species.

so an extra neutrino species gives



$$\frac{\Delta\Gamma_Z}{\Gamma_Z} = +6.6\%$$

$$\frac{\Delta\sigma}{\sigma} = -2\left(\frac{\Delta\Gamma_Z}{\Gamma_Z}\right) = -13\%$$

Current measurements give

$$\Gamma_Z = 2.4952 \pm .0023 \text{ GeV}$$

$$\sigma_{had.,Z} = 41.541 \pm .037 \text{ nb}$$ hadronic cross section at the Z^0 peak

both in good agreement w. 3 neutrino species. These measurements exclude contributions to the invisible decays of the Z^0 larger than 1% of the contribution of a ν species.

2.) a.) The charged-current weak interaction is mediated by:

$$= \frac{g^2}{2m_W^2}\left(\bar\mu\gamma^\mu P_L\nu\right)\left(\bar u\gamma_\mu P_L d\right)$$

$$= \frac{4G_F}{\sqrt{2}}\left(\bar\mu\gamma^\mu P_L\nu\right)\left(\bar u\gamma_\mu P_L d\right)$$

we saw that this leads to the formulas

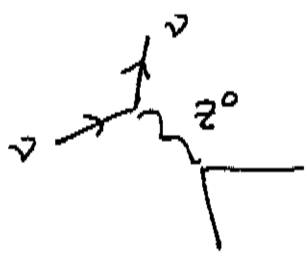
$$\frac{d^2\sigma}{dx\,dy}(\nu p\to\mu^- X) = \frac{G_F^2 s}{\pi}\left[x f_d(x) + (1-y)^2 x f_{\bar u}(x)\right]$$

$$\frac{d^2\sigma}{dx\,dy}(\bar\nu p\to\mu^+ X) = \frac{G_F^2 s}{\pi}\left[x f_u(x)(1-y)^2 + x f_{\bar d}(x)\right]$$

The factor $(1-y)^2$ appears when chiralities do not match

$\nu_L\bar u_R$ or $\bar\nu_R u_L$ scattering

The neutral-current interaction is mediated by



$$= \frac{e^2}{c_w^2 s_w^2}\frac{1}{m_Z^2}\left(\bar\nu\gamma^\mu I^3\nu\right)$$

$$\cdot\,\bar f\gamma^\mu\left(I^3 - s_w^2 Q\right)f$$

$$= \frac{g^2}{2m_W^2} \bar{\nu}\gamma^\mu P_L \nu \cdot \bar{f}\gamma^\mu(I^3 - \sin^2\theta_w Q) f$$

using $\frac{e^2}{s_w^2} = g^2 \qquad c_w^2 m_Z^2 = m_W^2$

$$= \frac{4G_F}{\sqrt{2}} (\bar{\nu}\gamma^\mu P_L \nu) \quad \bar{f}\gamma^\mu(I^3 - \sin^2\theta_w Q) f$$

the Z couples to $u_L \quad u_R \quad d_L \quad d_R$

$\bar{u}_L \quad \bar{u}_R \quad \bar{d}_L \quad \bar{d}_R$

adding all of these terms:

$$\frac{d\sigma}{dx\,dy}(\nu p \to \nu X) = \frac{G_F^2 s}{\pi} \Big[x f_u(x) \big[\overset{L}{(\tfrac{1}{2} - \tfrac{2}{3}s_w^2)^2} + \overset{R}{(\tfrac{2}{3}s_w^2)^2}(1-y)^2 \big]$$

$$+ x f_d(x) \big[(-\tfrac{1}{2} + \tfrac{1}{3}s_w^2)^2 + (\tfrac{1}{3}s_w^2)^2(1-y)^2 \big]$$

$$+ x f_{\bar{u}}(x) \big[(\tfrac{2}{3}s_w^2)^2 + (\tfrac{1}{2} - \tfrac{2}{3}s_w^2)^2(1-y)^2 \big]$$

$$+ x f_{\bar{d}}(x) \big[(\tfrac{1}{3}s_w^2)^2 + (-\tfrac{1}{2} + \tfrac{1}{3}s_w^2)(1-y)^2 \big] \Big]$$

for νn scattering, exchange $u \leftrightarrow d$

for $\bar{\nu} p$ scattering, exchange $1 \leftrightarrow (1-y)^2$

b.) If $f_u(x) = f_d(x)$ for a heavy nuclear target A

$$\frac{d^2\sigma}{dx\,dy}(\nu A \to \nu X) = \frac{G_F^2 s}{\pi}\Big[\, x f_q(x) \Big\{ (\tfrac{1}{2} - \tfrac{2}{3} s_W^2)^2 + (\tfrac{1}{2} - \tfrac{1}{3} s_W^2)^2$$
$$+ \big[(\tfrac{2}{3} s_W^2)^2 + (\tfrac{1}{3} s_W^2)^2 \big] (1-y)^2 \Big\}$$

