Physics 331 – Problem Set # 2

(due Wednesday, February 1)

1. Peskin and Schroeder, Problem 9.1.

2. Let Φ be a linear combination of free fields: $\Phi = \int d^4x g(x)\phi(x)$, where $g(x)$ is a fixed function and $\phi(x)$ is a free Klein-Gordon field.

   (a) First, look at the evaluation of products of Φ's in canonical quantization. Time-ordered expectation values of Φ are evaulated as sums of contractions. Show that

   $$\left\langle \Phi^4 \right\rangle = 3 \cdot \left\langle \Phi^2 \right\rangle^2 \quad \left\langle \Phi^4 \right\rangle = 5 \cdot 3 \cdot \left\langle \Phi^2 \right\rangle^3 \quad etc. \tag{1}$$

   where $\langle \cdots \rangle$ denotes the time-ordered expectation value and $\langle \Phi^2 \rangle$ is the contraction. Using these results, show that

   $$\langle \exp[\Phi] \rangle = \exp[\left\langle \Phi^2 \right\rangle /2] \tag{2}$$

   (b) Rederive (2) using the functional integral to define the expectation values of $\phi(x)$.

3. Peskin and Schroeder, Problem 15.3. Compare these results to Problem 2 of the previous problem set.

4. Peskin and Schroeder, Problem 15.4. The formula at the top of p. 504 should read:

$$D_F(x, y) = \int_0^\infty dT \int \mathcal{D}x \; \exp\left[i \int dt \, \frac{1}{2}(-(\frac{dx^\mu}{dt})^2 - m^2) - ie \int dt \, \frac{dx^\mu}{dt} A_\mu(x)\right] \tag{3}$$

which is correct, because $(d\vec{x}/dt)^2$ (the square of the space components of x^μ) should have a positive coefficient.