Physics 331 – Problem Set # 5

(due Wednesday, February 22)

1. The effective interaction used in class to compute the cross section for neutrino deep inelastic scattering can be tested in purely leptonic processes, in particular, in muon decay $\mu^- \to e^- \overline{\nu}_e \nu_\mu$. From the vertex

$$\Delta\mathcal{L} = \frac{4G_F}{\sqrt{2}} \overline{e}\gamma^\mu P_L \nu_e \; \overline{\nu}_\mu \gamma_\mu P_L \mu \tag{1}$$

where $P_L = (1-\gamma^5)/2$, and ignoring the masses of the electron and the neutrinos:

(a) Compute the muon decay rate Γ_μ. The measured muon lifetime, $\tau_\mu = 2.19703(4) \times 10^{-6}$ sec, gives the most accurate determination of G_F. Compute G_F (2 significant figures suffice).

(b) Compute the electron energy distribution $d\Gamma/dE(e^-)$ in the muon rest frame.

(c) (extra credit) For a muon at rest with spin oriented along the $+\hat{z}$ axis, compute the electron energy and angular distribution. When this distribution is averaged with that for a muon with spin oriented in the $-\hat{z}$ direction, you should find an angle-independent result that agrees with the answer in (b).

Parts (a) and (b) are quite straightforward with the use of the tricks for 3-body phase space described in Problem Set # 8 of Physics 330. Part (c) is more difficult; I have made it optional. It might be useful to use the identity for integrating over the phase space of massless vectors k and q such that $(k+q) = P$:

$$\int \frac{d^3k}{(2\pi)^3 2k} \int \frac{d^3q}{(2\pi)^3 2q} (2\pi)^4 \delta^{(4)}(k+q-P) \cdot k^\alpha q^\beta = \frac{1}{96\pi}(2P^\alpha P^\beta + g^{\alpha\beta}P^2) \tag{2}$$

2. Peskin and Schroeder, Problem 17.4.

3. Peskin and Schroeder, Problem 17.5. Work out both the total cross section and the differential cross section

$$E\frac{d\sigma}{d^3p} \tag{3}$$

where p, E are the energy and momentum of the heavy quark Q. You can work in the γ-p CM frame, though the quantity in (3) is actually invariant to longitudinal boosts.