$$+ \; x f_{\bar{q}}(x) \Big\{ \big[(\tfrac{2}{3} s_W^2)^2 + (\tfrac{1}{3} s_W^2)^2 \big]$$
$$+ \big[(\tfrac{1}{2} - \tfrac{2}{3} s_W^2)^2 + (\tfrac{1}{2} - \tfrac{1}{3} s_W^2)^2 \big] (1-y)^2 \Big\} \Big]$$

$$= \frac{G_F^2 s}{\pi}\Big[\, x f_q(x) \big[(\tfrac{1}{2} - s_W^2 + \tfrac{5}{9} s_W^4) + \tfrac{5}{9} s_W^4 (1-y)^2 \big]$$
$$+ \; x f_{\bar{q}}(x) \big[\tfrac{5}{9} s_W^4 (1-y)^2 + (\tfrac{1}{2} - s_W^2 + \tfrac{5}{9} s_W^4)(1-y)^2 \big] \Big]$$

$$\frac{d^2\sigma}{dx\,dy}(\bar{\nu} A \to \bar{\nu} X) = \frac{G_F^2 s}{\pi}\Big[\, x f_q(x) \big[(\tfrac{1}{2} - s_W^2 + \tfrac{5}{9} s_W^4)(1-y)^2 + \tfrac{5}{9} s_W^4 \big]$$
$$+ \; x f_{\bar{q}}(x) \big[\tfrac{5}{9} s_W^4 (1-y)^2 + (\tfrac{1}{2} - s_W^2 + \tfrac{5}{9} s_W^4) \big] \Big]$$

$$\frac{d^2\sigma}{dx\,dy}(\nu A \to \mu^- X) = \frac{G_F^2 s}{\pi}\big[x f_q(x) + (1-y)^2 x f_{\bar{q}}(x) \big]$$

$$\frac{d^2\sigma}{dx\,dy}(\bar{\nu} A \to \mu^+ X) = \frac{G_F^2 s}{\pi}\big[x f_q(x) (1-y)^2 + x f_{\bar{q}}(x) \big]$$

$$r = \frac{\frac{d\sigma}{dxdy}(\bar{\nu} A \to \mu^+ X)}{\frac{d\sigma}{dxdy}(\nu A \to \mu^- X)} = \frac{x f_q(x)(1-y)^2 + x f_{\bar{q}}(x)}{x f_q(x) + (1-y)^2 x f_{\bar{q}}(x)}$$

then

$$R^\nu = \frac{\frac{d^2\sigma}{dxdy}(\nu A \to \nu X)}{\frac{d\sigma}{dxdy}(\nu A \to \mu^- X)} = \left[\frac{1}{2} - s_w^2 + \frac{5}{9} s_w^4\right] + \frac{5}{9} s_w^4 \left[\frac{x f_q(x)(1-y)^2 + x f_{\bar{q}}(x)}{x f_q(x) + x f_{\bar{q}}(x)(1-y)^2}\right]$$

