

## Physics 331 – Problem Set # 3

(due Wednesday, February 8)

1. Peskin and Schroeder, Problem 9.2.
2. In class, we computed the matrix element for the process  $q\bar{q} \rightarrow gg$ , where  $q$  is a massless fermion in the representation  $r$  of a Yang-Mills gauge group  $G$  and  $g$  is the Yang-Mills gauge boson. Our result had the form:

$$i\mathcal{M} = \bar{v}(\bar{p})\gamma \cdot \epsilon^*(k_1)t^a \cdots \gamma \cdot \epsilon^*(k_2)t^b u(p) + \cdots \quad (1)$$

In QED, we were typically interested in cross sections summed over final spins and averaged over initial spins. In Yang-Mills theory, we might also wish to sum over final gauge indices ('colors') and average over initial colors.

- (a) For the term written out in (1), show that the group theory factor corresponding to this color average and sum of the squared matrix elements is

$$\frac{1}{d_r^2} \text{tr}[t^a t^b t^b t^a] \quad (2)$$

- (b) Show that this factor evaluates to

$$\frac{1}{d_r} [C_2(r)]^2 \quad (3)$$

- (c) Evaluate

$$\frac{1}{d_r^2} \text{tr}[t^a t^b t^a t^b] \quad (4)$$

3. Compute the differential cross section  $d\sigma/d\cos\theta$  for  $q\bar{q} \rightarrow gg$ , averaged over initial spins and colors and summed over final spins and colors. Use the method of Peskin and Schroeder, Problem 17.3(a). Use the same explicit spinors and polarization vectors that appeared in the Problem Sets 6 and 7 of Physics 330. Compute the color averages and sums using the results of Problem 2.