

Physics 331 – Problem Set # 1

(due Wednesday, January 25)

1. Consider the set of $SU(3)$ generators in the $\mathbf{3}$ representation given in Peskin and Schroeder, Problem 15.1. Compute the structure constants f^{abc} . You may use the fact that f^{abc} is totally antisymmetric. (Why?). Show explicitly that

$$f^{abc} f^{abd} = C_2(G) \delta^{cd} = 3\delta^{cd} \quad (1)$$

as claimed in class.

2. Review the discussion from Physics 330 of the nonrelativistic potential between heavy fermions due to photon exchange. Then:

- (a) Show that the nonrelativistic potential between two electrons is

$$V(R) = +\frac{e^2}{4\pi R} \quad (2)$$

- (b) A field of charge Q transforms under local $U(1)$ transformations as

$$\Psi(x) \rightarrow e^{-iQ\alpha(x)}\Psi(x) \quad (3)$$

Show that this implies that the covariant derivative on this field is

$$D_\mu = \partial_\mu - ieQA_\mu \quad (4)$$

and that the coupling to the photon field is the photon-electron coupling multiplied by $(-Q)$. (The electron has charge $Q = -1$.)

- (c) Show that the nonrelativistic potential between two heavy fermions with charges Q_1 and Q_2 is

$$V(R) = +\frac{e^2 Q_1 Q_2}{4\pi R} \quad (5)$$

- (d) Now generalize this result to Yang-Mills theory. Assume that the propagator for a Yang-Mills field is that given in Peskin and Schroeder, problem 15.3(c). Show that the nonrelativistic potential between two heavy fermions, one in the representation r_1 of the gauge group G and the other in the representation r_2 , has the form

$$V(R) = +\frac{g^2}{4\pi R} \cdot t_{r_1}^a \otimes t_{r_2}^a \quad (6)$$

- (e) The matrix $t_{r_1}^a \otimes t_{r_2}^a$ is a matrix of dimension $d_{r_1} d_{r_2} \times d_{r_1} d_{r_2}$. To diagonalize this matrix, represent the product representation $r_1 \times r_2$ in terms of its irreducible components

$$r_1 \times r_2 = \sum_i r_i . \quad (7)$$

A state in which r_1 and r_2 are combined into a given irreducible representation r_I is an eigenstate of this matrix. Show this, and show that the eigenvalue is given by a simple combination of $C_2(r_1)$, $C_2(r_2)$, and $C_2(r_I)$.

- (f) Show that, if $r_1 = r$, $r_2 = \bar{r}$, the complex conjugate representation, and the two fermions are in a state of zero (singlet) total charge ($r_I = 1$), then

$$V(R) = -\frac{g^2 C_2(r)}{4\pi R} \quad (8)$$

3. Peskin and Schroeder, Problem 15.5.