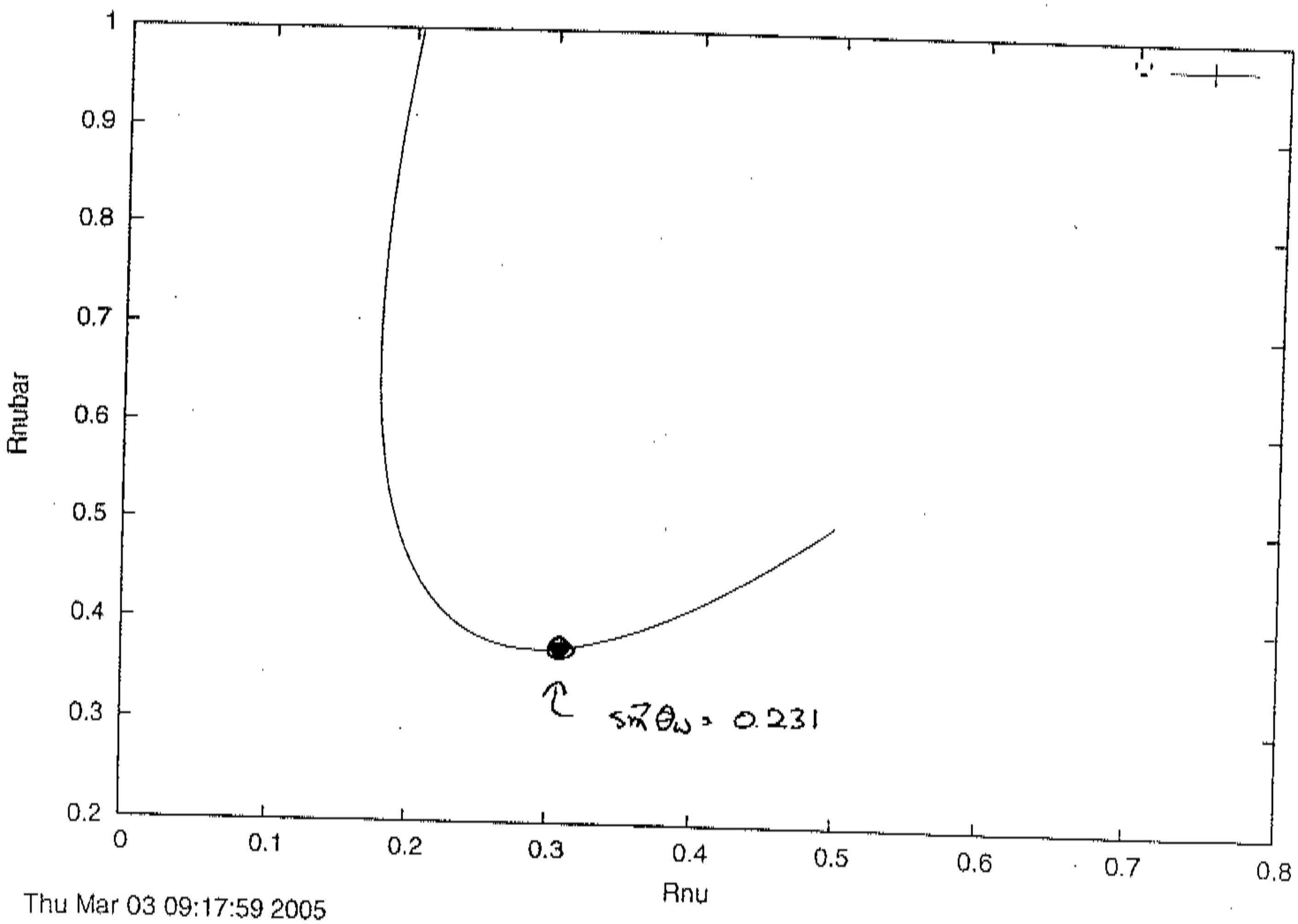
$$= \frac{1}{2} - s_w^2 + \frac{5}{9} s_w^4 + \frac{5}{9} s_w^4 \, r$$

$$R^{\bar{\nu}} = \frac{\frac{d^2\sigma}{dxdy}(\bar{\nu} A \to \bar{\nu} X)}{\frac{d\sigma}{dxdy}(\bar{\nu} A \to \mu^+ X)} = \frac{1}{2} - s_w^2 + \frac{5}{9} s_w^4 + \frac{5}{9} s_w^4 \left[\frac{x f_q(x) + x f_{\bar{q}}(x)(1-y)^2}{x f_q(x)(1-y)^2 + x f_{\bar{q}}(x)}\right]$$

$$= \frac{1}{2} - s_w^2 + \frac{5}{9} s_w^4 + \frac{5}{9} s_w^4 \, \frac{1}{r}$$

e.) A sketch of the curve R^ν vs. $R^{\bar{\nu}}$ is given on the next page

$\longrightarrow$



1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
Rnubar
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
Rnu
$\sin^2\theta_W = 0.231$
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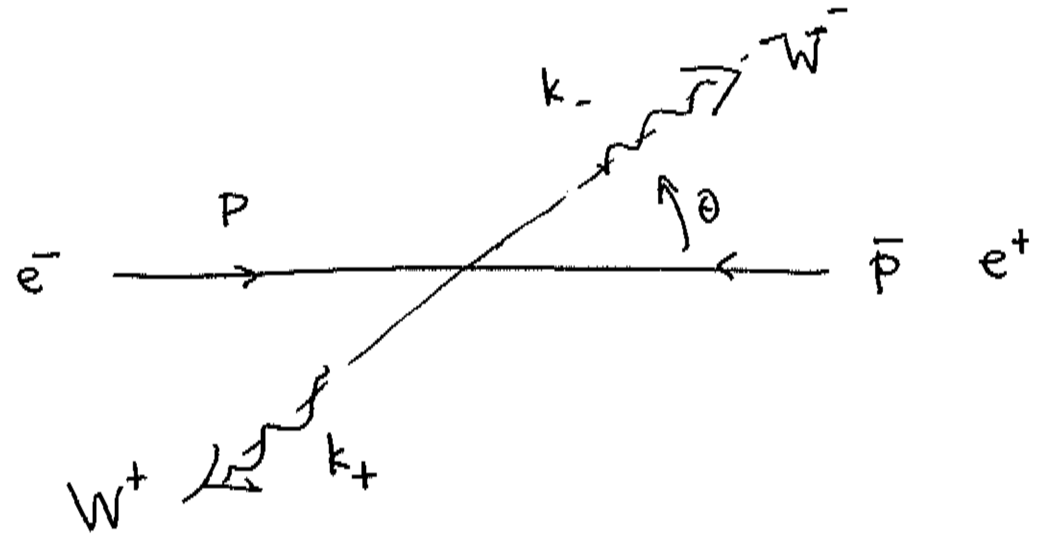
3.) For a longitudinally polarized W boson moving in the $\hat{3}$ direction

