Physics 331 – Final Exam

This exam is due at noon on Friday, March 24. Please hand it in to Wu-Yen Chuang (the officemate of Alex Giryavets) in Varian 363. If you have any questions about the exam, please contact me at mpeskin@slac.stanford.edu or 926-3250. If errata are reported, I will announce them on the course Web page.

Please do not collaborate on this exam. Please return the exam in a blue book (or multiple blue books) with the honor code acknowledgement signed. Wu-Yen has blue books, if you need one.

The exam is worth a total of 100 points. The distribution of points is indicated below.

It will quickly be apparent that the model described in this exam is not easy to restructure as a true model of Nature. So you will probably not find a good reference to help you do the calculations. However, the exam is open-book. If you find a useful reference other than the class textbook and notes, feel free to use it, as long as you cite the reference in your solution.

Consider an $SU(4)$ gauge theory with fermions in the two-index antisymmetric tensor representation, which is 6 dimensional. It is possible to break the gauge symmetry down to $SU(3)$; then the representation 6 of $SU(4)$ becomes the representation $3 + \overline{3}$ of $SU(3)$. We can identify this with the QCD representation of a quark. The theory contains some new particles, whose properties are explored in this exam.

There is a problem including electroweak interactions in this model. It will become obvious that the heavy vector bosons do not have a consistent assignment of electric charge, and that it is very difficult to assign the quarks the correct weak interaction quantum numbers. Fortunately, this is just an exam, not a serious proposal, so we do not have worry too much about this. Just ignore electroweak interactions and proceed.

For the purpose of this exam, ignore the running of the QCD coupling. When a value of α_s is required, set $\alpha_s = 0.12$.

a. (5 points) Let ϕ be a Higgs field in the fundamental representation of $SU(4)$:

$$V = -\mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2 \tag{1}$$

Show that the solution

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ v \end{pmatrix} \tag{2}$$

minimizes this potential, and find v in terms of μ, λ.

b. (5 points) The Lie algebra of $SU(4)$ has 15 generators. These can be represented as 4×4 traceless Hermitian matrices. Show that 8 generators, corresponding to the Lie algebra of $SU(3)$, annihilate $\langle \phi \rangle$. The other 7 generators correspond to spontaneously broken symmetries. Show that these can be grouped into a multiplet that transforms as a 3 of $SU(3)$, a multiplet that transforms as a $\overline{3}$ of $SU(3)$, and a singlet of $SU(3)$. Show that the Goldstone bosons transform under $SU(3)$ in the same way, so that when we gauge the $SU(4)$ symmetry, each broken gauge boson can eat a Goldstone boson.

c. (5 points) Couple ϕ to an $SU(4)$ gauge theory

$$\mathcal{L} = |D_\mu \phi|^2 + \cdots \tag{3}$$

and compute the masses obtained by the gauge bosons through the Higgs mechanism in terms of v and the $SU(4)$ gauge coupling g. It is convenient to group the massive bosons into two groups of three C, $\overline{C}$ which transform as $\overline{3}$ and 3 of $SU(3)$, respectively. and one $SU(3)$-singlet boson D. You should find $m(D)/2 < m(C) < m(D)$.

d. (5 points) Now couple the $SU(4)$ gauge theory to two massless left-handed fermions U, D in the 6-dimensional antisymmetric tensor representation of $SU(4)$. The antisymmetric tensor representation is the representation of antisymmetric matrices A_{ij} where each index is in the 4 of $SU(4)$. If ξ_i is a complex vector in the 4, then the transformation law is:

$$\xi_i \to U_{ij}\xi_j \qquad A_{ij} \to U_{ik}U_{jl}A_{kl} \tag{4}$$

(We met antisymmetric tensor representations in $SU(3)$, where we saw that the antisymmetric matrices with two indices in the 3 is a 3-dimensional representation equivalent to the $\overline{3}$.)

Show that each fermion can be considered under the unbroken $SU(3)$ symmetry as a left-handed fermion in the 3 and a left-handed fermion in the $\overline{3}$ of $SU(3)$. That is, three components of the U can be identified with u_L and three components with $\overline{u}_L$, the antiparticle of u_R. Similarly D can be recast as $(d_L, \overline{d}_L)$. We thus have a theory of QCD with 2 massless flavors, with some extra massive gauge bosons and scalars. From here on, ignore the scalars, and ignore all heavier quarks. Show that the $SU(3)$ (QCD) coupling g_s is equal to the original $SU(4)$ coupling g.

e. (10 points) Compute the partial width for a C boson to decay to quark pairs. Note that C decays to qq, not $q\overline{q}$. Compute the partial width for a D boson to decay to $q\overline{q}$ pairs.

f. (5 points) Show that neither the C nor the D boson decays to gluon pairs at the leading order in g_s.

g. (5 points) For the case $m(C) = 700$ GeV, compute the total widths and branching ratios of the C and D bosons.

h. (5 points) Compute the cross sections for C and D bosons to be produced singly in proton-proton collisions—$\sigma(p\overline{p} \to C + X)$, $\sigma(p\overline{p} \to \overline{C} + X)$, $\sigma(p\overline{p} \to D + X)$—in terms of the proton's parton distributions.

i. (10 points) For the case of $m(C) = 700$ GeV, compute the total cross sections numerically. If the Tevatron $p\overline{p}$ collider ($E_{CM} = 2000$ GeV) provides 1 fb^{-1} of luminosity (that is, the luminosity such that a cross section of 1 fb $= 10^{-12}$ mbarns gives 1 event), how many C. $\overline{C}$, and D production events would be expected?

j. (5 points) Make a plot of the distribution of produced C, $\overline{C}$, and D bosons as a function of rapidity for $p\overline{p}$ collisions for the parameters of part (i).

k. (5 points) We might be interested in observing a C or D boson at large transverse momentum. To compute the cross section for this process, we would need to compute the parton-level cross sections for

$$\text{parton} + \text{parton} \to \text{parton} + D \tag{5}$$

Make a list of the required cross sections, and draw the corresponding Feynman diagrams.

l. (5 points) Repeat this analysis for

$$\text{parton} + \text{parton} \to \text{parton} + C \tag{6}$$

m. (10 points) Compute the cross sections listed in part (k). The method of computing the helicity amplitudes is probably the most efficient.

n. (10 points) Compute the cross sections listed in part (l).

o. (10 points) Compute numerically the cross section $d\sigma/dp_T$ for C and D boson production for $p_T > 100$ GeV using the parameters of part (i).

Self-evaluation: To record a satisfactory performance on this exam, please complete at least through part (j). Prospective theorists should slog through to the end.