$$\varepsilon_0^\mu = \frac{1}{m_W}(k, 0, 0, E)$$

set up $e^- e^+ \to W^- W^+$ as follows:



$$p = (E, 0, 0, E) \qquad \bar{p} = (E, 0, 0, -E)$$

$$k_- = (E, ks, 0, kc) \qquad k_+ = (E, -ks, 0, -kc)$$

$$s = \sin\theta \qquad c = \cos\theta$$

$$\varepsilon_0^\mu(-) = \frac{1}{m_W}(k, Es, 0, Ec) \qquad \varepsilon_0^\mu(+) = \frac{1}{m_W}(k, -Es, 0, -Ec)$$

spinors:

$$u_R(p) = \sqrt{2E}\begin{pmatrix} 0 \\ 0 \\ \hline 1 \\ 0 \end{pmatrix} \qquad v_L(\bar{p}) = \sqrt{2E}\begin{pmatrix} 0 \\ 0 \\ \hline 0 \\ -1 \end{pmatrix}$$

$$u_L(p) = \sqrt{2E}\begin{pmatrix} 0 \\ 1 \\ \hline 0 \\ 0 \end{pmatrix} \qquad v_R(\bar{p}) = \sqrt{2E}\begin{pmatrix} -1 \\ 0 \\ \hline 0 \\ 0 \end{pmatrix}$$

We need:

$$= ie \left[\varepsilon^*(-)\,\varepsilon^*(+)\,(k_- - k_+)^\lambda + \varepsilon^{\lambda *}(-)\,(-q - k_-)\cdot\varepsilon^*(+) + \varepsilon^{\lambda *}(+)\,(q + k_+)\cdot\varepsilon^*(-) \right]$$

$$= ie \left[\varepsilon^*(-)\cdot\varepsilon^*(+)\,(k_- - k_+)^\lambda - 2\,k_-\cdot\varepsilon^*(+)\,\varepsilon^{\lambda *}(-) + 2\,k_+\cdot\varepsilon^*(-)\,\varepsilon^{\lambda}(+) \right]$$

since $k_-\cdot\varepsilon^*(-) = 0 = k_+\cdot\varepsilon^*(+)$

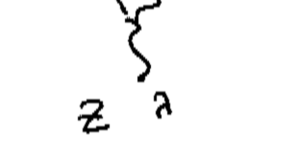
$$= ie \left[\frac{E^2+k^2}{m_W^2}(k_- - k_+)^\lambda - 2\,\frac{2Ek}{m_W^2}\,\varepsilon^{\lambda *}(-) + 2\,\frac{2Ek}{m_W^2}\,\varepsilon^{\lambda *}(+) \right]$$

$$= ie\,\frac{1}{m_W^2}\,(0,s,0,c)^\lambda \cdot \left[2k\,(E^2+k^2) - 4\,Ek\cdot E\cdot 2 \right]$$

$$= ie\,\frac{1}{m_W^2}\,(0\;s\;0\;c)^\lambda \left(-\frac{3E^2+k^2}{m_W^2}\right)\cdot 2k$$

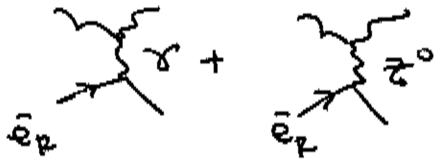
$$= -ie\,\frac{1}{m_W^2}\,(0\;s\;0\;c)^\lambda \cdot 2k \cdot \frac{2E^2+m_W^2}{m_W^2}$$

similarly,



$$= -ie\,\frac{c_w}{s_w}\,\frac{1}{m_W^2}\,(0,s,0,c)^\lambda\;2k\;\frac{2E^2+m_W^2}{m_W^2}$$

then



$$= (ie)\, \bar{v}\gamma_\lambda u \left[(-1)\,\frac{-i}{s}\,(-ie) + \frac{s_w^2}{s_w c_w}\,\frac{-i}{s-m_Z^2}\,\frac{-iec_w}{s_w} \right]$$

$$\cdot\ (0,s,0,c)^\lambda \cdot 2k\ \frac{2E^2+m_W^2}{m_W^2}$$

$$= ie^2\ \bar{v}\gamma_\lambda u\ (0,s,0,c)^\lambda \cdot 2k \cdot \frac{2E^2+m_W^2}{m_W^2}$$

$$\cdot \left[\frac{1}{s} - \frac{1}{s-m_Z^2} \right]$$

$$= ie^2\ \underbrace{\bar{v}\gamma_\lambda u\ (0,s,0,c)^\lambda}\ 2k\ \frac{2E^2+m_W^2}{m_W^2} \left(\frac{-m_Z^2}{s(s-m_Z^2)} \right)$$

this spinor element is

$$\bar{v}\gamma_\lambda u\ (0\, s\, 0\, c)^\lambda = 2E\ (0\ {-1})\ \sigma_\lambda (0\, s\, 0\, c)^\lambda \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= 2E\ (0\ {-1}) \begin{pmatrix} -c & -s \\ -s & c \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= 2Es$$

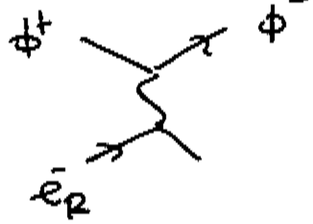
$$= ie^2\ (2E\sin\theta)\, 2k\ \frac{2E^2+m_W^2}{m_W^2} \left[\frac{-m_Z^2}{s(s-m_Z^2)} \right]$$

If $s \gg m_W^2 \; m_Z^2$ $\qquad 2E^2 \approx \frac{s}{2}$

$$\mathcal{M}(e_R^- e_L^+ \to W_0^- W_0^+) \longrightarrow -ie^2 (4Ek \sin\theta) \cdot \frac{s}{2} \frac{m_Z^2}{m_W^2} \frac{1}{s^2}$$

$$\longrightarrow -i \frac{e^2}{2c_W^2} \, 4Ek \sin\theta \, \frac{1}{s}$$

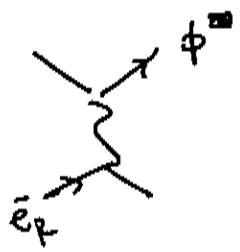
in QED



$$= (-ie)^2 \, \bar{v} \gamma_\lambda u \, (k_- - k_+)^\lambda \, \frac{-i}{s}$$

$$= ie^2 \; 2E \cdot 2k \sin\theta \; \frac{1}{s}$$

So in unbroken SU(2)×U(1) gauge theory



$$= i g'^2 (4Ek \sin\theta) \underset{Y_{e_R}}{(-1)} \underset{Y_\phi}{(-\tfrac{1}{2})} \frac{1}{s}$$

$$= +i \frac{e^2}{2c_W^2} (4Ek \sin\theta) \frac{1}{s}$$

in agreement with the result above.

Now consider $e^-_L e^+_R \to W^- W^+$

to compute the γ and Z diagrams, we need the spinor element

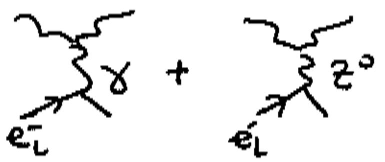
$$\bar{v}\gamma_\lambda u \, (0\, s, 0, c)^\lambda = \left(\sqrt{2E}\right)^2 (-1\ 0)\, \bar{\sigma}\cdot(0\, s\, 0\, c)\begin{pmatrix}0\\1\end{pmatrix}$$

$$= 2E\,(-1\ 0)\begin{pmatrix} c & s\\ s & -c\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}$$

$$= -2E\sin\theta$$

we also need to put in the Z^0 charge of the e^-_L : $\left(\frac{-\frac{1}{2}+s_w^2}{s_w c_w}\right)$

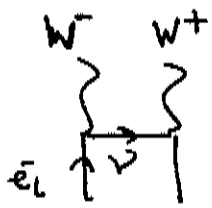
then

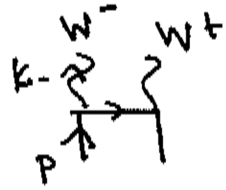


$$= ie^2\,(-2E\sin\theta)\;2k\;\frac{2E^2+m_W^2}{m_W^2}\left[\frac{1}{s} - \frac{(-\frac{1}{2}+s_w^2)}{s_w c_w}\frac{c_w}{s_w}\frac{1}{s-m_Z^2}\right]$$

$$= -ie^2\,(4Ek\sin\theta)\left(\frac{2E^2+m_W^2}{m_W^2}\right)\left[\frac{1}{2s_w^2}\frac{1}{s-m_Z^2} - \frac{m_Z^2}{s(s-m_Z^2)}\right]$$

to this, we must add the diagram:





$$= \left(\frac{ig}{\sqrt{2}}\right)^2 \bar{v}(\bar{p})\,\gamma\cdot\varepsilon^*(+)\,\frac{i\,\gamma\cdot(p-k_-)}{(p-k_-)^2}\,\gamma\cdot\varepsilon^*(-)\;u(p)$$

$$= -i\,\frac{e^2}{2s_w^2}\,\sqrt{2E}\,(-1\;\;0)\;\bar{\sigma}\cdot\varepsilon^*(+)\;\frac{\sigma\cdot(p-k_-)}{m_w^2-2p\cdot k_-}\;\bar{\sigma}\cdot\varepsilon^*(-)\begin{pmatrix}0\\1\end{pmatrix}\sqrt{2E}$$

$$= -i\,\frac{e^2}{2s_w^2}\;\frac{2E}{m_W^2}\;\frac{1}{[m_W^2-2E(E-kc)]}$$

$$\cdot\;(-1\;\;0)\begin{pmatrix}k-Ec & -Es\\ -Es & +(k+Ec)\end{pmatrix}\begin{pmatrix}kc-E & ks\\ ks & E-kc\end{pmatrix}\begin{pmatrix}k+Ec & Es\\ Es & k-Ec\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}$$

the 2^{nd} line is:

$$(Ec-k\quad Es)\begin{pmatrix}kc-E & ks\\ ks & E-kc\end{pmatrix}\begin{pmatrix}Es\\ k-Ec\end{pmatrix}$$

$$= (Ec-k)(kc-E)\,Es + Es(E-kc)(k-Ec) + Es\cdot ks\cdot Es + (Ec-k)\,ks\,(k-Ec)$$

$$= 2\,Es\,(k-Ec)(E-kc) + ks\,(k-Ec)(-k+Ec) + E^2ks^3$$

$$= 2E^2ks - 2E^3sc - \cancel{2Ek^2sc} + \cancel{2E^2ksc^2} - k^3s + \cancel{2Ek^2sc} - \cancel{E^2ksc^2} + E^2ks - \cancel{E^2ksc^2}$$

$$= (3E^2-k^2)\,ks - 2E^3sc$$

so

$$= +i \frac{e^2}{2s_w^2} \frac{2E}{m_w} 2 (\sin\theta) \left(\frac{[(3E^2-k^2)k - 2E^3 c]}{[2E^2 - m_w^2 - 2Ekc]} \right)$$

try to make the last factor look as much like 1 as possible:

$$(\quad) = \frac{E}{k} \frac{(3Ek^2 - k^4/E - 2E^2kc)}{(E^2+k^2-2Ekc)}$$

$$= \frac{E}{k} \frac{1}{(E^2+k^2-2Ekc)} \cdot \left[(E^2+k^2-2Ekc) + [-E^2 + 2k^2 - k^4/E^2] \right]$$

$$= \frac{E}{k} \frac{1}{(E^2+k^2-2Ekc)} \left[(E^2+k^2-2Ekc) + \frac{(E^2-k^2)^2}{E^2} \right]$$

$$= \frac{E}{k} + \frac{1}{(E^2+k^2-2Ekc)} \frac{m_w^4}{E^2}$$

so

$$= +i \frac{e^2}{2s_w^2} \frac{2E}{m_w^2} \sin\theta \, \frac{E}{k} \cdot \left(1 + \mathcal{O}\left(\frac{m_w^4}{E^4}\right) \right)$$

now we can combine the pieces!

$$= -ie^2\,(4Ek\sin\theta)\left(\frac{s}{2m_W^2}+1\right)\left[\frac{1}{2s_W^2}\,\frac{1}{s-m_Z^2} - \frac{m_Z^2}{s(s-m_Z^2)}\right]$$

$$+\,ie^2\,4Ek\sin\theta\;\frac{E^2}{2k^2}\,\frac{1}{m_W^2}\left(1+O\left(\frac{m_W^4}{E^4}\right)\right)$$

Now take the limit $E, k \gg m_W$ $\qquad \dfrac{E^2}{k^2} = 1 + \dfrac{m_W^2}{E^2} + \ldots$

$$= \left[-ie^2\,4Ek\sin\theta\right]\cdot\left\{-\frac{1}{2c_W^2}\,\frac{1}{s} + \frac{1}{2s_W^2}\left\{\cancel{\frac{1}{2m_W^2}} + \frac{m_Z^2}{2sm_W^2} + \frac{1}{s} - \cancel{\frac{1}{2m_W^2}} - \frac{1}{2m_W^2}\,\frac{4m_W^2}{s}\right\} + O\left(\frac{m_W^2}{s}\right)\right\}$$

$$= -ie^2\cdot(4Ek\sin\theta)$$

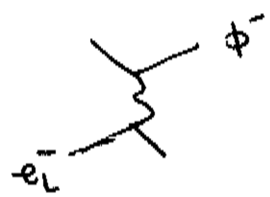
$$\cdot\left\{-\frac{1}{2c_W^2}\,\frac{1}{s} + \frac{1}{4c_W^2 s_W^2}\,\frac{1}{s} - \frac{1}{2s_W^2}\,\frac{1}{s}\right\}$$

$$= +ie^2\,(4Ek\sin\theta)\cdot\frac{1}{s}$$

$$\cdot\left\{\frac{1}{2c_w^2}+\frac{1}{2s_w^2}-\frac{1}{4}\left(\frac{1}{c_w^2}+\frac{1}{s_w^2}\right)\right\}$$

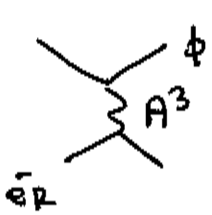
$$= ie^2\,(4Ek\sin\theta)\cdot\frac{1}{s}\cdot\left(\frac{1}{4c_w^2}+\frac{1}{4s_w^2}\right)$$

in QED



$$= -ie^2\;4Ek\sin\theta\cdot\frac{1}{s}$$

in unbroken SU(2) x U(1) gauge theory



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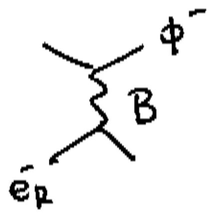


$$= \left(-ie^2\;4Ek\sin\theta\;\frac{1}{s}\right)$$

$\left(g=\frac{e}{s_w},\ g'=\frac{e}{c_w}\right)$ $$\cdot\left\{\frac{1}{s_w^2}\underbrace{\left(-\tfrac{1}{2}\right)}_{I^3_{e_L^-}}\underbrace{\left(-\tfrac{1}{2}\right)}_{I^3_{\phi^-}}+\frac{1}{c_w^2}\underbrace{\left(-\tfrac{1}{2}\right)}_{Y_{e_L^-}}\underbrace{\left(-\tfrac{1}{2}\right)}_{Y_{\phi^-}}\right\}$$

in agreement with the result above !

4.) The cross section for $q_{bf}\bar{q}_{bf} \to \mu^+\mu^-$ is

$$\frac{d\sigma}{d\cos\Theta_{cm}} = \frac{1}{4}\cdot\frac{1}{3}\cdot\frac{\pi\alpha^2}{2s}$$

$$\cdot\Big\{ (1+\cos\Theta)^2 \left| Q_f + \frac{(\tfrac{1}{2}-s_w^2)(I_f^3 - s_w^2 Q_f)}{s_w^2 c_w^2}\frac{s}{s-m_Z^2}\right|^2$$

$$+ (1-\cos\Theta)^2 \left| Q_f - \frac{s_w^2\,(I_f^3 - s_w^2 Q_f)}{s_w^2 c_w^2}\frac{s}{s-m_Z^2}\right|^2$$

$$+ (1-\cos\Theta)^2 \left| Q_f - \frac{(\tfrac{1}{2}-s_w^2)(s_w^2 Q_f)}{s_w^2 c_w^2}\frac{s}{s-m_Z^2}\right|^2$$

$$+ (1+\cos\Theta)^2 \left| Q_f + \frac{s_w^2 \cdot s_w^2 Q_f}{s_w^2 c_w^2}\frac{s}{s-m_Z^2}\right|^2 \Big\}$$

We need to implement this cross section in ppcollider:

$$\sigma = \int dx_1\, dx_2\, d\cos\Theta$$

$$\sum_{f=udscb} \left[f_f(x_1)\, f_{\bar{f}}(x_2) + f_{\bar{f}}(x_1)\, f_{f}(x_2)\right] \frac{d\sigma}{d\cos\Theta}(q_{bf}\bar{q}_{bf} \to \mu^+\mu^-)$$

In the program drellyan.cpp posted on the course Web site, I implent this. I use

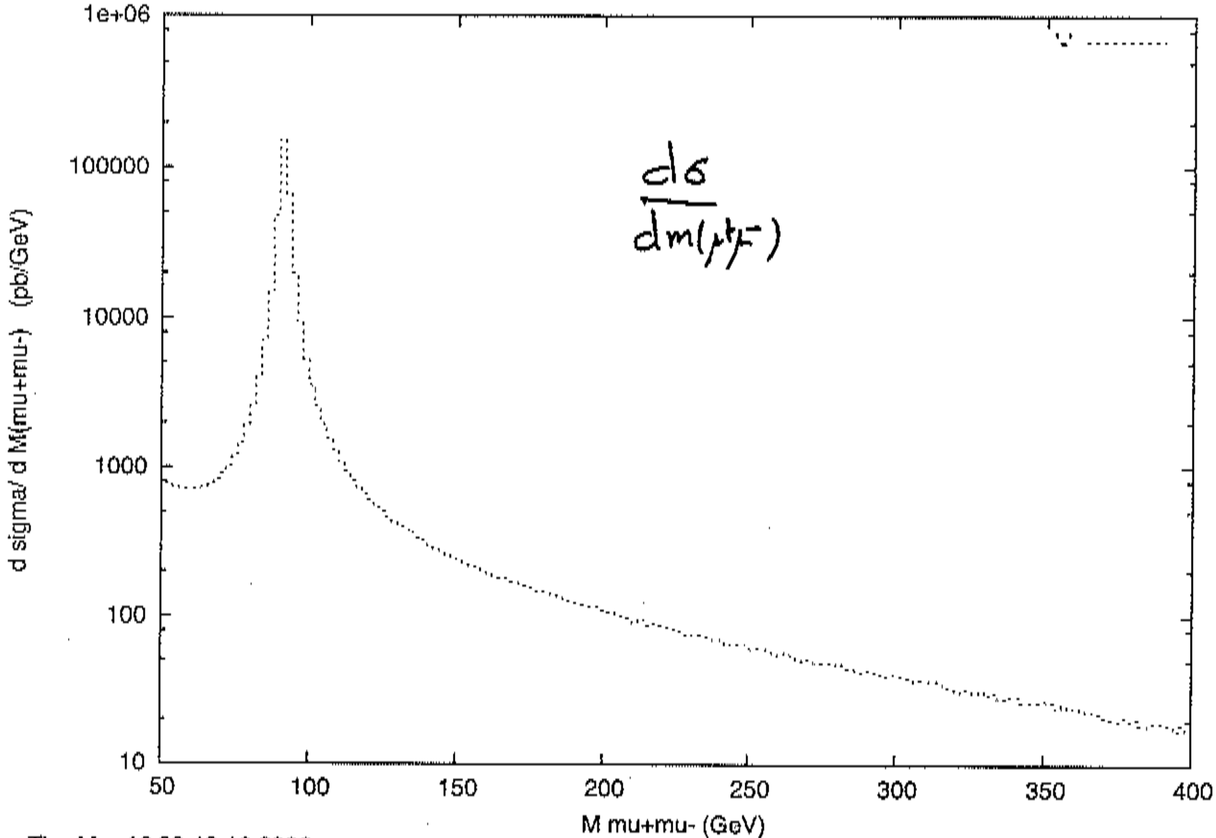
$$(1+\cos\theta)^2 = 4\,\frac{u^2}{s^2} \qquad (1-\cos\theta)^2 = 4\,\frac{t^2}{s^2}$$

drellyan makes a plot of $d\sigma/dm(\mu^+\mu^-)$ and $d\sigma/dp_\perp$. The Jacobian peak from

$$q\bar{q} \to Z^0 \to \mu^+\mu^-$$

is clearly seen in the latter plot.

Here are the plots ⟶



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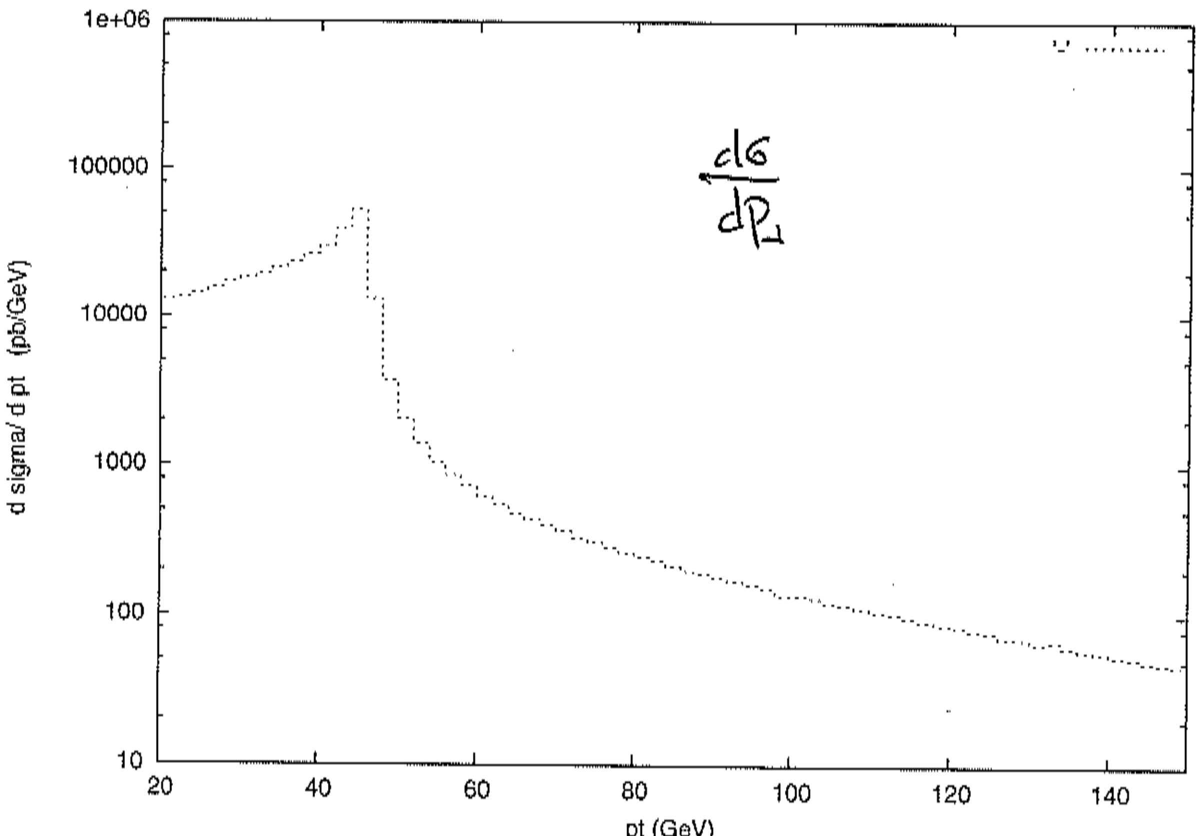

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